2020

01. Introduction. Electric charge. Coulomb force. Electric field

Gerhard Müller
University of Rhode Island, gmuller@uri.edu

Robert Coyne
University of Rhode Island, robcoyne@uri.edu

Mechanics

Kinematics:

- **position:** $\vec{r} = \int \vec{v}\, dt$
- **velocity:** $\vec{v} = \frac{d\vec{r}}{dt} = \int \vec{a}\, dt$
- **acceleration:** $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

Dynamics: $\vec{F} = m\vec{a}$ (cause and effect)

Modes of motion: translation, rotation, oscillation

Conservation laws: energy, momentum, angular momentum

Effective forces: elastic, contact, friction, weight, ...

Fundamental interaction: gravitational, electromagnetic, strong, weak

Particles generate fields:

- Massive particle generates gravitational field.
- Charged particle generates electric field.
- Moving charged particle generates electric field and magnetic field.

Fields exert force on particles:

- Gravitational field exerts force on massive particle.
- Electric field exerts force on charged particle.
- Magnetic field exerts force on moving charged particle.

Dynamics:

- Cause and effect between particles and fields and among fields.

Energy and momentum:

- Particles carry energy (kinetic, potential) and momentum.
- Fields carry energy (electric, magnetic) and momentum.

Atomic Structure of Matter

Periodic table: ∼100 elements.

Building blocks of atoms: fundamental particles.

particle	charge	mass
electron	$q_e = -e$	$m_e = 9.109 \times 10^{-31}\text{kg}$
proton	$q_p = +e$	$m_p = 1.673 \times 10^{-27}\text{kg}$
neutron	$q_n = 0$	$m_n = 1.675 \times 10^{-27}\text{kg}$

- SI unit of charge: 1C (Coulomb).
- Elementary charge: $e = 1.602 \times 10^{-19}\text{C}$.
- Atomic nuclei (protons, neutron) have a radius of $\sim 1\text{fm} = 10^{-15}\text{m}$.
- Atomic electron shells have a radius of $\sim 1\text{Å} = 10^{-10}\text{m}$.
- Atoms are electrically neutral (equal numbers of electrons and protons).
- Ions: atoms with one or several electrons added or removed.
- Isotopes: atoms differing in the number of neutrons.
- Positively (negatively) charged objects have a deficiency (surplus) of electrons.

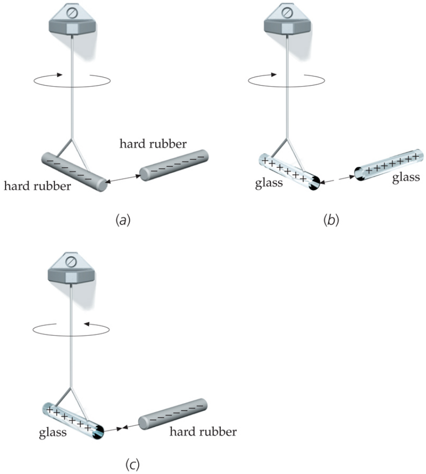
Periodic Table of Elements

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57–71 Rare Earths	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89–103 Actinides	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg							

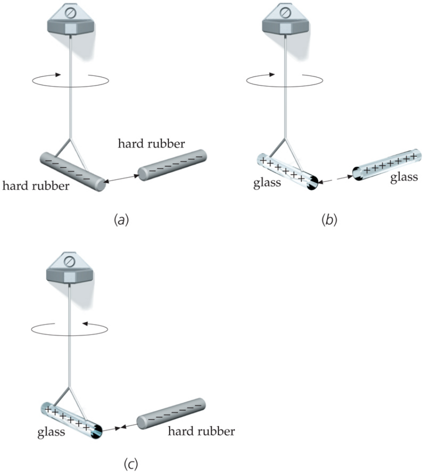
Rare Earths (Lanthanides)	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
Actinides	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

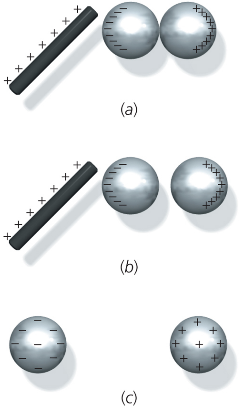
*The 1–18 group designation has been recommended by the International Union of Pure and Applied Chemistry (IUPAC).
Elements with atomic numbers 112, 114, and 116 have been reported but not fully authenticated as of September 2003.
From http://www.iupac.org/reports/periodic_table/IUPAC_Periodic_Table-3Oct05.pdf

