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E3. Previous Unit Exams 3

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Abstract

Exam slides 3 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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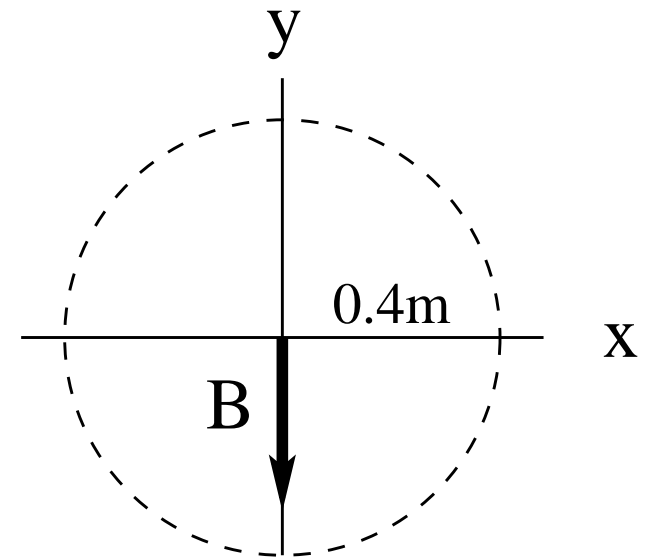
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Intermediate Exam III: Problem #1 (Spring '05)



An infinitely long straight current of magnitude $I = 6\text{A}$ is directed into the plane (\otimes) and located a distance $d = 0.4\text{m}$ from the coordinate origin (somewhere on the dashed circle). The magnetic field \vec{B} generated by this current is in the negative y -direction as shown.

- (a) Find the magnitude B of the magnetic field.
- (b) Mark the location of the position of the current \otimes on the dashed circle.



Intermediate Exam III: Problem #1 (Spring '05)



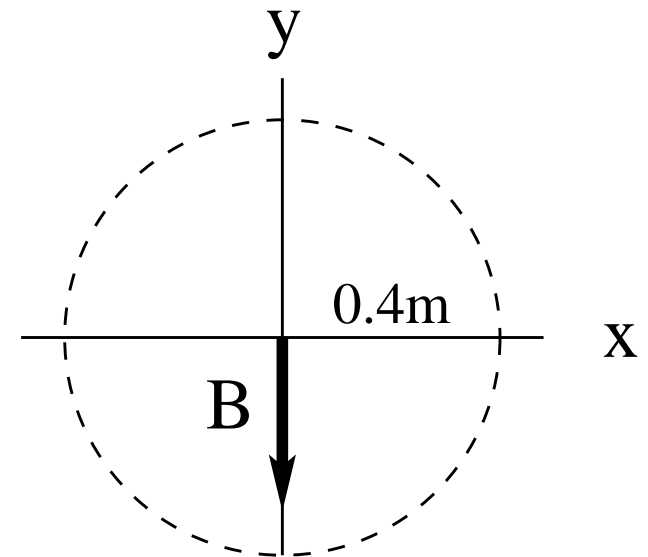
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- (a) Find the magnitude B of the magnetic field.
- (b) Mark the location of the position of the current \otimes on the dashed circle.

Solution:

(a) $B = \frac{\mu_0 I}{2\pi d} = 3\mu\text{T}.$

(b) Position of current \otimes is at $y = 0, x = -0.4\text{m}.$

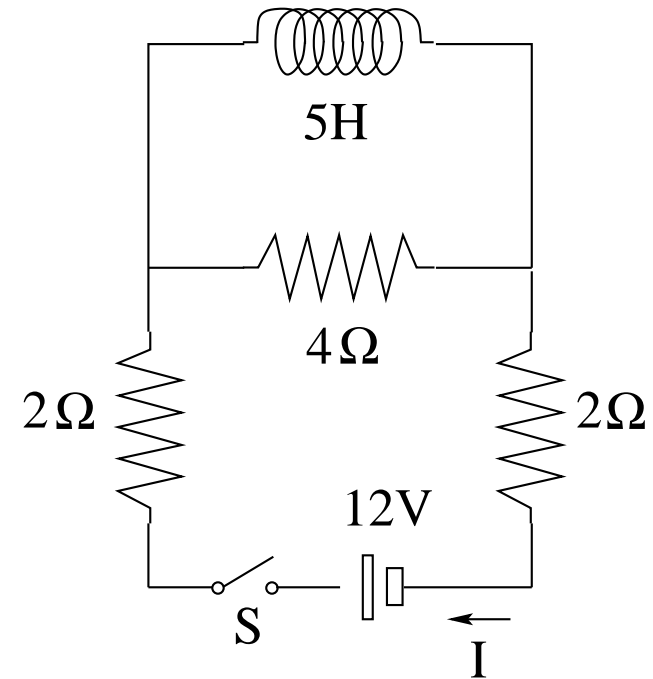


Intermediate Exam III: Problem #2 (Spring '05)



In the circuit shown we close the switch S at time $t = 0$. Find the current I through the battery and the voltage V_L across the inductor

- (a) immediately after the switch has been closed,
- (b) a very long time later.

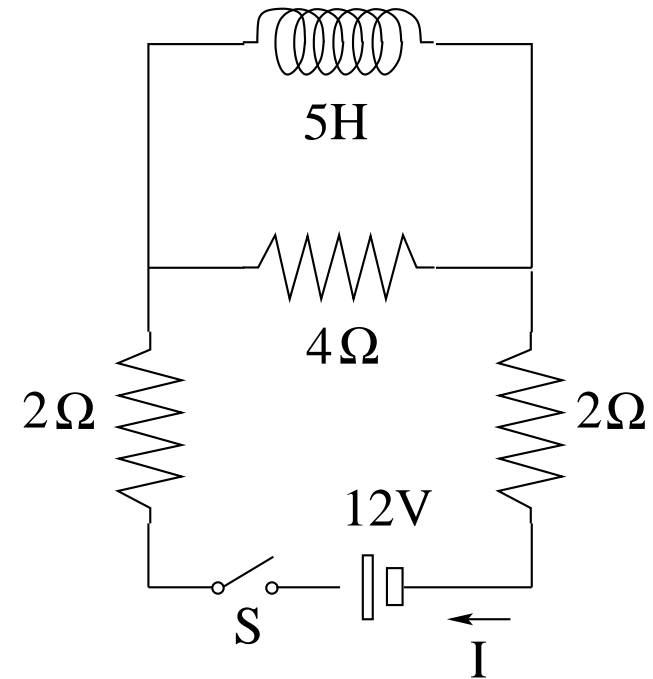


Intermediate Exam III: Problem #2 (Spring '05)



In the circuit shown we close the switch S at time $t = 0$. Find the current I through the battery and the voltage V_L across the inductor

- (a) immediately after the switch has been closed,
- (b) a very long time later.



Solution:

$$(a) \quad I = \frac{12V}{2\Omega + 4\Omega + 2\Omega} = 1.5A, \quad V_L = (4\Omega)(1.5A) = 6V.$$

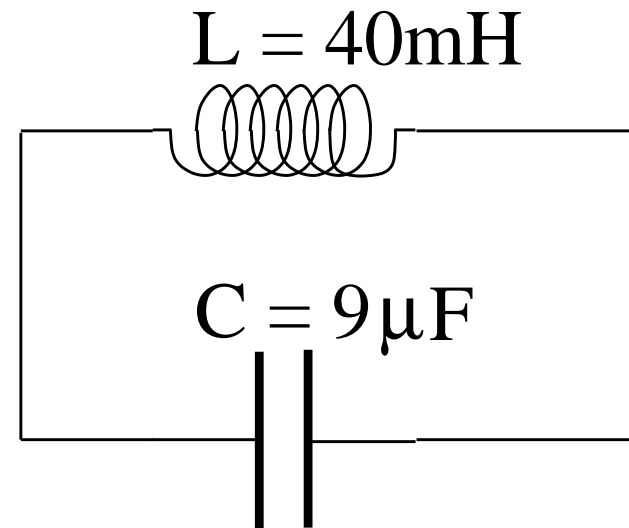
$$(b) \quad I = \frac{12V}{2\Omega + 2\Omega} = 3A, \quad V_L = 0.$$

Intermediate Exam III: Problem #3 (Spring '05)



At time $t = 0$ the capacitor is charged to $Q_{max} = 3\mu\text{C}$ and the current is instantaneously zero.

- (a) How much energy is stored in the capacitor at time $t = 0$?
- (b) At what time t_1 does the current reach its maximum value?
- (c) How much energy is stored in the inductor at time t_1 ?



Intermediate Exam III: Problem #3 (Spring '05)



At time $t = 0$ the capacitor is charged to $Q_{max} = 3\mu\text{C}$ and the current is instantaneously zero.

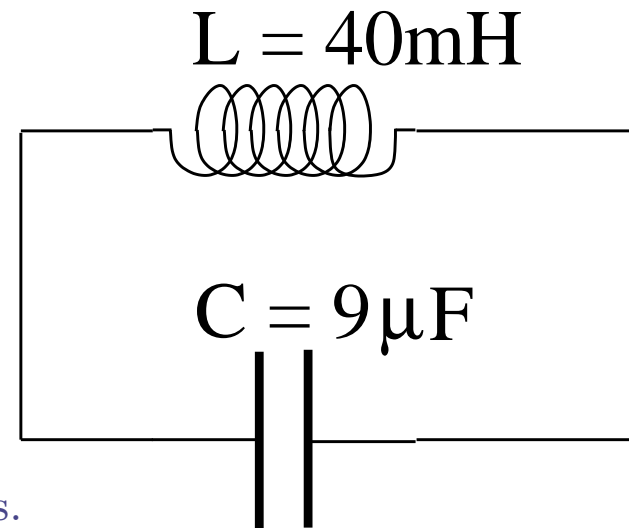
- (a) How much energy is stored in the capacitor at time $t = 0$?
- (b) At what time t_1 does the current reach its maximum value?
- (c) How much energy is stored in the inductor at time t_1 ?

Solution:

$$(a) U_C = \frac{Q_{max}^2}{2C} = 0.5\mu\text{J}.$$

$$(b) T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC} = 3.77\text{ms}, \quad t_1 = \frac{T}{4} = 0.942\text{ms}.$$

$$(c) U_L = U_C = 0.5\mu\text{J} \quad (\text{energy conservation.})$$

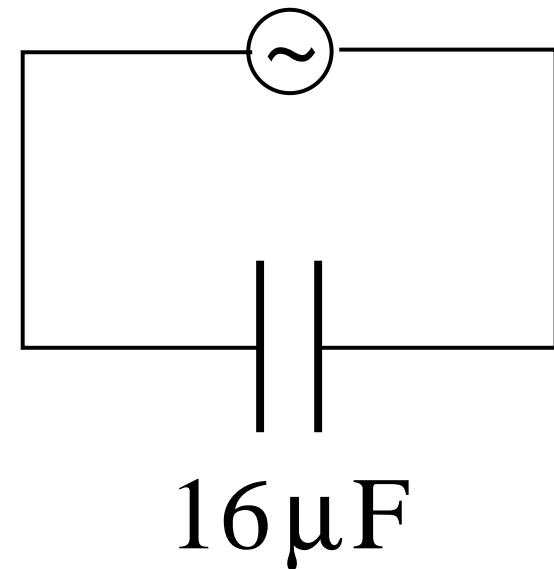


Intermediate Exam III: Problem #4 (Spring '05)



Consider the circuit shown. The *ac* voltage supplied is $\mathcal{E} = \mathcal{E}_{max} \cos(\omega t)$ with $\mathcal{E}_{max} = 170\text{V}$ and $\omega = 377\text{rad/s}$.

- (a) What is the maximum value I_{max} of the current?
- (b) What is the emf $\mathcal{E}(t)$ at $t = 0.01\text{s}$?
- (c) What is the current $I(t)$ at $t = 0.01\text{s}$?

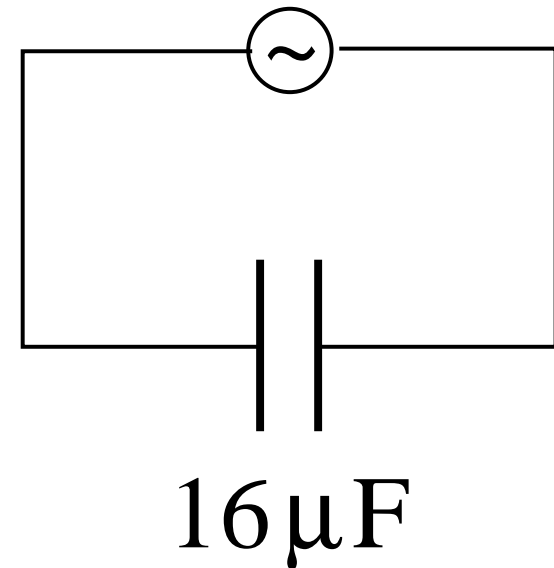


Intermediate Exam III: Problem #4 (Spring '05)



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- (a) What is the maximum value I_{max} of the current?
- (b) What is the emf $\mathcal{E}(t)$ at $t = 0.01\text{s}$?
- (c) What is the current $I(t)$ at $t = 0.01\text{s}$?



Solution:

$$(a) I_{max} = \frac{\mathcal{E}_{max}}{X_C} = \mathcal{E}_{max}\omega C = 1.03\text{A}.$$

$$(b) \mathcal{E} = (170\text{V}) \cos(3.77\text{rad}) = (170\text{V})(-0.809) = -138\text{V}.$$

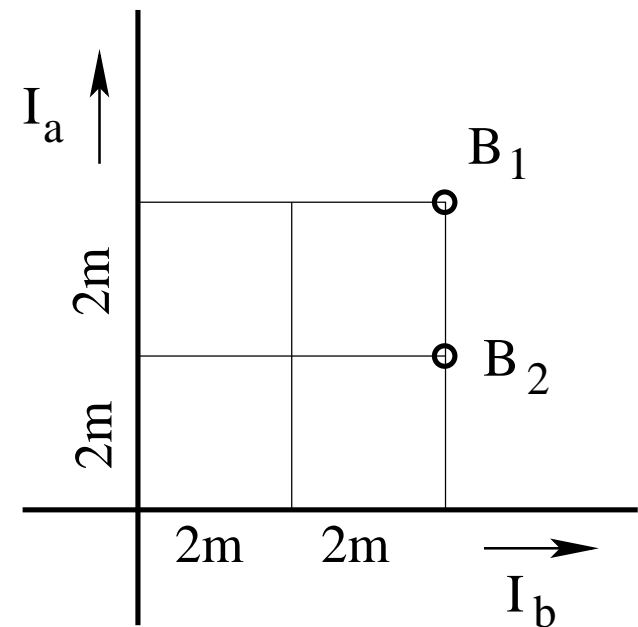
$$(c) I = \mathcal{E}_{max}\omega C \cos(3.77\text{rad} + \pi/2) = (1.03\text{A})(0.588) = 0.605\text{A}.$$

Intermediate Exam III: Problem #1 (Spring '06)



Consider two infinitely long, straight wires with currents of equal magnitude $I_1 = I_2 = 5\text{A}$ in the directions shown.

Find the direction (in/out) and the magnitude of the magnetic fields \mathbf{B}_1 and \mathbf{B}_2 at the points marked in the graph.



Intermediate Exam III: Problem #1 (Spring '06)



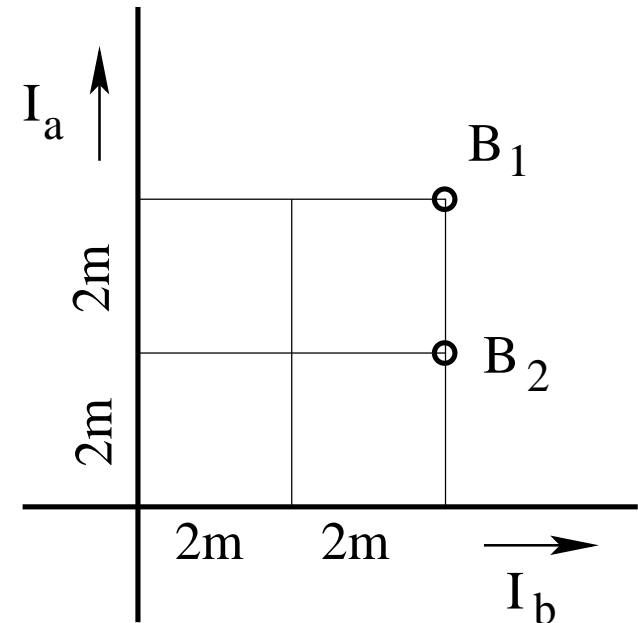
Consider two infinitely long, straight wires with currents of equal magnitude $I_1 = I_2 = 5\text{A}$ in the directions shown.

Find the direction (in/out) and the magnitude of the magnetic fields \mathbf{B}_1 and \mathbf{B}_2 at the points marked in the graph.

Solution:

- $B_1 = \frac{\mu_0}{2\pi} \left(\frac{5\text{A}}{4\text{m}} - \frac{5\text{A}}{4\text{m}} \right) = 0$ (no direction).

- $B_2 = \frac{\mu_0}{2\pi} \left(\frac{5\text{A}}{2\text{m}} - \frac{5\text{A}}{4\text{m}} \right) = 0.25\mu\text{T}$ (out of plane).

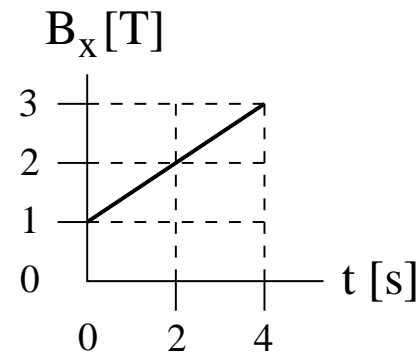
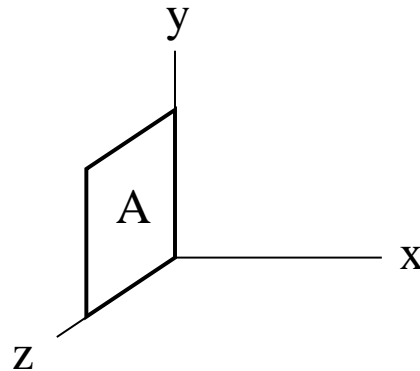


Intermediate Exam III: Problem #2 (Spring '06)



A conducting loop in the shape of a square with area $A = 4\text{m}^2$ and resistance $R = 5\Omega$ is placed in the yz -plane as shown. A time-dependent magnetic field $\mathbf{B} = B_x \hat{\mathbf{i}}$ is present. The dependence of B_x on time is shown graphically.

- Find the magnetic flux Φ_B through the loop at time $t = 0$.
- Find magnitude and direction (cw/ccw) of the induced current I at time $t = 2\text{s}$.

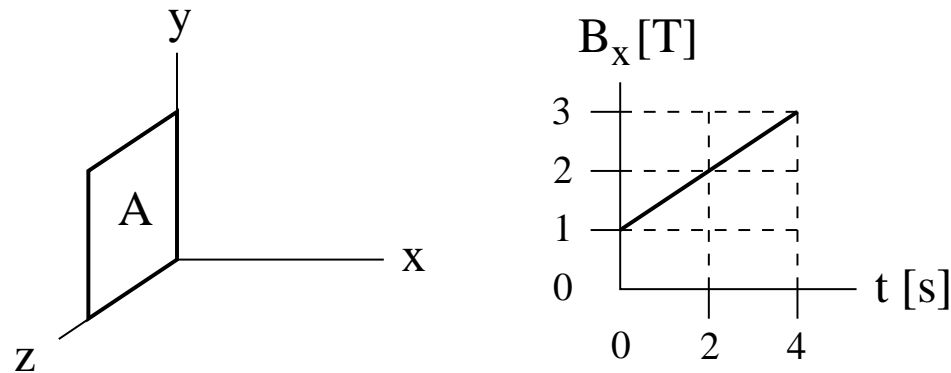


Intermediate Exam III: Problem #2 (Spring '06)



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- (a) Find the magnetic flux Φ_B through the loop at time $t = 0$.
(b) Find magnitude and direction (cw/ccw) of the induced current I at time $t = 2\text{s}$.



Choice of area vector: $\odot/\otimes \Rightarrow$ positive direction = ccw/cw.

(a) $\Phi_B = \pm(1\text{T})(4\text{m}^2) = \pm 4\text{Tm}^2$.

(b) $\frac{d\Phi_B}{dt} = \pm(0.5\text{T/s})(4\text{m}^2) = \pm 2\text{V} \quad \Rightarrow \quad \mathcal{E} = -\frac{d\Phi_B}{dt} = \mp 2\text{V}.$

$\Rightarrow I = \frac{\mathcal{E}}{R} = \mp \frac{2\text{V}}{5\Omega} = \mp 0.4\text{A} \quad (\text{cw}).$

Intermediate Exam III: Problem #3 (Spring '06)



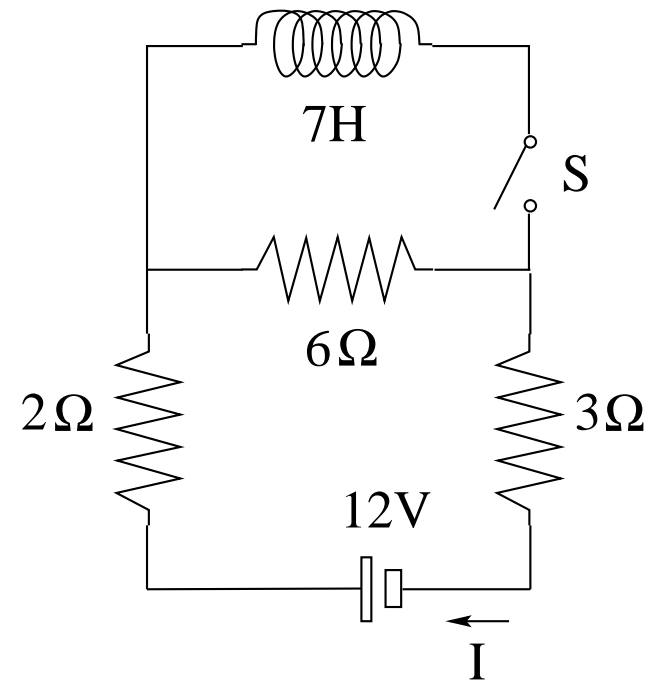
In the circuit shown the switch S is initially open.

Find the current I through the battery

(a) while the switch is open,

(b) immediately after the switch has been closed,

(c) a very long time later.

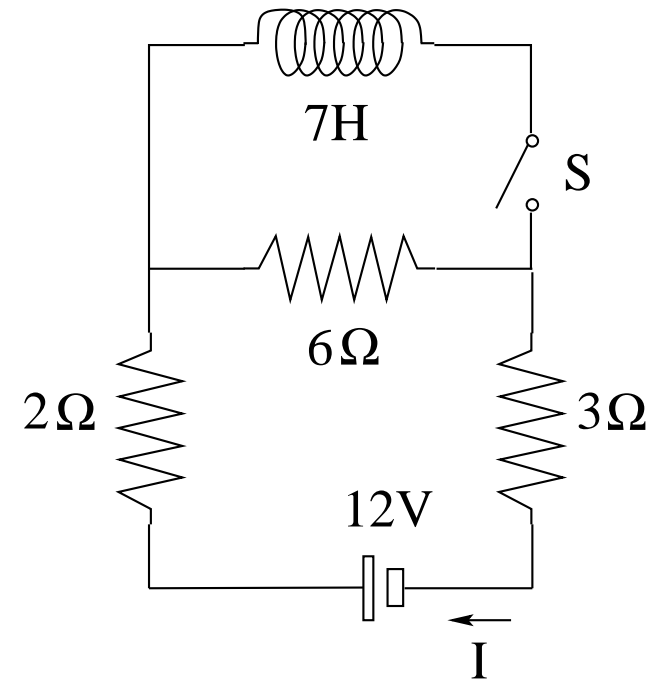


Intermediate Exam III: Problem #3 (Spring '06)



In the circuit shown the switch S is initially open.
Find the current I through the battery

- (a) while the switch is open,
- (b) immediately after the switch has been closed,
- (c) a very long time later.



$$(a) \quad I = \frac{12V}{2\Omega + 3\Omega + 6\Omega} = 1.09A.$$

$$(b) \quad I = \frac{12V}{2\Omega + 3\Omega + 6\Omega} = 1.09A.$$

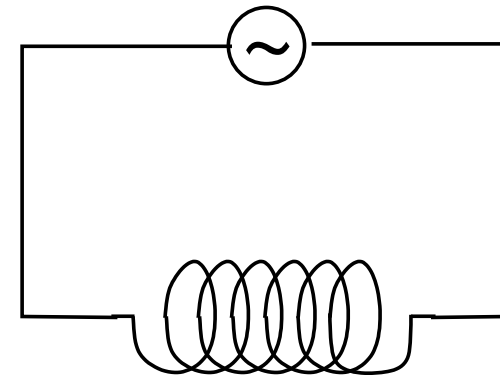
$$(c) \quad I = \frac{12V}{2\Omega + 3\Omega} = 2.4A.$$

Intermediate Exam III: Problem #4 (Spring '06)



Consider the circuit shown. The *ac* voltage supplied is $\mathcal{E} = \mathcal{E}_{max} \cos(\omega t)$ with $\mathcal{E}_{max} = 170\text{V}$ and $\omega = 377\text{rad/s}$.

- (a) What is the maximum value I_{max} of the current?
- (b) What is the emf \mathcal{E} at $t = 0.02\text{s}$?
- (c) What is the current I at $t = 0.02\text{s}$?



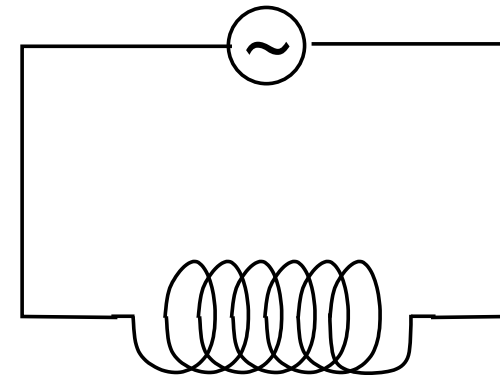
$$L = 30\text{mH}$$

Intermediate Exam III: Problem #4 (Spring '06)



Consider the circuit shown. The *ac* voltage supplied is $\mathcal{E} = \mathcal{E}_{max} \cos(\omega t)$ with $\mathcal{E}_{max} = 170\text{V}$ and $\omega = 377\text{rad/s}$.

- (a) What is the maximum value I_{max} of the current?
- (b) What is the emf \mathcal{E} at $t = 0.02\text{s}$?
- (c) What is the current I at $t = 0.02\text{s}$?



$$L = 30\text{mH}$$

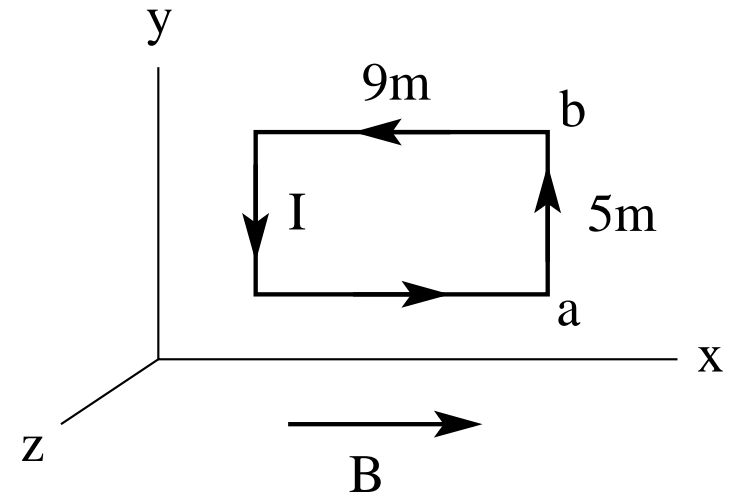
- (a) $I_{max} = \frac{\mathcal{E}_{max}}{X_L} = \frac{\mathcal{E}_{max}}{\omega L} = \frac{170\text{V}}{11.3\Omega} = 15.0\text{A}.$
- (b) $\mathcal{E} = \mathcal{E}_{max} \cos(7.54\text{rad}) = (170\text{V})(0.309) = 52.5\text{V}.$
- (c) $I = I_{max} \cos(7.54\text{rad} - \pi/2) = (15.0\text{A})(0.951) = 14.3\text{A}.$

Intermediate Exam III: Problem #1 (Spring '07)



Consider a rectangular conducting loop in the xy -plane with a counterclockwise current $I = 7\text{A}$ in a uniform magnetic field $\vec{B} = 3\text{T}\hat{i}$.

- (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
- (b) Find the force \vec{F} (magnitude and direction) acting on the side ab of the rectangle.
- (c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.

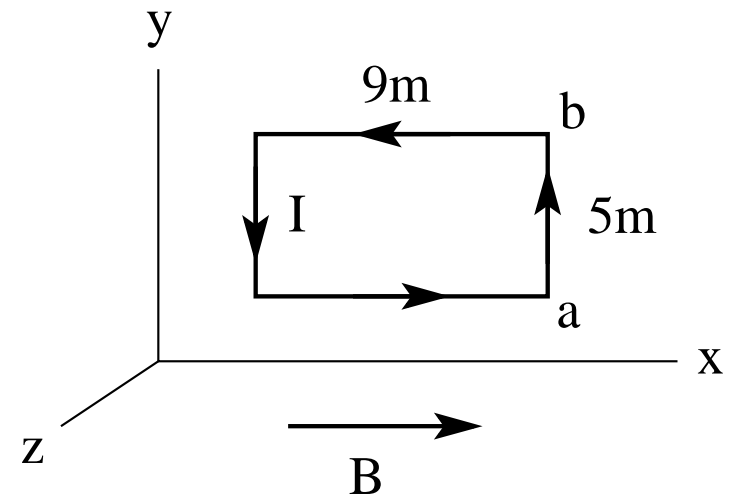


Intermediate Exam III: Problem #1 (Spring '07)



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Solution:

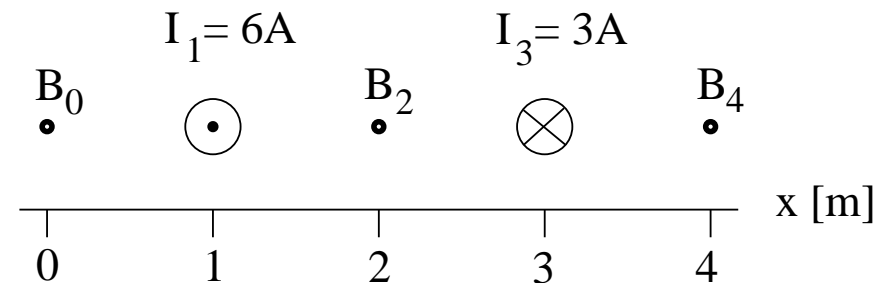
- $\vec{\mu} = (7\text{A})(45\text{m}^2)\hat{k} = 315\text{Am}^2\hat{k}$.
- $\vec{F} = I\vec{L} \times \vec{B} = (7\text{A})(5\text{m}\hat{j}) \times (3\text{T}\hat{i}) = -105\text{N}\hat{k}$.
- $\vec{\tau} = \vec{\mu} \times \vec{B} = (315\text{Am}^2\hat{k}) \times (3\text{T}\hat{i}) = 945\text{Nm}\hat{j}$

Intermediate Exam III: Problem #2 (Spring '07)



Consider two very long, straight wires with currents $I_1 = 6\text{A}$ at $x = 1\text{m}$ and $I_3 = 3\text{A}$ at $x = 3\text{m}$ in the directions shown. Find magnitude and direction (up/down) of the magnetic field

- (a) B_0 at $x = 0$,
- (b) B_2 at $x = 2\text{m}$,
- (c) B_4 at $x = 4\text{m}$.

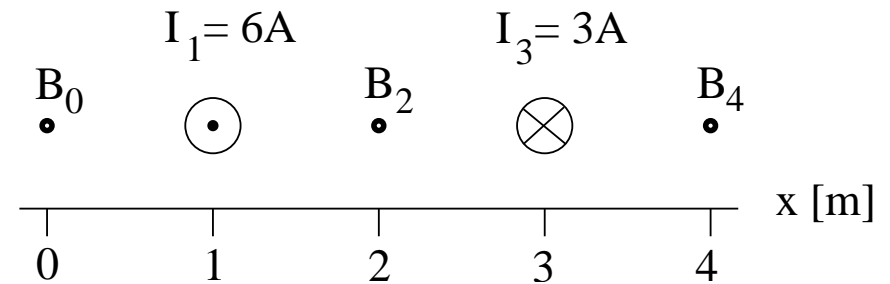


Intermediate Exam III: Problem #2 (Spring '07)



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- (a) B_0 at $x = 0$,
- (b) B_2 at $x = 2\text{m}$,
- (c) B_4 at $x = 4\text{m}$.



Solution:

$$(a) B_0 = -\frac{\mu_0(6\text{A})}{2\pi(1\text{m})} + \frac{\mu_0(3\text{A})}{2\pi(3\text{m})} = -1.2\mu\text{T} + 0.2\mu\text{T} = -1.0\mu\text{T} \quad (\text{down}),$$

$$(b) B_2 = \frac{\mu_0(6\text{A})}{2\pi(1\text{m})} + \frac{\mu_0(3\text{A})}{2\pi(1\text{m})} = 1.2\mu\text{T} + 0.6\mu\text{T} = 1.8\mu\text{T} \quad (\text{up}),$$

$$(c) B_4 = \frac{\mu_0(6\text{A})}{2\pi(3\text{m})} - \frac{\mu_0(3\text{A})}{2\pi(1\text{m})} = 0.4\mu\text{T} - 0.6\mu\text{T} = -0.2\mu\text{T} \quad (\text{down}).$$

Intermediate Exam III: Problem #3 (Spring '07)

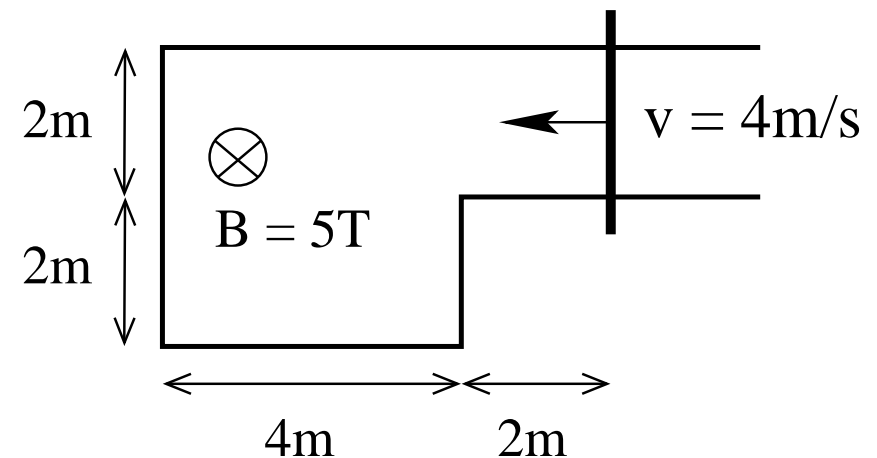


A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown.

(a) Find the magnetic flux Φ_B through the frame at the instant shown.

(b) Find the induced emf \mathcal{E} at the instant shown.

(c) Find the direction (cw/ccw) of the induced current.



Intermediate Exam III: Problem #3 (Spring '07)

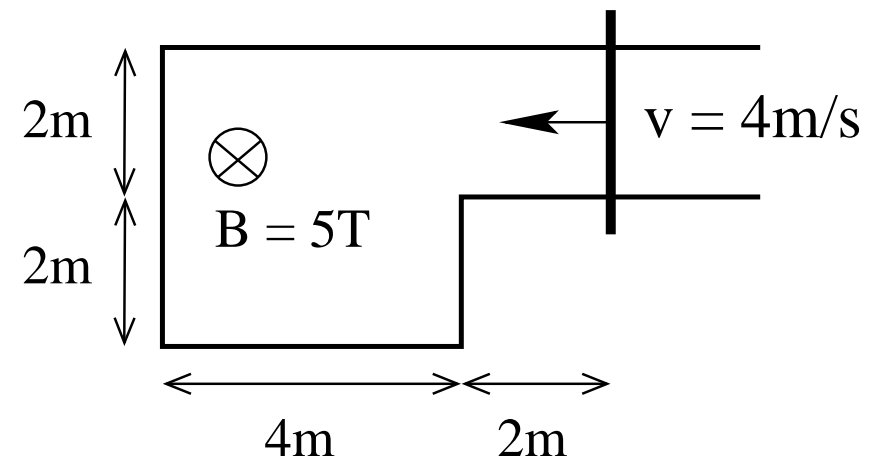


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(a) Find the magnetic flux Φ_B through the frame at the instant shown.

