11-19-2015

E1. Previous Unit Exam 1

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Abstract
Exam slides 1 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island

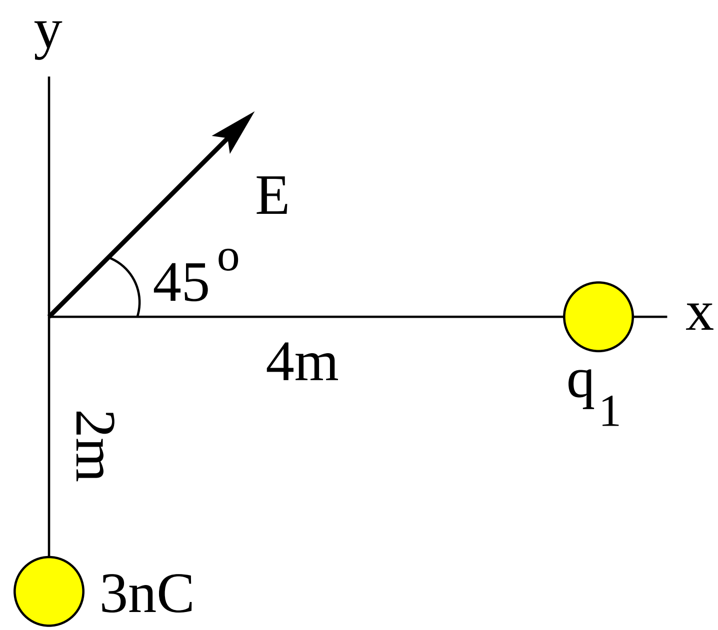
Some of the slides contain figures from the textbook, Paul A. Tipler and Gene Mosca. *Physics for Scientists and Engineers*, 5th/6th editions. The copyright to these figures is owned by W.H. Freeman. We acknowledge permission from W.H. Freeman to use them on this course web page. The textbook figures are not to be used or copied for any purpose outside this class without direct permission from W.H. Freeman.

Intermediate Exam I: Problem #1 (Spring '05)

The electric field $\vec{E}$ generated by the two point charges, 3nC and q_1 (unknown), has the direction shown.

(a) Find the magnitude of $\vec{E}$.

(b) Find the value of q_1.



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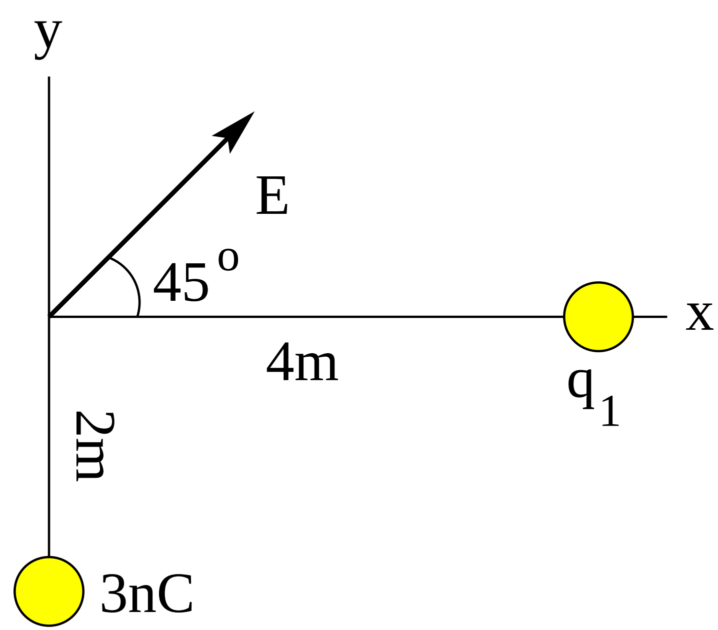


Solution:

(a) $E_y = k\dfrac{3\mathrm{nC}}{(2\mathrm{m})^2} = 6.75\mathrm{N/C},$

$E_x = E_y,$

$E = \sqrt{E_x^2 + E_y^2} = 9.55\mathrm{N/C}.$

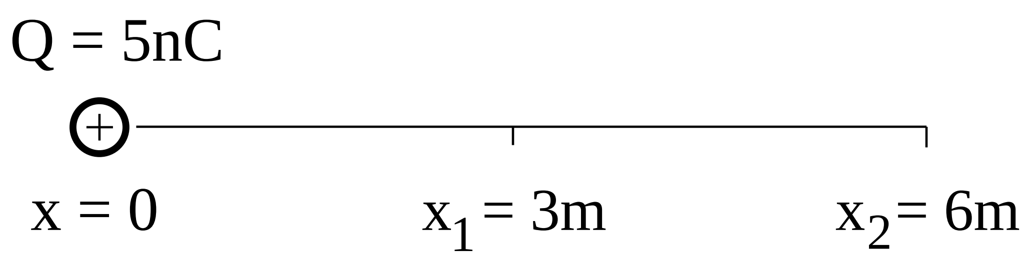
(b) $E_x = k\dfrac{(-q_1)}{(4\mathrm{m})^2},$

$$q_1 = -\frac{(6.75\mathrm{N/C})(16\mathrm{m}^2)}{k} = -12\mathrm{nC}.$$

Consider a point charge $Q = 5$nC fixed at position $x = 0$.

(a) Find the electric potential V_1 at position $x_1 = 3$m
and the electric potiential V_2 at position $x_2 = 6$m.

(b) If a charged particle ($q = 4$nC, $m = 1.5$ng) is released from rest at x_1,
what are its kinetic energy K_2 and its velocity v_2 when it reaches position x_2?



Intermediate Exam I: Problem #2 (Spring ’05)

Consider a point charge $Q = 5\text{nC}$ fixed at position $x = 0$.

(a) Find the electric potential V_1 at position $x_1 = 3\text{m}$
and the electric potiential V_2 at position $x_2 = 6\text{m}$.