Charging Up Insulators and Conductors



Charging Up Insulators and Conductors



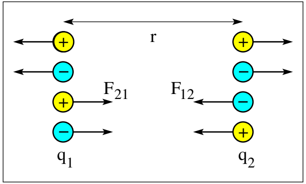


Coulomb's Law (1)

Electrostatic force between two charged particles:

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = k\frac{|q_1 q_2|}{r^2}$$

Permittivity constant: $\epsilon_0 = 8.854 \times 10^{-12}\mathrm{C}^2\mathrm{N}^{-1}\mathrm{m}^{-2}$
Coulomb constant: $k = 8.99 \times 10^9\mathrm{Nm}^2\mathrm{C}^{-2}$



Action-reaction pair of forces: $\vec{F}_{21} = -\vec{F}_{12}$.

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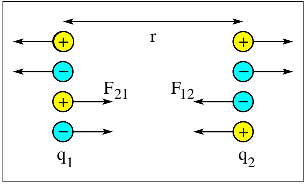
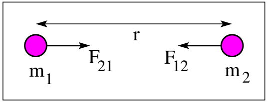


Action-reaction pair of forces: $\vec{F}_{21} = -\vec{F}_{12}$.

Newton's law of gravitation (for comparison)

Gravitational force between two massive particles:

$$F = G\frac{m_1 m_2}{r^2}$$



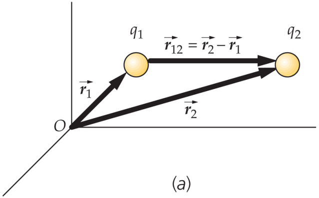
Gravitational constant: $G = 6.673 \times 10^{-11} \mathrm{Nm}^2\mathrm{kg}^{-2}$

Coulomb's Law (2)

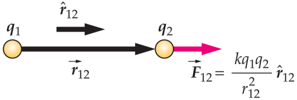
Coulomb's law for electrostatic force in vector forn

$$\vec{F}_{12} = k\,\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12},$$

$$\vec{r}_{12} \doteq \vec{r}_2 - \vec{r}_1, \quad \hat{r}_{12} \doteq \frac{\vec{r}_{12}}{r_{12}}.$$



(a)



(b)

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Coulomb's law for electrostatic force in vector forn

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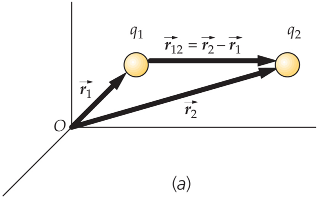

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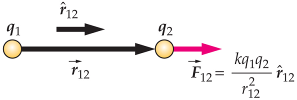
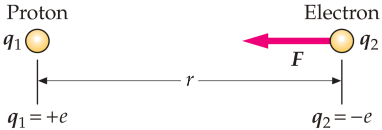


Electric force in hydrogen atom:

Average distance: $r = 5.3 \times 10^{-11}$m.
Elementary charge: $e = 1.60 \times 10^{-19}$C.

$$F = k\,\frac{|q_1 q_2|}{r^2} = \frac{(8.99 \times 10^9 \mathrm{Nm}^2/\mathrm{C}^2)(1.60 \times 10^{-19}\mathrm{C})^2}{(5.3 \times 10^{-11}\mathrm{m})^2} = 8.2 \times 10^{-8}\mathrm{N}.$$



Coulomb Force in One Dimension (1)

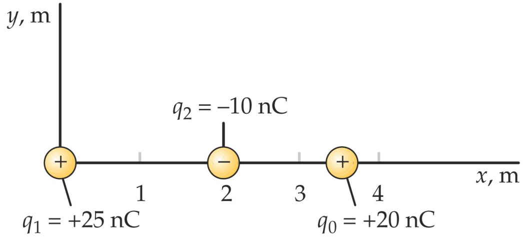
Find net force on charge q_0 due to charges q_1 and q_2.

Consider x-component of force.