(b) Find the induced emf \mathcal{E} at the instant shown.

(c) Find the direction (cw/ccw) of the induced current.



Solution:

(a) $\Phi_B = \vec{A} \cdot \vec{B} = \pm(20\text{m}^2)(5\text{T}) = \pm 100\text{Wb}.$

(b) $\mathcal{E} = -\frac{d\Phi_B}{dt} = \pm(5\text{T})(2\text{m})(4\text{m/s}) = \pm 40\text{V}.$

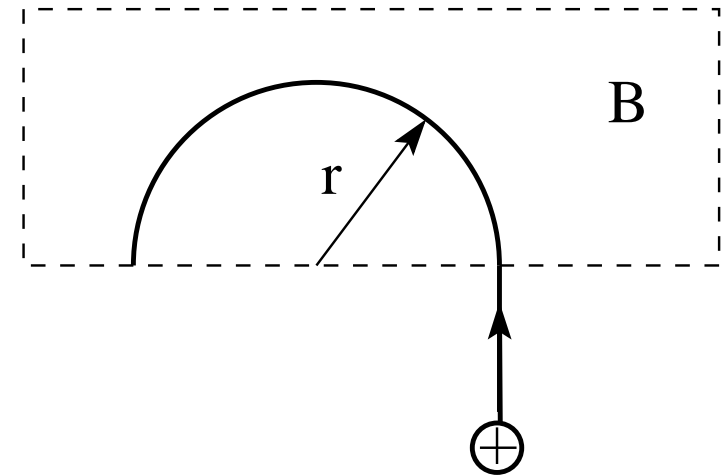
(c) clockwise.

Unit Exam III: Problem #4 (Spring '07)



A proton ($m = 1.67 \times 10^{-27} \text{ kg}$, $q = 1.60 \times 10^{-19} \text{ C}$) with velocity $v = 3.7 \times 10^4 \text{ m/s}$ enters a region of magnetic field B directed perpendicular to the plane of the sheet. The field bends the path of the proton into a semicircle of radius $r = 19 \text{ cm}$ as shown.

- Find the force necessary to keep the proton moving on the circle
- Find the direction (\odot or \otimes) and the magnitude of the magnetic field B that provides this force.
- Find the time t it takes the proton to complete the semicircular motion.



Unit Exam III: Problem #4 (Spring '07)



A proton ($m = 1.67 \times 10^{-27}$ kg, $q = 1.60 \times 10^{-19}$ C) with velocity $v = 3.7 \times 10^4$ m/s enters a region of magnetic field B directed perpendicular to the plane of the sheet. The field bends the path of the proton into a semicircle of radius $r = 19$ cm as shown.

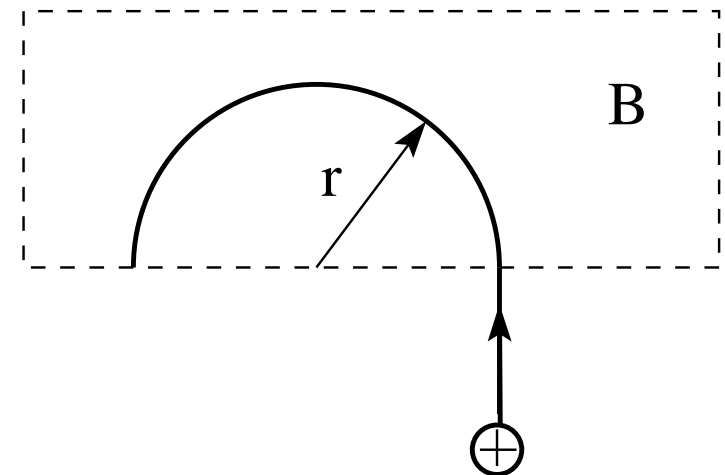
- Find the force necessary to keep the proton moving on the circle
- Find the direction (\odot or \otimes) and the magnitude of the magnetic field B that provides this force.
- Find the time t it takes the proton to complete the semicircular motion.

Solution:

$$(a) \quad F = \frac{mv^2}{r} = 1.20 \times 10^{-17} \text{ N.}$$

$$(b) \quad F = qvB \quad \Rightarrow \quad B = \frac{F}{qv} = 2.03 \times 10^{-3} \text{ T.} \quad \otimes$$

$$(c) \quad vt = \pi r \quad \Rightarrow \quad t = \frac{\pi r}{v} = 1.61 \times 10^{-5} \text{ s.}$$

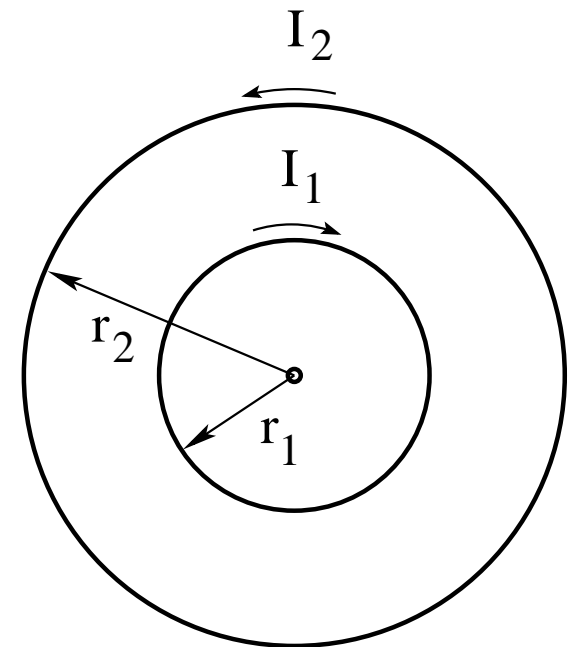


Unit Exam III: Problem #1 (Spring '08)



Consider two circular currents $I_1 = 3\text{A}$ at radius $r_1 = 2\text{m}$ and $I_2 = 5\text{A}$ at radius $r_2 = 4\text{m}$ in the directions shown.

- Find magnitude B and direction (\odot , \otimes) of the resultant magnetic field at the center.
- Find magnitude μ and direction (\odot , \otimes) of the magnetic dipole moment generated by the two currents.



Unit Exam III: Problem #1 (Spring '08)



Consider two circular currents $I_1 = 3\text{A}$ at radius $r_1 = 2\text{m}$ and $I_2 = 5\text{A}$ at radius $r_2 = 4\text{m}$ in the directions shown.

- (a) Find magnitude B and direction (\odot , \otimes) of the resultant magnetic field at the center.
(b) Find magnitude μ and direction (\odot , \otimes) of the magnetic dipole moment generated by the two currents.

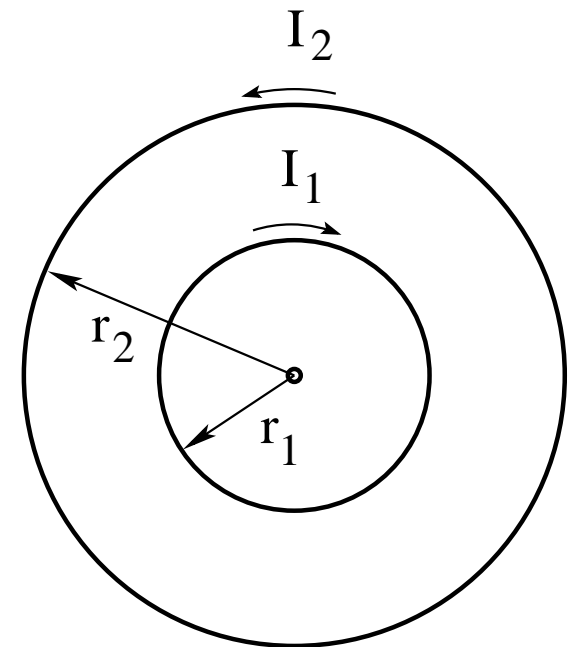
Solution:

$$(a) \quad B = \frac{\mu_0(3\text{A})}{2(2\text{m})} - \frac{\mu_0(5\text{A})}{2(4\text{m})} = (9.42 - 7.85) \times 10^{-7}\text{T}$$

$$\Rightarrow B = 1.57 \times 10^{-7}\text{T} \quad \otimes$$

$$(b) \quad \mu = \pi(4\text{m})^2(5\text{A}) - \pi(2\text{m})^2(3\text{A}) = (251 - 38)\text{Am}^2$$

$$\Rightarrow \mu = 213\text{Am}^2 \quad \odot$$



Unit Exam III: Problem #2 (Spring '08)



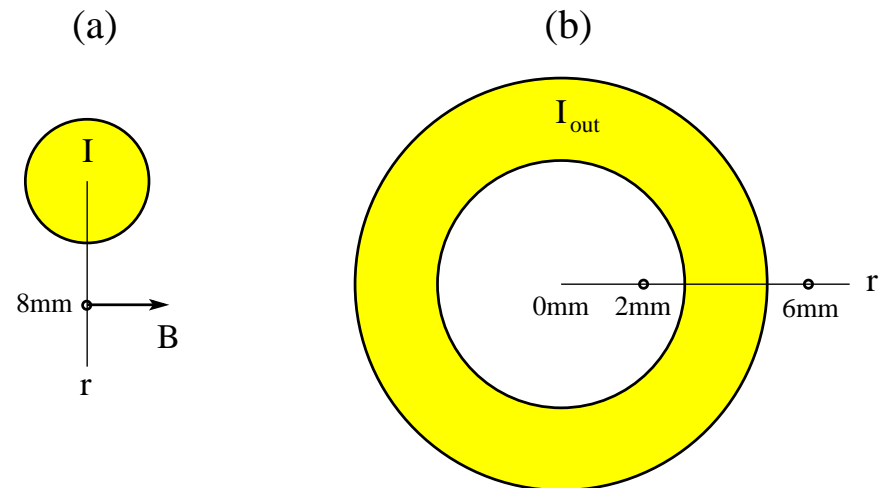
(a) Consider a solid wire of radius $R = 3\text{mm}$.

Find magnitude I and direction (in/out) that produces a magnetic field $B = 7\mu\text{T}$ at radius $r = 8\text{mm}$.

(b) Consider a hollow cable with inner radius $R_{int} = 3\text{mm}$ and outer radius $R_{ext} = 5\text{mm}$.

A current $I_{out} = 0.9\text{A}$ is directed out of the plane.

Find direction (up/down) and magnitude B_2, B_6 of the magnetic field at radius $r_2 = 2\text{mm}$ and $r_6 = 6\text{mm}$, respectively.



Unit Exam III: Problem #2 (Spring '08)



(a) Consider a solid wire of radius $R = 3\text{mm}$.

Find magnitude I and direction (in/out) that produces a magnetic field $B = 7\mu\text{T}$ at radius $r = 8\text{mm}$.

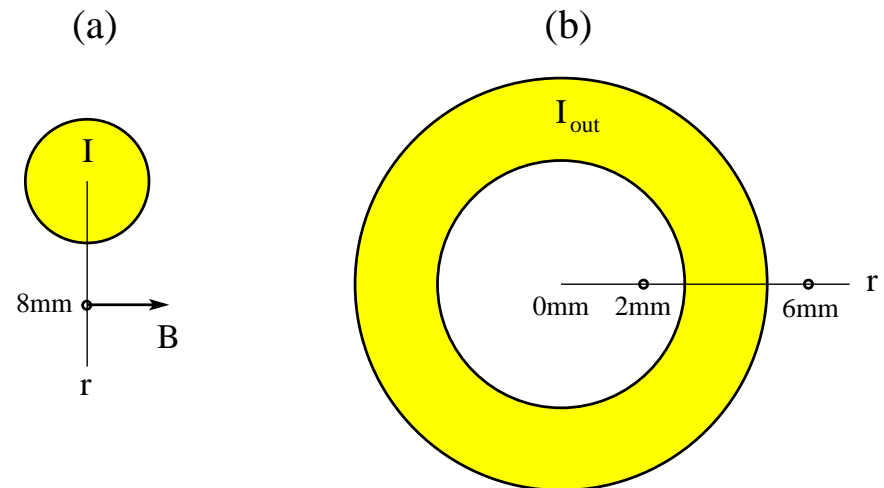
(b) Consider a hollow cable with inner radius $R_{int} = 3\text{mm}$ and outer radius $R_{ext} = 5\text{mm}$. A current $I_{out} = 0.9\text{A}$ is directed out of the plane.

Find direction (up/down) and magnitude B_2, B_6 of the magnetic field at radius $r_2 = 2\text{mm}$ and $r_6 = 6\text{mm}$, respectively.

Solution:

$$(a) \quad 7\mu\text{T} = \frac{\mu_0 I}{2\pi(8\text{mm})} \Rightarrow I = 0.28\text{A} \quad (\text{out}).$$

$$(b) \quad B_2 = 0, \quad B_6 = \frac{\mu_0(0.9\text{A})}{2\pi(6\text{mm})} = 30\mu\text{T} \quad (\text{up}).$$



Unit Exam III: Problem #3 (Spring '08)



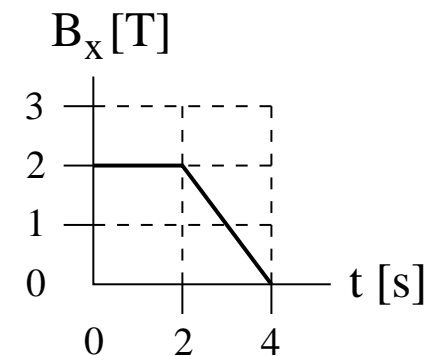
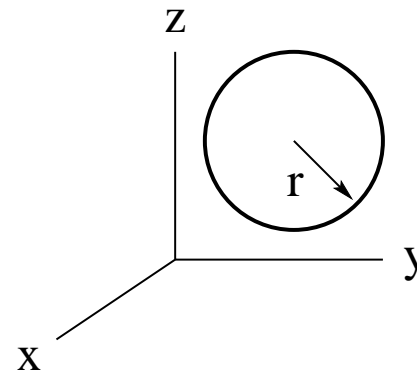
A circular wire of radius $r = 2.5\text{m}$ and resistance $R = 4.8\Omega$ is placed in the yz -plane as shown.

A time-dependent magnetic field $\mathbf{B} = B_x \hat{\mathbf{i}}$ is present.

The dependence of B_x on time is shown graphically.

(a) Find the magnitude $|\Phi_B^{(1)}|$ and $|\Phi_B^{(3)}|$ of the magnetic flux through the circle at times $t = 1\text{s}$ and $t = 3\text{s}$, respectively.

(b) Find magnitude I_1, I_3 and direction (cw/ccw) of the induced current at times $t = 1\text{s}$ and $t = 3\text{s}$, respectively.



Unit Exam III: Problem #3 (Spring '08)



A circular wire of radius $r = 2.5\text{m}$ and resistance $R = 4.8\Omega$ is placed in the yz -plane as shown.

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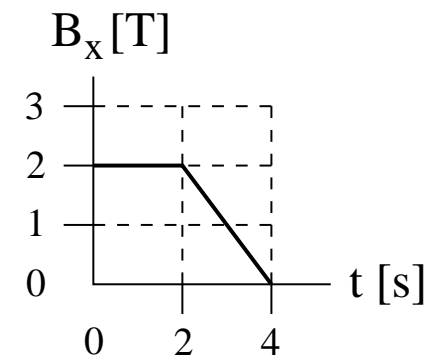
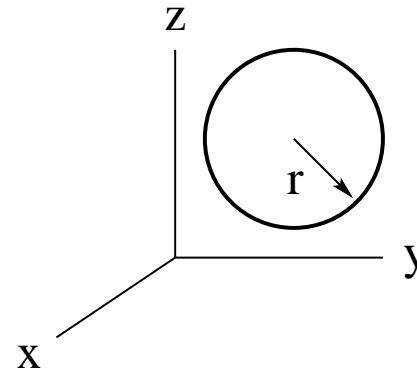
Solution:

$$(a) |\Phi_B^{(1)}| = \pi(2.5\text{m})^2(2\text{T}) = 39.3 \text{ Wb},$$

$$|\Phi_B^{(3)}| = \pi(2.5\text{m})^2(1\text{T}) = 19.6 \text{ Wb}.$$

$$(b) \left| \frac{d\Phi_B^{(1)}}{dt} \right| = 0 \Rightarrow I_1 = 0,$$

$$\left| \frac{d\Phi_B^{(3)}}{dt} \right| = |\pi(2.5\text{m})^2(-1\text{T/s})| = 19.6\text{V} \Rightarrow I_3 = \frac{19.6\text{V}}{4.8\Omega} = 4.1\text{A} \quad (\text{ccw}).$$

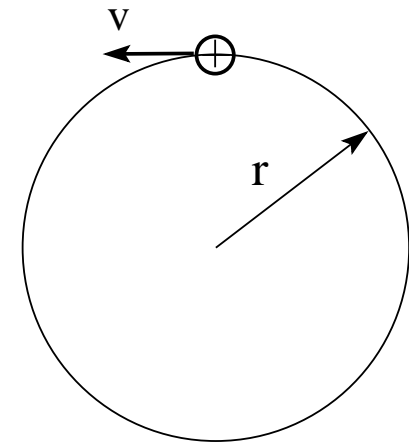


Unit Exam III: Problem #3 (Spring '08)



A proton ($m = 1.67 \times 10^{-27} \text{ kg}$, $q = 1.60 \times 10^{-19} \text{ C}$) with velocity $v = 3.7 \times 10^4 \text{ m/s}$ moves on a circle of radius $r = 0.49 \text{ m}$ in a counterclockwise direction.

- Find the centripetal force F needed to keep the proton on the circle.
- Find direction (\odot or \otimes) and magnitude of the field \mathbf{B} that provides the centripetal force F .
- Find the electric current I produced by the rotating proton.

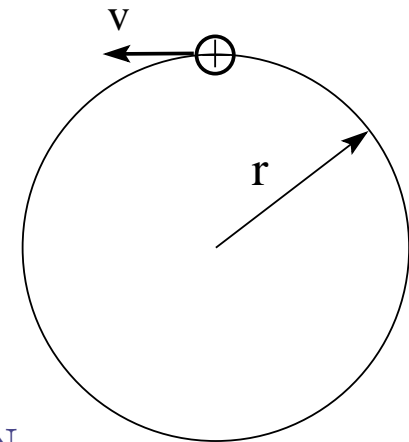


Unit Exam III: Problem #3 (Spring '08)



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- Find the centripetal force F needed to keep the proton on the circle.
- Find direction (\odot or \otimes) and magnitude of the field B that provides the centripetal force F .
- Find the electric current I produced by the rotating proton.



Solution:

$$(a) \quad F = \frac{mv^2}{r} = \frac{(1.67 \times 10^{-27} \text{ kg})(3.7 \times 10^4 \text{ m/s})^2}{0.49 \text{ m}} = 4.67 \times 10^{-18} \text{ N}.$$

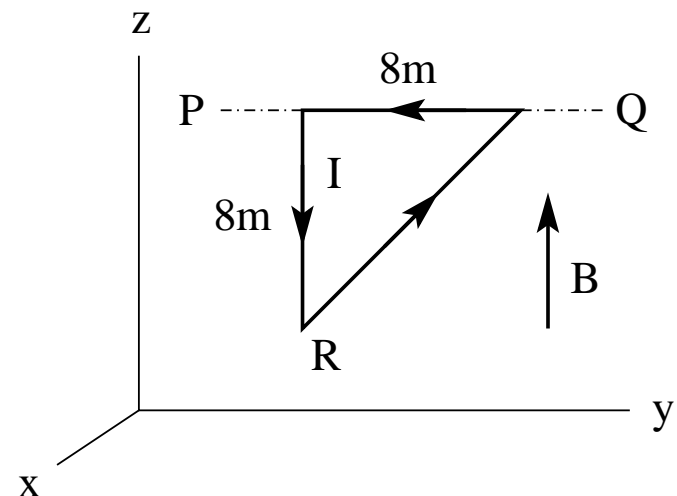
$$(b) \quad F = qvB \quad \Rightarrow \quad B = \frac{F}{qv} = \frac{4.67 \times 10^{-18} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(3.7 \times 10^4 \text{ m/s})} = 0.788 \text{ mT} \quad \otimes \text{ (in)}.$$

$$(c) \quad I = \frac{q}{\tau}, \quad \tau = \frac{2\pi r}{v} \quad \Rightarrow \quad I = \frac{qv}{2\pi r} = 1.92 \times 10^{-15} \text{ A}.$$

Unit Exam III: Problem #1 (Spring '09)



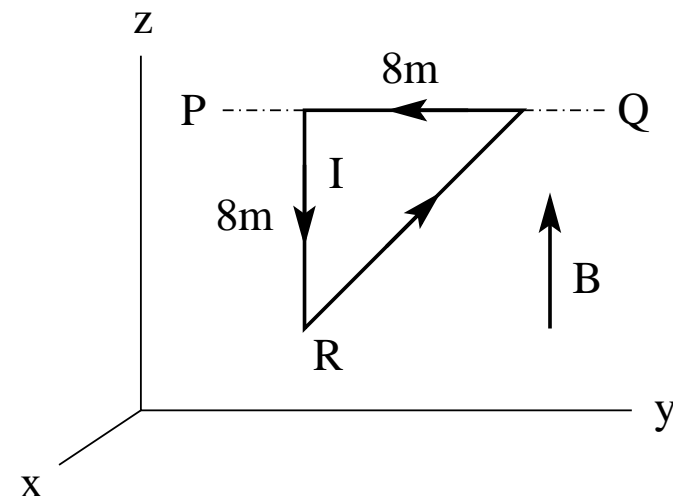
- A triangular conducting loop in the yz -plane with a counterclockwise current $I = 3\text{A}$ is free to rotate about the axis PQ . A uniform magnetic field $\vec{B} = 0.5\text{T}\hat{k}$ is present. (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the triangle. (b) Find the magnetic torque $\vec{\tau}$ (magnitude and direction) acting on the triangle. (c) Find the magnetic force \vec{F}_H (magnitude and direction) acting on the long side (hypotenuse) of the triangle. (d) Find the force \vec{F}_R (magnitude and direction) that must be applied to the corner R to keep the triangle from rotating.



Unit Exam III: Problem #1 (Spring '09)



- A triangular conducting loop in the yz -plane with a counterclockwise current $I = 3\text{A}$ is free to rotate about the axis PQ . A uniform magnetic field $\vec{B} = 0.5\text{T}\hat{k}$ is present. (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the triangle. (b) Find the magnetic torque $\vec{\tau}$ (magnitude and direction) acting on the triangle. (c) Find the magnetic force \vec{F}_H (magnitude and direction) acting on the long side (hypotenuse) of the triangle. (d) Find the force \vec{F}_R (magnitude and direction) that must be applied to the corner R to keep the triangle from rotating.



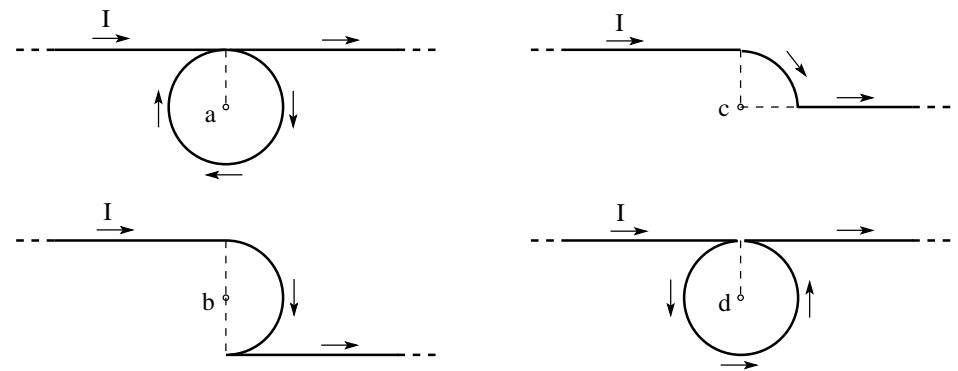
Solution:

- (a) $\vec{\mu} = (3\text{A})(32\text{m}^2)\hat{i} = 96\text{Am}^2\hat{i}$.
- (b) $\vec{\tau} = \vec{\mu} \times \vec{B} = (96\text{Am}^2\hat{i}) \times (0.5\text{T}\hat{k}) = -48\text{Nm}\hat{j}$.
- (c) $F_H = (3\text{A})(8\sqrt{2}\text{m})(0.5\text{T})(\sin 45^\circ) = 12\text{N} \quad \odot$.
- (d) $(-8\text{m}\hat{k}) \times \vec{F}_R = -\vec{\tau} = 48\text{Nm}\hat{j} \quad \Rightarrow \quad \vec{F}_R = -6\text{N}\hat{i}$.

Unit Exam III: Problem #2 (Spring '09)



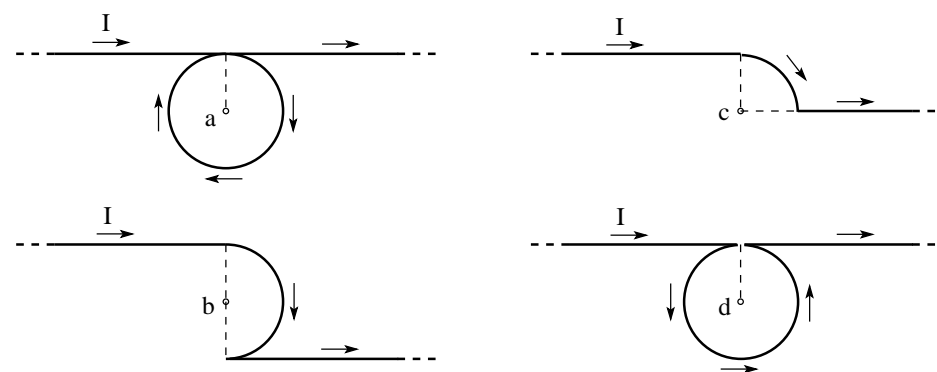
Two semi-infinite straight wires are connected to a curved wire in the form of a full circle, quarter circle, or half circle of radius $R = 1\text{m}$ in four different configurations. A current $I = 1\text{A}$ flows in the directions shown. Find magnitude B_a, B_b, B_c, B_d and direction (\odot/\otimes) of the magnetic field thus generated at the points a, b, c, d .



Unit Exam III: Problem #2 (Spring '09)



Two semi-infinite straight wires are connected to a curved wire in the form of a full circle, quarter circle, or half circle of radius $R = 1\text{m}$ in four different configurations. A current $I = 1\text{A}$ flows in the directions shown. Find magnitude B_a, B_b, B_c, B_d and direction (\odot/\otimes) of the magnetic field thus generated at the points a, b, c, d .



Solution:

$$B_a = \left| \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R} \right| = |100\text{nT} + 628\text{nT} + 100\text{nT}| = 828\text{nT} \quad \otimes$$

$$B_b = \left| \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4R} - \frac{\mu_0 I}{4\pi R} \right| = |100\text{nT} + 314\text{nT} - 100\text{nT}| = 314\text{nT} \quad \otimes$$

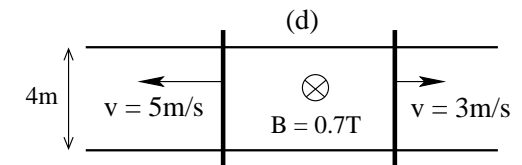
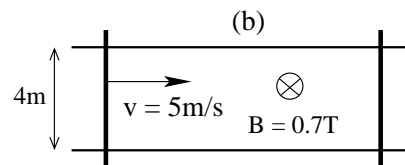
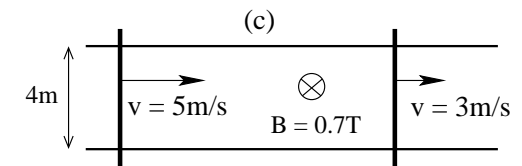
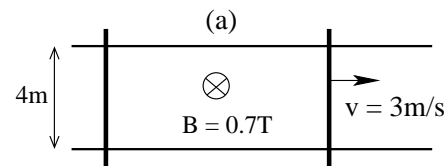
$$B_c = \left| \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{8R} + 0 \right| = |100\text{nT} + 157\text{nT}| = 257\text{nT} \quad \otimes$$

$$B_d = \left| \frac{\mu_0 I}{4\pi R} - \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R} \right| = |100\text{nT} - 628\text{nT} + 100\text{nT}| = 428\text{nT} \quad \odot$$

Unit Exam III: Problem #3 (Spring '09)



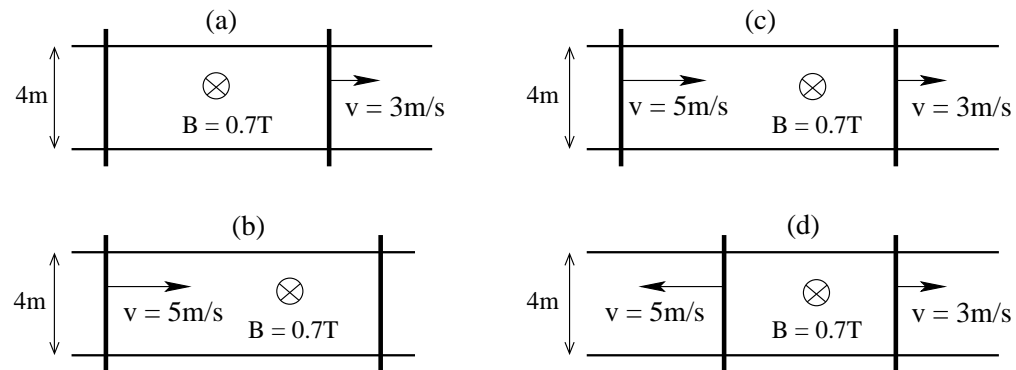
A pair of rails are connected by two mobile rods. A uniform magnetic field B directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is $R = 0.2\Omega$ in each case. Find magnitude I and direction (cw/ccw) of the induced current in each case.



Unit Exam III: Problem #3 (Spring '09)



A pair of rails are connected by two mobile rods. A uniform magnetic field B directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is $R = 0.2\Omega$ in each case. Find magnitude I and direction (cw/ccw) of the induced current in each case.



Solution:

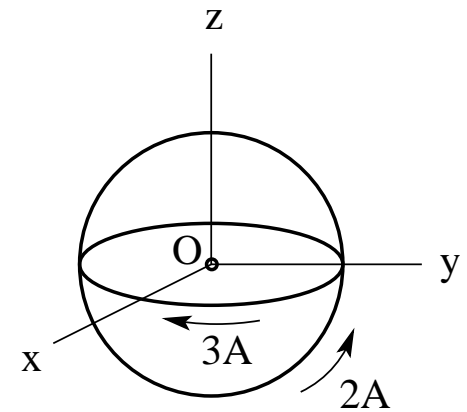
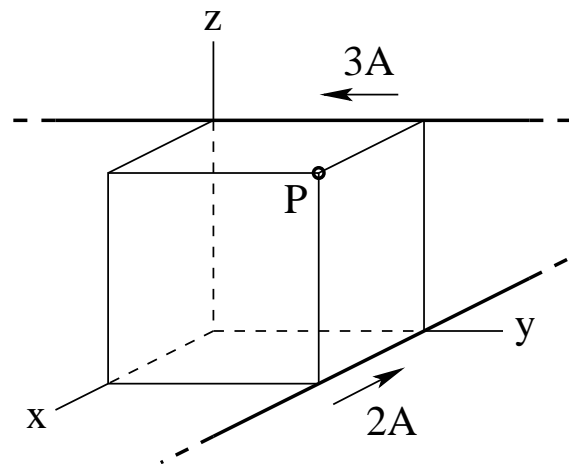
$$\begin{aligned}
 \text{(a)} \quad |\mathcal{E}| &= (3\text{m/s})(0.7\text{T})(4\text{m}) = 8.4\text{V}, & I &= \frac{8.4\text{V}}{0.2\Omega} = 42\text{A} & \text{CCW} \\
 \text{(b)} \quad |\mathcal{E}| &= (5\text{m/s})(0.7\text{T})(4\text{m}) = 14\text{V}, & I &= \frac{14\text{V}}{0.2\Omega} = 70\text{A} & \text{CW} \\
 \text{(c)} \quad |\mathcal{E}| &= (5\text{m/s} - 3\text{m/s})(0.7\text{T})(4\text{m}) = 5.6\text{V}, & I &= \frac{5.6\text{V}}{0.2\Omega} = 28\text{A} & \text{CW} \\
 \text{(d)} \quad |\mathcal{E}| &= (5\text{m/s} + 3\text{m/s})(0.7\text{T})(4\text{m}) = 22.4\text{V}, & I &= \frac{22.4\text{V}}{0.2\Omega} = 112\text{A} & \text{CCW}
 \end{aligned}$$

Unit Exam III: Problem #1 (Spring '11)



(a) Two very long straight wires carry currents as shown. A cube with edges of length 8cm serves as scaffold. Find the magnetic field at point P in the form $\mathbf{B} = B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}$ with B_x, B_y, B_z in SI units.

(b) Two circular currents of radius 5cm, one in the xy -lane and the other in the yz -plane, carry currents as shown. Both circles are centered at point O . Find the magnetic field at point O in the form $\mathbf{B} = B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}$ with B_x, B_y, B_z in SI units.

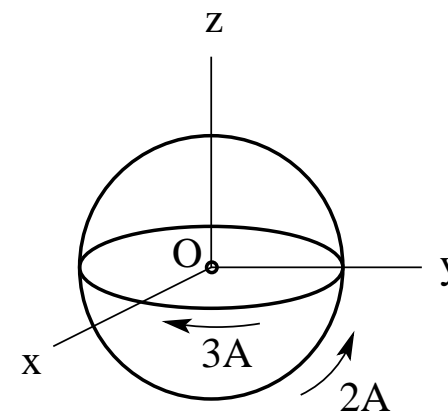
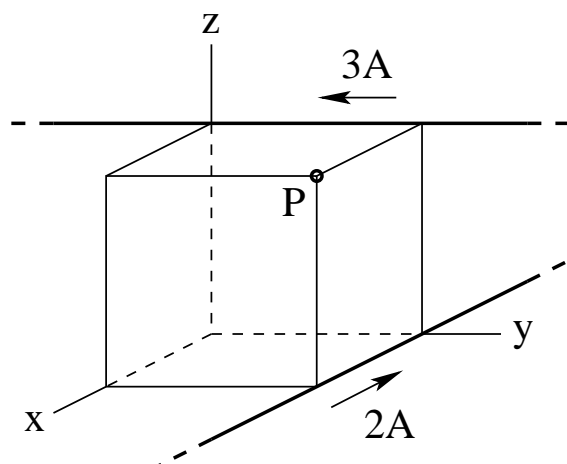


Unit Exam III: Problem #1 (Spring '11)



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Solution:

$$(a) \quad B_x = 0, \quad B_y = \frac{\mu_0(2A)}{2\pi(0.08\text{m})} = 5\mu\text{T}, \quad B_z = \frac{\mu_0(3A)}{2\pi(0.08\text{m})} = 7.5\mu\text{T}.$$

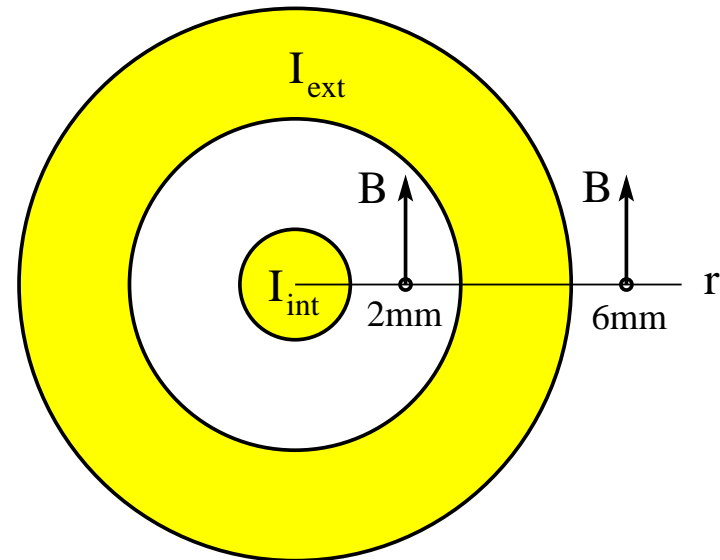
$$(b) \quad B_x = \frac{\mu_0(2A)}{2(0.05\text{m})} = 25.1\mu\text{T}, \quad B_y = 0, \quad B_z = -\frac{\mu_0(3A)}{2(0.05\text{m})} = -37.7\mu\text{T}.$$

Unit Exam III: Problem #2 (Spring '11)



The coaxial cable shown has surfaces at radii 1mm, 3mm, and 5mm. The magnetic field is the same at radii 2mm and 6mm, namely $B = 7\mu\text{T}$ in the direction shown.