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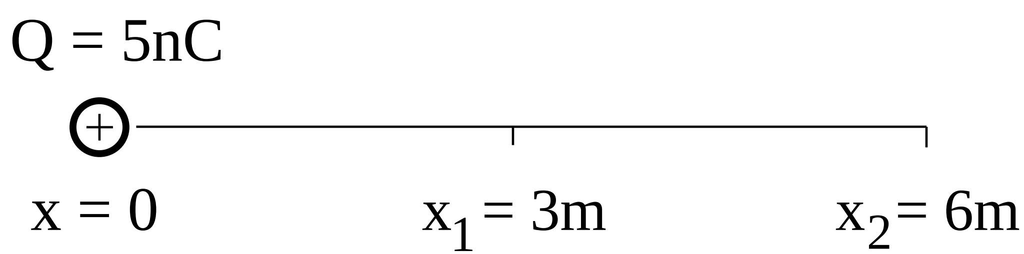


Solution:

(a) $V_1 = k\dfrac{Q}{x_1} = 15\text{V}, \quad V_2 = k\dfrac{Q}{x_2} = 7.5\text{V}.$

(b) $\Delta U = q(V_2 - V_1) = (4\text{nC})(-7.5\text{V}) = -30\text{nJ} \quad \Rightarrow \Delta K = -\Delta U = 30\text{nJ}.$

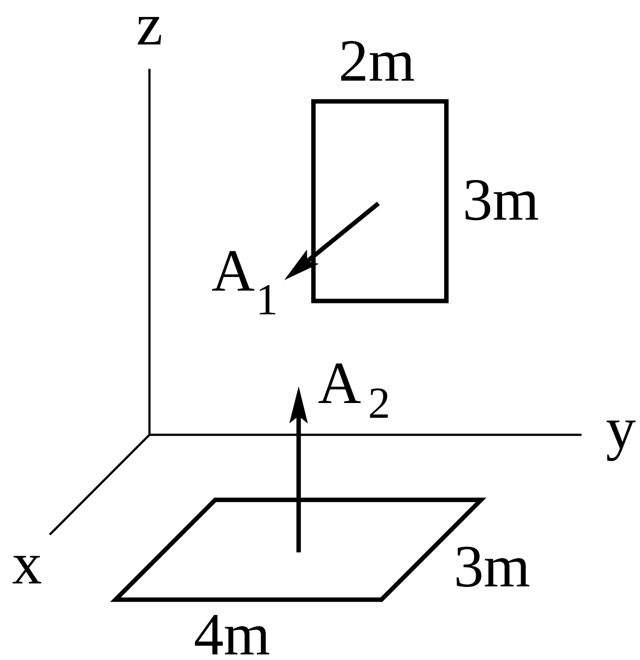
$\Delta K = K_2 = \dfrac{1}{2}mv_2^2 \quad \Rightarrow \; v_2 = \sqrt{\dfrac{2K_2}{m}} = 200\text{m/s}.$

Intermediate Exam I: Problem #3 (Spring '05)

Consider two plane surfaces with area vectors $\vec{A}_1$ (pointing in positive x-direction) and $\vec{A}_2$ (pointing in positive z-direction). The region is filled with a uniform electric field $\vec{E} = (2\hat{i} + 7\hat{j} - 3\hat{k})$N/C.

(a) Find the electric flux $\Phi_E^{(1)}$ through area A_1.

(b) Find the electric flux $\Phi_E^{(2)}$ through area A_2.



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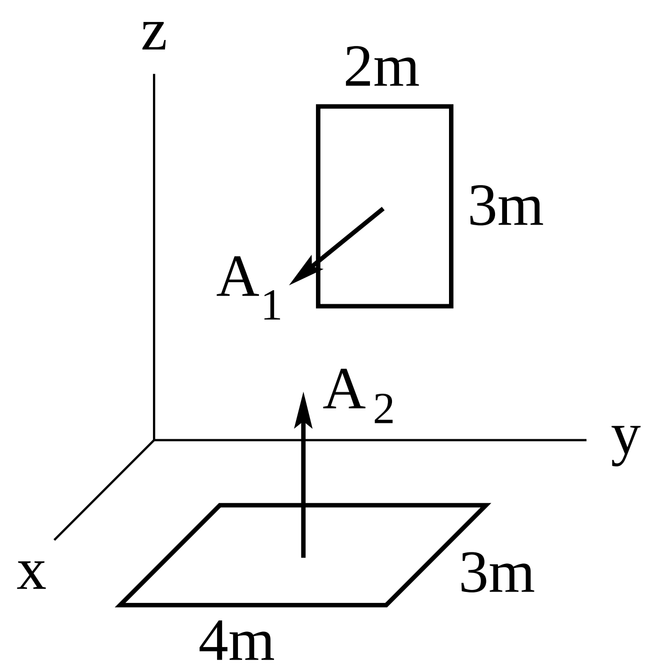


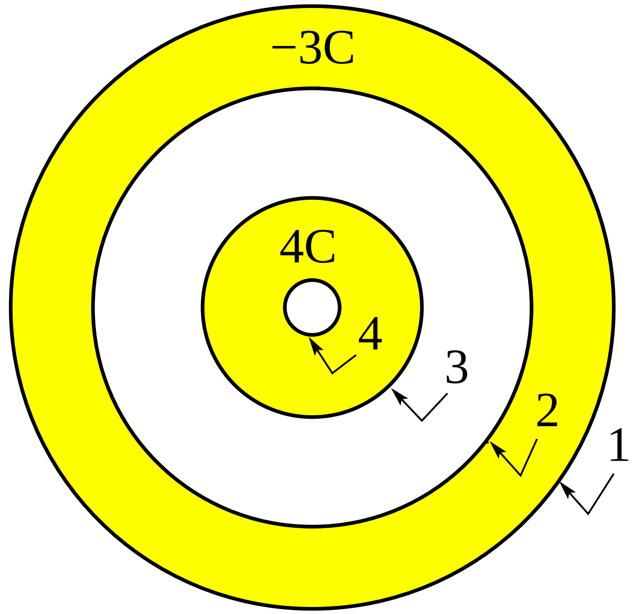
Solution:

(a) $\vec{A}_1 = 6\hat{\imath}\,\mathrm{m}^2$,
$\Phi_E^{(1)} = \vec{E}\cdot\vec{A}_1 = (2\mathrm{N/C})(6\mathrm{m}^2) = 12\mathrm{Nm}^2/\mathrm{C}.$

(b) $\vec{A}_2 = 12\hat{k}\,\mathrm{m}^2$,
$\Phi_E^{(2)} = \vec{E}\cdot\vec{A}_2 = (-3\mathrm{N/C})(12\mathrm{m}^2) = -36\mathrm{Nm}^2/\mathrm{C}.$

Intermediate Exam I: Problem #4 (Spring '05)

Consider two concentric conducting spherical shells. The total electric charge on the inner shell is 4C and the total electric charge on the outer shell is -3C. Find the electric charges q_1, q_2, q_3, q_4 on each surface of both shells as identified in the figure.