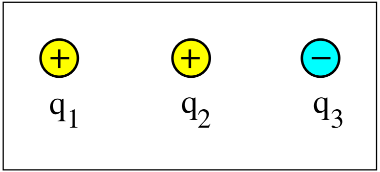
$$F_0 = +k\frac{|q_1 q_0|}{(3.5\text{m})^2} - k\frac{|q_2 q_0|}{(1.5\text{m})^2} = +3.67\times10^{-7}\text{N} - 7.99\times10^{-7}\text{N} = -4.32\times10^{-7}\text{N}.$$

Find net force on charge q_2 due to charges q_1 and q_0.

$$F_2 = -k\frac{|q_1 q_2|}{(2.0\text{m})^2} + k\frac{|q_2 q_0|}{(1.5\text{m})^2} = -5.62\times10^{-7}\text{N} + 7.99\times10^{-7}\text{N} = +2.37\times10^{-7}\text{N}.$$



Three particles with charges of magnitude 1C are positioned on a straight line with two equal spacings.



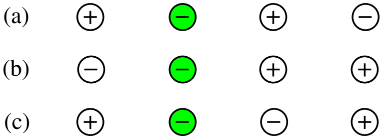
(a) Find the direction (left/right) of the net forces $\vec{F}_1, \vec{F}_2, \vec{F}_3$ on each particle.

(b) Which force is the strongest and which force is the weakest?

Coulomb Force in One Dimension (3)

Four point charges equal magnitude are lined up in three different configurations.

The Coulomb force between nearest neighbors is 4N.

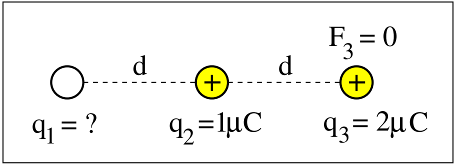


Find direction and magnitude of the net force experienced by the green particle in each configuration.

Coulomb Force in One Dimension (4)

Three charged particles are positioned along a straight line with two equal spacings .

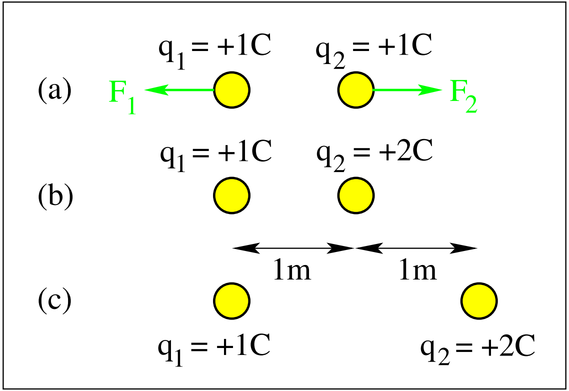
The net Coulomb force on charge q_3 happens to vanish.



What is the value of q_1?

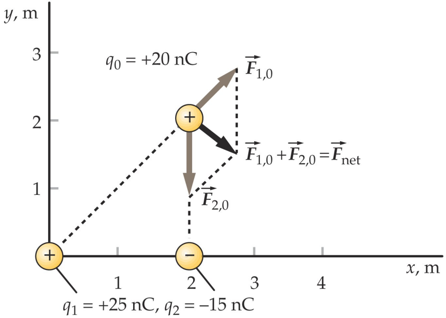
Coulomb Force in One Dimension (5)

How are the forces $\vec{F}_1$ and $\vec{F}_2$ in (a) affected by the changes made in (b) and (c)?



What changes if the charge q_2 is made negative?

Coulomb Force in Two Dimensions (1a)



Find the magnitude and direction of the resultant force on charge q_0.

- Magnitude of individual forces:

$$F_{1,0} = k\frac{|q_1 q_0|}{r_{1,0}^2} = 5.62 \times 10^{-7}\text{N}, \quad F_{2,0} = k\frac{|q_2 q_0|}{r_{2,0}^2} = 6.74 \times 10^{-7}\text{N}.$$

- Components of individual forces:

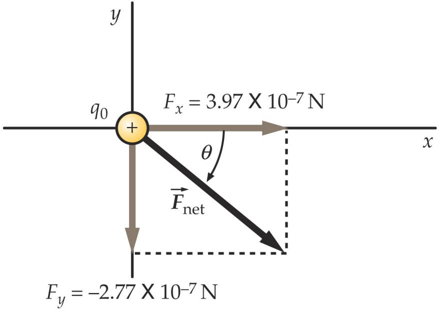
$$F_{1,0}^x = F_{1,0}\cos 45^\circ, \quad F_{1,0}^y = F_{1,0}\sin 45^\circ, \quad F_{2,0}^x = 0, \quad F_{2,0}^y = -F_{2,0}.$$