- (a) Find magnitude (in SI units) and direction (in/out) of the current I_{int} flowing through the inner conductor.
- (b) Find magnitude (in SI units) and direction (in/out) of the current I_{ext} flowing through the outer conductor.

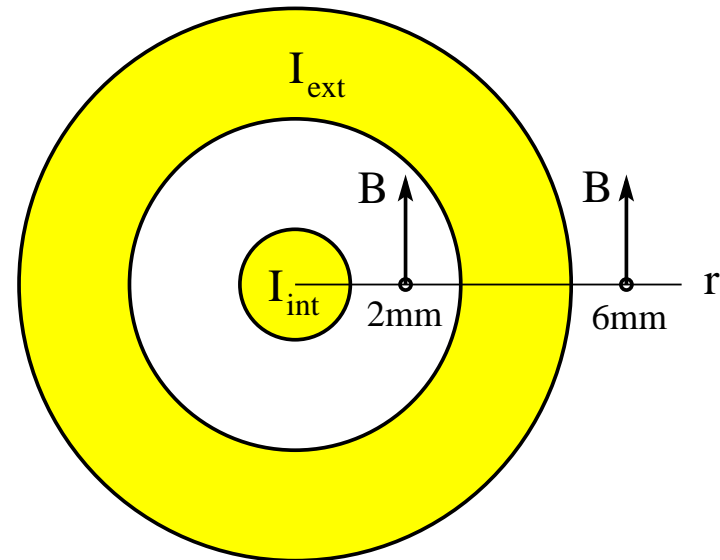


Unit Exam III: Problem #2 (Spring '11)



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- (b) Find magnitude (in SI units) and direction (in/out) of the current I_{ext} flowing through the outer conductor.



Solution:

$$(a) (7\mu\text{T})(2\pi)(0.002\text{m}) = \mu_0 I_{\text{int}} \Rightarrow I_{\text{int}} = 0.07\text{A} \quad (\text{out})$$

$$(b) (7\mu\text{T})(2\pi)(0.006\text{m}) = \mu_0(I_{\text{int}} + I_{\text{ext}}) \Rightarrow I_{\text{int}} + I_{\text{ext}} = 0.21\text{A} \quad (\text{out})$$
$$\Rightarrow I_{\text{ext}} = 0.14\text{A} \quad (\text{out})$$

Unit Exam III: Problem #3 (Spring '11)

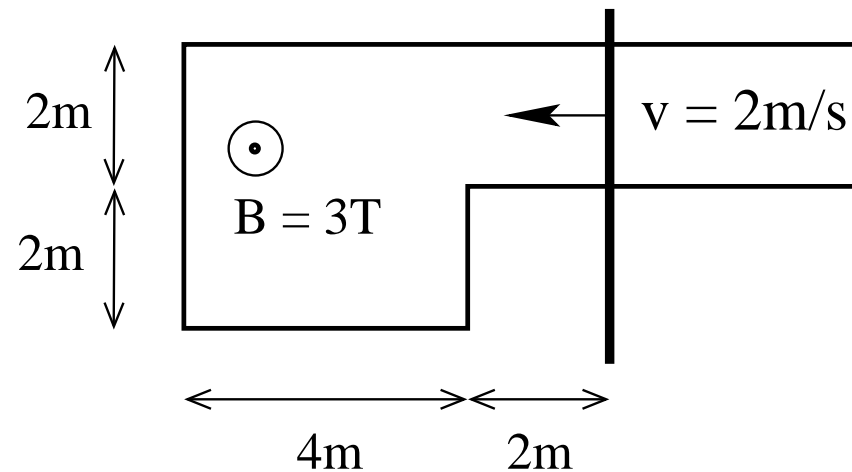


A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown. The rod moves at constant velocity.

(a) Find the magnetic flux Φ_B through the frame and the induced emf \mathcal{E} around the frame at the instant shown.

(b) Find the magnetic flux Φ_B through the frame and the induced emf \mathcal{E} around the frame two seconds later.

Write magnitudes only (in SI units), no directions.



Unit Exam III: Problem #3 (Spring '11)

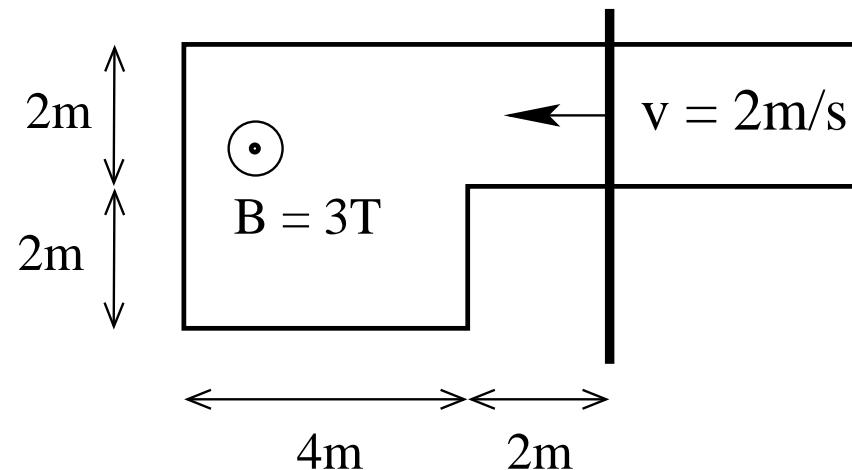


A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown. The rod moves at constant velocity.

(a) Find the magnetic flux Φ_B through the frame and the induced emf \mathcal{E} around the frame at the instant shown.

(b) Find the magnetic flux Φ_B through the frame and the induced emf \mathcal{E} around the frame two seconds later.

Write magnitudes only (in SI units), no directions.



Solution:

(a) $\Phi_B = (20\text{m}^2)(3\text{T}) = 60\text{Wb}$, $\mathcal{E} = (2\text{m/s})(3\text{T})(2\text{m}) = 12\text{V}$.

(b) $\Phi_B = (8\text{m}^2)(3\text{T}) = 24\text{Wb}$, $\mathcal{E} = (2\text{m/s})(3\text{T})(4\text{m}) = 24\text{V}$.

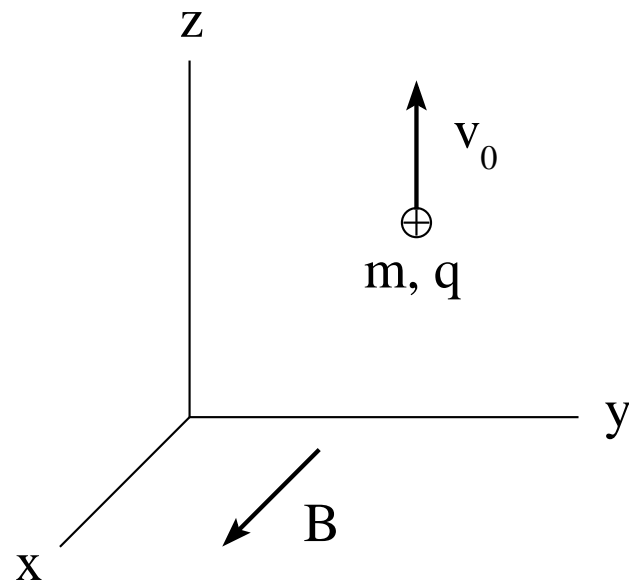
Unit Exam III: Problem #1 (Spring '12)



In a region of uniform magnetic field $\mathbf{B} = 5\text{mT}\hat{\mathbf{i}}$, a proton

($m = 1.67 \times 10^{-27}\text{kg}$, $q = 1.60 \times 10^{-19}\text{C}$) is launched with velocity $\mathbf{v}_0 = 4000\text{m/s}\hat{\mathbf{k}}$.

- (a) Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.
- (b) Calculate the radius r of the circular path.
- (c) Calculate the time T it takes the proton to go around that circle once.
- (d) Sketch the circular path of the proton in the graph.



Unit Exam III: Problem #1 (Spring '12)



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- Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.
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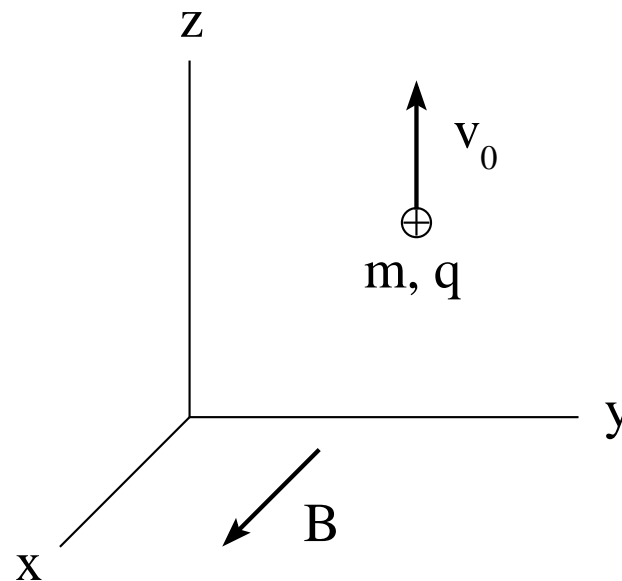
Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\text{N}$.

(b) $\frac{mv_0^2}{r} = qv_0B \Rightarrow r = \frac{mv_0}{qB} = 8.35\text{mm}$.

(c) $T = \frac{2\pi r}{v_0} = \frac{2\pi m}{qB} = 13.1\mu\text{s}$.

(d) Center of circle to the right of proton's initial position (cw motion).

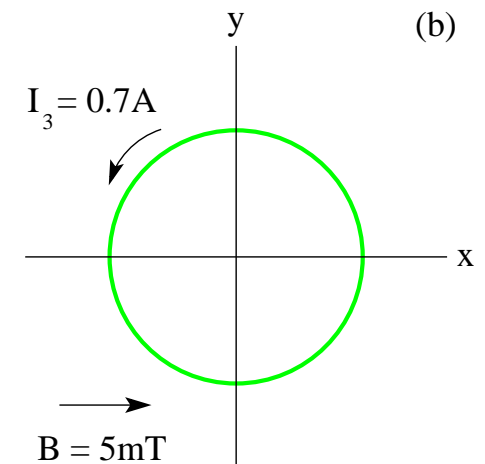
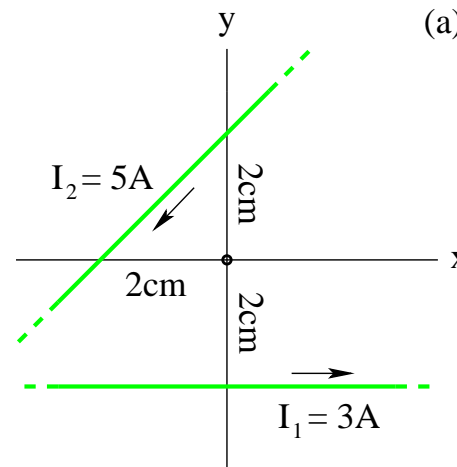


Unit Exam III: Problem #2 (Spring '12)



(a) Two very long straight wires positioned in the xy -plane carry electric currents I_1, I_2 as shown. Calculate magnitude (B_1, B_2) and direction (\odot, \otimes) of the magnetic field produced by each current at the origin of the coordinate system.

(b) A conducting loop of radius $r = 3\text{cm}$ placed in the xy -plane carries a current $I_3 = 0.7\text{A}$ in the direction shown. Find direction and magnitude of the torque $\vec{\tau}$ acting on the loop if it is placed in a magnetic field $\mathbf{B} = 5\text{mT}\hat{i}$.

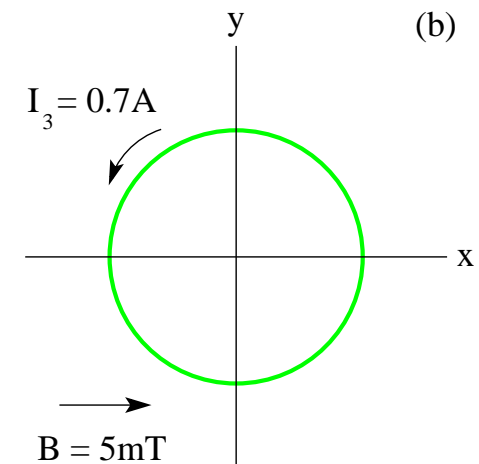
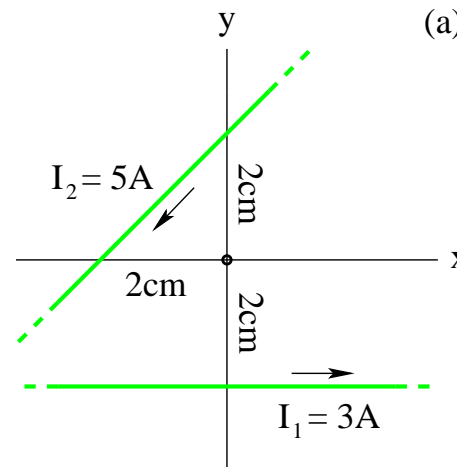


Unit Exam III: Problem #2 (Spring '12)



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Solution:

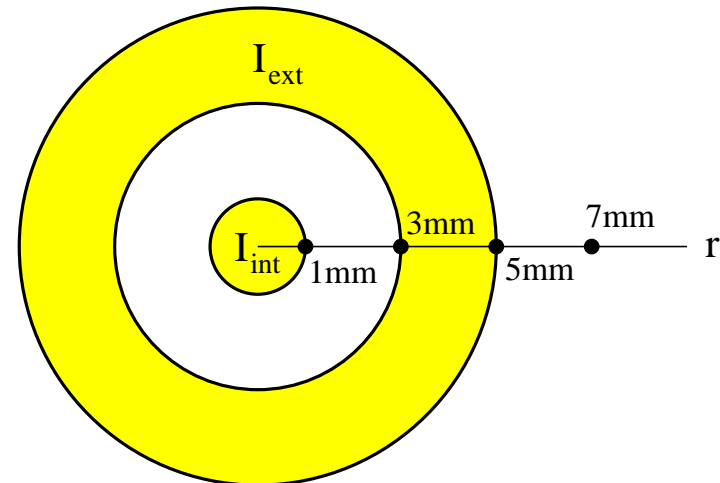
$$(a) \quad B_1 = \frac{\mu_0(3\text{A})}{2\pi(2\text{cm})} = 30\mu\text{T}. \quad \odot \quad B_2 = \frac{\mu_0(5\text{A})}{2\pi(1.41\text{cm})} = 70.9\mu\text{T}. \quad \odot$$

$$(b) \quad \vec{\mu} = \pi(3\text{cm})^2(0.7\text{A})\hat{\mathbf{k}} = 1.98 \times 10^{-3} \text{Am}^2\hat{\mathbf{k}} \quad \Rightarrow \quad \vec{\tau} = \vec{\mu} \times \mathbf{B} = 9.90 \times 10^{-6} \text{Nm}\hat{\mathbf{j}}.$$

Unit Exam III: Problem #3 (Spring '12)



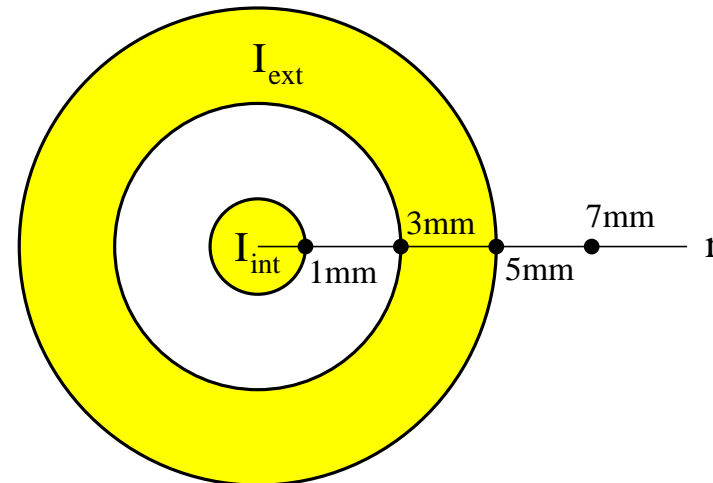
The coaxial cable shown in cross section has surfaces at radii 1mm, 3mm, and 5mm. Equal currents flow through both conductors: $I_{int} = I_{ext} = 0.03\text{A} \odot$ (out). Find direction (\uparrow, \downarrow) and magnitude (B_1, B_3, B_5, B_7) of the magnetic field at the four radii indicated (\bullet).



Unit Exam III: Problem #3 (Spring '12)



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Solution:

$$2\pi(1\text{mm})B_1 = \mu_0(0.03\text{A}) \Rightarrow B_1 = 6\mu\text{T} \uparrow$$

$$2\pi(3\text{mm})B_1 = \mu_0(0.03\text{A}) \Rightarrow B_1 = 2\mu\text{T} \uparrow$$

$$2\pi(5\text{mm})B_1 = \mu_0(0.06\text{A}) \Rightarrow B_1 = 2.4\mu\text{T} \uparrow$$

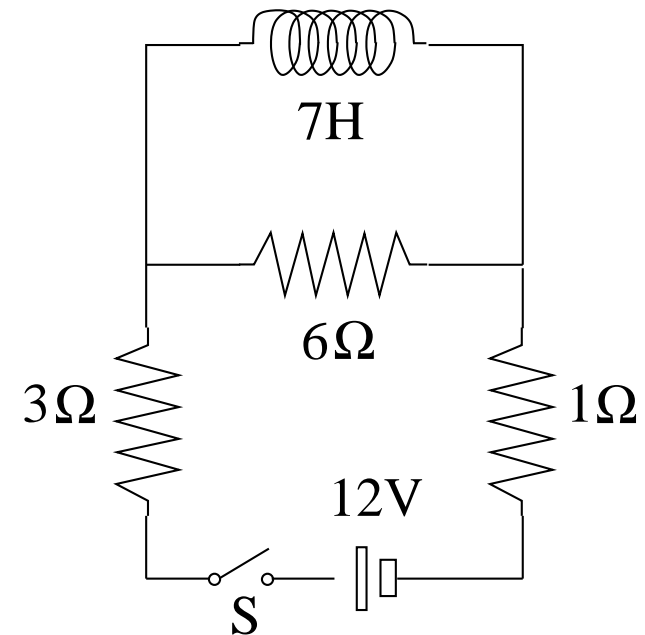
$$2\pi(7\text{mm})B_1 = \mu_0(0.06\text{A}) \Rightarrow B_1 = 1.71\mu\text{T} \uparrow$$

Unit Exam IV: Problem #1 (Spring '12)



In the circuit shown we close the switch S at time $t = 0$. Find the current I_L through the inductor and the voltage V_6 across the 6Ω -resistor

- (a) immediately after the switch has been closed,
- (b) a very long time later.



Unit Exam IV: Problem #1 (Spring '12)



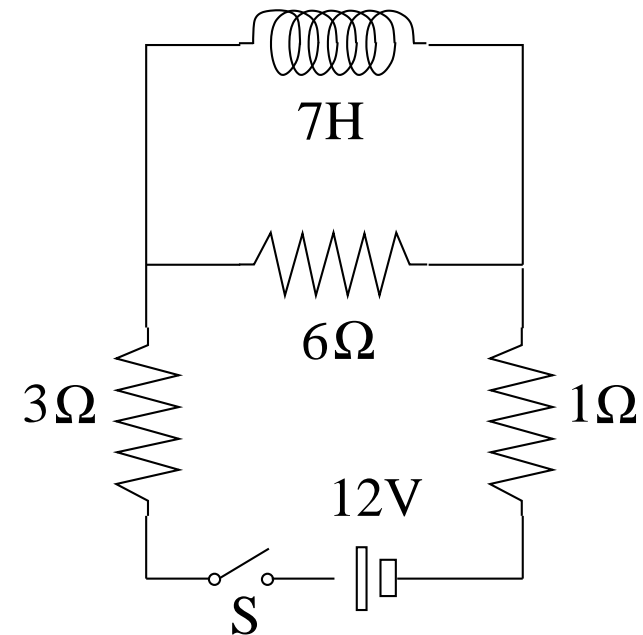
In the circuit shown we close the switch S at time $t = 0$. Find the current I_L through the inductor and the voltage V_6 across the 6Ω -resistor

- (a) immediately after the switch has been closed,
- (b) a very long time later.

Solution:

(a) $I_L = 0, \quad I_6 = \frac{12V}{10\Omega} = 1.2A, \quad V_6 = (6\Omega)(1.2A) = 7.2V.$

(b) $I_L = \frac{12V}{4\Omega} = 3A, \quad V_6 = 0.$

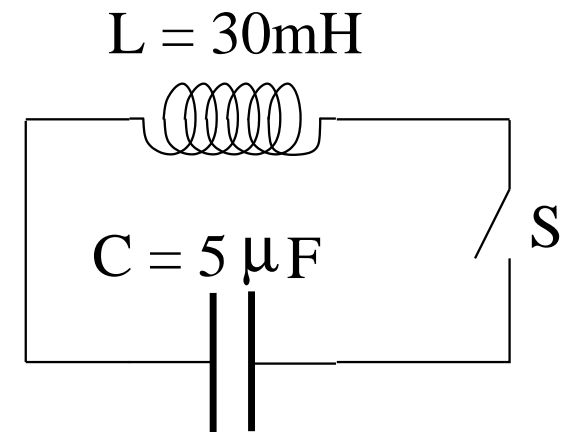


Unit Exam IV: Problem #2 (Spring '12)



At time $t = 0$ the capacitor is charged to $Q_{max} = 4\mu\text{C}$ and the switch is being closed. The charge on the capacitor begins to decrease and the current through the inductor begins to increase.

- (a) At what time t_1 is the capacitor discharged for the first time?
- (b) At what time t_2 has the current through the inductor returned to zero for the first time?
- (c) What is the maximum energy stored in the capacitor at any time?
- (d) What is the maximum energy stored in the inductor at any time?



Unit Exam IV: Problem #2 (Spring '12)



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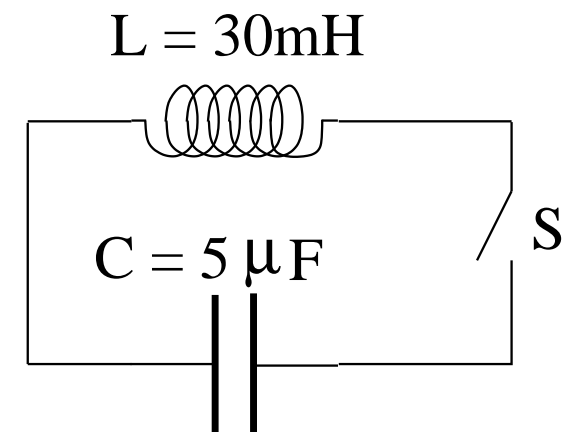
Solution:

$$(a) \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC} = 2.43\text{ms}, \quad t_1 = \frac{T}{4} = 0.608\text{ms}.$$

$$(b) \quad t_2 = \frac{T}{2} = 1.22\text{ms}.$$

$$(c) \quad U_C^{max} = \frac{Q_{max}^2}{2C} = 1.6\mu\text{J}.$$

$$(d) \quad U_L^{max} = U_C^{max} = 1.6\mu\text{J} \quad (\text{energy conservation.})$$

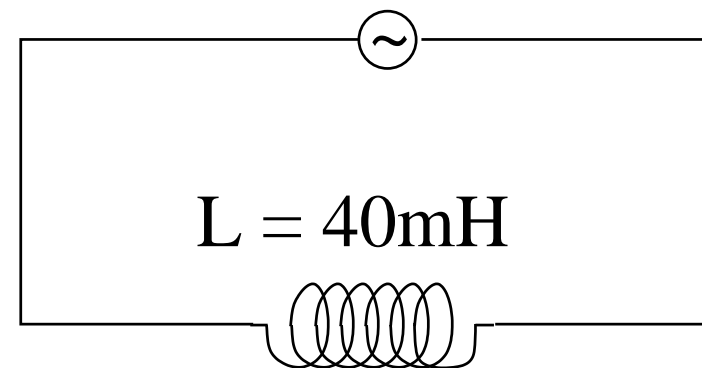


Unit Exam IV: Problem #3 (Spring '12)



The *ac* voltage supplied in the circuit shown is $\mathcal{E} = \mathcal{E}_{max} \cos(\omega t)$ with $\mathcal{E}_{max} = 170\text{V}$ and $\omega = 377\text{rad/s}$.

- (a) What is the maximum value I_{max} of the current?
- (b) What is the emf $\mathcal{E}(t)$ at $t = 5\text{ms}$?
- (c) What is the current $I(t)$ at $t = 5\text{ms}$?
- (d) What is the power transfer $P(t)$ between *ac* source and device at $t = 5\text{ms}$?

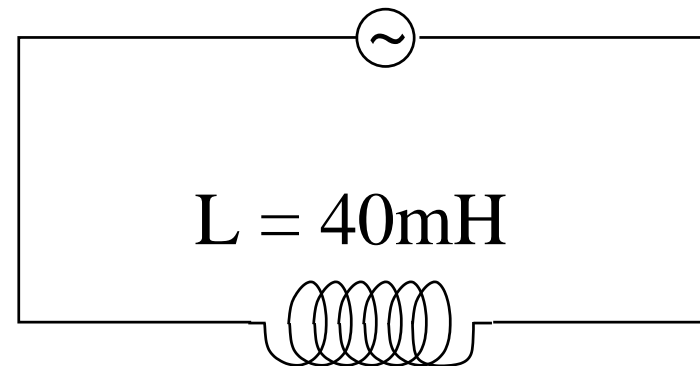


Unit Exam IV: Problem #3 (Spring '12)



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- (d) What is the power transfer $P(t)$ between *ac* source and device at $t = 5\text{ms}$?



Solution:

$$(a) I_{max} = \frac{\mathcal{E}_{max}}{\omega L} = \frac{170\text{V}}{(377\text{rad/s})(40\text{mH})} = 11.3\text{A}.$$

$$(b) \mathcal{E} = (170\text{V}) \cos(1.885\text{rad}) = (170\text{V})(-0.309) = -52.5\text{V}.$$

$$(c) I = (11.3\text{A}) \cos(1.885\text{rad} - \pi/2) = (11.3\text{A}) \cos(0.314) = (11.3\text{A})(0.951) = 10.7\text{A}.$$

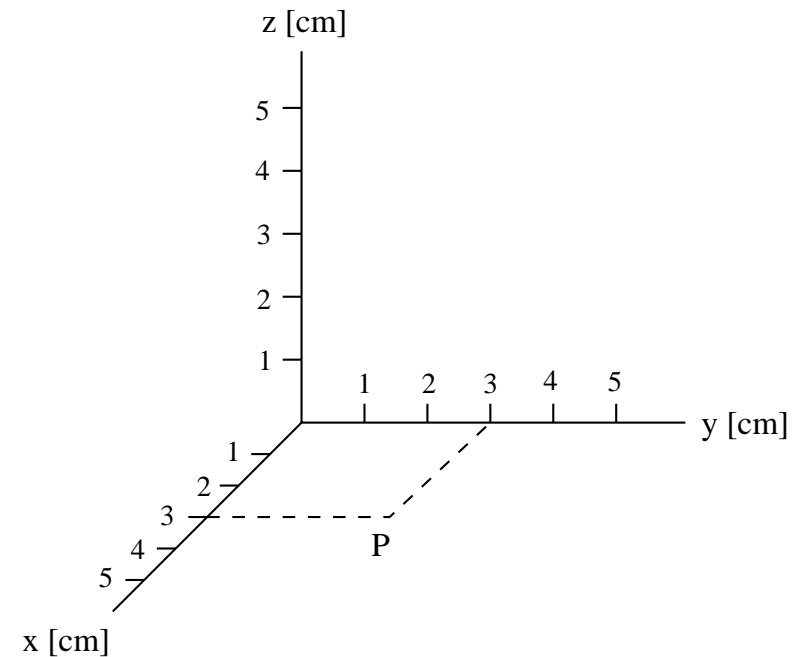
$$(d) P = \mathcal{E}I = (-52.5\text{V})(10.7\text{A}) = -562\text{W}.$$

Unit Exam III: Problem #1 (Spring '13)



In a region of uniform magnetic field \mathbf{B} a proton ($m = 1.67 \times 10^{-27}\text{kg}$, $q = 1.60 \times 10^{-19}\text{C}$) experiences a force $\mathbf{F} = 9.0 \times 10^{-19}\text{N}\hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 3000\text{m/s}\hat{\mathbf{j}}$ on a circular path.

- (a) Find the magnetic field \mathbf{B} (magnitude and direction).
- (b) Calculate the radius r of the circular path.
- (c) Locate the center C of the circular path in the coordinate system on the page.



Unit Exam III: Problem #1 (Spring '13)



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Solution:

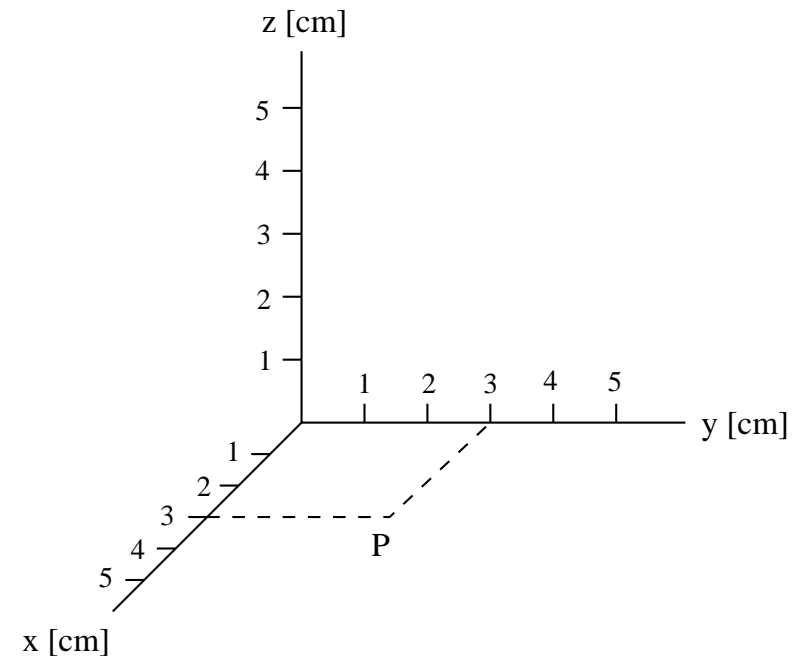
$$(a) \quad B = \frac{F}{qv_0} = 1.88 \times 10^{-3} \text{T}, \quad \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{k}}$$

$$\Rightarrow \mathbf{B} = 1.88 \times 10^{-3} \text{T} \hat{\mathbf{k}}.$$

$$(b) \quad F = \frac{mv_0^2}{r} = qv_0 B$$

$$\Rightarrow r = \frac{mv_0^2}{F} = \frac{mv_0}{qB} = 1.67 \text{cm}.$$

$$(c) \quad C = 4.67 \text{cm} \hat{\mathbf{i}} + 3.00 \text{cm} \hat{\mathbf{j}}.$$

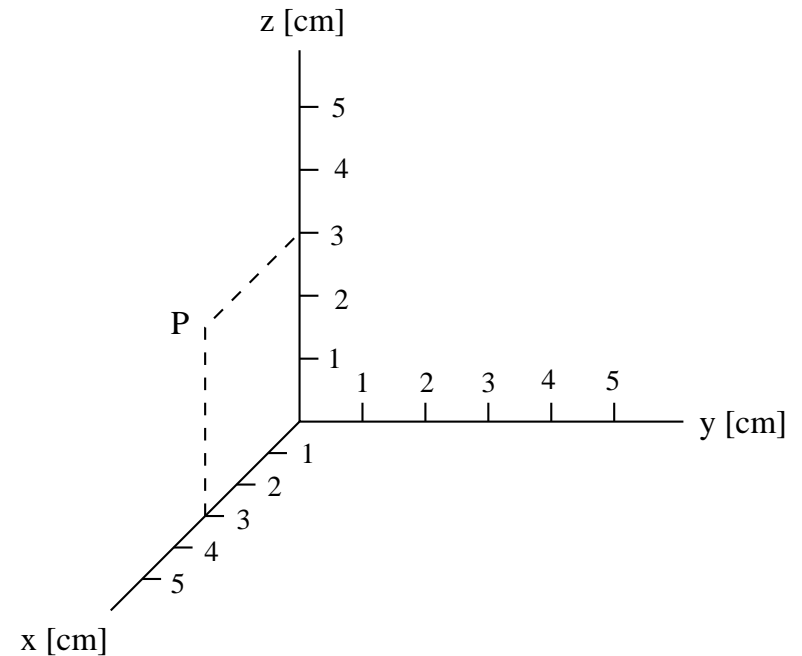


Unit Exam III: Problem #1 (Spring '13)



In a region of uniform magnetic field \mathbf{B} a proton ($m = 1.67 \times 10^{-27} \text{kg}$, $q = 1.60 \times 10^{-19} \text{C}$) experiences a force $\mathbf{F} = 8.0 \times 10^{-19} \text{N} \hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 2000 \text{m/s} \hat{\mathbf{k}}$ on a circular path.

- (a) Find the magnetic field \mathbf{B} (magnitude and direction).
- (b) Calculate the radius r of the circular path.
- (c) Locate the center C of the circular path in the coordinate system on the page.



Unit Exam III: Problem #1 (Spring '13)



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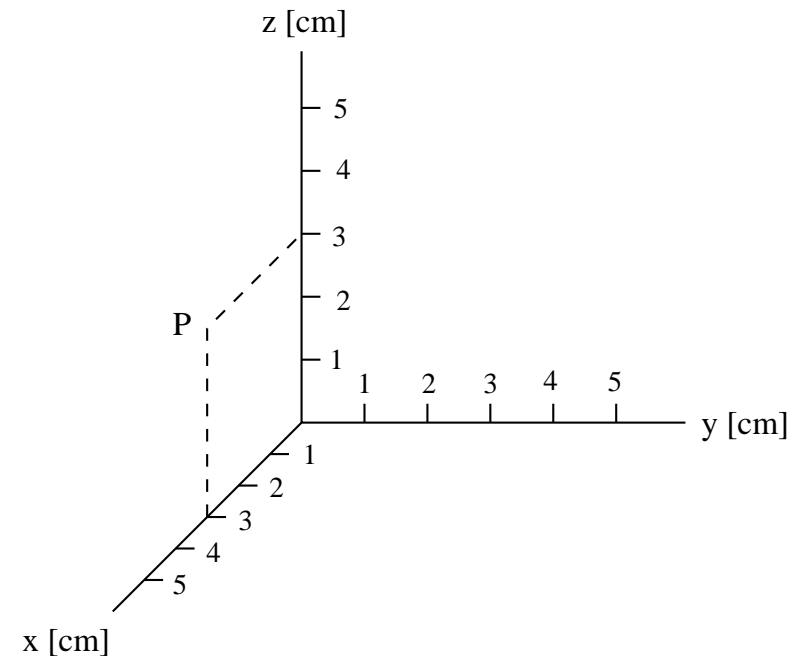
- (a) Find the magnetic field \mathbf{B} (magnitude and direction).
- (b) Calculate the radius r of the circular path.
- (c) Locate the center C of the circular path in the coordinate system on the page.