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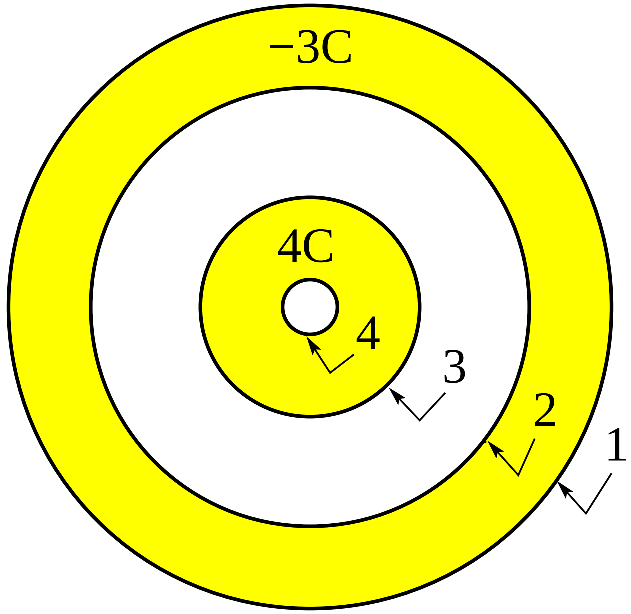


Solution:

Start with the innermost surface.
Note that any excess charge is located at the surface of a conductor.
Note also that the electric field inside a conductor at equilibrium vanishes.

- Gauss’s law predicts $q_4 = 0$.
- Charge conservation then predicts $q_3 + q_4 = 4$C. Hence $q_3 = 4$C.
- Gauss’s law predicts $q_2 = -(q_3 + q_4) = -4$C.
- Charge conservation then predicts $q_1 + q_2 = -3$C. Hence $q_1 = +1$C.

Intermediate Exam I: Problem #1 (Spring '06)

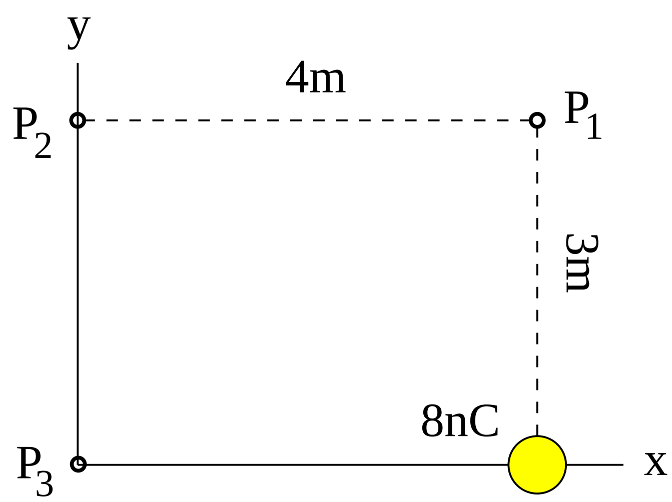
Consider a point charge $q = +8$nC at position $x = 4$m, $y = 0$ as shown.

(a) Find the electric field components E_x and E_y at point P_1.

(b) Find the electric field components E_x and E_y at point P_2.

(c) Find the electric potential V at point P_3.

(d) Find the electric potential V at point P_2.



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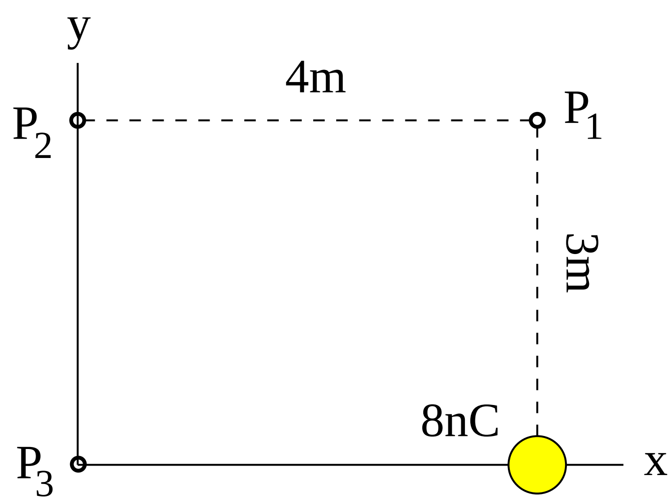


Solution:

(a) $E_x = 0, \qquad E_y = k\dfrac{8\text{nC}}{(3\text{m})^2} = 7.99\text{N/C}.$

(b) $E_x = -k\dfrac{8\text{nC}}{(5\text{m})^2}\cos\theta = -2.88\text{N/C} \times \dfrac{4}{5} = -2.30\text{N/C}.$

$E_y = k\dfrac{8\text{nC}}{(5\text{m})^2}\sin\theta = 2.88\text{N/C} \times \dfrac{3}{5} = 1.73\text{N/C}.$

(c) $V = k\dfrac{8\text{nC}}{4\text{m}} = 17.98\text{V}.$

(d) $V = k\dfrac{8\text{nC}}{5\text{m}} = 14.38\text{V}.$

Intermediate Exam I: Problem #2 (Spring ’06)

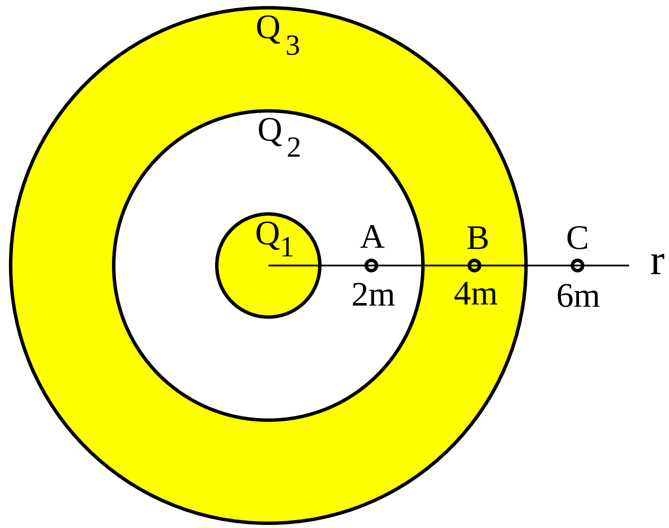
Consider a conducting sphere of radius $r_1 = 1$m and a conducting spherical shell of inner radius $r_2 = 3$m and outer radius $r_3 = 5$m. The charge on the inner sphere is $Q_1 = -0.6\mu$C. The net charge on the shell is zero.

(a) Find the charge Q_2 on the inner surface and the charge Q_3 on the outer surface of the shell.

(b) Find magnitude and direction of the electric field at point A between the sphere and the shell.