Coulomb Force in Two Dimensions (1b)

- Components of resultant force:

$$F_x = F^x_{1,0} + F^x_{2,0} = 3.97 \times 10^{-7}\mathrm{N}, \quad F_y = F^y_{1,0} + F^y_{2,0} = -2.77 \times 10^{-7}\mathrm{N}.$$

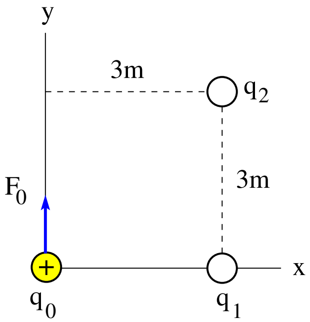
- Magnitude of resultant force: $F = \sqrt{F_x^2 + F_y^2} = 4.84 \times 10^{-7}\mathrm{N}$.
- Direction of resultant force: $\theta = \arctan\left(F_y/F_x\right) = -34.9^\circ$.



Coulomb Force in Two Dimensions (2)

The unknwon point charges q_1, q_2 exert a force $F_0 = 2\text{N}$ on the known point charge $q_0 = 1\text{nC}$. This force is directed in the positive y-direction as shown.

Determine first whether q_1, q_2 are positive or negative. Then determine the values of the two point charges.



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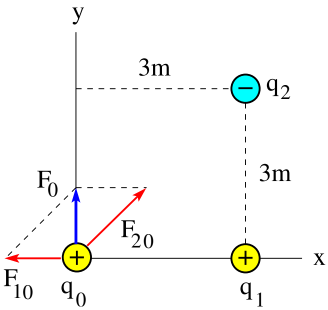
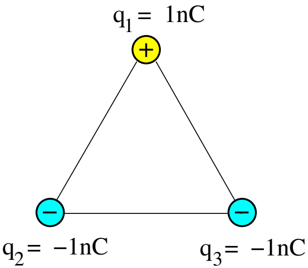


Coulomb Force in Two Dimensions (3)

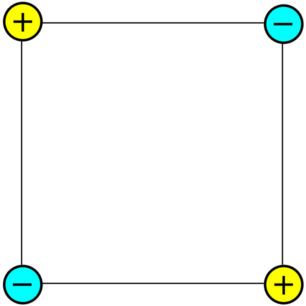
Point charges of equal magnitude are positioned at the corners of an equilateral triangle.



- Copy this configuration and indicate by arrows the direction of the resultant force on each point charge.
- Which point charge experiences the strongest force?

Coulomb Force in Two Dimensions (4)

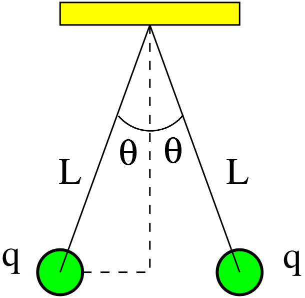
Point charges of equal magnitude are positioned at the corners of a square.



- Copy this configuration and indicate by arrows the direction of the resultant force on each point charge.
- If the force between nearest-neighbor charges is 1N, what is the strength of the resultant force on each charge?

Coulomb Force in Two Dimensions (5)

Two identical small charged spheres, each having a mass $m = 30\text{g}$, hang in equilibrium at an anlge of $\theta = 5^{\circ}$ from the vertical. The length of the strings is $L = 15\text{cm}$.



- Identify all forces acting on each sphere.
- Find the magnitude of the charge q on each sphere.

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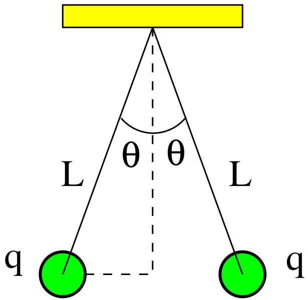

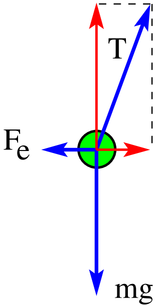

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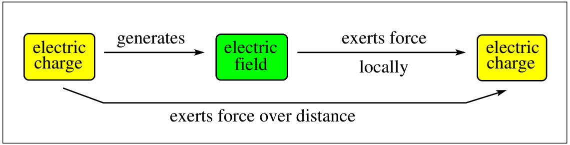
Electric Field of a Point Charge