Solution:

$$(a) \quad B = \frac{F}{qv_0} = 2.50 \times 10^{-3} \text{T}, \quad \hat{\mathbf{i}} = \hat{\mathbf{k}} \times (-\hat{\mathbf{j}})$$
$$\Rightarrow \mathbf{B} = -2.50 \times 10^{-3} \text{T} \hat{\mathbf{j}}.$$

$$(b) \quad F = \frac{mv_0^2}{r} = qv_0 B$$
$$\Rightarrow r = \frac{mv_0^2}{F} = \frac{mv_0}{qB} = 0.835 \text{cm}.$$

$$(c) \quad C = 3.84 \text{cm} \hat{\mathbf{i}} + 3.00 \text{cm} \hat{\mathbf{k}}.$$

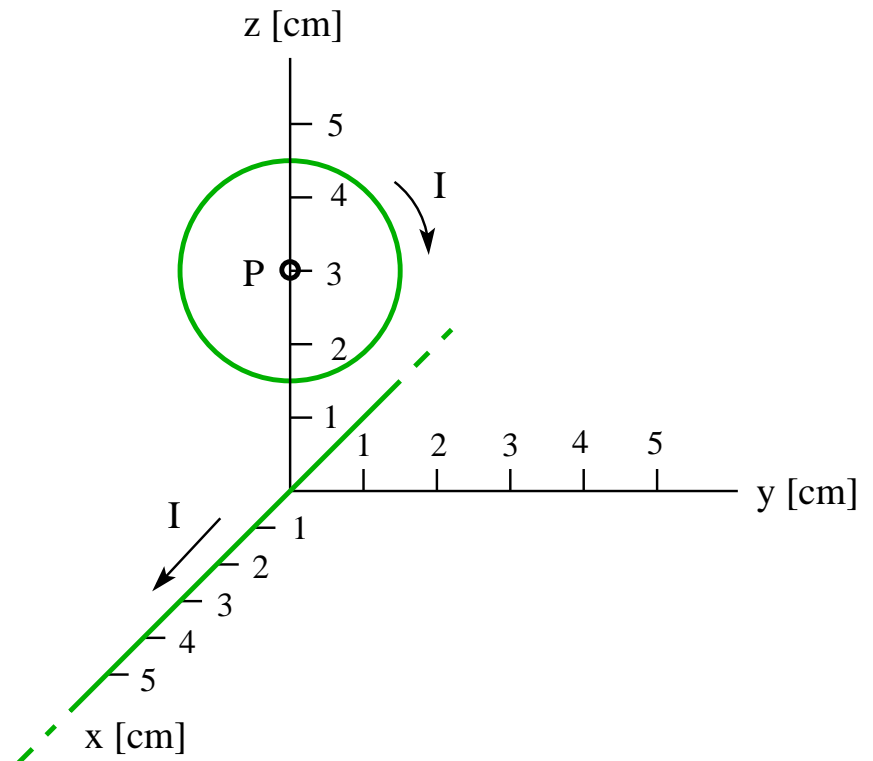


Unit Exam III: Problem #2 (Spring '13)



A very long, straight wire is positioned along the x -axis and a circular wire of 1.5cm radius in the yz plane with its center P on the z -axis as shown. Both wires carry a current $I = 0.6\text{A}$ in the directions shown.

- Find the magnetic field \mathbf{B}_c (magnitude and direction) generated at point P by the current in the circular wire.
- Find the magnetic field \mathbf{B}_s (magnitude and direction) generated at point P by the current in the straight wire.
- Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the circular current.



Unit Exam III: Problem #2 (Spring '13)



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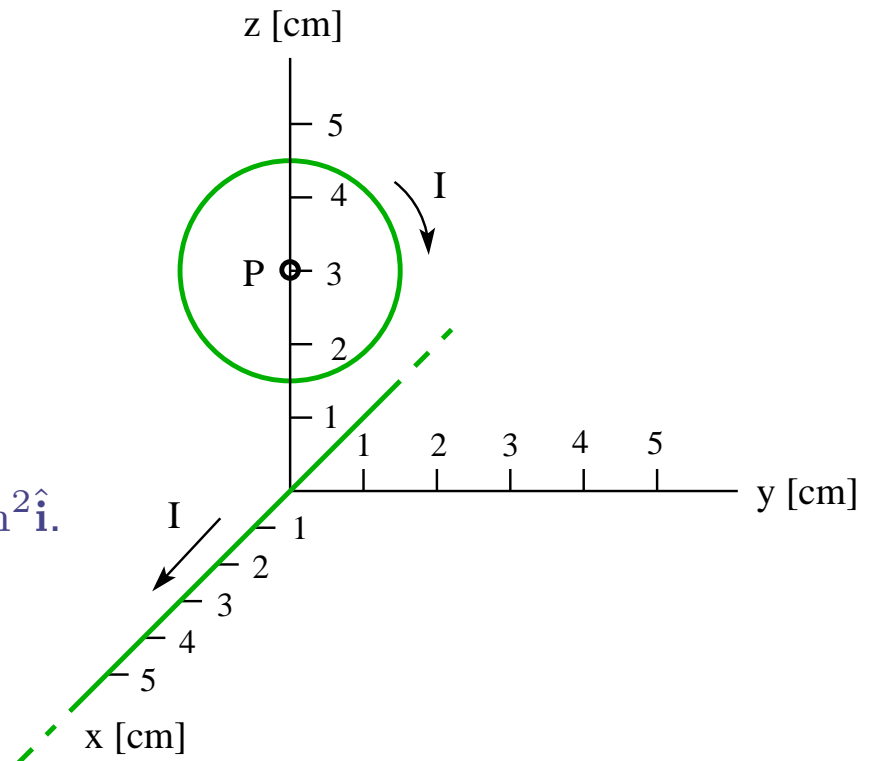
- (a) Find the magnetic field \mathbf{B}_c (magnitude and direction) generated at point P by the current in the circular wire.
- (b) Find the magnetic field \mathbf{B}_s (magnitude and direction) generated at point P by the current in the straight wire.
- (c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the circular current.

Solution:

$$(a) \mathbf{B}_c = \frac{\mu_0(0.6\text{A})}{2(0.015\text{m})}(-\hat{\mathbf{i}}) = -2.51 \times 10^{-5}\text{T}\hat{\mathbf{i}}.$$

$$(b) \mathbf{B}_s = \frac{\mu_0(0.6\text{A})}{2\pi(0.03\text{m})}(-\hat{\mathbf{j}}) = -4.00 \times 10^{-6}\text{T}\hat{\mathbf{j}}.$$

$$(c) \vec{\mu} = \pi(0.015\text{m})^2(0.6\text{A})(-\hat{\mathbf{i}}) = -4.24 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}.$$

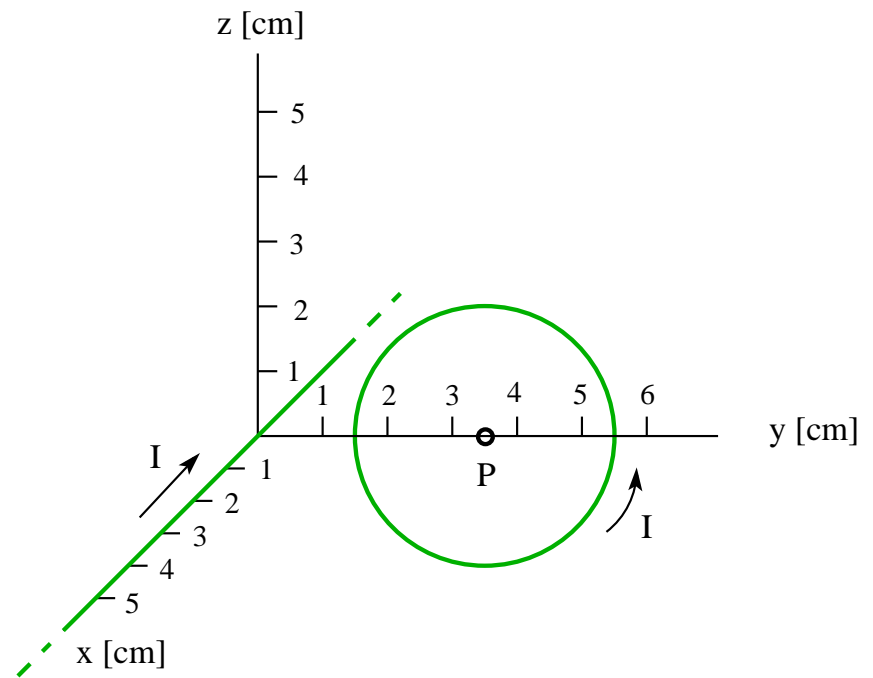


Unit Exam III: Problem #2 (Spring '13)



A very long straight wire is positioned along the x -axis and a circular wire of 2.0cm radius in the yz plane with its center P on the y -axis as shown. Both wires carry a current $I = 0.5\text{A}$ in the directions shown.

- Find the magnetic field \mathbf{B}_c (magnitude and direction) generated at point P by the current in the circular wire.
- Find the magnetic field \mathbf{B}_s (magnitude and direction) generated at point P by the current in the straight wire.
- Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the circular current.



Unit Exam III: Problem #2 (Spring '13)



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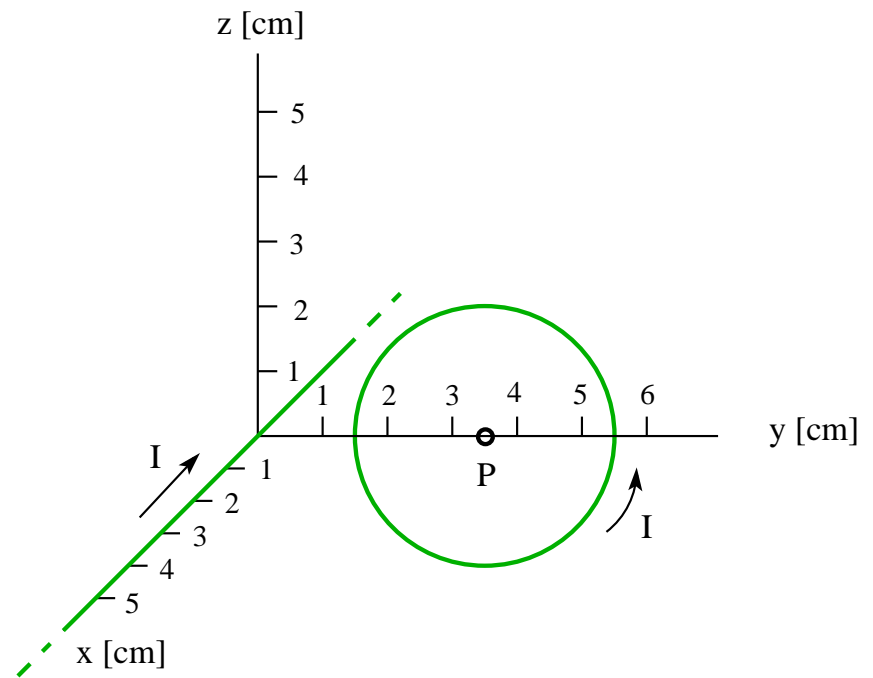
- Find the magnetic field \mathbf{B}_c (magnitude and direction) generated at point P by the current in the circular wire.
- Find the magnetic field \mathbf{B}_s (magnitude and direction) generated at point P by the current in the straight wire.
- Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the circular current.

Solution:

$$(a) \mathbf{B}_c = \frac{\mu_0(0.5\text{A})}{2(0.02\text{m})} \hat{\mathbf{i}} = 1.57 \times 10^{-5} \text{T} \hat{\mathbf{i}}.$$

$$(b) \mathbf{B}_s = \frac{\mu_0(0.5\text{A})}{2\pi(0.035\text{m})} (-\hat{\mathbf{k}}) = -2.86 \times 10^{-6} \text{T} \hat{\mathbf{k}}.$$

$$(c) \vec{\mu} = \pi(0.02\text{m})^2(0.5\text{A}) \hat{\mathbf{i}} = 6.28 \times 10^{-4} \text{Am}^2 \hat{\mathbf{i}}.$$

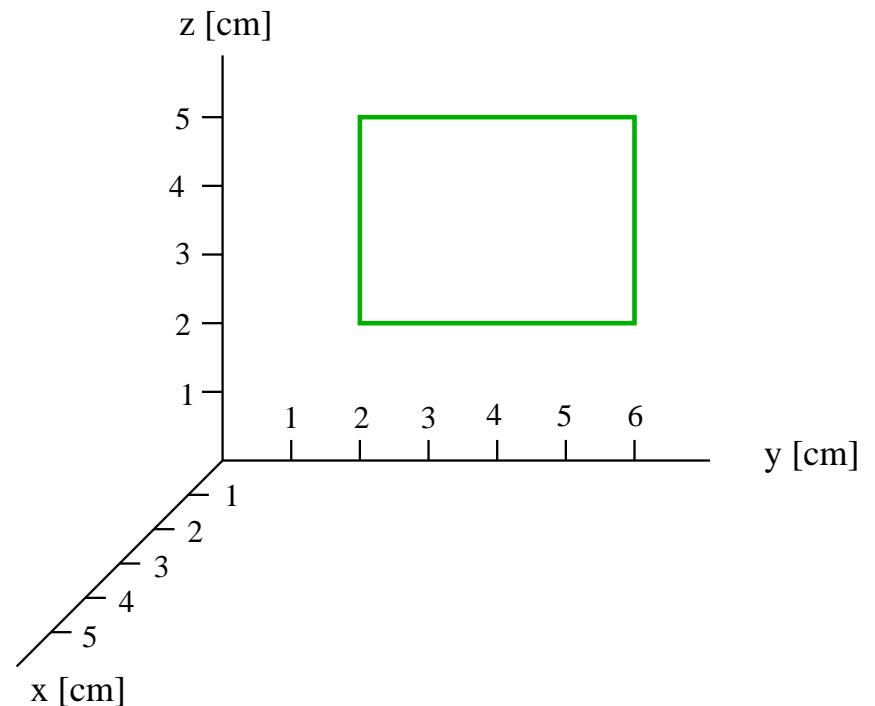


Unit Exam III: Problem #3 (Spring '13)



Consider a wire with a resistance per unit length of $1\Omega/\text{cm}$ bent into a rectangular loop and placed into the yz -plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (2\hat{\mathbf{i}} + 1\hat{\mathbf{j}} + 3\hat{\mathbf{k}})t\text{T/s}$, where t is the time in seconds.

- (a) Find the magnetic flux Φ_B through the rectangle at time $t = 2\text{s}$.
- (b) Find magnitude and direction (cw/ccw) of the induced EMF \mathcal{E} around the rectangle at time $t = 2\text{s}$.
- (c) Infer the induced current I from the induced EMF.



Unit Exam III: Problem #3 (Spring '13)



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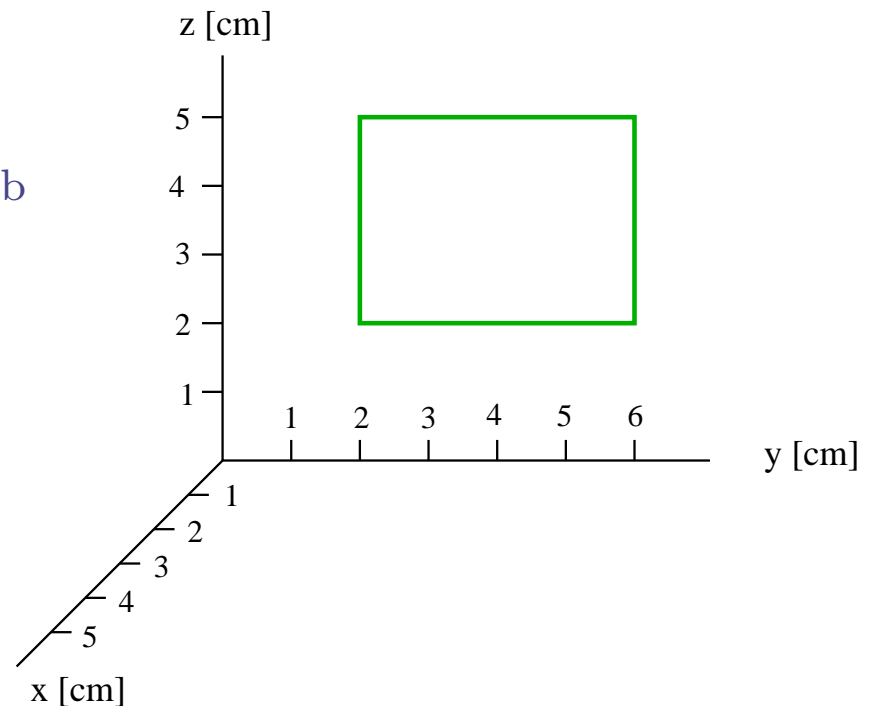
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- (b) Find magnitude and direction (cw/ccw) of the induced EMF \mathcal{E} around the rectangle at time $t = 2\text{s}$.
- (c) Infer the induced current I from the induced EMF.

Solution:

(a) $\Phi_B = \pm(4\text{cm})(3\text{cm})(2\text{T/s})(2\text{s}) = \pm 4.8 \times 10^{-3}\text{Wb}$

(b) $\mathcal{E} = \mp(4\text{cm})(3\text{cm})(2\text{T/s}) = \mp 2.4\text{mV}$ (cw)

(c) $I = \frac{2.4\text{mV}}{(1\Omega/\text{cm})(14\text{cm})} = 0.171\text{mA}$

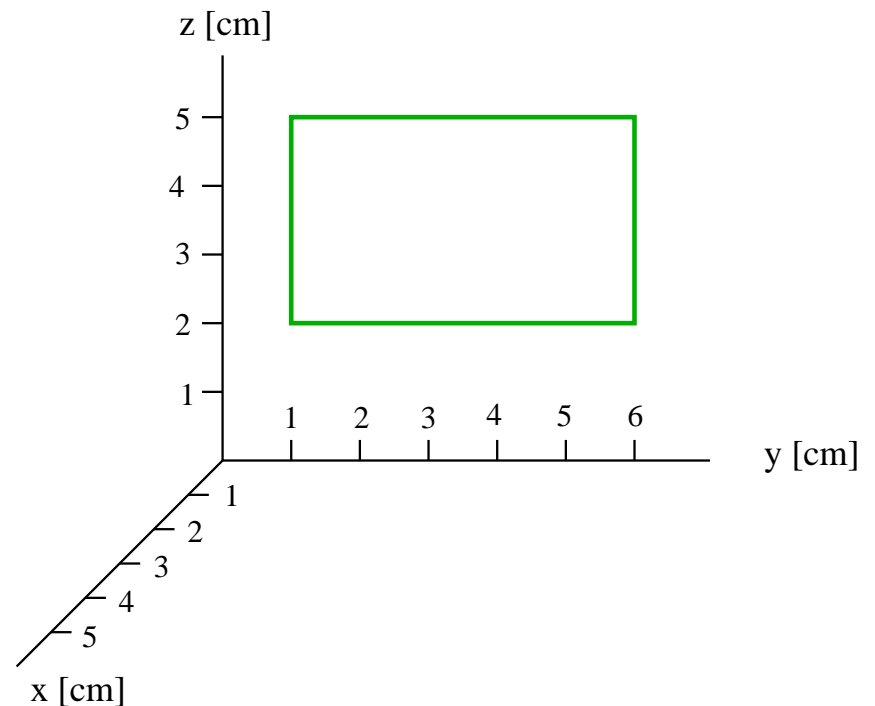


Unit Exam III: Problem #3 (Spring '13)



Consider a wire with a resistance per unit length of $1\Omega/\text{cm}$ bent into a rectangular loop and placed into the yz -plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (3\hat{\mathbf{i}} + 1\hat{\mathbf{j}} + 2\hat{\mathbf{k}})t\text{T/s}$, where t is the time in seconds.

- (a) Find the magnetic flux Φ_B through the rectangle at time $t = 2\text{s}$.
- (b) Find magnitude and direction (cw/ccw) of the induced EMF \mathcal{E} around the rectangle at time $t = 2\text{s}$.
- (c) Infer the induced current I from the induced EMF.



Unit Exam III: Problem #3 (Spring '13)



Consider a wire with a resistance per unit length of $1\Omega/\text{cm}$ bent into a rectangular loop and placed into the yz -plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (3\hat{\mathbf{i}} + 1\hat{\mathbf{j}} + 2\hat{\mathbf{k}})t\text{T/s}$, where t is the time in seconds.

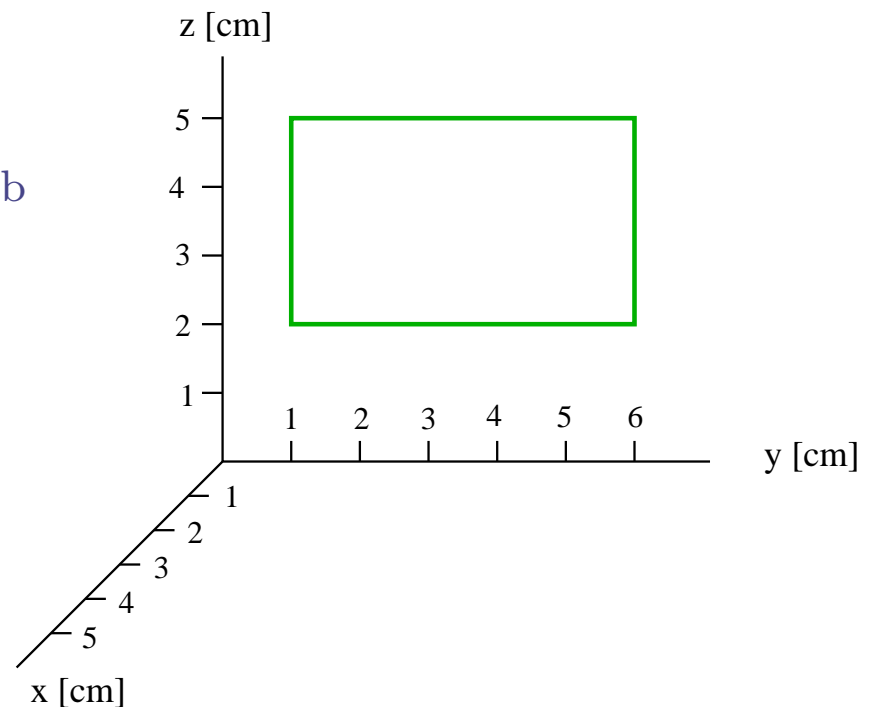
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- (c) Infer the induced current I from the induced EMF.

Solution:

(a) $\Phi_B = \pm(5\text{cm})(3\text{cm})(3\text{T/s})(2\text{s}) = \pm 9.0 \times 10^{-3}\text{Wb}$

(b) $\mathcal{E} = \mp(5\text{cm})(3\text{cm})(3\text{T/s}) = \mp 4.5\text{mV}$ (cw)

(c) $I = \frac{4.5\text{mV}}{(1\Omega/\text{cm})(16\text{cm})} = 0.281\text{mA}$

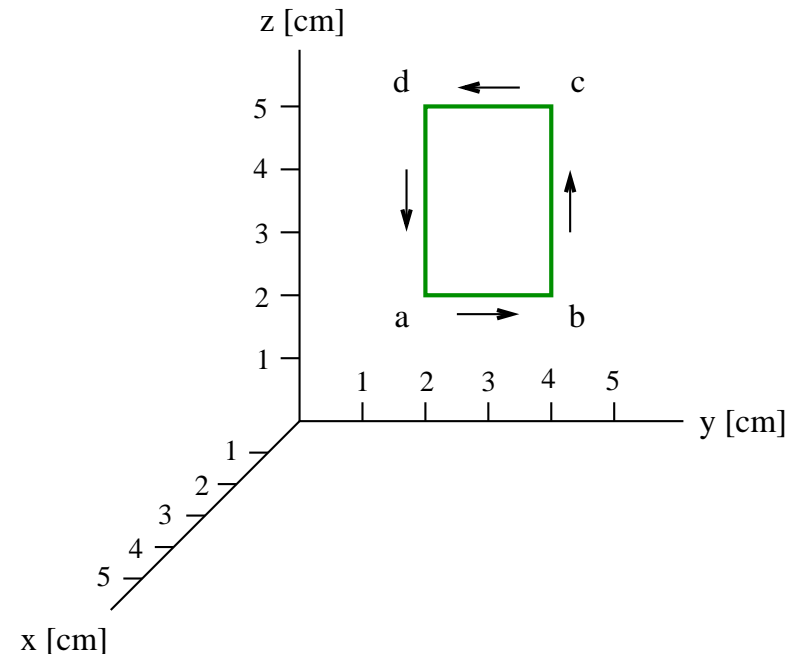


Unit Exam III: Problem #1 (Spring '14)



A counterclockwise current $I = 1.7\text{A}$ [$I = 1.3\text{A}$] is flowing through the conducting rectangular frame shown in a region of magnetic field $\mathbf{B} = 6\text{mT}\hat{\mathbf{j}}$ [$\mathbf{B} = 6\text{mT}\hat{\mathbf{k}}$].

- (a) Find the force \mathbf{F}_{bc} [\mathbf{F}_{ab}] (magnitude and direction) acting on side bc [ab] of the rectangle.
- (b) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
- (c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the current loop.

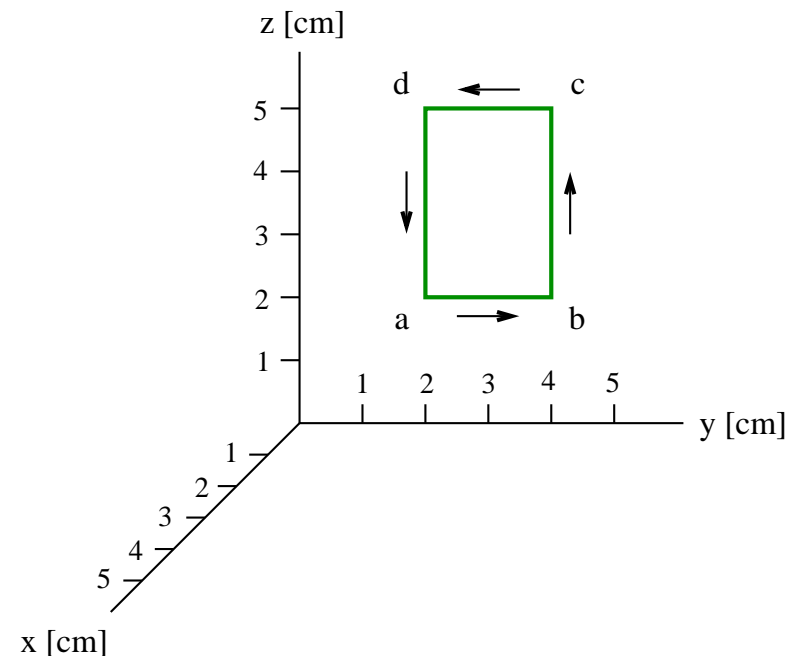


Unit Exam III: Problem #1 (Spring '14)



A counterclockwise current $I = 1.7\text{A}$ [$I = 1.3\text{A}$] is flowing through the conducting rectangular frame shown in a region of magnetic field $\mathbf{B} = 6\text{mT}\hat{\mathbf{j}}$ [$\mathbf{B} = 6\text{mT}\hat{\mathbf{k}}$].

- Find the force \mathbf{F}_{bc} [\mathbf{F}_{ab}] (magnitude and direction) acting on side bc [ab] of the rectangle.
- Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
- Find the torque $\vec{\tau}$ (magnitude and direction) acting on the current loop.



Solution:

$$(a) \mathbf{F}_{bc} = (1.7\text{A})(3\text{cm}\hat{\mathbf{k}}) \times (6\text{mT}\hat{\mathbf{j}}) = -3.06 \times 10^{-4}\text{N}\hat{\mathbf{i}}$$

$$[\mathbf{F}_{ab} = (1.3\text{A})(2\text{cm}\hat{\mathbf{j}}) \times (6\text{mT}\hat{\mathbf{k}}) = 1.56 \times 10^{-4}\text{N}\hat{\mathbf{i}}]$$

$$(b) \vec{\mu} = [(2\text{cm})(3\text{cm})\hat{\mathbf{i}}](1.7\text{A}) = 1.02 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}$$

$$[\vec{\mu} = [(2\text{cm})(3\text{cm})\hat{\mathbf{i}}](1.3\text{A}) = 7.8 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}]$$

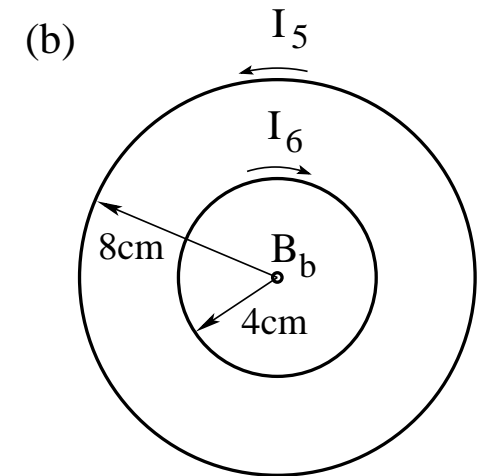
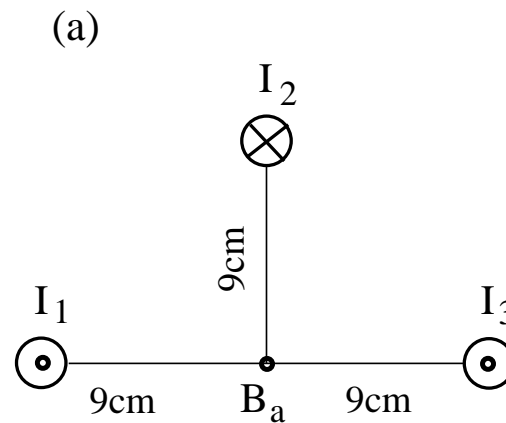
$$(c) \vec{\tau} = (1.02 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}) \times (6\text{mT}\hat{\mathbf{j}}) = 6.12 \times 10^{-6}\text{Nm}\hat{\mathbf{k}}$$

$$[\vec{\tau} = (7.8 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}) \times (6\text{mT}\hat{\mathbf{k}}) = -4.68 \times 10^{-6}\text{Nm}\hat{\mathbf{j}}]$$

Unit Exam III: Problem #2 (Spring '14)



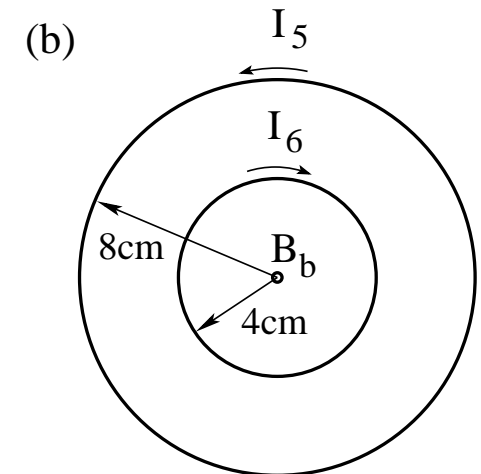
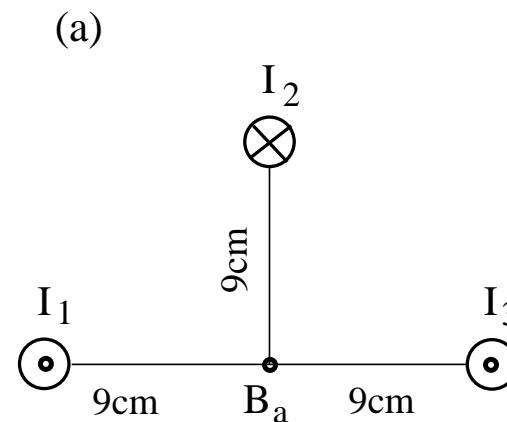
- (a) Find the magnetic field \mathbf{B}_a (magnitude and direction) generated by the three long, straight currents $I_1 = I_2 = I_3 = 1.8\text{mA}$ [2.7mA] in the directions shown.
- (b) Find the magnetic field \mathbf{B}_b (magnitude and direction) generated by the two circular currents $I_5 = I_6 = 1.5\text{mA}$ [2.5mA] in the directions shown.



Unit Exam III: Problem #2 (Spring '14)



- (a) Find the magnetic field B_a (magnitude and direction) generated by the three long, straight currents $I_1 = I_2 = I_3 = 1.8\text{mA}$ [2.7mA] in the directions shown.
- (b) Find the magnetic field B_b (magnitude and direction) generated by the two circular currents $I_5 = I_6 = 1.5\text{mA}$ [2.5mA] in the directions shown.



Solution:

$$(a) B_a = \frac{\mu_0(1.8\text{mA})}{2\pi(9\text{cm})} = 4 \times 10^{-9}\text{T} \quad (\text{directed } \leftarrow)$$

$$[B_a = \frac{\mu_0(2.7\text{mA})}{2\pi(9\text{cm})} = 6 \times 10^{-9}\text{T} \quad (\text{directed } \leftarrow)]$$

$$(b) B_b = \frac{\mu_0(1.5\text{mA})}{2(4\text{cm})} - \frac{\mu_0(1.5\text{mA})}{2(8\text{cm})} = 1.18 \times 10^{-8}\text{T} \quad (\text{directed } \otimes)$$

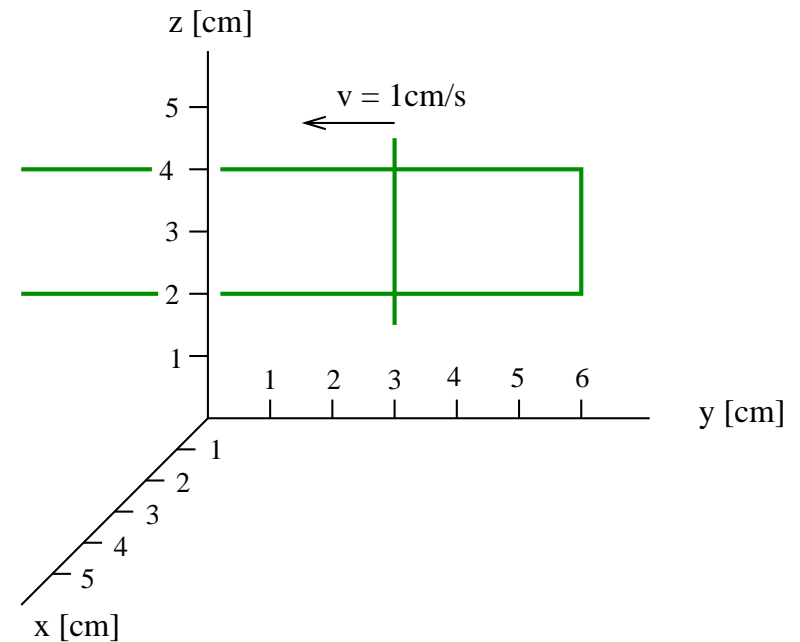
$$[B_b = \frac{\mu_0(2.5\text{mA})}{2(4\text{cm})} - \frac{\mu_0(2.5\text{mA})}{2(8\text{cm})} = 1.96 \times 10^{-8}\text{T} \quad (\text{directed } \otimes)]$$

Unit Exam III: Problem #3 (Spring '14)



Consider a region of uniform magnetic field $\mathbf{B} = (3\hat{i} + 2\hat{j} + 1\hat{k})\text{mT}$ [$\mathbf{B} = (2\hat{i} + 3\hat{j} + 1\hat{k})\text{mT}$]. A conducting rod slides along conducting rails in the yz -plane as shown. The rails are connected on the right. The clock is set to $t = 0$ at the instant shown.

- (a) Find the magnetic flux Φ_B through the conducting loop at $t = 0$.
- (b) Find the magnetic flux Φ_B through the conducting loop at $t = 1\text{s}$.
- (c) Find the induced EMF.
- (d) Find the direction (cw/ccw) of the induced current.



Unit Exam III: Problem #3 (Spring '14)

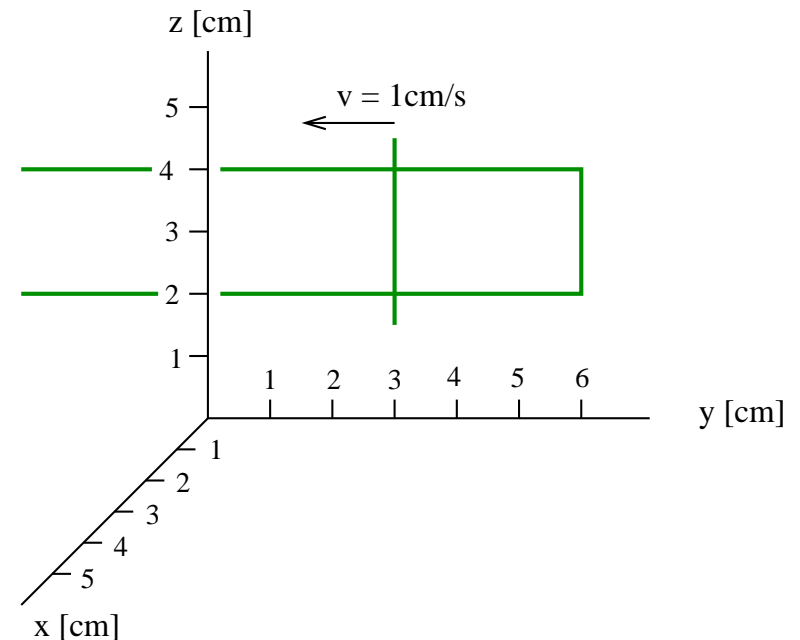


Consider a region of uniform magnetic field $\mathbf{B} = (3\hat{i} + 2\hat{j} + 1\hat{k})\text{mT}$ [$\mathbf{B} = (2\hat{i} + 3\hat{j} + 1\hat{k})\text{mT}$]. A conducting rod slides along conducting rails in the yz -plane as shown. The rails are connected on the right. The clock is set to $t = 0$ at the instant shown.

- Find the magnetic flux Φ_B through the conducting loop at $t = 0$.
- Find the magnetic flux Φ_B through the conducting loop at $t = 1\text{s}$.
- Find the induced EMF.
- Find the direction (cw/ccw) of the induced current.