(c) Find magnitude and direction of the electric field at point B inside the shell.

(d) Find magnitude and direction of the electric field at point C outside the shell.



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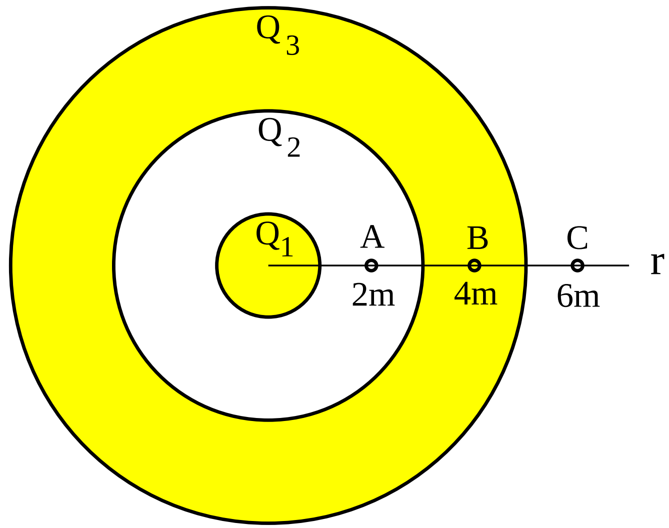
Solution:

(a) Gauss's law implies that $Q_2 = -Q_1 = +0.6\mu\text{C}$.
Given that $Q_2 + Q_3 = 0$ we infer $Q_3 = -0.6\mu\text{C}$.

(b) $E_A = k\dfrac{0.6\mu\text{C}}{(2\text{m})^2} = 1349\text{N/C}$ (inward).

(c) $E_B = 0$ inside conductor.

(d) $E_C = k\dfrac{0.6\mu\text{C}}{(6\text{m})^2} = 150\text{N/C}$ (inward).



Intermediate Exam I: Problem #3 (Spring '06)

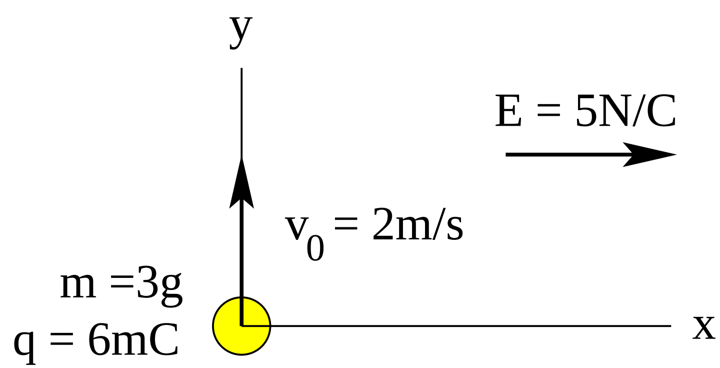
Consider a region of uniform electric field as shown. A charged particle is projected at time $t = 0$ with initial velocity as shown. Ignore gravity.

(a) Find the components a_x and a_y of the acceleration at time $t = 0$.

(b) Find the components v_x and v_y of the velocity at time $t = 0$.

(c) Find the components v_x and v_y of the velocity at time $t = 1.2$s.

(d) Find the components x and y of the position at time $t = 1.2$s.



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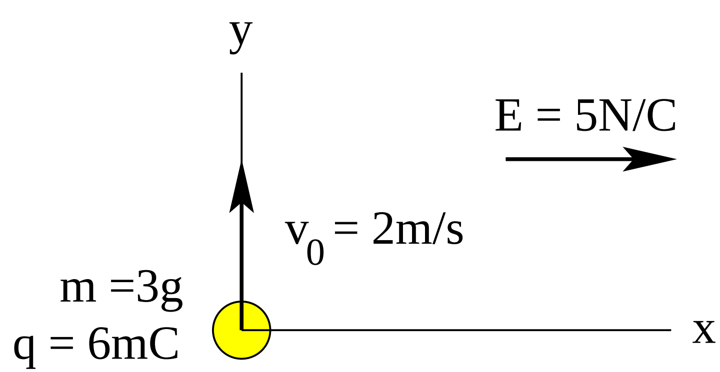


Solution:

(a) $a_x = \frac{q}{m}E = \frac{6 \times 10^{-3}\text{C}}{3 \times 10^{-3}\text{kg}}(5\text{N/C}) = 10\text{m/s}^2, \quad a_y = 0.$

(b) $v_x = 0, \quad v_y = v_0 = 2\text{m/s}.$

(c) $v_x = a_x t = (10\text{m/s}^2)(1.2\text{s}) = 12\text{m/s}, \quad v_y = v_0 = 2\text{m/s}.$

(d) $x = \frac{1}{2}a_x t^2 = 0.5(10\text{m/s}^2)(1.2\text{s})^2 = 7.2\text{m}, \quad y = v_y t = (2\text{m/s})(1.2\text{s}) = 2.4\text{m}.$

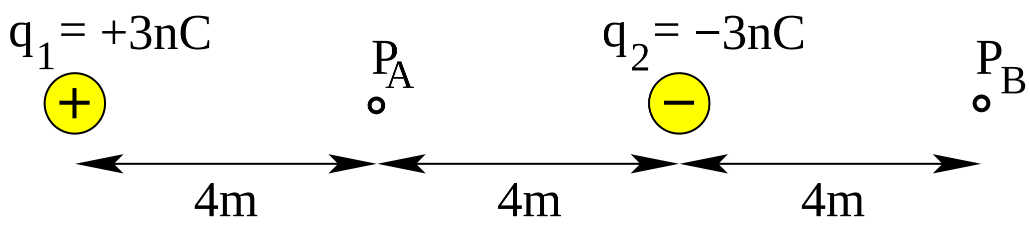
Unit Exam I: Problem #1 (Spring '07)

Consider the configuration of two point charges as shown.

(a) Find magnitude and direction of the force $\mathbf{F}_{21}$ exerted by q_2 on q_1.

(b) Find magnitude and direction of the electric field $\mathbf{E}_A$ at point P_A.

(c) Find the electric potential V_B at point P_B.