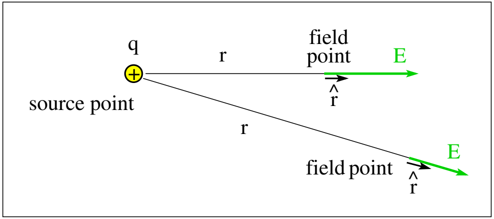


(1) Electric field $\vec{E}$ generated by point charge q: $\vec{E} = k\,\frac{q}{r^2}\hat{r}$

(2) Force $\vec{F}_1$ exerted by field $\vec{E}$ on point charge q_1: $\vec{F}_1 = q_1\vec{E}$

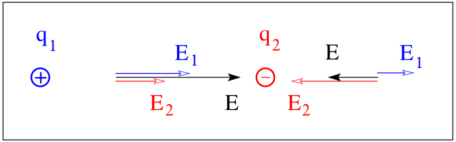
(1+2) Force $\vec{F}_1$ exerted by charge q on charge q_1: $\vec{F}_1 = k\,\frac{qq_1}{r^2}\hat{r}$ (static conditions)

- $\epsilon_0 = 8.854 \times 10^{-12}\mathrm{C}^2\mathrm{N}^{-1}\mathrm{m}^{-2}$
- $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \mathrm{Nm}^2\mathrm{C}^{-2}$

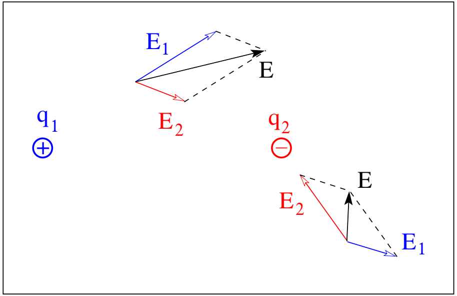


Superposition Principle for Electric Field

Electric field on line connecting two point charges:



Electric field of point charges in plane:



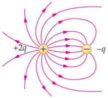
Vector Field and Electric Field Lines

- The electric field is a vector field:

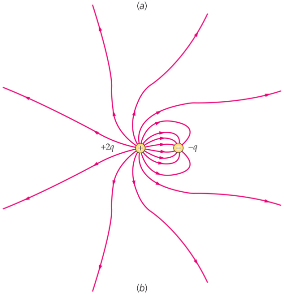
$$\vec{E}(\vec{r}) = \vec{E}(x,y,z) = E_x(x,y,z)\hat{i} + E_y(x,y,z)\hat{j} + E_z(x,y,z)\hat{k}$$

- Electric field lines: graphical representation of

- Properties of electric field lines (electrostatics):
  - Electric field lines begin at positive charges or at infinity.
  - Electric field lines end at negative charges or at infinity.
  - The direction of $\vec{E}$ is tangential to the field line going through the field point.
  - Electric field lines bunched together indicate a strong field.
  - Electric field lines far apart indicate a weak field.
  - Field lines do not intersect.



(a)



(b)

Electric Field on Line Connecting Point Charges (1)

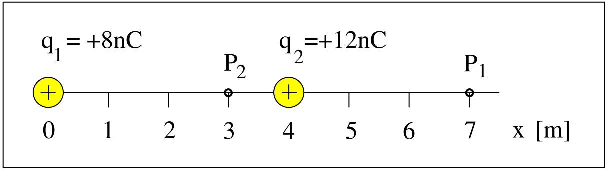
Consider the x-component of the electric field.

- Electric field at point P_1:

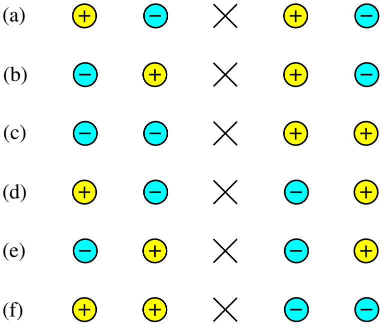
$$E = E_1 + E_2 = \frac{kq_1}{(7\mathrm{m})^2} + \frac{kq_2}{(3\mathrm{m})^2} = 1.47\mathrm{N/C} + 12.0\mathrm{N/C} = 13.5\mathrm{N/C}.$$