Solution:

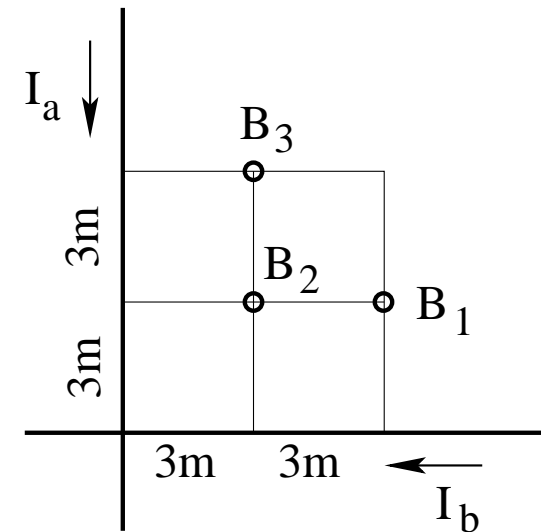
- $\Phi_B = (3\text{cm})(2\text{cm})(3\text{mT}) = 1.8 \times 10^{-6}\text{Wb}$
 $[\Phi_B = (3\text{cm})(2\text{cm})(2\text{mT}) = 1.2 \times 10^{-6}\text{Wb}]$
- $\Phi_B = (4\text{cm})(2\text{cm})(3\text{mT}) = 2.4 \times 10^{-6}\text{Wb}$
 $[\Phi_B = (4\text{cm})(2\text{cm})(2\text{mT}) = 1.6 \times 10^{-6}\text{Wb}]$
- $\mathcal{E} = (1\text{cm/s})(3\text{mT})(2\text{cm}) = 6 \times 10^{-7}\text{V}$
 $[\mathcal{E} = (1\text{cm/s})(2\text{mT})(2\text{cm}) = 4 \times 10^{-7}\text{V}]$
- cw [cw]



Unit Exam III: Problem #1 (Fall '14)



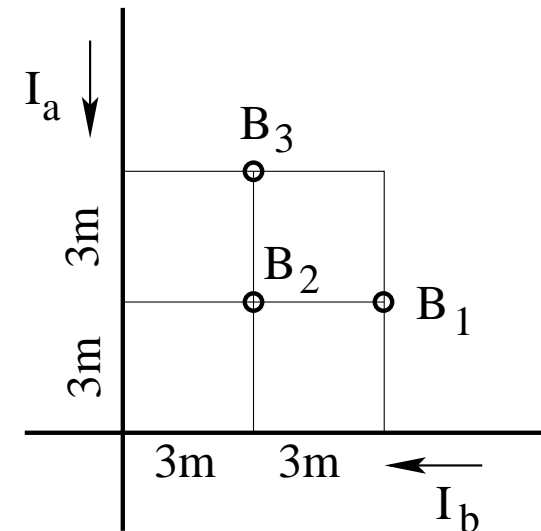
Consider two infinitely long, straight wires with currents $I_a = 7\text{A}$, $I_b = 9\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields \mathbf{B}_1 , \mathbf{B}_2 , \mathbf{B}_3 at the points marked in the graph.



Unit Exam III: Problem #1 (Fall '14)



Consider two infinitely long, straight wires with currents $I_a = 7\text{A}$, $I_b = 9\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields B_1 , B_2 , B_3 at the points marked in the graph.



Solution:

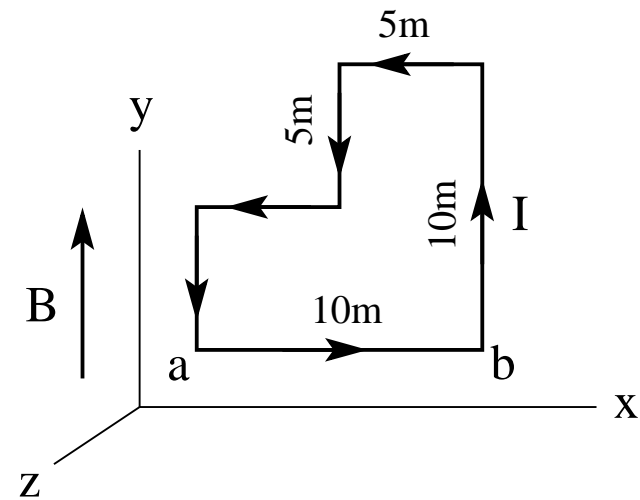
- Convention used: out = positive, in = negative
- $B_1 = \frac{\mu_0}{2\pi} \left(\frac{7\text{A}}{6\text{m}} - \frac{9\text{A}}{3\text{m}} \right) = -0.367\mu\text{T}$ (in).
- $B_2 = \frac{\mu_0}{2\pi} \left(\frac{7\text{A}}{3\text{m}} - \frac{9\text{A}}{3\text{m}} \right) = -0.133\mu\text{T}$ (in).
- $B_3 = \frac{\mu_0}{2\pi} \left(\frac{7\text{A}}{3\text{m}} - \frac{9\text{A}}{6\text{m}} \right) = +0.167\mu\text{T}$ (out).

Unit Exam III: Problem #2 (Fall '14)



Consider the (piecewise rectangular) conducting loop in the xy -plane as shown with a counterclockwise current $I = 4\text{A}$ in a uniform magnetic field $\vec{B} = 2\text{T}\hat{j}$.

- Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
- Find the force \vec{F} (magnitude and direction) acting on the side ab of the rectangle.
- Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.

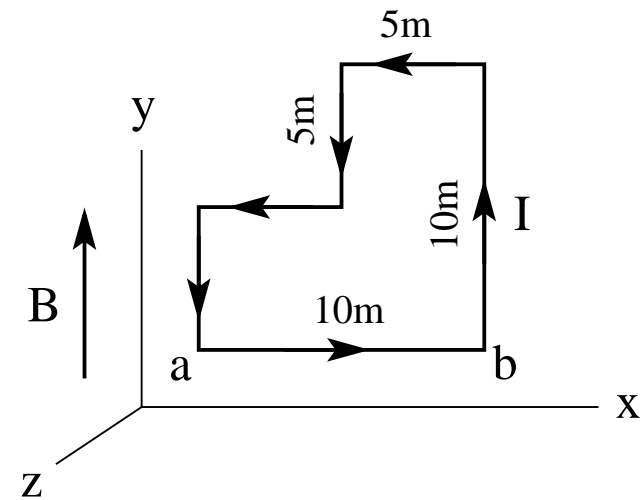


Unit Exam III: Problem #2 (Fall '14)



Consider the (piecewise rectangular) conducting loop in the xy -plane as shown with a counterclockwise current $I = 4\text{A}$ in a uniform magnetic field $\vec{B} = 2\text{T}\hat{j}$.

- Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
- Find the force \vec{F} (magnitude and direction) acting on the side ab of the rectangle.
- Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



Solution:

$$(a) \vec{\mu} = (4\text{A})(75\text{m}^2)\hat{k} = 300\text{Am}^2\hat{k}.$$

$$(b) \vec{F} = I\vec{L} \times \vec{B} = (4\text{A})(10\text{m}\hat{i}) \times (2\text{T}\hat{j}) = 80\text{N}\hat{k}.$$

$$(c) \vec{\tau} = \vec{\mu} \times \vec{B} = (300\text{Am}^2\hat{k}) \times (2\text{T}\hat{j}) = -600\text{Nm}\hat{i}$$

Unit Exam III: Problem #3 (Fall '14)

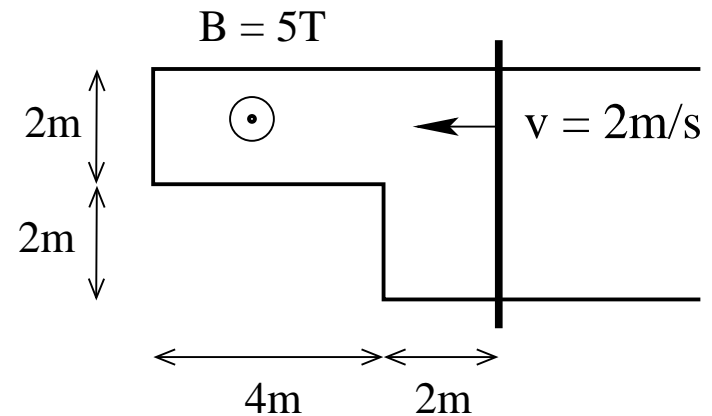


A conducting frame with a moving conducting rod is located in a uniform magnetic field directed out of the plane as shown. The rod moves at constant velocity.

(a) Find the magnetic flux Φ_B through the frame and the induced emf \mathcal{E} around the frame at the instant shown.

(b) Find the magnetic flux Φ_B through the frame and the induced emf \mathcal{E} around the frame two seconds later.

Write magnitudes only (in SI units), no directions.



Unit Exam III: Problem #3 (Fall '14)

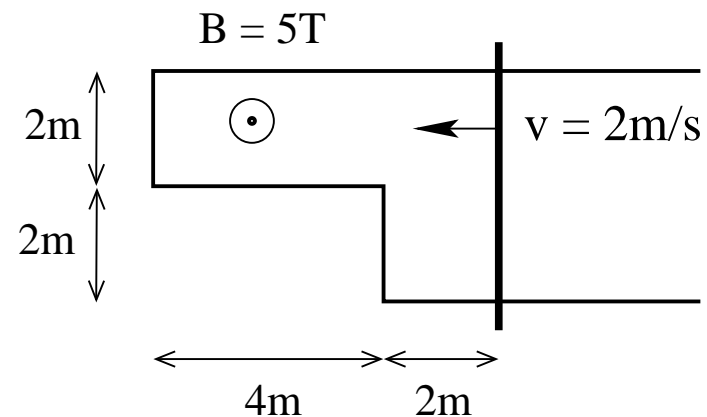


A conducting frame with a moving conducting rod is located in a uniform magnetic field directed out of the plane as shown. The rod moves at constant velocity.

(a) Find the magnetic flux Φ_B through the frame and the induced emf \mathcal{E} around the frame at the instant shown.

(b) Find the magnetic flux Φ_B through the frame and the induced emf \mathcal{E} around the frame two seconds later.

Write magnitudes only (in SI units), no directions.



Solution:

(a) $\Phi_B = (16\text{m}^2)(5\text{T}) = 80\text{Wb}$, $\mathcal{E} = (2\text{m/s})(5\text{T})(4\text{m}) = 40\text{V}$.

(b) $\Phi_B = (4\text{m}^2)(5\text{T}) = 20\text{Wb}$, $\mathcal{E} = (2\text{m/s})(5\text{T})(2\text{m}) = 20\text{V}$.

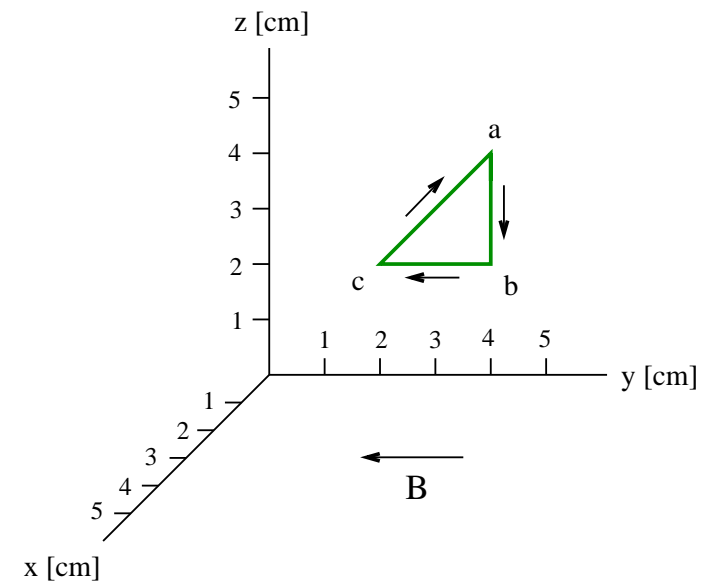
Unit Exam III: Problem #1 (Spring '15)



A clockwise current $I = 2.1\text{A}$ is flowing around the conducting triangular frame shown in a region of uniform magnetic field $\vec{B} = -3\text{mT}\hat{j}$.

- (a) Find the force \vec{F}_{ab} acting on side ab of the triangle.
- (b) Find the force \vec{F}_{bc} acting on side bc of the triangle.
- (c) Find the magnetic moment $\vec{\mu}$ of the current loop.
- (d) Find the torque $\vec{\tau}$ acting on the current loop.

Remember that vectors have components or magnitude and direction.



Unit Exam III: Problem #1 (Spring '15)



A clockwise current $I = 2.1\text{A}$ is flowing around the conducting triangular frame shown in a region of uniform magnetic field $\vec{B} = -3\text{mT}\hat{\mathbf{j}}$.

- (a) Find the force \vec{F}_{ab} acting on side ab of the triangle.
 - (b) Find the force \vec{F}_{bc} acting on side bc of the triangle.
 - (c) Find the magnetic moment $\vec{\mu}$ of the current loop.
 - (d) Find the torque $\vec{\tau}$ acting on the current loop.
- Remember that vectors have components or magnitude and direction.

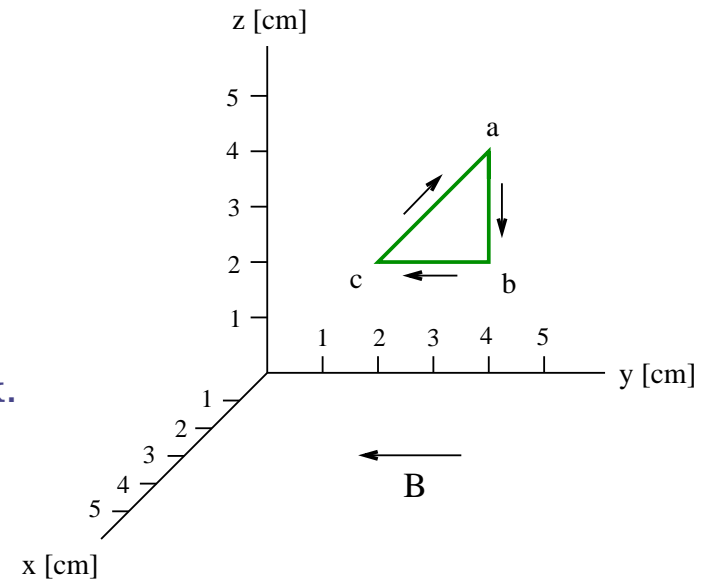
Solution:

(a) $\vec{F}_{ab} = (2.1\text{A})(-2\text{cm}\hat{\mathbf{k}}) \times (-3\text{mT}\hat{\mathbf{j}}) = -1.26 \times 10^{-4}\text{N}\hat{\mathbf{i}}$.

(b) $\vec{F}_{bc} = 0$.

(c) $\vec{\mu} = \left[-\frac{1}{2}(2\text{cm})(2\text{cm})\hat{\mathbf{i}} \right] (2.1\text{A}) = -4.2 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}$.

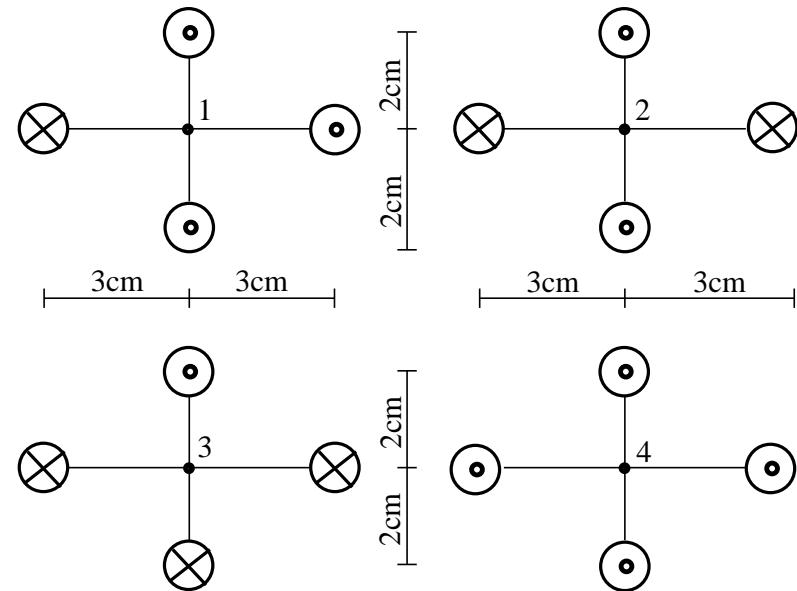
(d) $\vec{\tau} = (-4.2 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}) \times (-3\text{mT}\hat{\mathbf{j}}) = 1.26 \times 10^{-6}\text{Nm}\hat{\mathbf{k}}$.



Unit Exam III: Problem #2 (Spring '15)



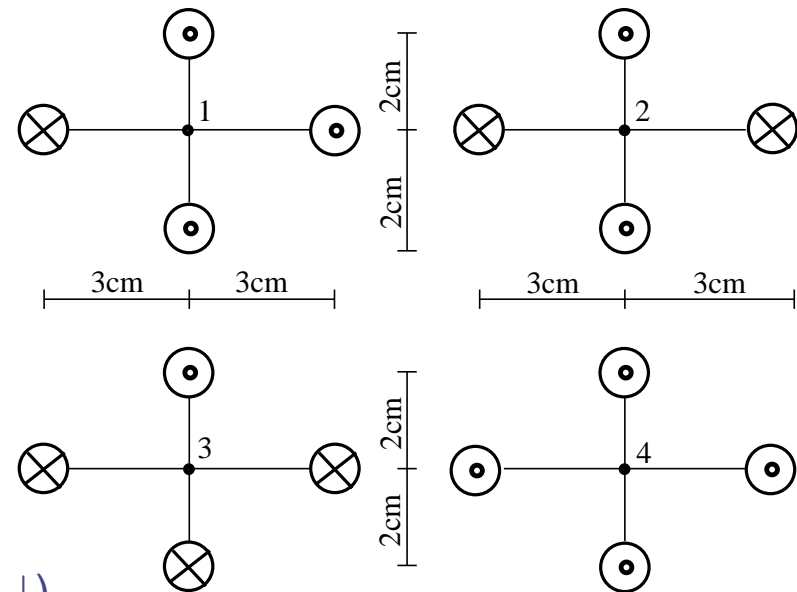
Consider four long, straight currents in four different configurations. All currents are $I = 4\text{mA}$ in the directions shown (\otimes = in, \odot = out). Find the magnitude (in SI units) and the direction (\leftarrow , \rightarrow , \uparrow , \downarrow) of the magnetic fields \mathbf{B}_1 , \mathbf{B}_2 , \mathbf{B}_3 , \mathbf{B}_4 generated at the points 1, \dots , 4, respectively.



Unit Exam III: Problem #2 (Spring '15)



Consider four long, straight currents in four different configurations. All currents are $I = 4\text{mA}$ in the directions shown (\otimes = in, \odot = out). Find the magnitude (in SI units) and the direction (\leftarrow , \rightarrow , \uparrow , \downarrow) of the magnetic fields \mathbf{B}_1 , \mathbf{B}_2 , \mathbf{B}_3 , \mathbf{B}_4 generated at the points 1, \dots , 4, respectively.



Solution:

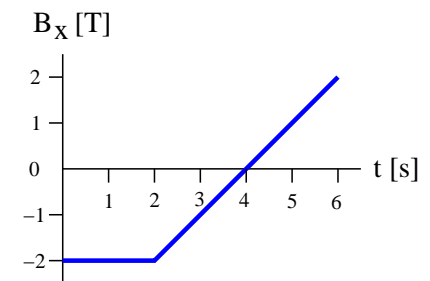
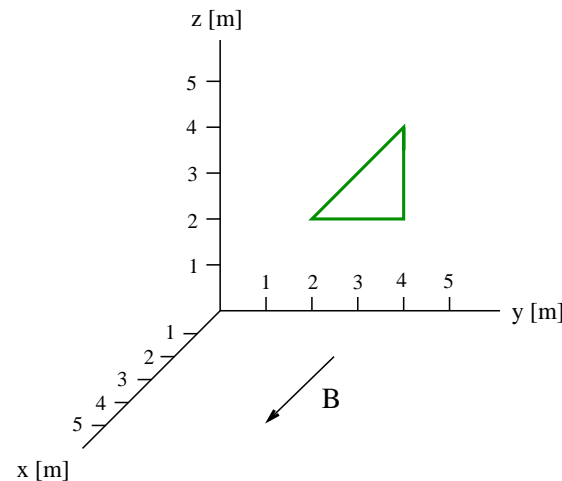
- $B_1 = 2 \frac{\mu_0(4\text{mA})}{2\pi(3\text{cm})} = 5.33 \times 10^{-8}\text{T}$ (directed \downarrow).
- $B_2 = 0$ (no direction).
- $B_3 = 2 \frac{\mu_0(4\text{mA})}{2\pi(2\text{cm})} = 8.00 \times 10^{-8}\text{T}$ (directed \rightarrow).
- $B_4 = 0$ (no direction).

Unit Exam III: Problem #3 (Spring '15)



A wire shaped into a triangle has resistance $R = 3.5\Omega$ and is placed in the yz -plane as shown. A uniform time-dependent magnetic field $\mathbf{B} = B_x(t)\hat{\mathbf{i}}$ is present. The dependence of B_x on time is shown graphically.

- (a) Find magnitude $|\Phi_B^{(1)}|$ and $|\Phi_B^{(4)}|$ of the magnetic flux through the triangle at times $t = 1\text{s}$ and $t = 4\text{s}$, respectively.
- (b) Find magnitude I_1, I_4 and direction (cw/ccw) of the induced current at times $t = 1\text{s}$ and $t = 4\text{s}$, respectively.



Unit Exam III: Problem #3 (Spring '15)



A wire shaped into a triangle has resistance $R = 3.5\Omega$ and is placed in the yz -plane as shown. A uniform time-dependent magnetic field $\mathbf{B} = B_x(t)\hat{\mathbf{i}}$ is present. The dependence of B_x on time is shown graphically.

(a) Find magnitude $|\Phi_B^{(1)}|$ and $|\Phi_B^{(4)}|$ of the magnetic flux through the triangle at times $t = 1\text{s}$ and $t = 4\text{s}$, respectively.

(b) Find magnitude I_1, I_4 and direction (cw/ccw) of the induced current at times $t = 1\text{s}$ and $t = 4\text{s}$, respectively.

Solution:

$$(a) |\Phi_B^{(1)}| = |(2\text{m}^2)(-2\text{T})| = 4.0 \text{ Wb},$$

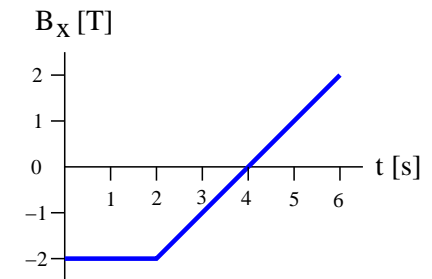
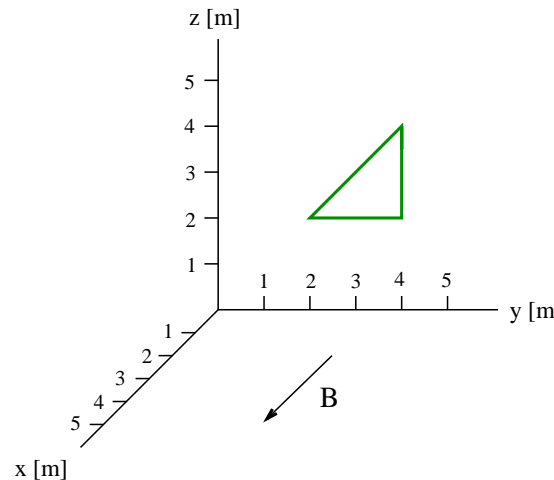
$$|\Phi_B^{(4)}| = |(2\text{m}^2)(0\text{T})| = 0.$$

$$(b) \left| \frac{d\Phi_B^{(1)}}{dt} \right| = \left| A \frac{dB}{dt} \right| = |(2\text{m}^2)(0\text{T/s})| = 0$$

$$\Rightarrow I_1 = 0,$$

$$\left| \frac{d\Phi_B^{(4)}}{dt} \right| = \left| A \frac{dB}{dt} \right| = |(2\text{m}^2)(1\text{T/s})| = 2.0\text{V}$$

$$\Rightarrow I_4 = \frac{2.0\text{V}}{3.5\Omega} = 0.571\text{A} \quad (\text{cw}).$$

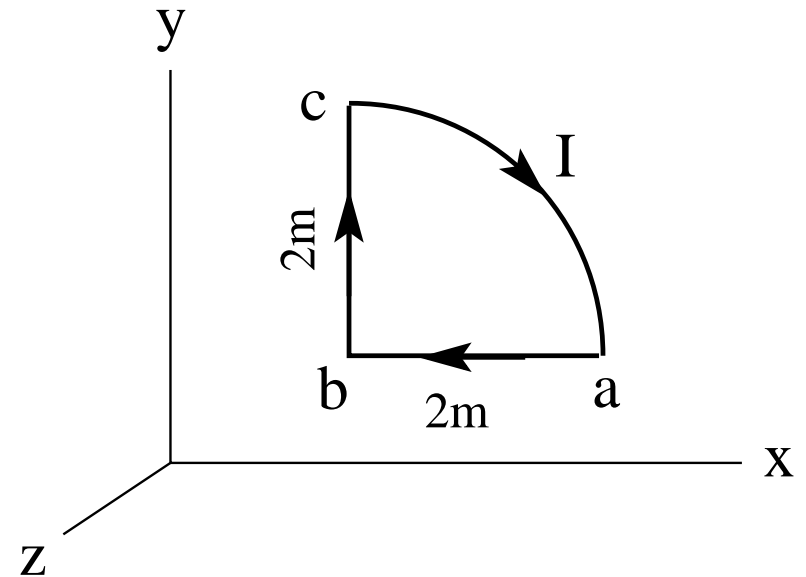


Unit Exam III: Problem #1 (Fall '15)



Consider a region with uniform magnetic field (i) $\vec{B} = 5T\hat{j}$, (ii) $\vec{B} = -6T\hat{i}$. A conducting loop in the xy -plane has the shape of a quarter circle with a clockwise current (i) $I = 4A$, (ii) $I = 3A$.

- (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
- (b) Find the force \vec{F} (magnitude and direction) acting on the side (i) ab , (ii) bc of the loop.
- (c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



Unit Exam III: Problem #1 (Fall '15)



Consider a region with uniform magnetic field (i) $\vec{B} = 5T\hat{j}$, (ii) $\vec{B} = -6T\hat{i}$. A conducting loop in the xy -plane has the shape of a quarter circle with a clockwise current (i) $I = 4A$, (ii) $I = 3A$.

- (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
- (b) Find the force \vec{F} (magnitude and direction) acting on the side (i) ab , (ii) bc of the loop.
- (c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.

Solution:

$$(ia) \quad \vec{\mu} = (4A)(3.14m^2)(-\hat{k}) = -12.6Am^2\hat{k}.$$

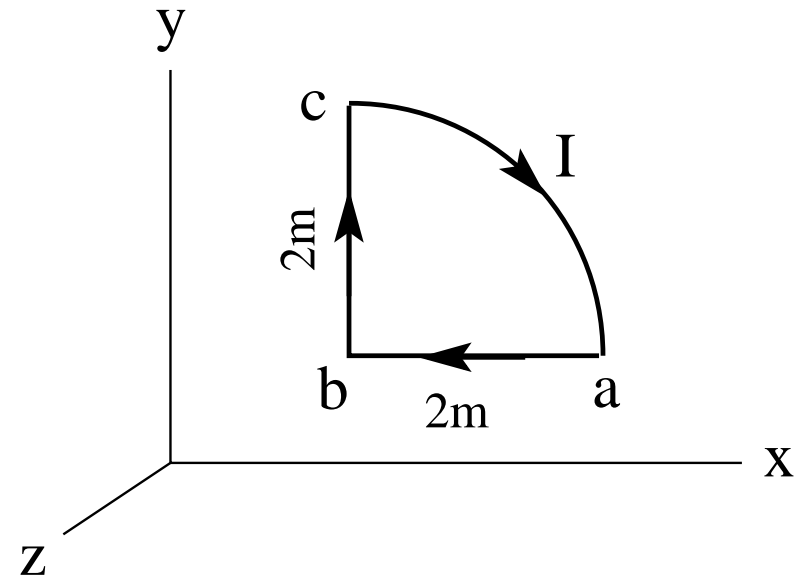
$$(ib) \quad \vec{F}_{ab} = (4A)(-2m\hat{i}) \times (5T\hat{j}) = -40N\hat{k}.$$

$$(ic) \quad \vec{\tau} = (-12.6Am^2\hat{k}) \times (5T\hat{j}) = 63.0Nm\hat{i}$$

$$(iia) \quad \vec{\mu} = (3A)(3.14m^2)(-\hat{k}) = -9.42Am^2\hat{k}.$$

$$(iib) \quad \vec{F}_{bc} = (3A)(2m\hat{j}) \times (-6T\hat{i}) = 36N\hat{k}.$$

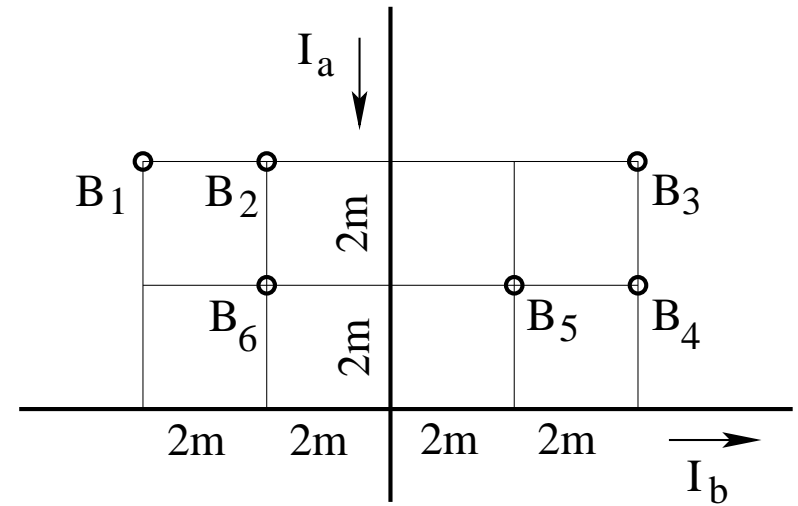
$$(iic) \quad \vec{\tau} = (-9.42Am^2\hat{k}) \times (-6T\hat{i}) = 56.5Nm\hat{j}$$



Unit Exam III: Problem #2 (Fall '15)



Consider two infinitely long, straight wires with currents of equal magnitude $I_a = I_b = 6A$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields B_1, \dots, B_6 at the points marked in the graph.



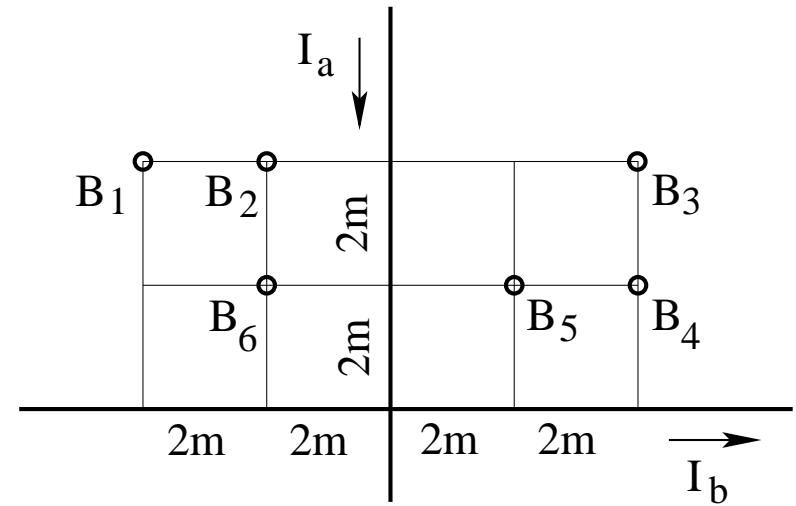
Unit Exam III: Problem #2 (Fall '15)



Consider two infinitely long, straight wires with currents of equal magnitude $I_a = I_b = 6\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields B_1, \dots, B_6 at the points marked in the graph.

Solution:

- $B_1 = \frac{\mu_0}{2\pi} \left(\frac{6\text{A}}{4\text{m}} - \frac{6\text{A}}{4\text{m}} \right) = 0$ (no direction).
- $B_2 = \frac{\mu_0}{2\pi} \left(\frac{6\text{A}}{4\text{m}} - \frac{6\text{A}}{2\text{m}} \right) = -0.3\mu\text{T}$ (into plane).
- $B_3 = \frac{\mu_0}{2\pi} \left(\frac{6\text{A}}{4\text{m}} + \frac{6\text{A}}{4\text{m}} \right) = +0.6\mu\text{T}$ (out of plane).
- $B_4 = \frac{\mu_0}{2\pi} \left(\frac{6\text{A}}{2\text{m}} + \frac{6\text{A}}{4\text{m}} \right) = 0.9\mu\text{T}$ (out of plane).
- $B_5 = \frac{\mu_0}{2\pi} \left(\frac{6\text{A}}{2\text{m}} + \frac{6\text{A}}{2\text{m}} \right) = 1.2\mu\text{T}$ (out of plane).
- $B_6 = \frac{\mu_0}{2\pi} \left(\frac{6\text{A}}{2\text{m}} - \frac{6\text{A}}{2\text{m}} \right) = 0$ (no direction).



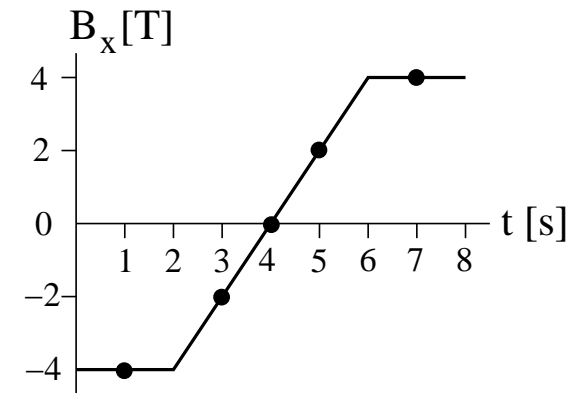
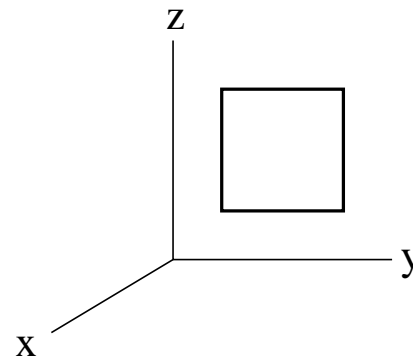
Unit Exam III: Problem #3 (Fall '15)



A conducting wire bent into a square of side (i) 1.2m, (ii) 1.3m is placed in the yz -plane. The time-dependence of the magnetic field $\mathbf{B}(t) = B_x(t)\hat{\mathbf{i}}$ is shown graphically.

- (a) Find the magnitude $|\Phi_B|$ of the magnetic flux through the square at times (i) $t = 1\text{s}$, $t = 3\text{s}$, and $t = 4\text{s}$, (ii) $t = 4\text{s}$, $t = 5\text{s}$, and $t = 7\text{s}$.
- (b) Find the magnitude $|\mathcal{E}|$ of the induced EMF at the above times.
- (c) Find the direction (cw, ccw, zero) of the induced current at the above times.

Solution:



Unit Exam III: Problem #3 (Fall '15)



A conducting wire bent into a square of side (i) 1.2m, (ii) 1.3m is placed in the yz -plane. The time-dependence of the magnetic field $\mathbf{B}(t) = B_x(t)\hat{\mathbf{i}}$ is shown graphically.

- (a) Find the magnitude $|\Phi_B|$ of the magnetic flux through the square at times (i) $t = 1\text{s}$, $t = 3\text{s}$, and $t = 4\text{s}$, (ii) $t = 4\text{s}$, $t = 5\text{s}$, and $t = 7\text{s}$.
- (b) Find the magnitude $|\mathcal{E}|$ of the induced EMF at the above times.
- (c) Find the direction (cw, ccw, zero) of the induced current at the above times.