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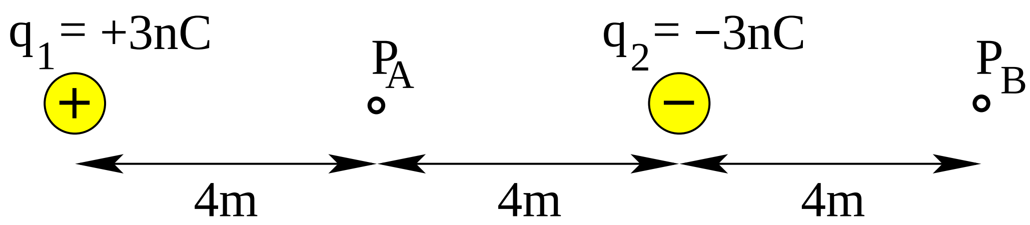


Solution:

(a) $F_{12} = k\frac{|3\text{nC}|^2}{(8\text{m})^2} = 1.27\text{nN}$ (directed right).

(b) $E_A = 2k\frac{|3\text{nC}|}{(4\text{m})^2} = 3.38\text{N/C}$ (directed right).

(c) $V_B = k\frac{(+3\text{nC})}{12\text{m}} + k\frac{(-3\text{nC})}{4\text{m}} = -4.50\text{V}.$

Unit Exam I: Problem #2 (Spring '07)

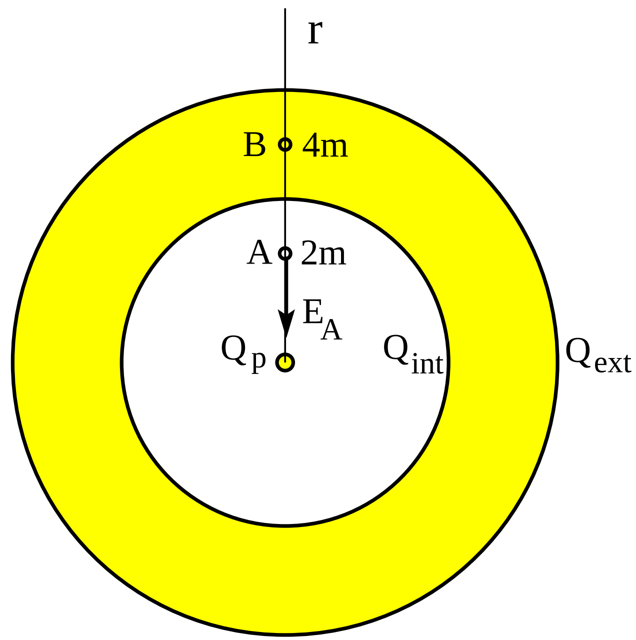
A point charge Q_p is positioned at the center of a conducting spherical shell of inner radius $r_2 = 3.00$m and outer radius $r_3 = 5.00$m. The total charge on the shell $Q_s = +7.00$nC. The electric field at point A has strength $E_A = 6.75$N/C and is pointing radially inward.

(a) Find the value of Q_p (point charge).

(b) Find the charge Q_{int} on the inner surface of the shell.

(c) Find the charge Q_{ext} on the outer surface of the shell.

(d) Find the electric field at point B.



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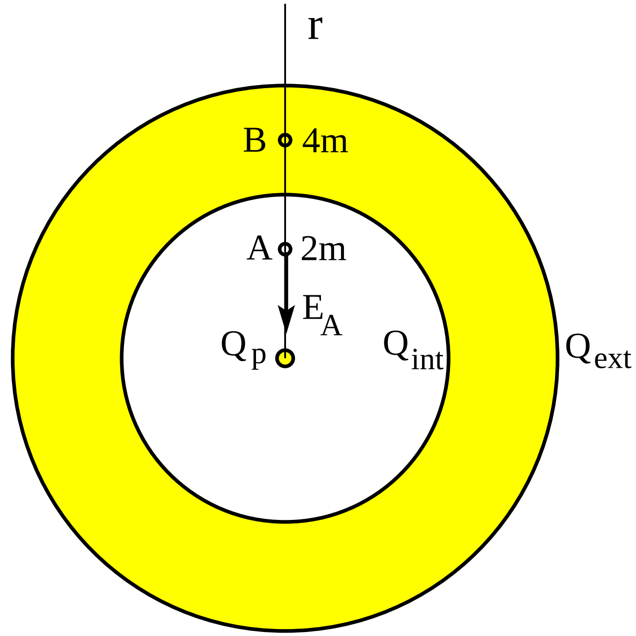


Solution:

(a) Gauss's law implies that $-E_A(4\pi r_A^2) = \dfrac{Q_p}{\epsilon_0}$

$\Rightarrow\; Q_p = -3.00$nC.

(b) Gauss's law implies that $Q_{int} = -Q_p = +3.00$nC.

(c) Charge conservation, $Q_{int} + Q_{ext} = Q_s = 7.00$nC, then implies that $Q_{ext} = +4.00$nC.

(d) $E_B = 0$ inside conductor.

Unit Exam I: Problem #3 (Spring '07)

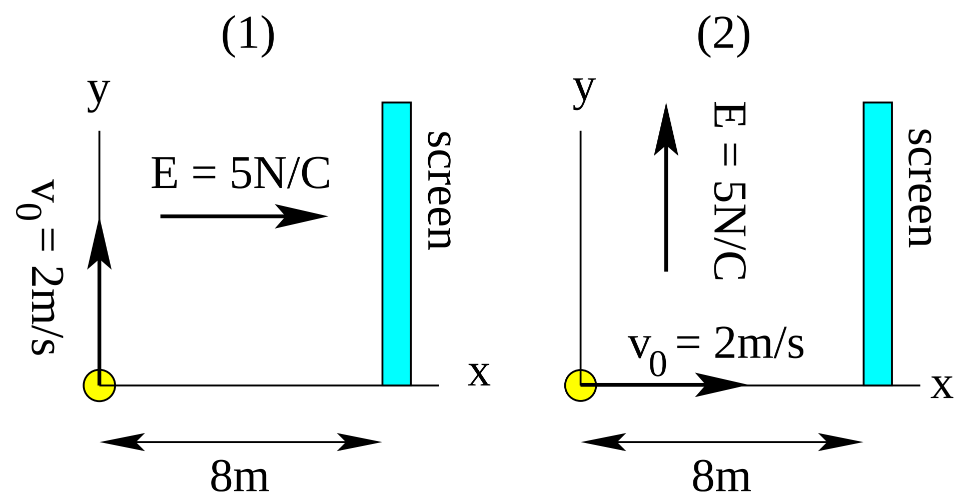
Consider two regions of uniform electric field as shown. Charged particles of mass $m = 2$kg and charge $q = 1$C are projected at time $t = 0$ with initial velocities as shown. Both particles will hit the screen eventually. Ignore gravity.

(a) At what time t_1 does the particle in region (1) hit the screen?

(b) At what height y_1 does the particle in region (1) hit the screen?

(c) At what time t_2 does the particle in region (2) hit the screen?