- Electric field at point P_2:

$$E = E_1 + E_2 = \frac{kq_1}{(3\mathrm{m})^2} - \frac{kq_2}{(1\mathrm{m})^2} = 7.99\mathrm{N/C} - 108\mathrm{N/C} = -100\mathrm{N/C}.$$



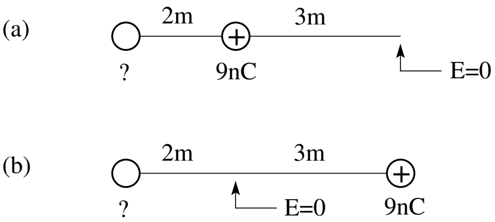
Four particles with charges of equal magnitude are positioned on a horizontal line in six different configurations.



Determine for each configuration the direction of the resultant electric field (left/right/zero) at the location indicated by $\times$.

Electric Field on Line Connecting Point Charges (3)

- Is the unknown charge positive or negative?
- What is the value of the unknown charge?



Electric Field of Point Charges in Plane (1)

Determine magnitude of $\vec{E}_1$ and $\vec{E}_2$
and identify directions in plane:

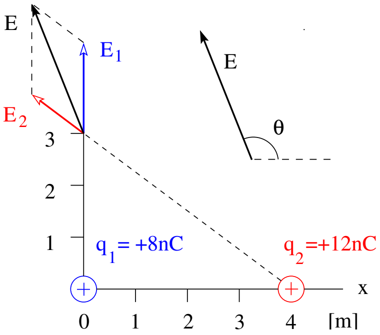
$$E_1 = \frac{k|q_1|}{(3\text{m})^2} = 7.99\text{N/C}, \; E_2 = \frac{k|q_2|}{(5\text{m})^2} = 4.32\text{N/C}.$$

Determine x- and y-components of $\vec{E}_1$ and $\vec{E}_2$
and of the resultant field $\vec{E}$:

$$E_1^x = 0, \quad E_1^y = 7.99\text{N/C};$$
$$E_2^x = -3.46\text{N/C}, \quad E_2^y = 2.59\text{N/C};$$
$$E_x = -3.46\text{N/C}, \quad E_y = 10.6\text{N/C}.$$

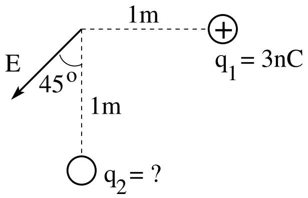
Determine magnitude and direction of $\vec{E}$:

$$E = \sqrt{E_x^2 + E_y^2} = 11.2\text{N/C}, \quad \theta = \arctan\left(\frac{E_y}{E_x}\right) = 108^\circ.$$

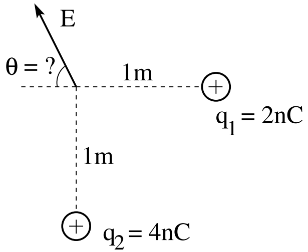


Electric Field of Point Charges in Plane (2)

(a) Find the electric charge q_2.



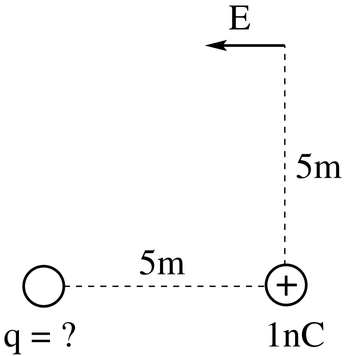
(b) Find the angle θ.



Electric Field of Point Charges in Plane (3)

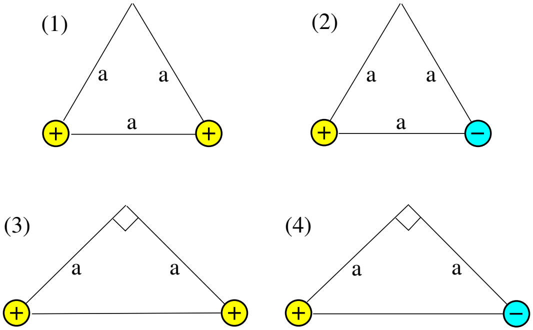
Two point charges, one known and the other unknown, produce a horizontal electric field as shown.

What is the value of the unknown charge?