Solution:

$$(ia) \quad |\Phi_B^{(1)}| = (1.44\text{m}^2)(4\text{T}) = 5.76 \text{ Wb}$$

$$|\Phi_B^{(3)}| = (1.44\text{m}^2)(2\text{T}) = 2.88 \text{ Wb}$$

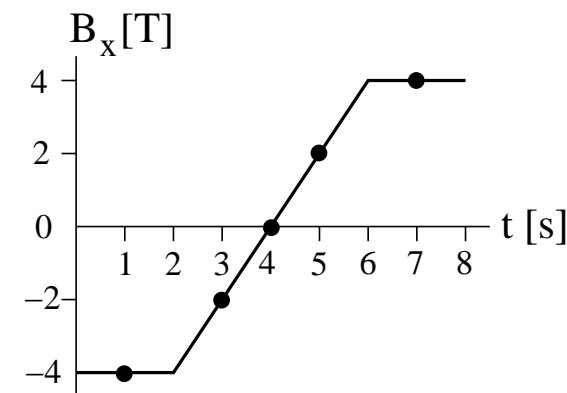
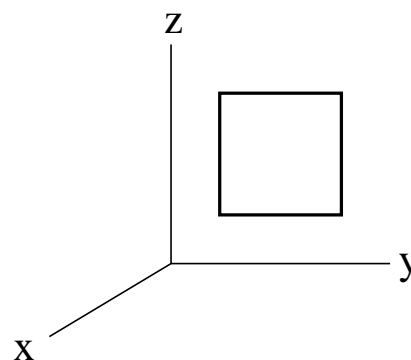
$$|\Phi_B^{(4)}| = (1.44\text{m}^2)(0\text{T}) = 0$$

$$(ib) \quad \mathcal{E}^{(1)} = (1.44\text{m}^2)(0\text{T/s}) = 0$$

$$\mathcal{E}^{(3)} = (1.44\text{m}^2)(2\text{T/s}) = 2.88\text{V}$$

$$\mathcal{E}^{(4)} = (1.44\text{m}^2)(2\text{T/s}) = 2.88\text{V}$$

(ic) zero, cw, cw



Unit Exam III: Problem #3 (Fall '15)



A conducting wire bent into a square of side (i) 1.2m, (ii) 1.3m is placed in the yz -plane. The time-dependence of the magnetic field $\mathbf{B}(t) = B_x(t)\hat{\mathbf{i}}$ is shown graphically.

- (a) Find the magnitude $|\Phi_B|$ of the magnetic flux through the square at times (i) $t = 1\text{s}$, $t = 3\text{s}$, and $t = 4\text{s}$, (ii) $t = 4\text{s}$, $t = 5\text{s}$, and $t = 7\text{s}$.
- (b) Find the magnitude $|\mathcal{E}|$ of the induced EMF at the above times.
- (c) Find the direction (cw, ccw, zero) of the induced current at the above times.

Solution:

(iia) $|\Phi_B^{(4)}| = (1.69\text{m}^2)(0\text{T}) = 0$

$$|\Phi_B^{(5)}| = (1.69\text{m}^2)(2\text{T}) = 3.38 \text{ Wb}$$

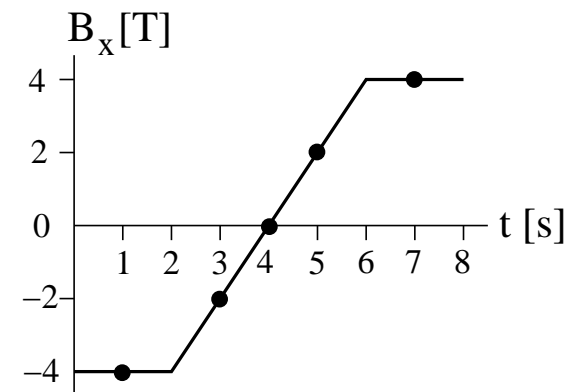
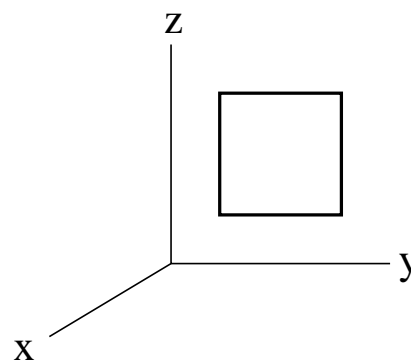
$$|\Phi_B^{(7)}| = (1.69\text{m}^2)(4\text{T}) = 6.76 \text{ Wb}$$

(iib) $\mathcal{E}^{(4)} = (1.69\text{m}^2)(2\text{T/s}) = 3.38\text{V}$

$$\mathcal{E}^{(5)} = (1.69\text{m}^2)(2\text{T/s}) = 3.38\text{V}$$

$$\mathcal{E}^{(7)} = (1.69\text{m}^2)(0\text{T/s}) = 0$$

(iic) cw, cw, zero

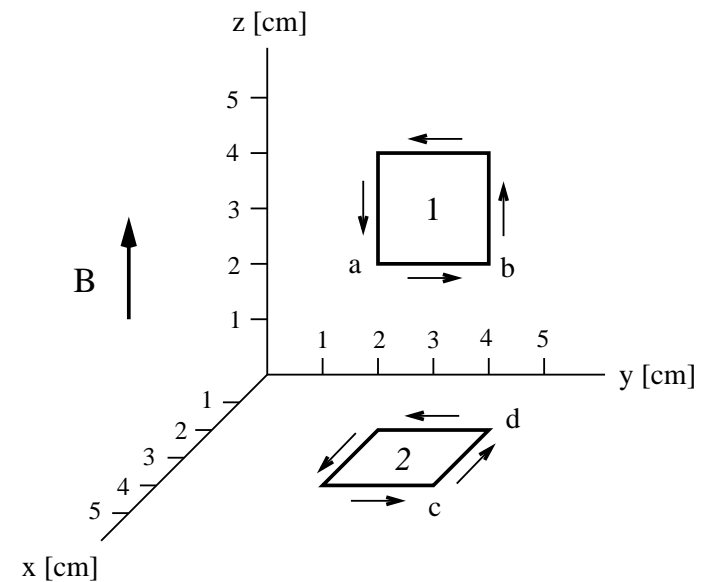


Unit Exam III: Problem #1 (Spring '16)



Conducting squares 1 and 2, each of side 2cm, are positioned as shown. A current $I = 3A$ is flowing around each square in the direction shown. A uniform magnetic field $\vec{B} = 5mT\hat{k}$ exists in the entire region.

- (a) Find the forces \vec{F}_{ab} and \vec{F}_{cd} acting on sides ab and cd , respectively.
 - (b) Find the magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ of squares 1 and 2, respectively.
 - (c) Find the torques $\vec{\tau}_1$ and $\vec{\tau}_2$ acting on squares 1 and 2, respectively.
- Remember that vectors have components or magnitude and direction.



Unit Exam III: Problem #1 (Spring '16)



Conducting squares 1 and 2, each of side 2cm, are positioned as shown. A current $I = 3A$ is flowing around each square in the direction shown. A uniform magnetic field $\vec{B} = 5mT\hat{k}$ exists in the entire region.

- Find the forces \vec{F}_{ab} and \vec{F}_{cd} acting on sides ab and cd , respectively.
 - Find the magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ of squares 1 and 2, respectively.
 - Find the torques $\vec{\tau}_1$ and $\vec{\tau}_2$ acting on squares 1 and 2, respectively.
- Remember that vectors have components or magnitude and direction.

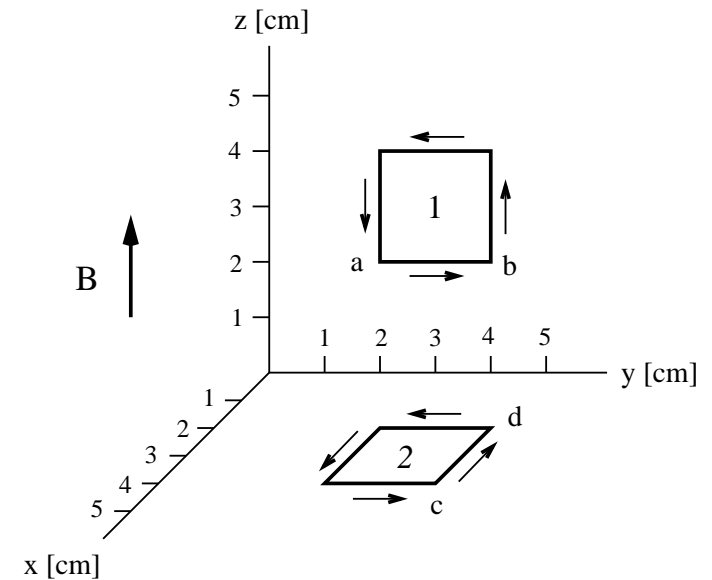
Solution:

- $$\vec{F}_{ab} = (3A)(2cm\hat{j}) \times (5mT\hat{k}) = 3 \times 10^{-4}N\hat{i}.$$

$$\vec{F}_{cd} = (3A)(-2cm\hat{i}) \times (5mT\hat{k}) = 3 \times 10^{-4}N\hat{j}.$$
- $$\vec{\mu}_1 = (2cm)^2(3A)\hat{i} = 1.2 \times 10^{-3}Am^2\hat{i}.$$

$$\vec{\mu}_2 = (2cm)^2(3A)\hat{k} = 1.2 \times 10^{-3}Am^2\hat{k}.$$
- $$\vec{\tau}_1 = (1.2 \times 10^{-3}Am^2\hat{i}) \times (5mT\hat{k}) = -6 \times 10^{-6}Nm\hat{j}.$$

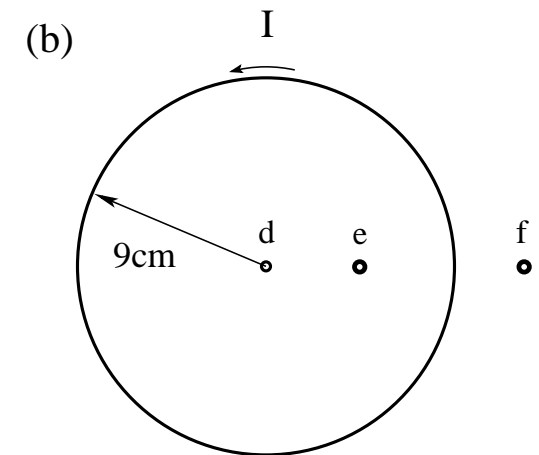
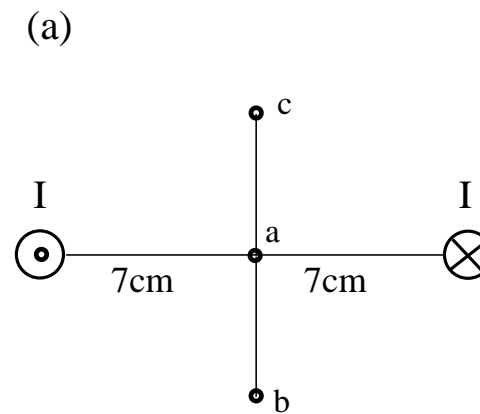
$$\vec{\tau}_2 = (1.2 \times 10^{-3}Am^2\hat{k}) \times (5mT\hat{k}) = 0.$$



Unit Exam III: Problem #2 (Spring '16)



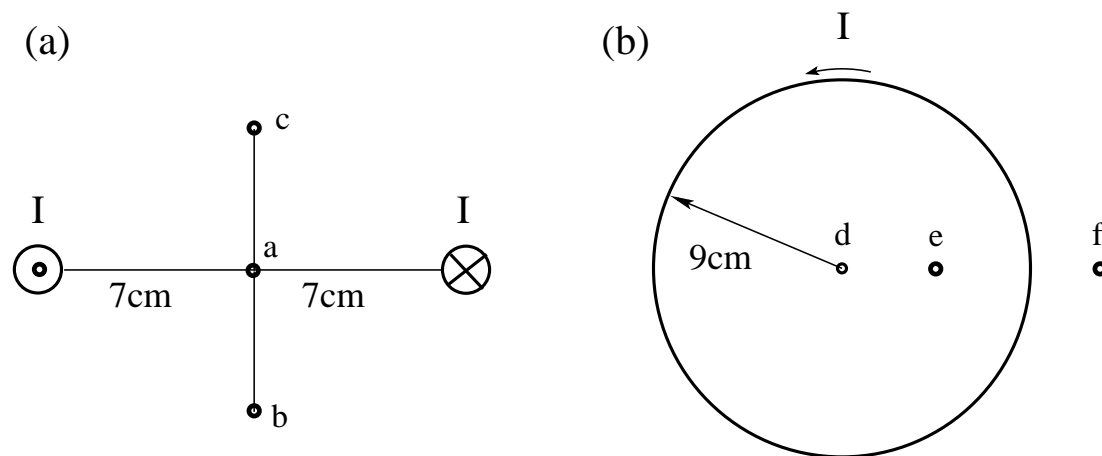
- (a) Consider two long, straight currents $I = 3\text{mA}$ in the directions shown. Find the magnitude of the magnetic field at point a . Find the directions (\leftarrow , \rightarrow , \uparrow , \downarrow) of the magnetic field at points b and c .
- (b) Consider a circular current $I = 3\text{mA}$ in the direction shown. Find the magnitude of the magnetic field at point d . Find the directions (\otimes , \odot) of the magnetic field at points e and f .



Unit Exam III: Problem #2 (Spring '16)



- (a) Consider two long, straight currents $I = 3\text{mA}$ in the directions shown. Find the magnitude of the magnetic field at point a . Find the directions ($\leftarrow, \rightarrow, \uparrow, \downarrow$) of the magnetic field at points b and c .
- (b) Consider a circular current $I = 3\text{mA}$ in the direction shown. Find the magnitude of the magnetic field at point d . Find the directions (\otimes, \odot) of the magnetic field at points e and f .



Solution:

$$(a) B_a = 2 \frac{\mu_0(3\text{mA})}{2\pi(7\text{cm})} = 1.71 \times 10^{-8}\text{T} \quad B_b \uparrow, \quad B_c \uparrow.$$

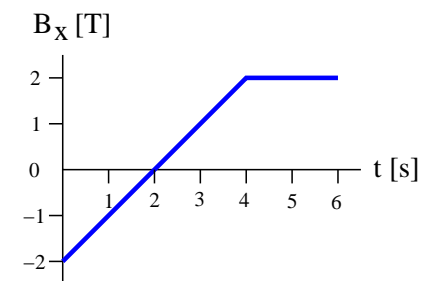
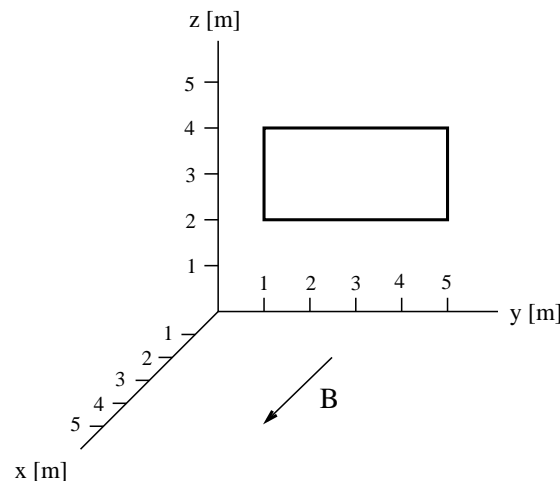
$$(b) B_d = \frac{\mu_0(3\text{mA})}{2(9\text{cm})} = 2.09 \times 10^{-8}\text{T}, \quad B_e \odot, \quad B_f \otimes.$$

Unit Exam III: Problem #3 (Spring '16)



A wire shaped into a rectangular loop as shown is placed in the yz -plane. A uniform time-dependent magnetic field $\mathbf{B} = B_x(t)\hat{\mathbf{i}}$ is present. The dependence of B_x on time is shown graphically.

- (a) Find magnitude $|\Phi_B^{(2)}|$ of the magnetic flux through the loop at time $t = 2\text{s}$.
- (b) Find magnitude $|\Phi_B^{(5)}|$ of the magnetic flux through the loop at time $t = 5\text{s}$.
- (c) Find magnitude $|\mathcal{E}^{(2)}|$ of the induced EMF at time $t = 2\text{s}$.
- (d) Find magnitude $|\mathcal{E}^{(5)}|$ of the induced EMF at time $t = 5\text{s}$.
- (e) Find the direction (cw/ccw) and magnitude I of the induced current at time $t = 2\text{s}$ if the wire has resistance 1Ω per meter of length.



Unit Exam III: Problem #3 (Spring '16)



A wire shaped into a rectangular loop as shown is placed in the yz -plane. A uniform time-dependent magnetic field $\mathbf{B} = B_x(t)\hat{\mathbf{i}}$ is present. The dependence of B_x on time is shown graphically.

- (a) Find magnitude $|\Phi_B^{(2)}|$ of the magnetic flux through the loop at time $t = 2\text{s}$.
- (b) Find magnitude $|\Phi_B^{(5)}|$ of the magnetic flux through the loop at time $t = 5\text{s}$.
- (c) Find magnitude $|\mathcal{E}^{(2)}|$ of the induced EMF at time $t = 2\text{s}$.
- (d) Find magnitude $|\mathcal{E}^{(5)}|$ of the induced EMF at time $t = 5\text{s}$.
- (e) Find the direction (cw/ccw) and magnitude I of the induced current at time $t = 2\text{s}$ if the wire has resistance 1Ω per meter of length.

Solution:

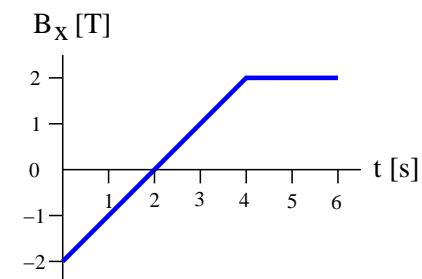
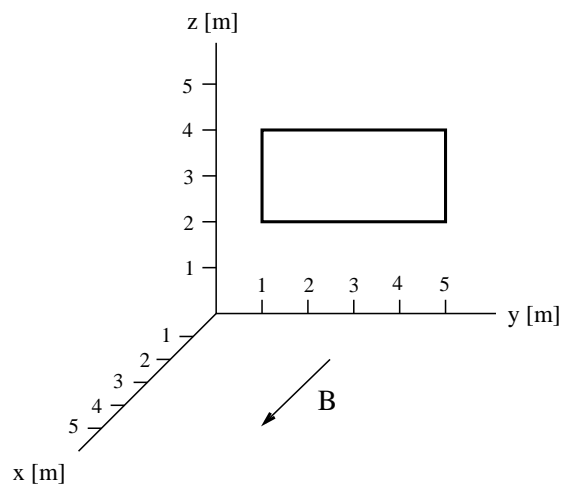
(a) $|\Phi_B^{(2)}| = |(8\text{m}^2)(0\text{T})| = 0,$

(b) $|\Phi_B^{(5)}| = |(8\text{m}^2)(2\text{T})| = 16 \text{ Wb},$

(c) $|\mathcal{E}^{(2)}| = \left| A \frac{dB}{dt} \right| = |(8\text{m}^2)(1\text{T/s})| = 8\text{V}$

(d) $|\mathcal{E}^{(5)}| = \left| A \frac{dB}{dt} \right| = |(8\text{m}^2)(0\text{T/s})| = 0$

(e) $I^{(2)} = \frac{8\text{V}}{12\Omega} = 0.667\text{A}. \quad (\text{cw}).$

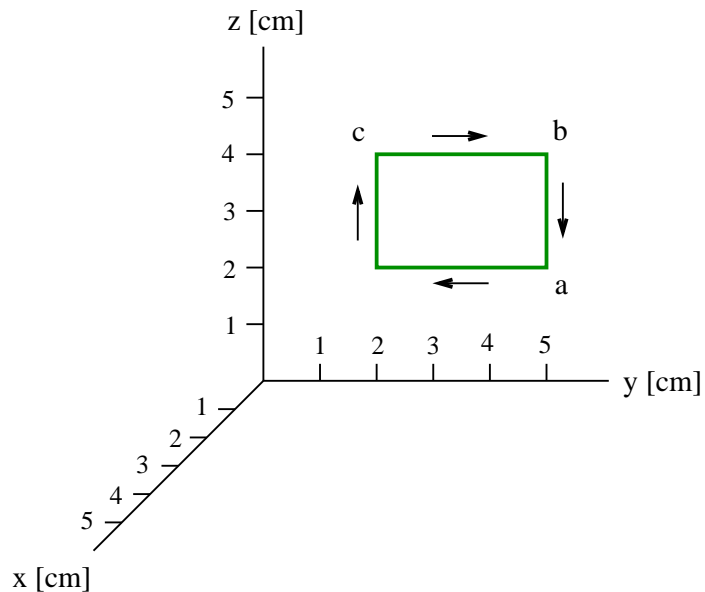


Unit Exam III: Problem #1 (Fall '16)



A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field \mathbf{B} .

- (a) Find the force \mathbf{F}_{ab} (magnitude and direction) acting on side ab .
- (b) Find the force \mathbf{F}_{bc} (magnitude and direction) acting on side bc .
- (c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
- (d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.

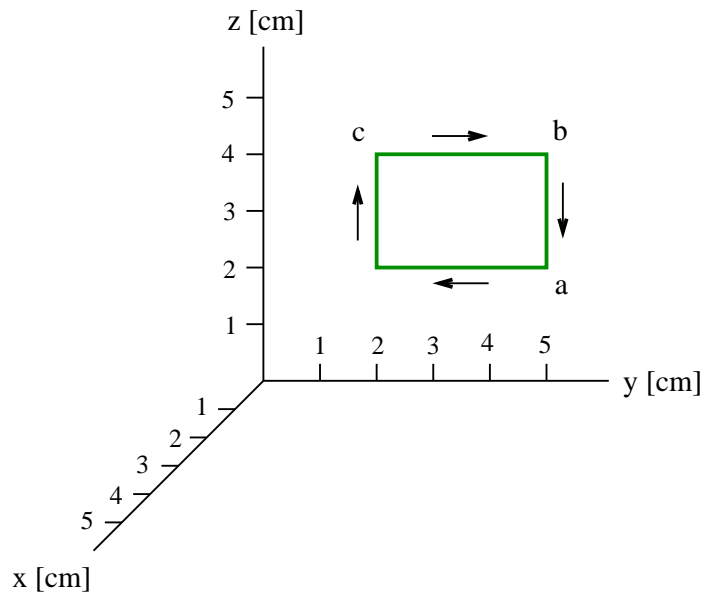


Unit Exam III: Problem #1 (Fall '16)



A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field \mathbf{B} .

- (a) Find the force \mathbf{F}_{ab} (magnitude and direction) acting on side ab .
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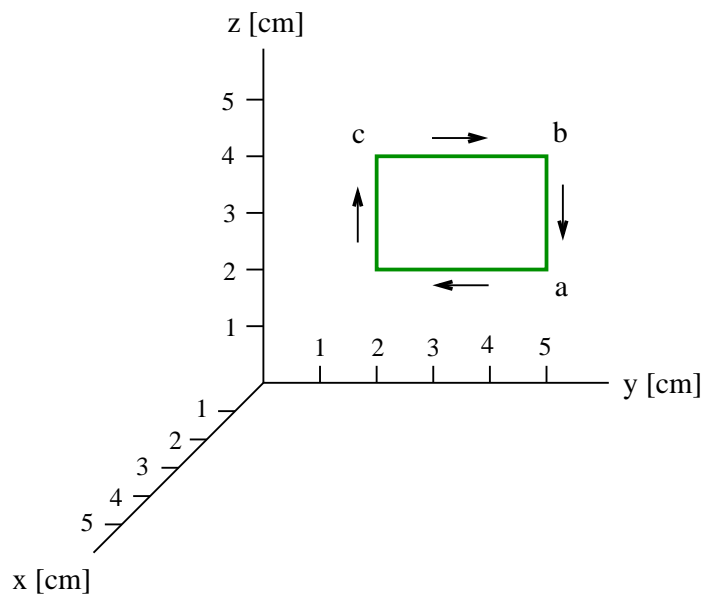
Solution for $I = 1.2\text{A}$, $\mathbf{B} = 0.7\text{mT}\hat{\mathbf{k}}$:

Unit Exam III: Problem #1 (Fall '16)



A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field \mathbf{B} .

- (a) Find the force \mathbf{F}_{ab} (magnitude and direction) acting on side ab .
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- (c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
- (d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.



(a) $\mathbf{F}_{ab} = (1.2\text{A})(-2\text{cm}\hat{\mathbf{k}}) \times (0.7\text{mT}\hat{\mathbf{k}}) = 0.$

(b) $\mathbf{F}_{bc} = (1.2\text{A})(3\text{cm}\hat{\mathbf{j}}) \times (0.7\text{mT}\hat{\mathbf{k}}) = 2.52 \times 10^{-5}\text{N}\hat{\mathbf{i}}.$

(c) $\vec{\mu} = (2\text{cm})(3\text{cm})(1.2\text{A})(-\hat{\mathbf{i}}) = -7.2 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}.$

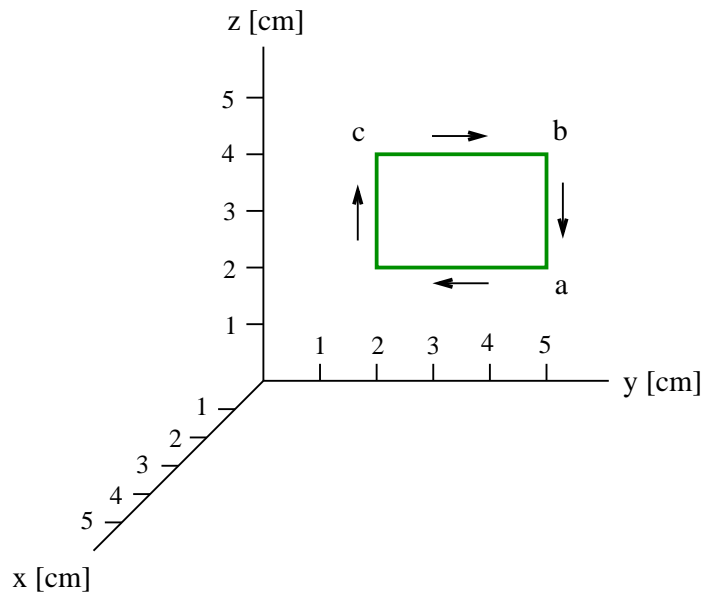
(d) $\vec{\tau} = (-7.2 \times 10^{-4}\text{Am}^2\hat{\mathbf{i}}) \times (0.7\text{mT}\hat{\mathbf{k}}) = 5.04 \times 10^{-7}\text{Nm}\hat{\mathbf{j}}.$

Unit Exam III: Problem #1 (Fall '16)



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- (a) Find the force \mathbf{F}_{ab} (magnitude and direction) acting on side ab .
- (b) Find the force \mathbf{F}_{bc} (magnitude and direction) acting on side bc .
- (c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
- (d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.



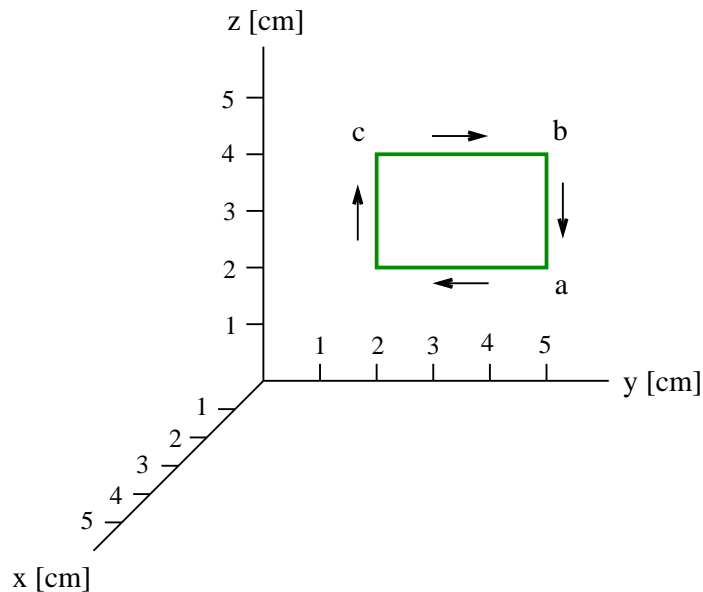
Solution for $I = 2.1\text{A}$, $\mathbf{B} = 0.8\text{mT}\hat{\mathbf{j}}$

Unit Exam III: Problem #1 (Fall '16)



A current I is flowing around the conducting rectangular frame in the direction shown. The frame is located in a region of uniform magnetic field \mathbf{B} .

- Find the force \mathbf{F}_{ab} (magnitude and direction) acting on side ab .
- Find the force \mathbf{F}_{bc} (magnitude and direction) acting on side bc .
- Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
- Find the torque $\vec{\tau}$ (magnitude and direction) acting on the frame.



$$(a) \mathbf{F}_{ab} = (2.1\text{A})(-2\text{cm}\hat{\mathbf{k}}) \times (0.8\text{mT}\hat{\mathbf{j}}) = 3.36 \times 10^{-5}\text{N}\hat{\mathbf{i}}.$$

$$(b) \mathbf{F}_{bc} = (2.1\text{A})(3\text{cm}\hat{\mathbf{j}}) \times (0.8\text{mT}\hat{\mathbf{j}}) = 0.$$

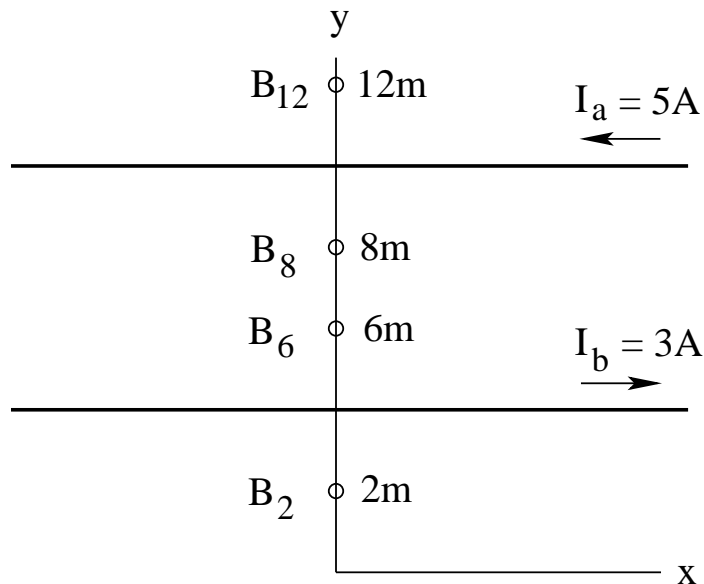
$$(c) \vec{\mu} = (2\text{cm})(3\text{cm})(2.1\text{A})(-\hat{\mathbf{i}}) = -1.26 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}.$$

$$(d) \vec{\tau} = (-1.26 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}) \times (0.8\text{mT}\hat{\mathbf{j}}) = -1.01 \times 10^{-6}\text{Nm}\hat{\mathbf{k}}.$$

Unit Exam III: Problem #2 (Fall '16)



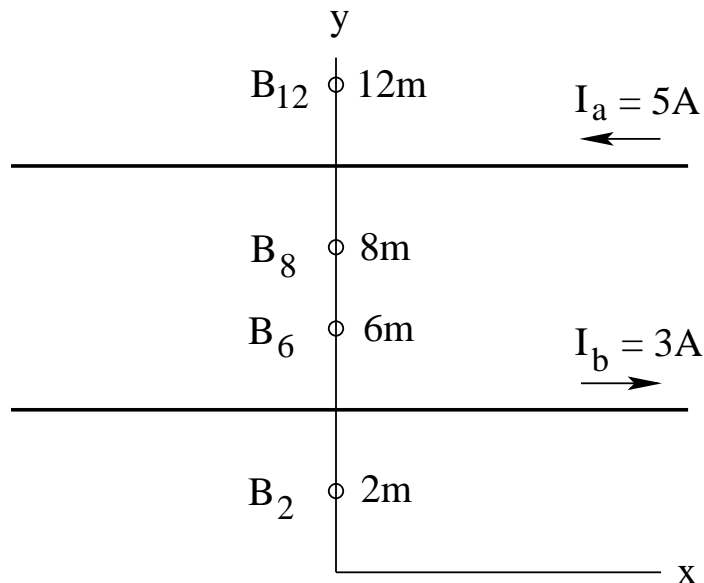
Two infinitely long, straight wires at positions $y = 10\text{m}$ and $y = 4\text{m}$ carry currents I_a and I_b , respectively. Find direction (in/out) and magnitude of the magnetic fields B_{12} , B_8 , B_6 , and B_2 at the points marked in the graph.



Unit Exam III: Problem #2 (Fall '16)



Two infinitely long, straight wires at positions $y = 10\text{m}$ and $y = 4\text{m}$ carry currents I_a and I_b , respectively. Find direction (in/out) and magnitude of the magnetic fields B_{12} , B_8 , B_6 , and B_2 at the points marked in the graph.



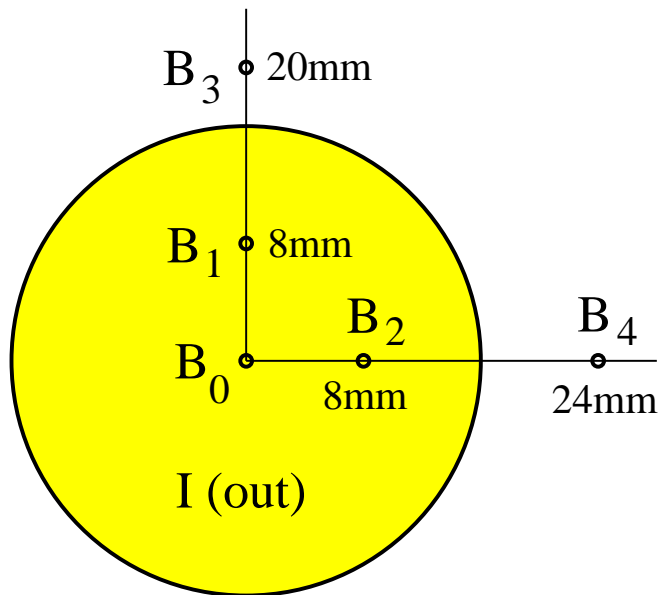
Solution:

- $B_{12} = \frac{\mu_0}{2\pi} \left(-\frac{5\text{A}}{2\text{m}} + \frac{3\text{A}}{8\text{m}} \right) = -4.25 \times 10^{-7}\text{T}$ (in).
- $B_8 = \frac{\mu_0}{2\pi} \left(\frac{5\text{A}}{2\text{m}} + \frac{3\text{A}}{4\text{m}} \right) = 6.50 \times 10^{-7}\text{T}$ (out).
- $B_6 = \frac{\mu_0}{2\pi} \left(\frac{5\text{A}}{4\text{m}} + \frac{3\text{A}}{2\text{m}} \right) = 5.50 \times 10^{-7}\text{T}$ (out).
- $B_2 = \frac{\mu_0}{2\pi} \left(\frac{5\text{A}}{8\text{m}} - \frac{3\text{A}}{2\text{m}} \right) = -1.75 \times 10^{-7}\text{T}$ (in).

Unit Exam III: Problem #3 (Fall '16)



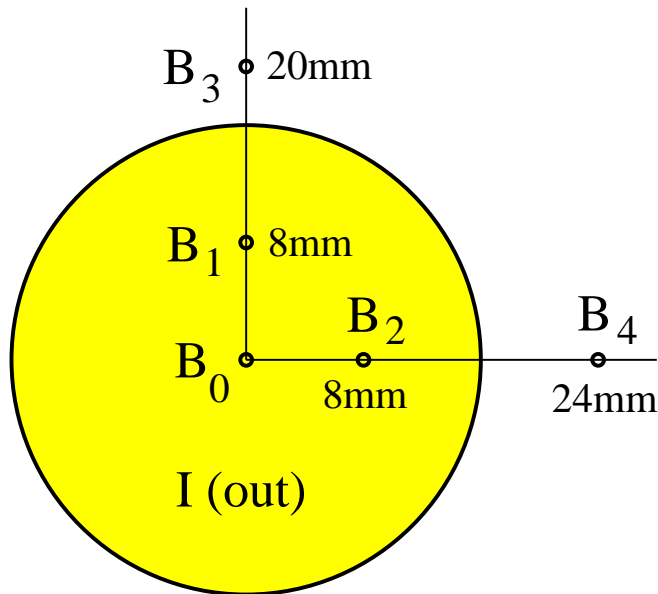
A conducting wire of 16mm radius carries a current I that is uniformly distributed over its cross section and directed out of the plane. Find direction (left/right/up/down) and magnitude of the magnetic fields \mathbf{B}_0 , \mathbf{B}_1 , \mathbf{B}_2 , \mathbf{B}_3 , and \mathbf{B}_4 at the positions indicated if the current is $I = 2.5\text{A}$.