(d) At what height y_2 does the particle in region (2) hit the screen?



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(c) At what time t_2 does the particle in region (2) hit the screen?

(d) At what height y_2 does the particle in region (2) hit the screen?

Solution:

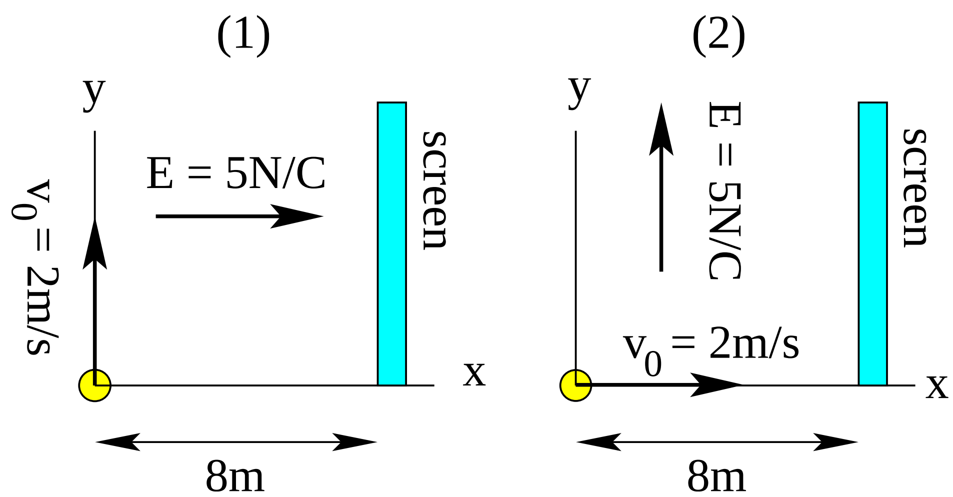
(a) $x_1 = \frac{1}{2}at_1^2 \quad \text{with} \quad a = \frac{q}{m}E = 2.5\text{m/s}^2,$

$x_1 = 8\text{m} \quad \Rightarrow \; t_1 = 2.53\text{s}.$

(b) $y_1 = v_0 t_1 = 5.06\text{m}.$

(c) $x_2 = v_0 t_2 \quad \Rightarrow \; t_2 = \frac{8\text{m}}{2\text{m/s}} = 4\text{s}.$

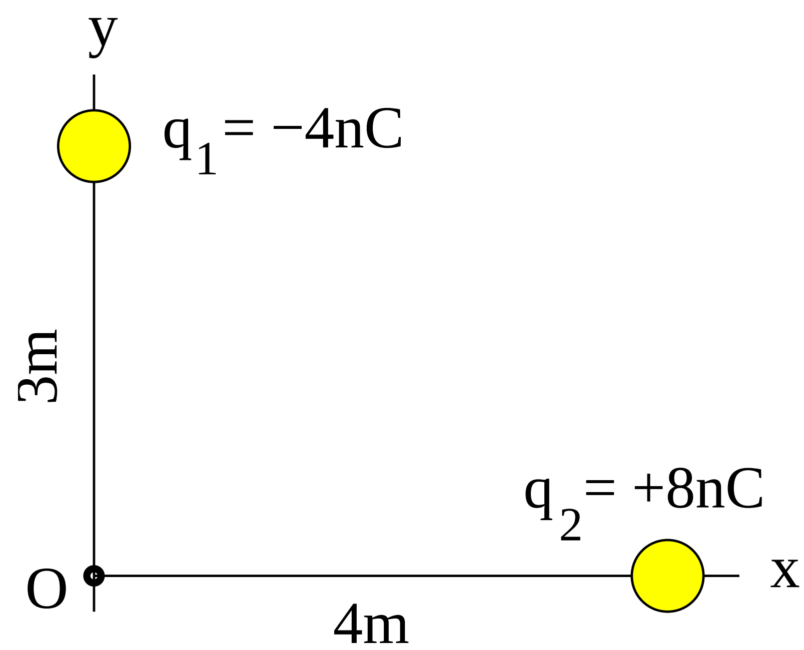
(d) $y_2 = \frac{1}{2}at_2^2 = 20\text{m}.$



Unit Exam I: Problem #1 (Spring ’08)

Consider two point charges positioned in the xy-plane as shown.

(a) Find the magnitude F of the force between the two charges.
(b) Find the components E_x and E_y of the electric field at point O.
(c) Find the electric potential V at point O.
(d) Find the potential energy U of charge q_2 in the presence of charge q_1.



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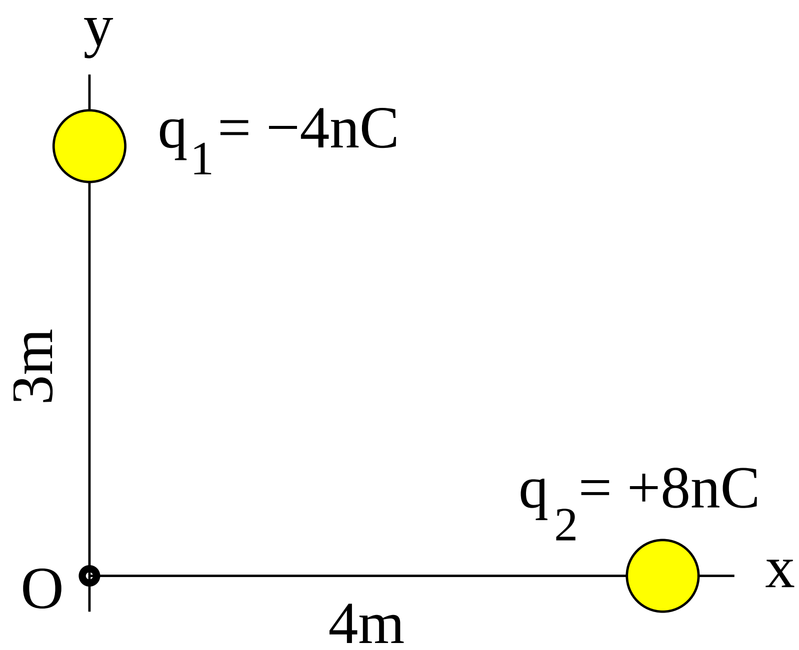


Solution:

(a) $F = k\dfrac{|q_1 q_2|}{(5\text{m})^2} = 1.15 \times 10^{-8}\text{N}.$

(b) $E_x = -k\dfrac{|q_2|}{(4\text{m})^2} = -4.5\,\text{N/C},$

$E_y = +k\dfrac{|q_1|}{(3\text{m})^2} = +4.0\,\text{N/C}.$

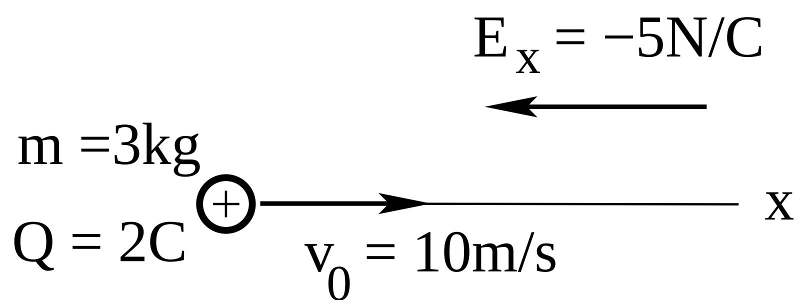
(c) $V = k\dfrac{q_2}{4\text{m}} + k\dfrac{q_1}{3\text{m}} = 18\text{V} - 12\text{V} = 6\text{V}.$

(d) $U = k\dfrac{q_1 q_2}{5\text{m}} = -57.6\text{nJ}.$

Unit Exam I: Problem #2 (Spring '08)

Consider a region of uniform electric field $E_x = -5$N/C. A charged particle (charge $Q = 2$C, mass $m = 3$kg) is launched from initial position $x = 0$ with velocity $v_0 = 10$m/s in the positive x-direction.

(a) Find the (negative) acceleration a_x experienced by the particle.
(b) Find the time t_s it takes the particle to come to a stop.
(c) Find the position x_s of the particle at time t_s.
(d) Find the work W done by the electric field to bring the particle to a stop.



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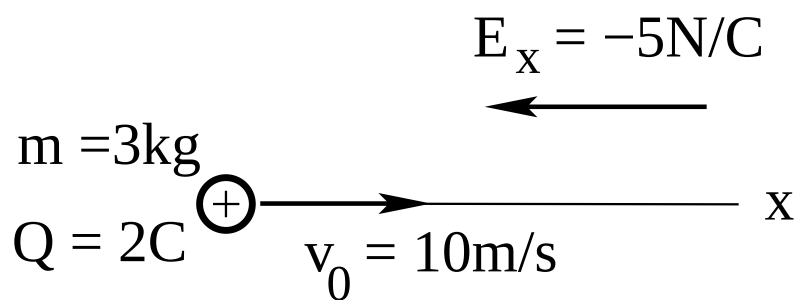


(a) $a_x = \frac{2\text{C}}{3\text{kg}}(-5\text{N/C}) = -3.33\text{m/s}^2.$

(b) $t_s = \frac{v_0}{|a_x|} = 3.00\text{s}.$

(c) $x_s = \frac{v_0^2}{2|a_x|} = 15.0\text{m}.$

(d) $W = \Delta K = -\frac{1}{2}mv_0^2 = -150\text{J}.$

Unit Exam I: Problem #3 (Spring '08)

Consider a conducting spherical shell of inner radius $r_{int} = 3$m and outer radius $r_{ext} = 5$m. The net charge on the shell is $Q_{shell} = 7\mu$C.

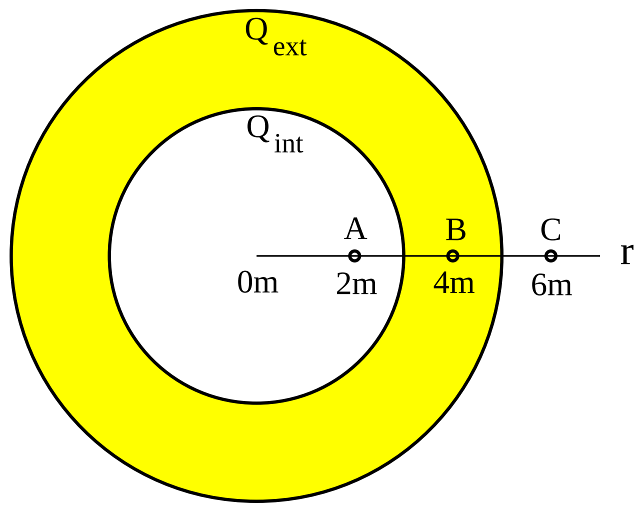
(a) Find the charge Q_{int} on the inner surface and the charge Q_{ext} on the outer surface of the shell.

(b) Find the direction (left/right/none) of the electric field at points A, B, C.

Now place a point charge $Q_{point} = -3\mu$C into the center of the shell ($r = 0$m).

(c) Find the charge Q_{int} on the inner surface and the charge Q_{ext} on the outer surface of the shell.

(d) Find the direction (left/right/none) of the electric field at points A, B, C.



Consider a conducting spherical shell of inner radius $r_{int} = 3$m and outer radius $r_{ext} = 5$m. The net charge on the shell is $Q_{shell} = 7\mu$C.

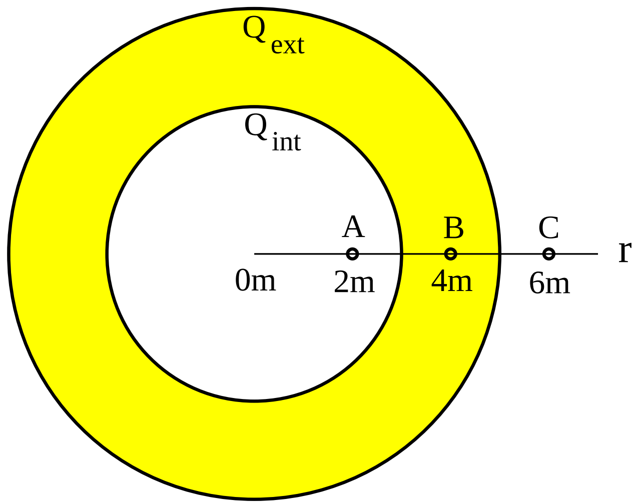
(a) Find the charge Q_{int} on the inner surface and the charge Q_{ext} on the outer surface of the shell.

(b) Find the direction (left/right/none) of the electric field at points A, B, C.

Now place a point charge $Q_{point} = -3\mu$C into the center of the shell ($r = 0$m).

(c) Find the charge Q_{int} on the inner surface and the charge Q_{ext} on the outer surface of the shell.

(d) Find the direction (left/right/none) of the electric field at points A, B, C.



Solution:

(a) $Q_{int} = 0, \quad Q_{ext} = 7\mu\text{C}.$

(b) A: none, B: none, C: right.