Electric Field of Point Charges in Plane (4)

Consider four triangles with point charges of equal magnitude at two of the three corners.



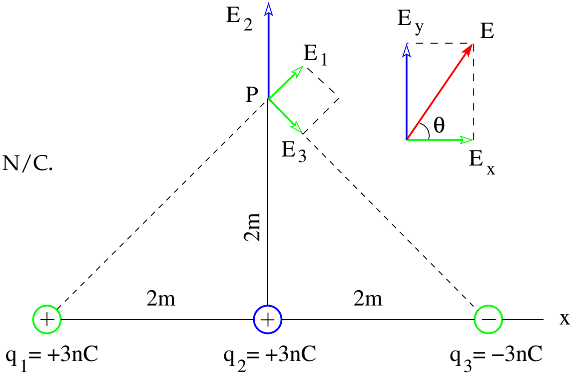
(a) Determine the direction of the electric field $\vec{E}_i$ at the third corner of triangle (i).

(b) Rank the fields E_i according to strength.

Electric Field of Point Charges in Plane (5)

Find magnitude and direction of the resultant electric field at point P.

- $E_1 = \dfrac{k|q_1|}{8\text{m}^2} = 3.38\,\text{N/C}.$
- $E_2 = \dfrac{k|q_2|}{4\text{m}^2} = 6.75\,\text{N/C}.$
- $E_3 = \dfrac{k|q_3|}{8\text{m}^2} = 3.38\,\text{N/C}.$
- $E_x = E_1\cos 45^\circ + E_3\cos 45^\circ = 4.78\,\text{N/C}.$
- $E_y = E_2 = 6.75\,\text{N/C}.$
- $E = \sqrt{E_x^2 + E_y^2} = 8.27\,\text{N/C}.$
- $\tan\theta = \dfrac{E_y}{E_x} = 1.41.$
- $\theta = \arctan 1.41 = 54.7^\circ.$

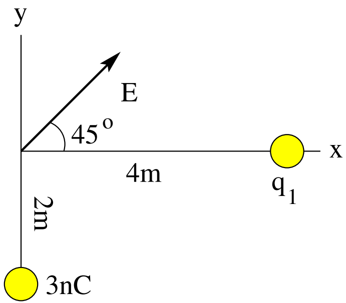


Intermediate Exam I: Problem #1 (Spring '05)

The electric field $\vec{E}$ generated by the two point charges, 3nC and q_1 (unknown), has the direction shown.

(a) Find the magnitude of $\vec{E}$.

(b) Find the value of q_1.



The electric field $\vec{E}$ generated by the two point charges, 3nC and q_1 (unknown), has the direction shown.

(a) Find the magnitude of $\vec{E}$.

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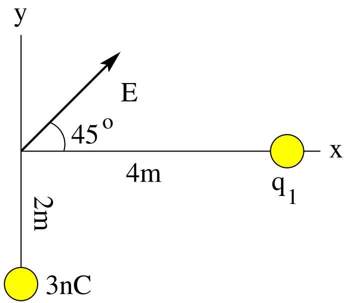

Solution:

(a) $E_y = k\dfrac{3\text{nC}}{(2\text{m})^2} = 6.75\text{N/C},$

$E_x = E_y,$

$E = \sqrt{E_x^2 + E_y^2} = 9.55\text{N/C}.$

Intermediate Exam I: Problem #1 (Spring '05)

The electric field $\vec{E}$ generated by the two point charges, 3nC and q_1 (unknown), has the direction shown.

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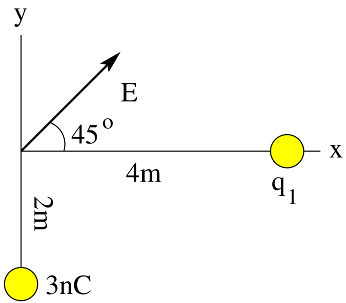

Solution:

(a) $E_y = k\dfrac{3\text{nC}}{(2\text{m})^2} = 6.75\text{N/C},$

$E_x = E_y,$

$E = \sqrt{E_x^2 + E_y^2} = 9.55\text{N/C}.$

(b) $E_x = k\dfrac{(-q_1)}{(4\text{m})^2},$

$$q_1 = -\frac{(6.75\text{N/C})(16\text{m}^2)}{k} = -12\text{nC}.$$