Unit Exam III: Problem #3 (Fall '16)



A conducting wire of 16mm radius carries a current I that is uniformly distributed over its cross section and directed out of the plane. Find direction (left/right/up/down) and magnitude of the magnetic fields \mathbf{B}_0 , \mathbf{B}_1 , \mathbf{B}_2 , \mathbf{B}_3 , and \mathbf{B}_4 at the positions indicated if the current is $I = 2.5\text{A}$.



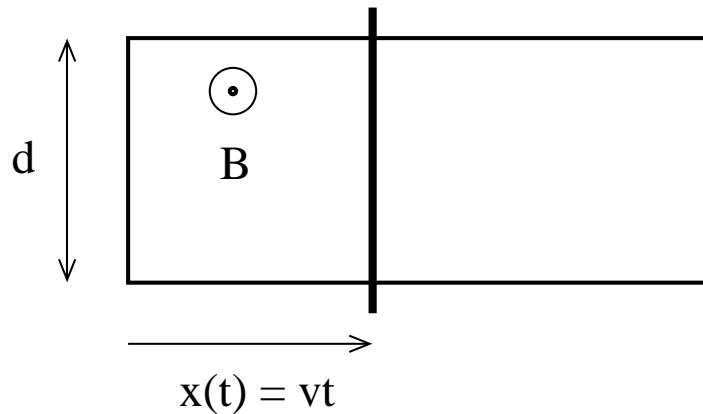
Solution:

- $B_0 = 0$
- $(B_1)(2\pi)(8\text{mm}) = \mu_0(I/4) \Rightarrow B_1 = 1.56 \times 10^{-5}\text{T}$ (left)
- $(B_2)(2\pi)(8\text{mm}) = \mu_0(I/4) \Rightarrow B_2 = 1.56 \times 10^{-5}\text{T}$ (up)
- $(B_3)(2\pi)(20\text{mm}) = \mu_0 I \Rightarrow B_3 = 2.5 \times 10^{-5}\text{T}$ (left)
- $(B_4)(2\pi)(24\text{mm}) = \mu_0 I \Rightarrow B_4 = 2.08 \times 10^{-5}\text{T}$ (up)

Unit Exam III: Problem #4 (Fall '16)



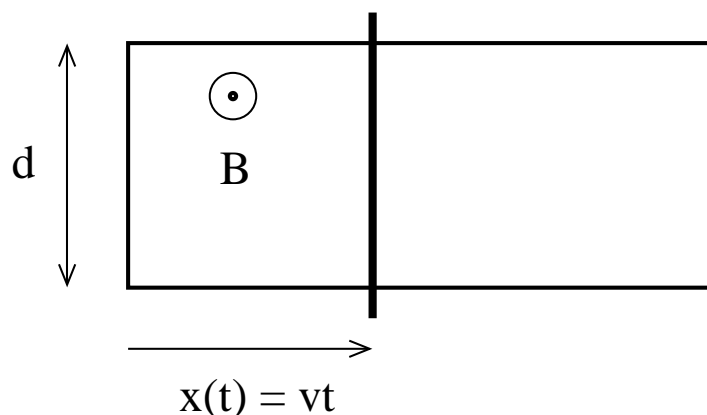
A conducting frame of width $d = 1.6\text{m}$ with a moving conducting rod is located in a uniform magnetic field $B = 3\text{T}$ directed out of the plane. The rod moves at constant velocity $v = 0.4\text{m/s}$ toward the right. Its instantaneous position is $x(t) = vt$. Find the magnetic flux Φ_B through the frame and the induced emf \mathcal{E} around the frame at times $t_2 = 2\text{s}$, $t_3 = 3\text{s}$, $t_4 = 4\text{s}$, and $t_5 = 5\text{s}$. Write magnitudes only (in SI units), no directions. Is the induced current directed clockwise or counterclockwise?



Unit Exam III: Problem #4 (Fall '16)



A conducting frame of width $d = 1.6\text{m}$ with a moving conducting rod is located in a uniform magnetic field $B = 3\text{T}$ directed out of the plane. The rod moves at constant velocity $v = 0.4\text{m/s}$ toward the right. Its instantaneous position is $x(t) = vt$. Find the magnetic flux Φ_B through the frame and the induced emf \mathcal{E} around the frame at times $t_2 = 2\text{s}$, $t_3 = 3\text{s}$, $t_4 = 4\text{s}$, and $t_5 = 5\text{s}$. Write magnitudes only (in SI units), no directions. Is the induced current directed clockwise or counterclockwise?



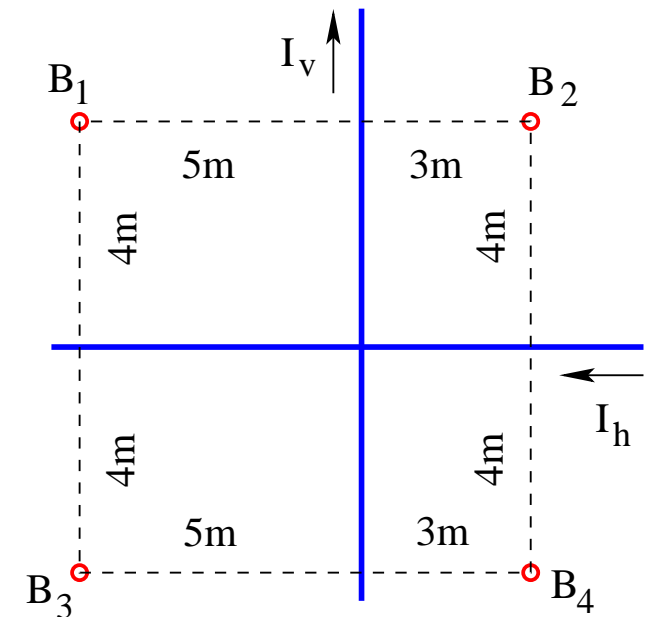
Solution:

- $\Phi_B^{(2)} = (1.6\text{m})(0.8\text{m})(3\text{T}) = 3.84\text{Wb}$,
 $\mathcal{E}^{(2)} = (0.4\text{m/s})(3\text{T})(1.6\text{m}) = 1.92\text{V}$.
- $\Phi_B^{(3)} = (1.6\text{m})(1.2\text{m})(3\text{T}) = 5.76\text{Wb}$,
 $\mathcal{E}^{(3)} = (0.4\text{m/s})(3\text{T})(1.6\text{m}) = 1.92\text{V}$.
- $\Phi_B^{(4)} = (1.6\text{m})(1.6\text{m})(3\text{T}) = 7.68\text{Wb}$,
 $\mathcal{E}^{(4)} = (0.4\text{m/s})(3\text{T})(1.6\text{m}) = 1.92\text{V}$.
- $\Phi_B^{(5)} = (1.6\text{m})(2.0\text{m})(3\text{T}) = 9.60\text{Wb}$,
 $\mathcal{E}^{(5)} = (0.4\text{m/s})(3\text{T})(1.6\text{m}) = 1.92\text{V}$.
- Clockwise current.

Unit Exam III: Problem #1 (Spring '17)



Consider two infinitely long, straight wires with currents $I_v = 6.9\text{A}$, $I_h = 7.2\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields \mathbf{B}_1 , \mathbf{B}_2 , \mathbf{B}_3 , \mathbf{B}_4 , at the points marked in the graph.



Unit Exam III: Problem #1 (Spring '17)



Consider two infinitely long, straight wires with currents $I_v = 6.9\text{A}$, $I_h = 7.2\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields \mathbf{B}_1 , \mathbf{B}_2 , \mathbf{B}_3 , \mathbf{B}_4 , at the points marked in the graph.

Solution:

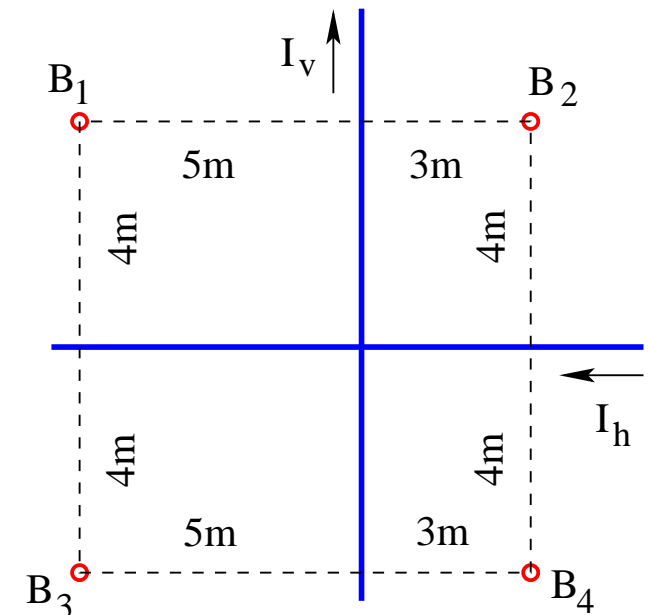
- Convention used: out = positive, in = negative

- $B_1 = \frac{\mu_0}{2\pi} \left(\frac{6.9\text{A}}{5\text{m}} - \frac{7.2\text{A}}{4\text{m}} \right) = -0.84 \times 10^{-7}\text{T}$ (in).

- $B_2 = \frac{\mu_0}{2\pi} \left(-\frac{6.9\text{A}}{3\text{m}} - \frac{7.2\text{A}}{4\text{m}} \right) = -8.20 \times 10^{-7}\text{T}$ (in).

- $B_3 = \frac{\mu_0}{2\pi} \left(\frac{6.9\text{A}}{5\text{m}} + \frac{7.2\text{A}}{4\text{m}} \right) = 6.36 \times 10^{-7}\text{T}$ (out).

- $B_4 = \frac{\mu_0}{2\pi} \left(-\frac{6.9\text{A}}{3\text{m}} + \frac{7.2\text{A}}{4\text{m}} \right) = -1.00 \times 10^{-7}\text{T}$ (in).

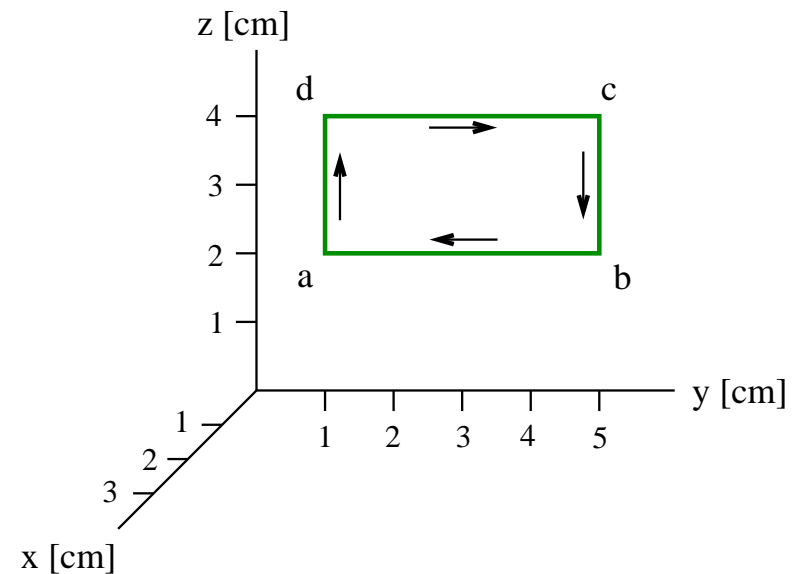


Unit Exam III: Problem #2 (Spring '17)



In a region of uniform magnetic field $\mathbf{B} = 4\text{mT}\hat{\mathbf{k}}$ [$\mathbf{B} = 5\text{mT}\hat{\mathbf{j}}$] a clockwise current $I = 1.4\text{A}$ [$I = 1.5\text{A}$] is flowing through the conducting rectangular frame.

- (i) Find the force \mathbf{F}_{dc} (magnitude and direction) acting on side dc of the rectangle. (ii) Find the force \mathbf{F}_{ad} (magnitude and direction) acting on side ad of the rectangle.
- (iii) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
- (iv) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the current loop.



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- (iii) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the current loop.
- (iv) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the current loop.

Solution:

$$(i) \mathbf{F}_{dc} = (1.4\text{A})(4\text{cm}\hat{\mathbf{j}}) \times (4\text{mT}\hat{\mathbf{k}}) = 2.24 \times 10^{-4}\text{N}\hat{\mathbf{i}}.$$

$$[\mathbf{F}_{dc} = (1.5\text{A})(4\text{cm}\hat{\mathbf{j}}) \times (5\text{mT}\hat{\mathbf{j}}) = 0.]$$

$$(ii) \mathbf{F}_{ad} = (1.4\text{A})(2\text{cm}\hat{\mathbf{k}}) \times (4\text{mT}\hat{\mathbf{k}}) = 0.$$

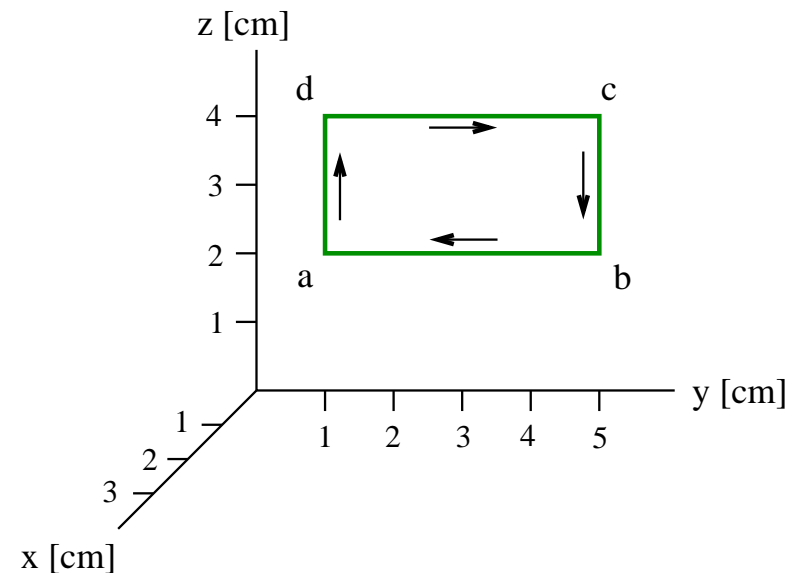
$$[\mathbf{F}_{ad} = (1.5\text{A})(2\text{cm}\hat{\mathbf{k}}) \times (5\text{mT}\hat{\mathbf{j}}) = -1.50 \times 10^{-4}\text{N}\hat{\mathbf{i}}.]$$

$$(iii) \vec{\mu} = [-(4\text{cm})(2\text{cm})\hat{\mathbf{i}}](1.4\text{A}) = -1.12 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}.$$

$$[\vec{\mu} = [-(4\text{cm})(2\text{cm})\hat{\mathbf{i}}](1.5\text{A}) = -1.20 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}.]$$

$$(iv) \vec{\tau} = (-1.12 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}) \times (4\text{mT}\hat{\mathbf{k}}) = 4.48 \times 10^{-6}\text{Nm}\hat{\mathbf{j}}.$$

$$[\vec{\tau} = (-1.20 \times 10^{-3}\text{Am}^2\hat{\mathbf{i}}) \times (5\text{mT}\hat{\mathbf{j}}) = -6.00 \times 10^{-6}\text{Nm}\hat{\mathbf{k}}.]$$



Unit Exam III: Problem #3 (Spring '17)



A conducting frame with a moving conducting rod is located in a uniform magnetic field directed out of the plane as shown. The rod moves at constant velocity.

Find the magnetic flux Φ_B through the frame and the induced emf \mathcal{E} around the frame when the rod is

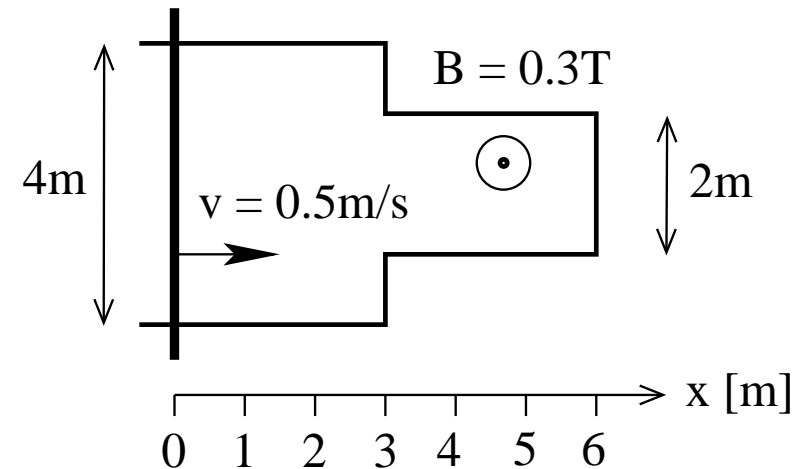
(a) at position $x = 1\text{m}$,

(b) at position $x = 4\text{m}$.

(c) at position $x = 2\text{m}$,

(d) at position $x = 5\text{m}$.

Write magnitudes only (in SI units), no directions.



Unit Exam III: Problem #3 (Spring '17)

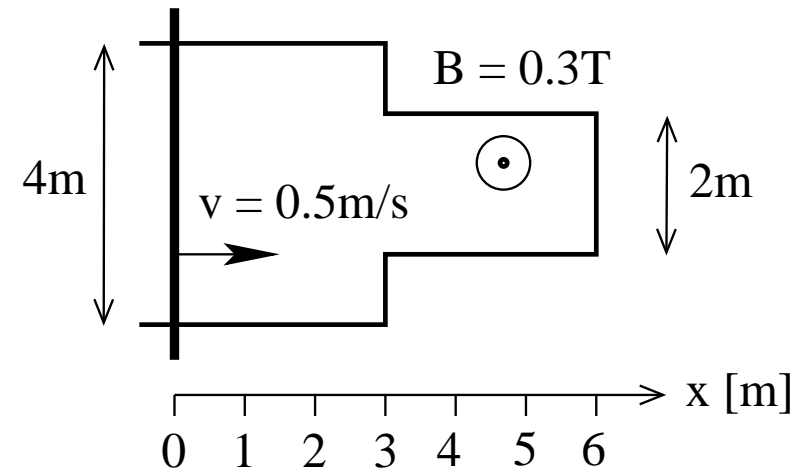


A conducting frame with a moving conducting rod is located in a uniform magnetic field directed out of the plane as shown. The rod moves at constant velocity.

Find the magnetic flux Φ_B through the frame and the induced emf \mathcal{E} around the frame when the rod is

- (a) at position $x = 1\text{m}$,
- (b) at position $x = 4\text{m}$.
- (c) at position $x = 2\text{m}$,
- (d) at position $x = 5\text{m}$.

Write magnitudes only (in SI units), no directions.



Solution:

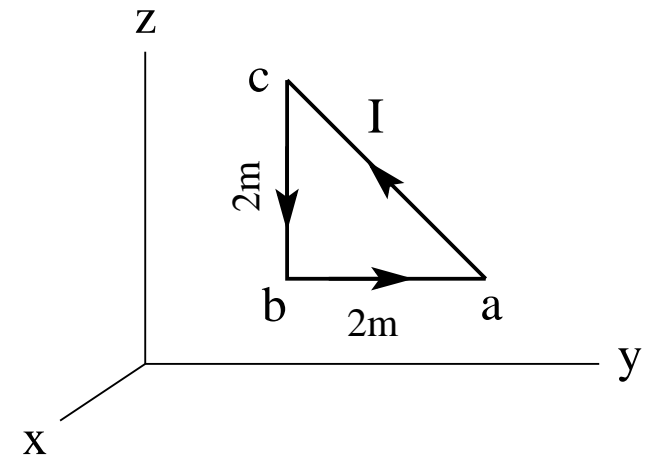
- (a) $\Phi_B = (8 + 6)\text{m}^2(0.3\text{T}) = 4.2\text{Wb}$, $\mathcal{E} = (0.5\text{m/s})(0.3\text{T})(4\text{m}) = 0.6\text{V}$.
- (b) $\Phi_B = (4\text{m}^2)(0.3\text{T}) = 1.2\text{Wb}$, $\mathcal{E} = (0.5\text{m/s})(0.3\text{T})(2\text{m}) = 0.3\text{V}$.
- (c) $\Phi_B = (4 + 6)\text{m}^2(0.3\text{T}) = 3.0\text{Wb}$, $\mathcal{E} = (0.5\text{m/s})(0.3\text{T})(4\text{m}) = 0.6\text{V}$.
- (d) $\Phi_B = (2\text{m}^2)(0.3\text{T}) = 0.6\text{Wb}$, $\mathcal{E} = (0.5\text{m/s})(0.3\text{T})(2\text{m}) = 0.3\text{V}$.

Unit Exam III: Problem #1 (Fall '17)



Consider a region with uniform magnetic field $\vec{B} = 4\text{T}\hat{j}$ [$\vec{B} = 5\text{T}\hat{k}$]. A conducting loop in the yz -plane has the shape of a right-angled triangle as shown with a counterclockwise current $I = 0.7\text{A}$ [$I = 0.9\text{A}$].

- (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
- (b) Find the force \vec{F}_{ab} (magnitude and direction) acting on the side ab of the loop.
- (c) Find the force \vec{F}_{bc} (magnitude and direction) acting on the side bc of the loop.
- (d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



Unit Exam III: Problem #1 (Fall '17)

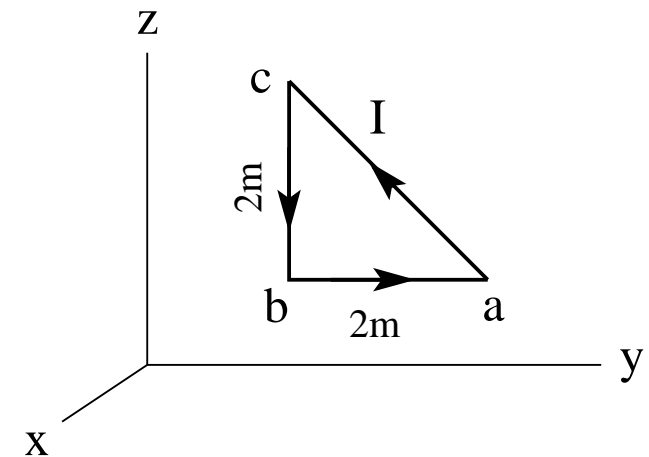


Consider a region with uniform magnetic field $\vec{B} = 4\text{T}\hat{j}$ [$\vec{B} = 5\text{T}\hat{k}$]. A conducting loop in the yz -plane has the shape of a right-angled triangle as shown with a counterclockwise current $I = 0.7\text{A}$ [$I = 0.9\text{A}$].

- (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
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- (d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.

Solution:

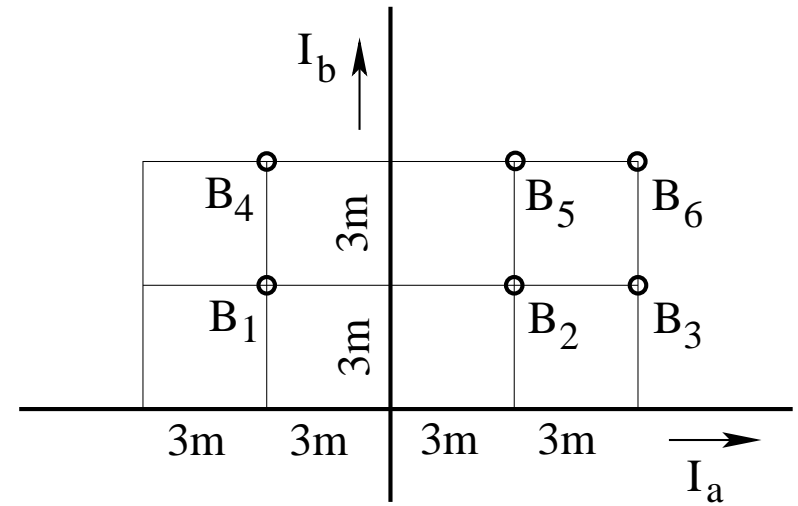
- (a) $\vec{\mu} = (0.7\text{A})(2\text{m}^2)\hat{i} = 1.4\text{Am}^2\hat{i}$
[$\vec{\mu} = (0.9\text{A})(2\text{m}^2)\hat{i} = 1.8\text{Am}^2\hat{i}$]
- (b) $\vec{F}_{ab} = 0$ [$\vec{F}_{ab} = (0.9\text{A})(2\text{m}\hat{j}) \times (5\text{T}\hat{k}) = 9.0\text{N}\hat{i}$]
- (c) $\vec{F}_{bc} = (0.7\text{A})(-2\text{m}\hat{k}) \times (4\text{T}\hat{j}) = 5.6\text{N}\hat{i}$ [$\vec{F}_{bc} = 0$]
- (d) $\vec{\tau} = (1.4\text{Am}^2\hat{i}) \times (4\text{T}\hat{j}) = 5.6\text{Nm}\hat{k}$
[$\vec{\tau} = (1.8\text{Am}^2\hat{i}) \times (5\text{T}\hat{k}) = -9.0\text{Nm}\hat{j}$]



Unit Exam III: Problem #2 (Fall '17)



Consider two infinitely long, straight wires with currents $I_a = I_b = 7\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3, \mathbf{B}_4, \mathbf{B}_5, \mathbf{B}_6$ at the points marked in the graph.



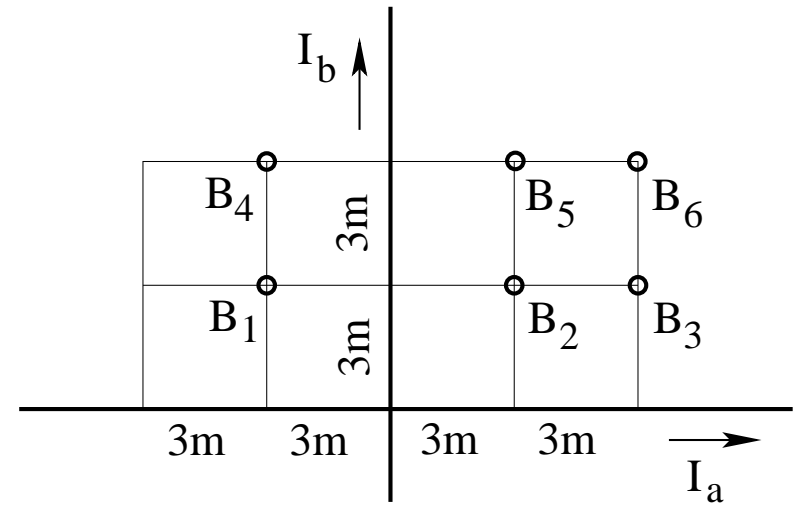
Unit Exam III: Problem #2 (Fall '17)



Consider two infinitely long, straight wires with currents $I_a = I_b = 7\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $B_1, B_2, B_3, B_4, B_5, B_6$ at the points marked in the graph.

Solution:

- $B_1 = \frac{\mu_0}{2\pi} \left(\frac{7\text{A}}{3\text{m}} + \frac{7\text{A}}{3\text{m}} \right) = +0.933\mu\text{T}$ (out of plane).
- $B_2 = \frac{\mu_0}{2\pi} \left(\frac{7\text{A}}{3\text{m}} - \frac{7\text{A}}{3\text{m}} \right) = 0$ (no direction).
- $B_3 = \frac{\mu_0}{2\pi} \left(\frac{7\text{A}}{3\text{m}} - \frac{7\text{A}}{6\text{m}} \right) = +0.233\mu\text{T}$ (out of plane).
- $B_4 = \frac{\mu_0}{2\pi} \left(\frac{7\text{A}}{6\text{m}} + \frac{7\text{A}}{3\text{m}} \right) = 0.7\mu\text{T}$ (out of plane).
- $B_5 = \frac{\mu_0}{2\pi} \left(\frac{7\text{A}}{6\text{m}} - \frac{7\text{A}}{3\text{m}} \right) = -0.233\mu\text{T}$ (into plane).
- $B_6 = \frac{\mu_0}{2\pi} \left(\frac{7\text{A}}{6\text{m}} - \frac{7\text{A}}{6\text{m}} \right) = 0$ (no direction).



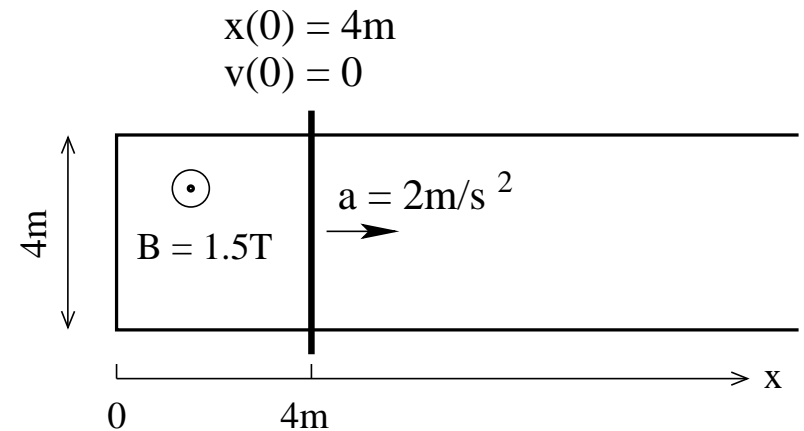
Unit Exam III: Problem #3 (Fall '17)



A conducting frame with a moving conducting rod is placed in a uniform magnetic field directed out of the plane. The rod starts from rest at time $t = 0$ at the position shown and moves with constant acceleration to the right.

- (a) Find the magnetic flux Φ_B through the conducting loop and the induced emf \mathcal{E} around the loop at $t = 0$.
- (b) Find the position $x(3s)$ and velocity $v(3s)$ of the rod at time $t = 3s$.
- (c) Find the magnetic flux Φ_B through the loop and the induced emf \mathcal{E} around the loop at time $t = 3s$.

Write magnitudes only (in SI units), no directions.



Unit Exam III: Problem #3 (Fall '17)



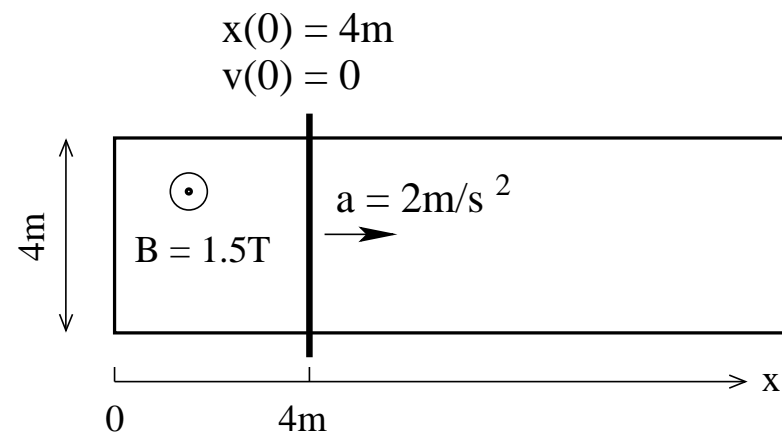
A conducting frame with a moving conducting rod is placed in a uniform magnetic field directed out of the plane. The rod starts from rest at time $t = 0$ at the position shown and moves with constant acceleration to the right.

(a) Find the magnetic flux Φ_B through the conducting loop and the induced emf \mathcal{E} around the loop at $t = 0$.

(b) Find the position $x(3s)$ and velocity $v(3s)$ of the rod at time $t = 3s$.

(c) Find the magnetic flux Φ_B through the loop and the induced emf \mathcal{E} around the loop at time $t = 3s$.

Write magnitudes only (in SI units), no directions.



Solution:

(a) $\Phi_B = (16\text{m}^2)(1.5\text{T}) = 24\text{Wb}, \quad \mathcal{E} = 0.$

(b) $x(3s) = 4\text{m} + \frac{1}{2}(2\text{m/s}^2)(3s)^2 = 13\text{m}, \quad v(3s) = (2\text{m/s}^2)(3s) = 6\text{m/s}.$

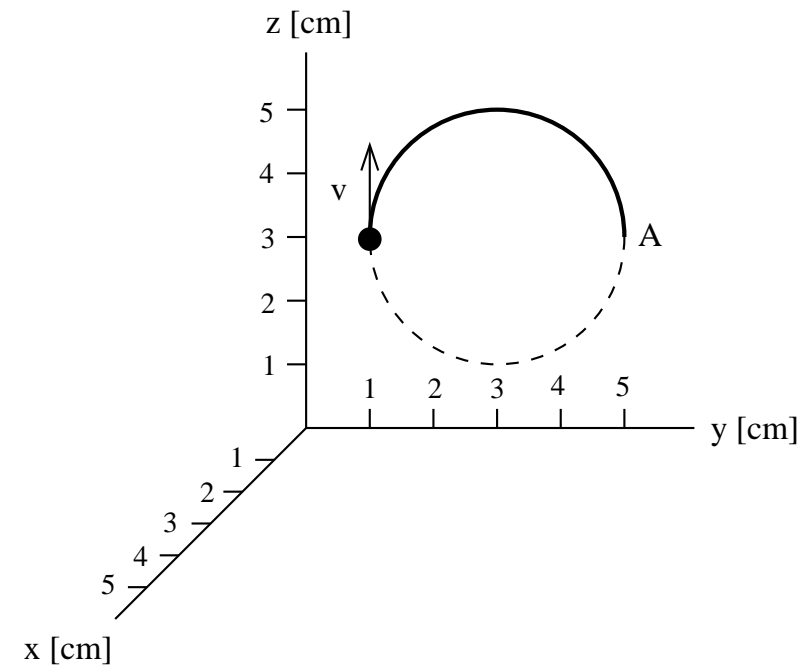
(c) $\Phi_B = (52\text{m}^2)(1.5\text{T}) = 78\text{Wb}, \quad \mathcal{E} = (6\text{m/s})(1.5\text{T})(4\text{m}) = 36\text{V}.$

Unit Exam III: Problem #1 (Spring '18)



In a uniform magnetic field of strength $B = 3.5\text{mT}$ [$B = 5.3\text{mT}$], a proton with specifications ($m = 1.67 \times 10^{-27}\text{kg}$, $q = 1.60 \times 10^{-19}\text{C}$) moves at speed v around a circle in the yz -plane as shown.

- (a) Show that the direction of the magnetic field must be $+\hat{i}$
- (b) What is the speed of the proton?
- (c) How long does it take the proton to reach point A from its current position?



Unit Exam III: Problem #1 (Spring '18)



In a uniform magnetic field of strength $B = 3.5\text{mT}$ [$B = 5.3\text{mT}$], a proton with specifications ($m = 1.67 \times 10^{-27}\text{kg}$, $q = 1.60 \times 10^{-19}\text{C}$) moves at speed v around a circle in the yz -plane as shown.

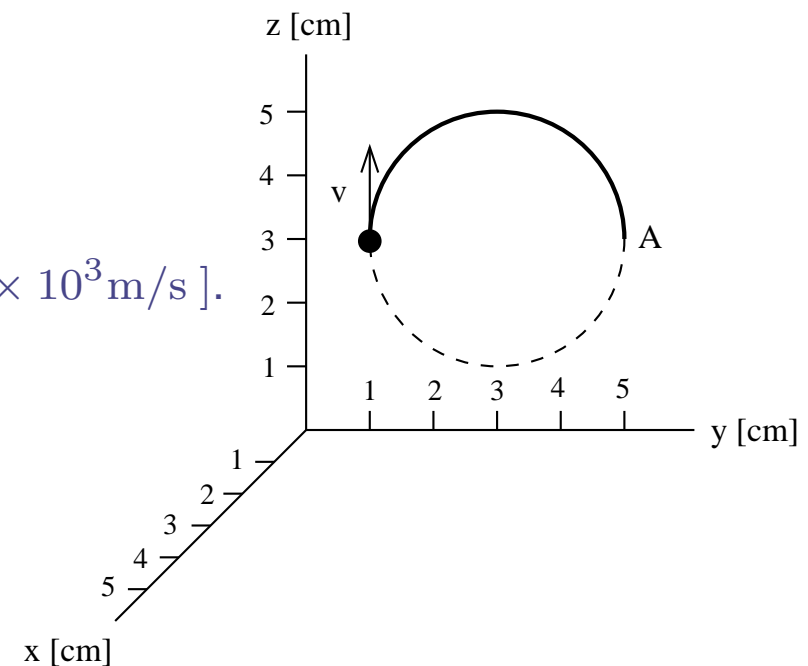
- (a) Show that the direction of the magnetic field must be $+\hat{i}$
- (b) What is the speed of the proton?
- (c) How long does it take the proton to reach point A from its current position?

Solution:

(a) $F\hat{j} = qv\hat{k} \times B\hat{i}$.

(b) $\frac{mv^2}{r} = qvB \Rightarrow v = \frac{qBr}{m} = 6.71 \times 10^3\text{m/s}$ [$10.2 \times 10^3\text{m/s}$].

(c) $t = \frac{\pi r}{v} = \frac{\pi m}{qB} = 9.37 \times 10^{-6}\text{s}$ [$6.19 \times 10^{-6}\text{s}$].

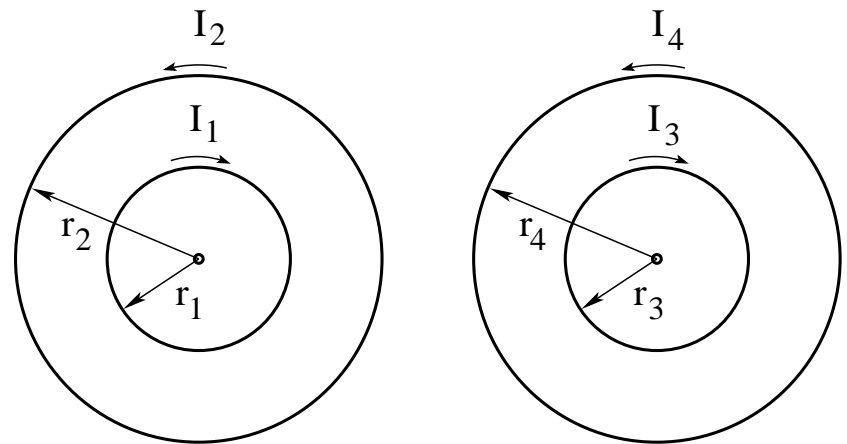


Unit Exam III: Problem #2a (Spring '18)



Consider two pairs of concentric circular currents in separate regions. The current directions are indicated by arrows. The radii are $r_1 = r_3 = 5\text{cm}$ and $r_2 = r_4 = 10\text{cm}$

- (a) Find magnitude B_1 and direction (\odot, \otimes) of the magnetic field produced by current $I_1 = 1.5\text{A}$ at the center.
- (b) Find magnitude μ_4 and direction (\odot, \otimes) of the magnetic dipole moment produced by current $I_4 = 4.5\text{A}$.
- (c) What must be the ratio I_2/I_1 such that the magnetic field at the center is zero?
- (d) What must be the ratio I_4/I_3 such that the magnetic dipole moment is zero?



Unit Exam III: Problem #2a (Spring '18)



Consider two pairs of concentric circular currents in separate regions. The current directions are indicated by arrows. The radii are $r_1 = r_3 = 5\text{cm}$ and $r_2 = r_4 = 10\text{cm}$

- (a) Find magnitude B_1 and direction (\odot, \otimes) of the magnetic field produced by current $I_1 = 1.5\text{A}$ at the center.
- (b) Find magnitude μ_4 and direction (\odot, \otimes) of the magnetic dipole moment produced by current $I_4 = 4.5\text{A}$.
- (c) What must be the ratio I_2/I_1 such that the magnetic field at the center is zero?
- (d) What must be the ratio I_4/I_3 such that the magnetic dipole moment is zero?

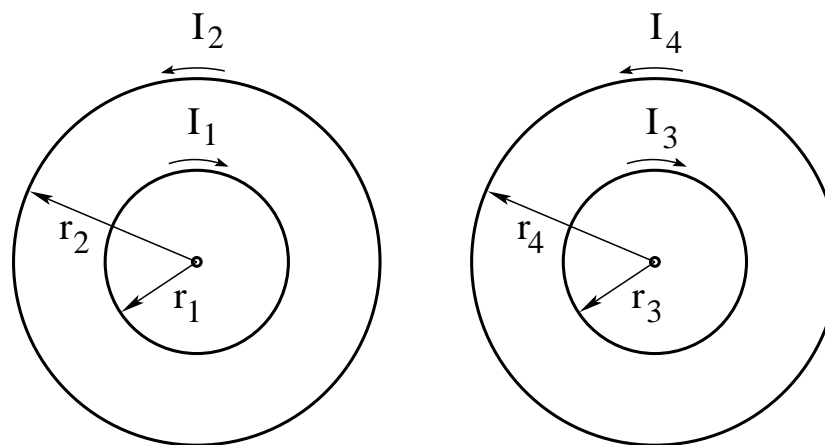
Solution:

(a) $B_1 = \frac{\mu_0(1.5\text{A})}{2(5\text{cm})} = 1.88 \times 10^{-5}\text{T} \quad \otimes$

(b) $\mu_4 = \pi(10\text{cm})^2(4.5\text{A}) = 1.41 \times 10^{-1}\text{Am}^2 \quad \odot$

(c) $B_1 = B_2 \Rightarrow \frac{I_2}{I_1} = \frac{r_2}{r_1} = 2.$

(d) $\mu_3 = \mu_4 \Rightarrow \frac{I_4}{I_3} = \frac{r_3^2}{r_4^2} = 0.25.$

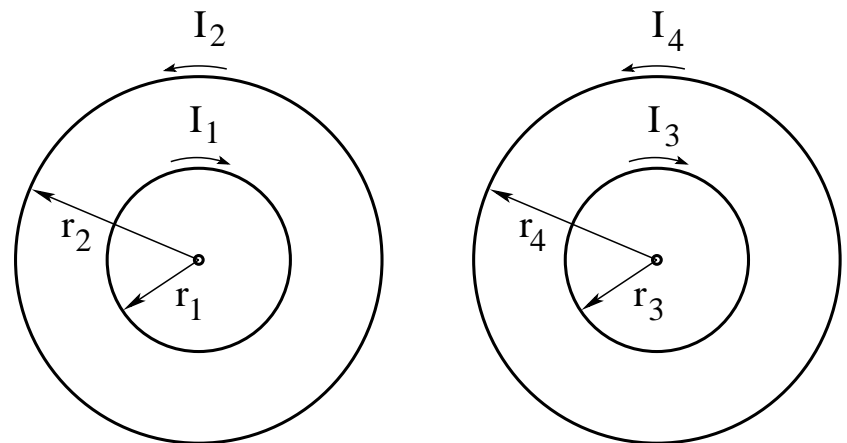


Unit Exam III: Problem #2b (Spring '18)



Consider two pairs of concentric circular currents in separate regions. The current directions are indicated by arrows. The radii are $r_1 = r_3 = 5\text{cm}$ and $r_2 = r_4 = 10\text{cm}$

- (a) Find magnitude B_2 and direction (\odot , \otimes) of the magnetic field produced by current $I_2 = 2.5\text{A}$ at the center.
- (b) Find magnitude μ_3 and direction (\odot , \otimes) of the magnetic dipole moment produced by current $I_3 = 3\text{A}$.
- (c) What must be the ratio I_2/I_1 such that the magnetic field at the center is zero?
- (d) What must be the ratio I_4/I_3 such that the magnetic dipole moment is zero?



Unit Exam III: Problem #2b (Spring '18)



Consider two pairs of concentric circular currents in separate regions. The current directions are indicated by arrows. The radii are $r_1 = r_3 = 5\text{cm}$ and $r_2 = r_4 = 10\text{cm}$

- (a) Find magnitude B_2 and direction (\odot, \otimes) of the magnetic field produced by current $I_2 = 2.5\text{A}$ at the center.
- (b) Find magnitude μ_3 and direction (\odot, \otimes) of the magnetic dipole moment produced by current $I_3 = 3\text{A}$.
- (c) What must be the ratio I_2/I_1 such that the magnetic field at the center is zero?
- (d) What must be the ratio I_4/I_3 such that the magnetic dipole moment is zero?

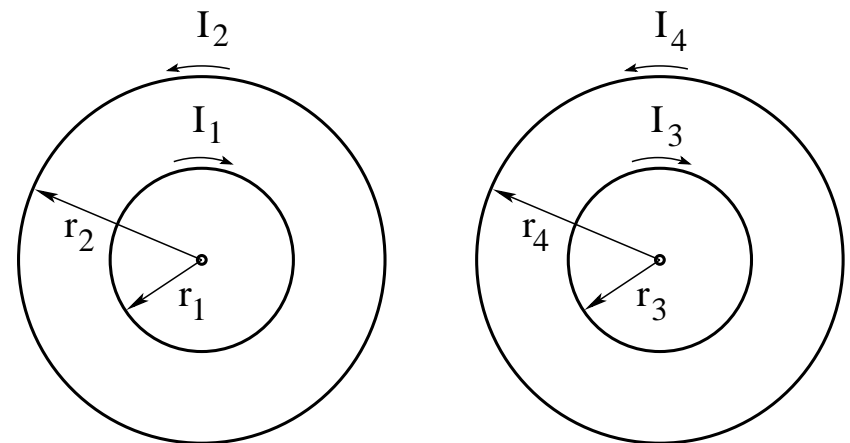
Solution:

(a) $B_2 = \frac{\mu_0(2.5\text{A})}{2(10\text{cm})} = 1.57 \times 10^{-5}\text{T} \quad \odot$

(b) $\mu_3 = \pi(5\text{cm})^2(3\text{A}) = 2.36 \times 10^{-2}\text{Am}^2 \quad \otimes$

(c) $B_1 = B_2 \Rightarrow \frac{I_2}{I_1} = \frac{r_2}{r_1} = 2.$

(d) $\mu_3 = \mu_4 \Rightarrow \frac{I_4}{I_3} = \frac{r_3^2}{r_4^2} = 0.25.$

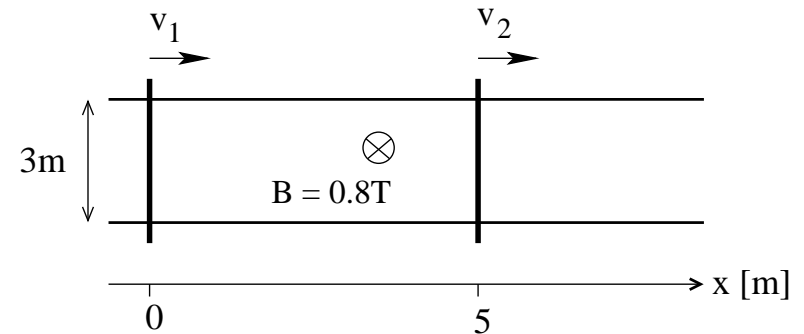


Unit Exam III: Problem #3 (Spring '18)



A pair of fixed rails are connected by two moving rods. A uniform magnetic field B is present. The positions of the rods at time $t = 0$ are as shown. The (constant) velocities are $v_1 = 0.5\text{m/s}$, $v_2 = 2.5\text{m/s}$ [$v_1 = 1.5\text{m/s}$, $v_2 = 0.5\text{m/s}$].

- (a) Find the magnetic flux Φ_0 at time $t = 0$ and Φ_1 at $t = 2\text{s}$ (magnitude only).
- (b) Find the induced emf \mathcal{E}_0 at time $t = 0$ and \mathcal{E}_1 at $t = 2\text{s}$ (magnitude only).
- (c) Find the direction (cw/ccw) of the induced current at $t = 0$.

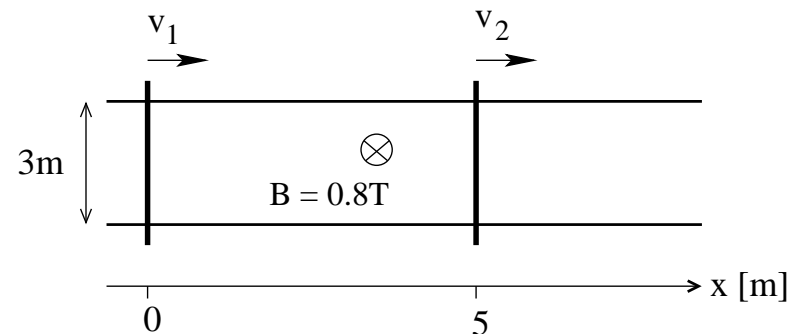


Unit Exam III: Problem #3 (Spring '18)



A pair of fixed rails are connected by two moving rods. A uniform magnetic field B is present. The positions of the rods at time $t = 0$ are as shown. The (constant) velocities are $v_1 = 0.5\text{m/s}$, $v_2 = 2.5\text{m/s}$ [$v_1 = 1.5\text{m/s}$, $v_2 = 0.5\text{m/s}$].

- Find the magnetic flux Φ_0 at time $t = 0$ and Φ_1 at $t = 2\text{s}$ (magnitude only).
- Find the induced emf \mathcal{E}_0 at time $t = 0$ and \mathcal{E}_1 at $t = 2\text{s}$ (magnitude only).
- Find the direction (cw/ccw) of the induced current at $t = 0$.



Solution:

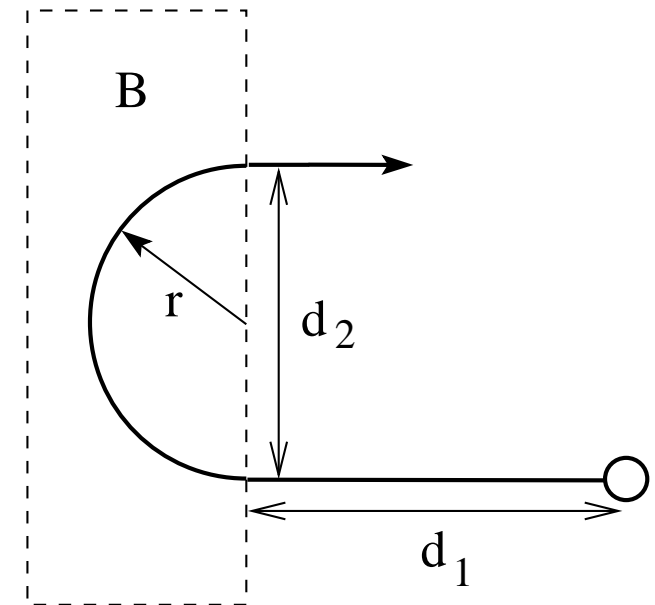
- $\Phi_0 = (5\text{m} - 0\text{m})(3\text{m})(0.8\text{T}) = 12\text{Wb}$, $\Phi_1 = (10\text{m} - 1\text{m})(3\text{m})(0.8\text{T}) = 21.6\text{Wb}$
[$\Phi_0 = (5\text{m} - 0\text{m})(3\text{m})(0.8\text{T}) = 12\text{Wb}$, $\Phi_1 = (6\text{m} - 3\text{m})(3\text{m})(0.8\text{T}) = 7.2\text{Wb}$]
- $|\mathcal{E}_0| = |\mathcal{E}_1| = (2.5\text{m/s} - 0.5\text{m/s})(0.8\text{T})(3\text{m}) = 4.8\text{V}$
[$|\mathcal{E}_0| = |\mathcal{E}_1| = (1.5\text{m/s} - 0.5\text{m/s})(0.8\text{T})(3\text{m}) = 2.4\text{V}$]
- ccw [cw]

Unit Exam III: Problem #1 (Fall '18)



A proton ($m = 1.67 \times 10^{-27} \text{kg}$, $q = 1.60 \times 10^{-19} \text{C}$), launched with initial speed $v_0 = 4000 \text{m/s}$ [3000m/s] a distance $d_1 = 25 \text{cm}$ [32cm] from a region of magnetic field, exits that region after a half-circle turn of diameter $d_2 = 30 \text{cm}$ [35cm].

- Find the centripetal force F provided by the magnetic field.
- Find magnitude and direction (\odot , \otimes) of the magnetic field \mathbf{B} .
- Find the time t_1 elapsed between launch and entrance into the region of field.
- Find the time t_2 elapsed between entrance and exit.



Unit Exam III: Problem #1 (Fall '18)



A proton ($m = 1.67 \times 10^{-27} \text{kg}$, $q = 1.60 \times 10^{-19} \text{C}$), launched with initial speed $v_0 = 4000 \text{m/s}$ [3000m/s] a distance $d_1 = 25 \text{cm}$ [32cm] from a region of magnetic field, exits that region after a half-circle turn of diameter $d_2 = 30 \text{cm}$ [35cm].

- Find the centripetal force F provided by the magnetic field.
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- Find the time t_1 elapsed between launch and entrance into the region of field.
- Find the time t_2 elapsed between entrance and exit.

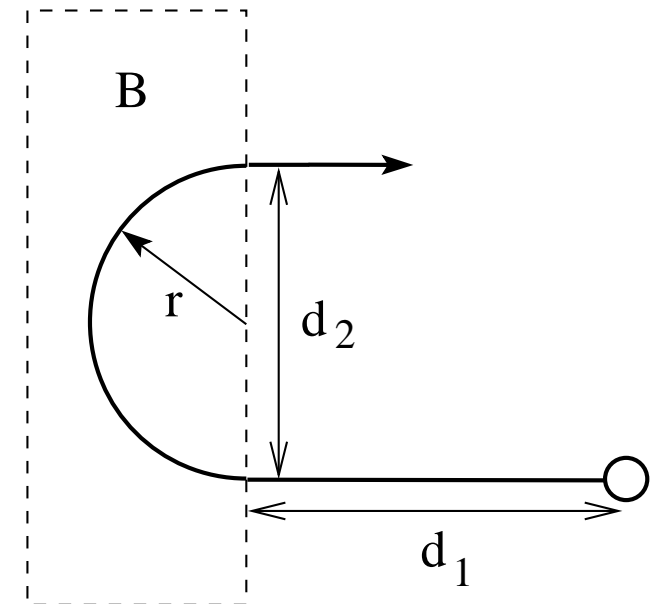
Solution:

$$(a) \frac{mv_0^2}{d_2/2} = 1.78 \times 10^{-19} \text{N} \quad [8.59 \times 10^{-20} \text{N}].$$

$$(b) B = \frac{F}{qv_0} = 2.78 \times 10^{-4} \text{T} \quad [1.79 \times 10^{-4} \text{T}] \quad \odot$$

$$(c) t_1 = \frac{d_1}{v_0} = 6.25 \times 10^{-5} \text{s} \quad [1.07 \times 10^{-4} \text{s}].$$

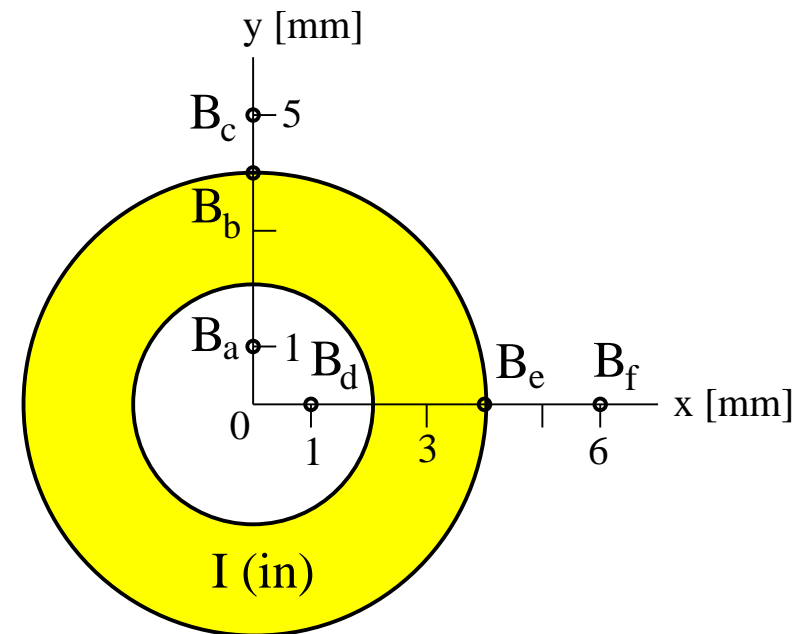
$$(d) t_2 = \frac{\pi d_2}{2v_0} = 1.18 \times 10^{-4} \text{s} \quad [1.83 \times 10^{-4} \text{s}].$$



Unit Exam III: Problem #2 (Fall '18)



A wire in the shape of a cylindrical shell with a 2mm inner radius and 4mm outer radius carries a current $I = 3.7\text{A}$ [4.1A] that is uniformly distributed over its cross section and directed into the plane. Find direction (left/right/up/down/in/out) and magnitude of the magnetic fields \mathbf{B}_a , \mathbf{B}_b , \mathbf{B}_c [\mathbf{B}_d , \mathbf{B}_e , \mathbf{B}_f] at the positions indicated.



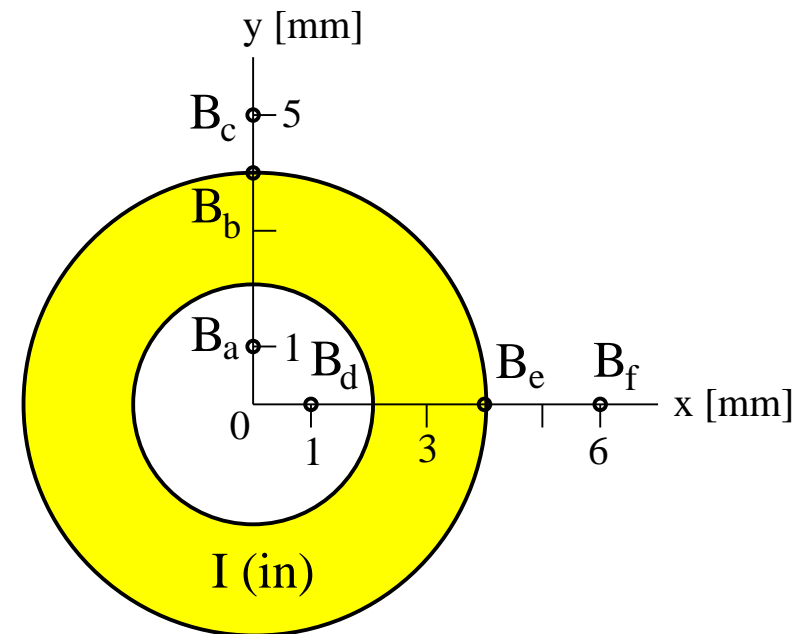
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Solution:

- $B_a = 0$
- $(B_b)(2\pi)(4\text{mm}) = \mu_0(3.7\text{A})$
 $\Rightarrow B_b = 1.85 \times 10^{-4}\text{T}$ (right)
- $(B_c)(2\pi)(5\text{mm}) = \mu_0(3.7\text{A})$
 $\Rightarrow B_c = 1.48 \times 10^{-4}\text{T}$ (right)
- $[B_d = 0]$
- $[(B_e)(2\pi)(4\text{mm}) = \mu_0(4.1\text{A}) \Rightarrow B_e = 2.05 \times 10^{-4}\text{T}$ (down)]
- $[(B_f)(2\pi)(6\text{mm}) = \mu_0(4.1\text{A}) \Rightarrow B_f = 1.37 \times 10^{-4}\text{T}$ (down)]



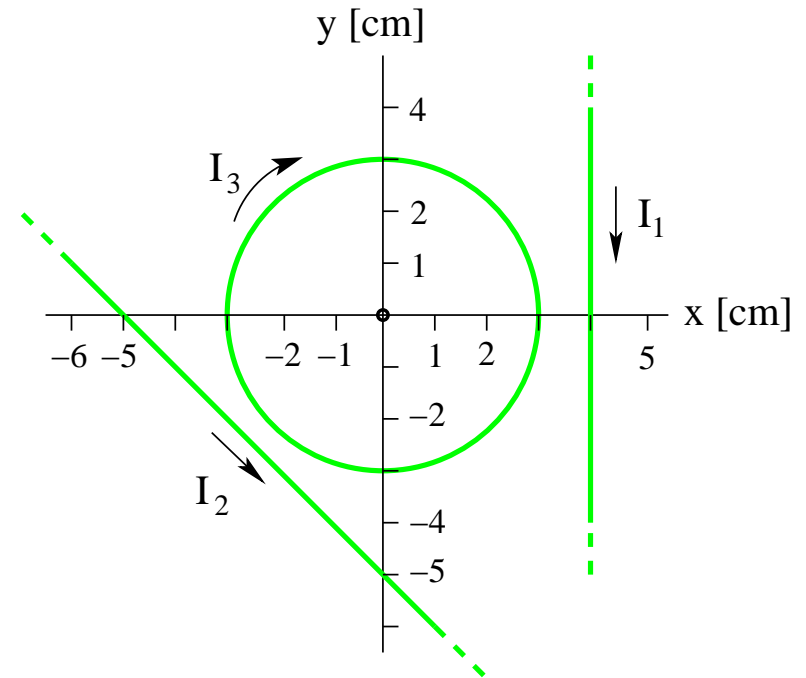
Unit Exam III: Problem #3 (Fall '18)



Two very long straight wires and a circular wire positioned in the xy -plane carry electric currents $I_1 = I_2 = I_3 = 1.3\text{A}$ [1.7A] in the directions shown.

(a) Calculate magnitude (B_1, B_2, B_2) and direction (left/right/up/down/in/out) of the magnetic field produced by each current at the origin of the coordinate system.

(b) Calculate magnitude μ and direction (left/right/up/down/in/out) of the magnetic dipole moment produced by the circular current.



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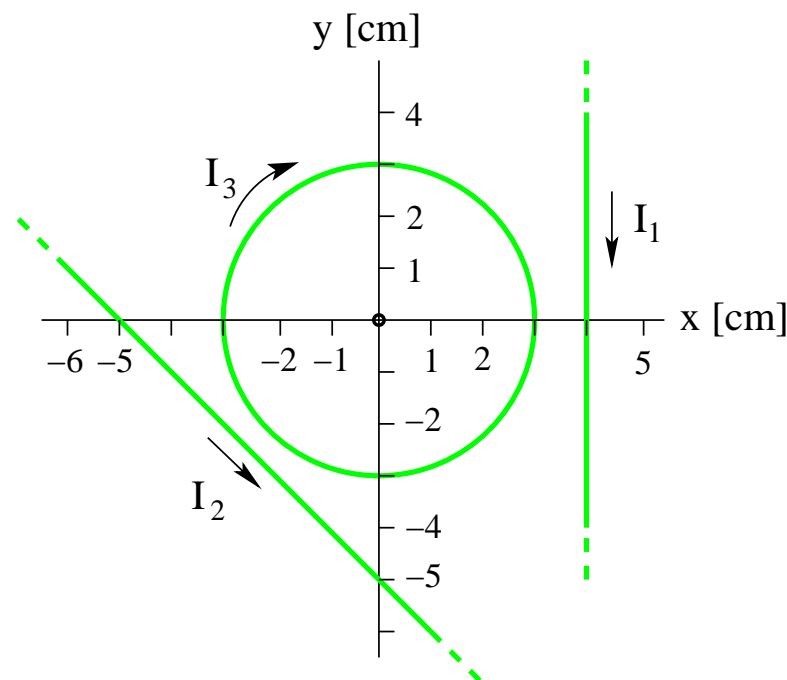
Solution:

$$(a) B_1 = \frac{\mu_0(I_1)}{2\pi(4\text{cm})} = 6.5\mu\text{T} \quad [8.5\mu\text{T}]. \quad (\text{in})$$

$$B_2 = \frac{\mu_0(I_2)}{2\pi(5\text{cm}/\sqrt{2})} = 7.35\mu\text{T} \quad [9.62\mu\text{T}] \quad (\text{out})$$

$$B_3 = \frac{\mu_0(I_3)}{2(3\text{cm})} = 27.2\mu\text{T} \quad [35.6\mu\text{T}] \quad (\text{in})$$

$$(b) \mu = \pi(3\text{cm})^2(I_3) = 3.68 \times 10^{-3}\text{Am}^2 \quad [4.81 \times 10^{-3}\text{Am}^2] \quad (\text{in})$$

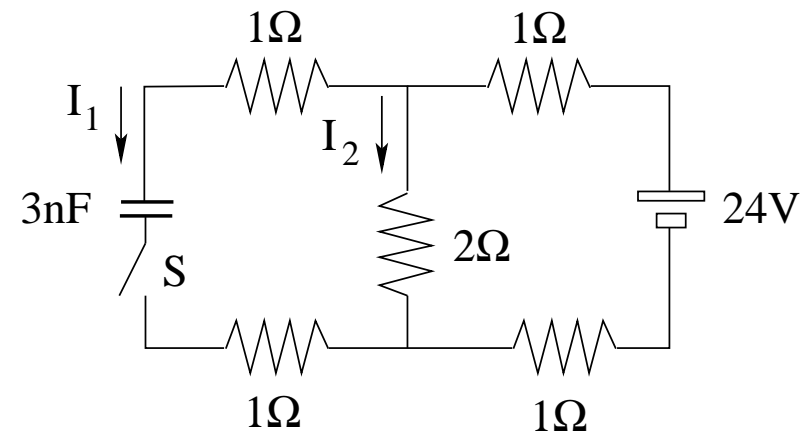


Unit Exam III: Problem #1 (Spring '19)



This circuit is in a steady state with the switch open and the capacitor discharged.

- (a) Find the currents I_1 and I_2 while the switch is still open.
- (b) Find the currents I_1 and I_2 right after the switch has been closed.
- (c) Find the currents I_1 and I_2 a long time later.
- (d) Find the voltage V across the capacitor, also a long time later.

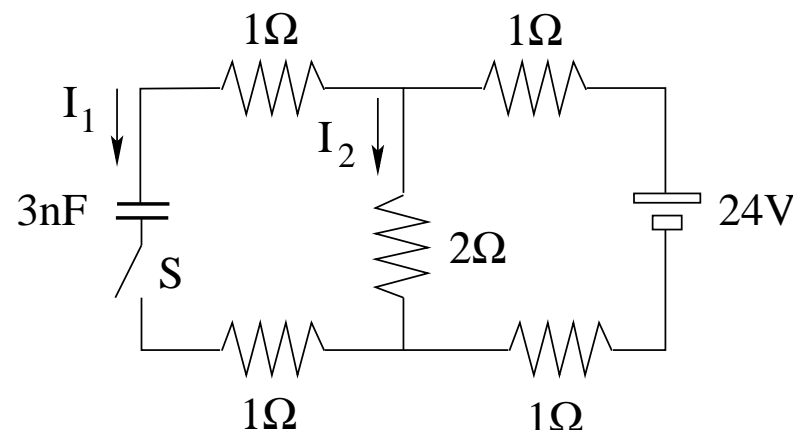


Unit Exam III: Problem #1 (Spring '19)



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- Find the voltage V across the capacitor, also a long time later.



Solution:

$$(a) \quad I_1 = 0, \quad I_2 = \frac{24V}{1\Omega + 2\Omega + 1\Omega} = 6A.$$

$$(b) \quad R_{eq} = 1\Omega + \left(\frac{1}{2\Omega} + \frac{1}{1\Omega + 1\Omega} \right)^{-1} + 1\Omega = 3\Omega \quad (\text{capacitor discharged})$$

$$\Rightarrow I_1 + I_2 = \frac{24V}{3\Omega} = 8A, \quad I_1 = I_2 = 4A.$$

$$(c) \quad \text{capacitor fully charged: } I_1 = 0, \quad I_2 = \frac{24V}{1\Omega + 2\Omega + 1\Omega} = 6A.$$

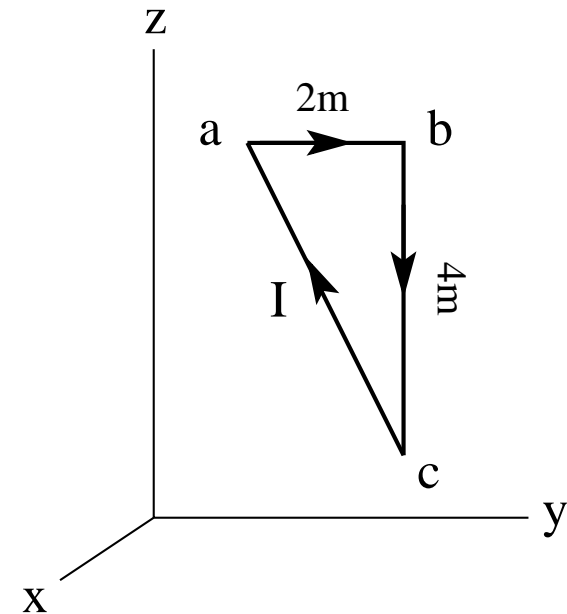
$$(d) \quad \text{loop rule: } (2\Omega)(6A) - (1\Omega)(0A) - V - (1\Omega)(0A) = 0 \Rightarrow V = 12V.$$

Unit Exam III: Problem #2 (Spring '19)



Consider a region with uniform magnetic field $\vec{B} = 3T\hat{j} + 5T\hat{k}$. A conducting loop positioned in the yz -plane has the shape of a right-angled triangle and carries a clockwise current $I = 2A$.

- Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
- Find the force \vec{F}_{ab} (magnitude and direction) acting on side ab .
- Find the force \vec{F}_{bc} (magnitude and direction) acting on side bc .
- Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



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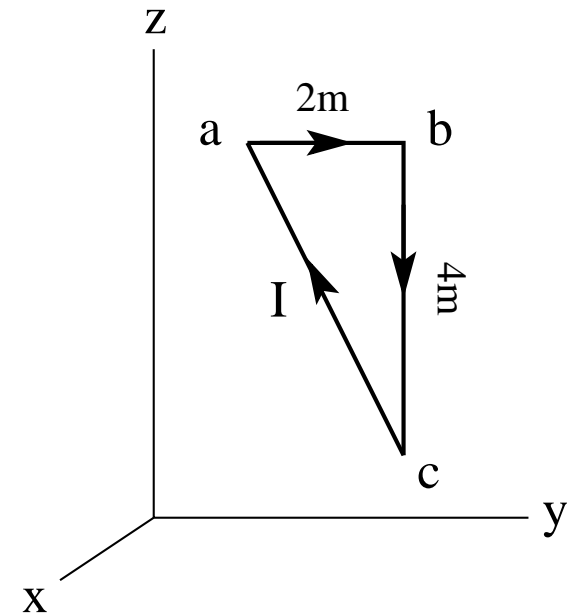
Solution:

$$(a) \vec{\mu} = -(2A)(4m^2)\hat{i} = -8Am^2\hat{i}.$$

$$(b) \vec{F}_{ab} = (2A)(2m\hat{j}) \times [3T\hat{j} + 5T\hat{k}] = 20N\hat{i}.$$

$$(c) \vec{F}_{bc} = (2A)(-4m\hat{k}) \times [3T\hat{j} + 5T\hat{k}] = 24N\hat{i}.$$

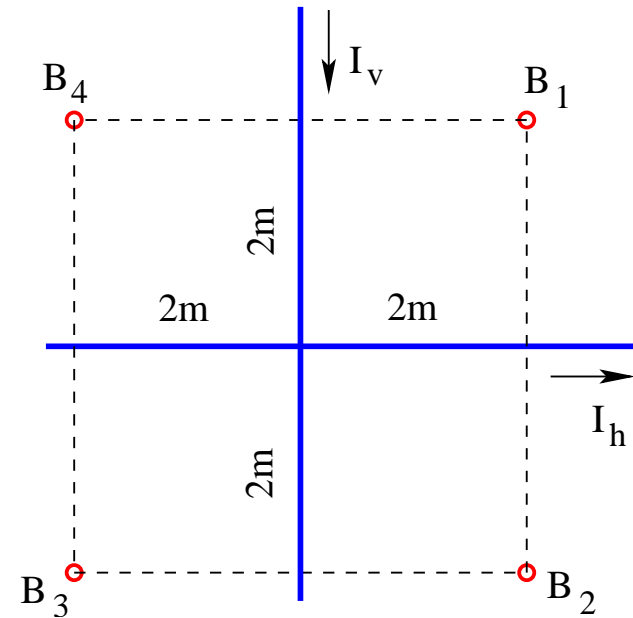
$$(d) \vec{\tau} = (-8Am^2\hat{i}) \times [3T\hat{j} + 5T\hat{k}] = -24Nm\hat{k} + 40Nm\hat{j}$$



Unit Exam III: Problem #3 (Spring '19)



Consider two infinitely long, straight wires with currents $I_v = 3\text{A}$, $I_h = 3\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields \mathbf{B}_1 , \mathbf{B}_2 , \mathbf{B}_3 , \mathbf{B}_4 , at the points marked in the graph.



Unit Exam III: Problem #3 (Spring '19)



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Solution:

- $B_1 = \frac{\mu_0}{2\pi} \left(\frac{I_v}{2\text{m}} + \frac{I_h}{2\text{m}} \right) = +6 \times 10^{-7} \text{T (out)}$.
- $B_2 = \frac{\mu_0}{2\pi} \left(\frac{I_v}{2\text{m}} - \frac{I_h}{2\text{m}} \right) = 0$.
- $B_3 = \frac{\mu_0}{2\pi} \left(-\frac{I_v}{2\text{m}} - \frac{I_h}{2\text{m}} \right) = -6 \times 10^{-7} \text{T (in)}$.
- $B_4 = \frac{\mu_0}{2\pi} \left(-\frac{I_v}{2\text{m}} + \frac{I_h}{2\text{m}} \right) = 0$.