(c) $Q_{int} = 3\mu\text{C}, \quad Q_{ext} = 4\mu\text{C}.$

(d) A: left, B: none, C: right.

Unit Exam I: Problem #1 (Spring '09)

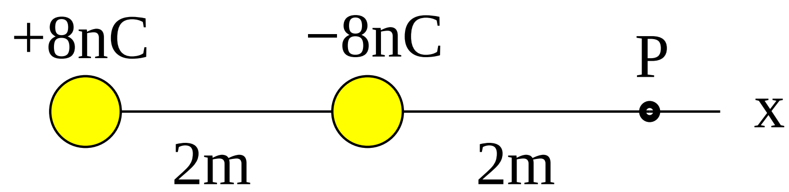
Consider two point charges positioned on the x-axis as shown.

(a) Find magnitude and direction of the electric field at point P.

(b) Find the electric potential at point P.

(c) Find the electric potential energy of an electron (mass $m = 9.1 \times 10^{-31}$kg, charge $q = -1.6 \times 10^{-19}$C) when placed at point P.

(d) Find magnitude and direction of the acceleration the electron experiences when released at point P.



Unit Exam I: Problem #1 (Spring '09)

Consider two point charges positioned on the x-axis as shown.

(a) Find magnitude and direction of the electric field at point P.

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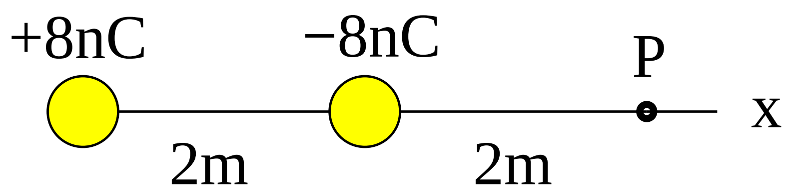


Solution:

(a) $E_x = +k\frac{8\text{nC}}{(4\text{m})^2} + k\frac{(-8\text{nC})}{(2\text{m})^2} = 4.5\text{N/C} - 18\text{N/C} = -13.5\text{N/C}$ (directed left).

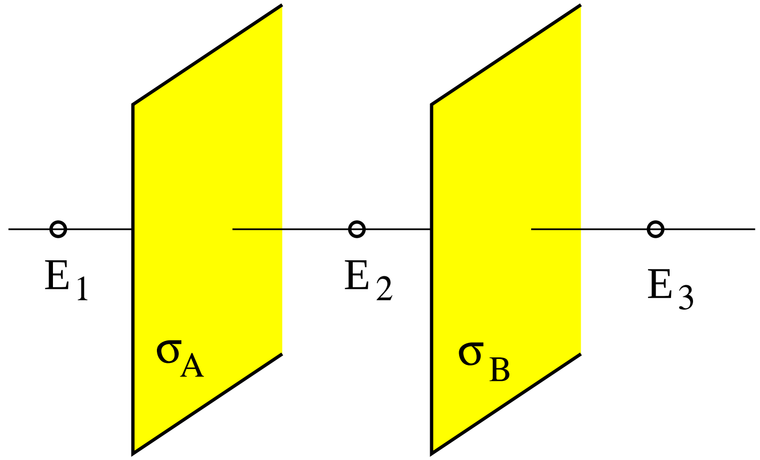
(b) $V = +k\frac{8\text{nC}}{4\text{m}} + k\frac{(-8\text{nC})}{2\text{m}} = 18\text{V} - 36\text{V} = -18\text{V}.$

(c) $U = qV = (-18\text{V})(-1.6 \times 10^{-19}\text{C}) = 2.9 \times 10^{-18}\text{J}.$

(d) $a_x = \frac{qE_x}{m} = \frac{(-1.6 \times 10^{-19}\text{C})(-13.5\text{N/C})}{9.1 \times 10^{-31}\text{kg}} = 2.4 \times 10^{12}\text{ms}^{-2}$ (directed right).

Unit Exam I: Problem #2 (Spring '09)

Consider two very large uniformly charged parallel sheets as shown. The charge densities are $\sigma_A = +7 \times 10^{-12}\mathrm{Cm}^{-2}$ and $\sigma_B = -4 \times 10^{-12}\mathrm{Cm}^{-2}$, respectively. Find magnitude and direction (left/right) of the electric fields $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$.



Unit Exam I: Problem #2 (Spring ’09)

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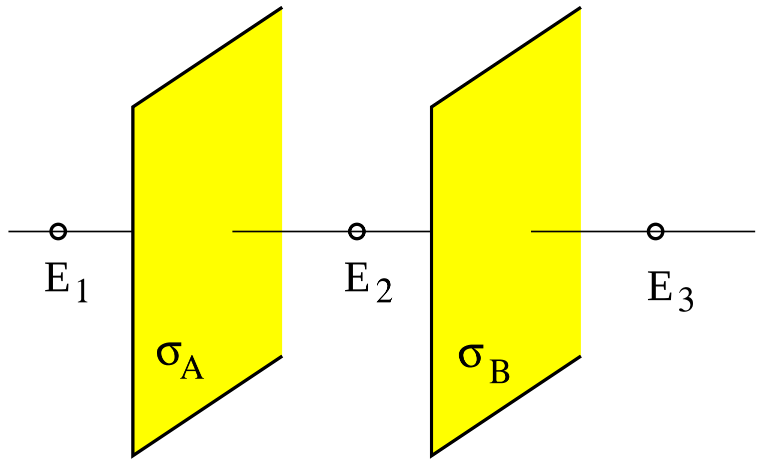


Solution:

$$E_A = \frac{|\sigma_A|}{2\epsilon_0} = 0.40\mathrm{N/C} \quad \text{(directed away from sheet A).}$$

$$E_B = \frac{|\sigma_B|}{2\epsilon_0} = 0.23\mathrm{N/C} \quad \text{(directed toward sheet B).}$$

$$E_1 = E_A - E_B = 0.17\mathrm{N/C} \quad \text{(directed left).}$$

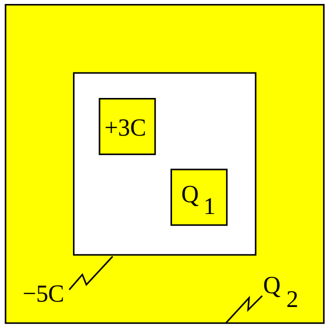
$$E_2 = E_A + E_B = 0.63\mathrm{N/C} \quad \text{(directed right).}$$

$$E_2 = E_A - E_B = 0.17\mathrm{N/C} \quad \text{(directed right).}$$

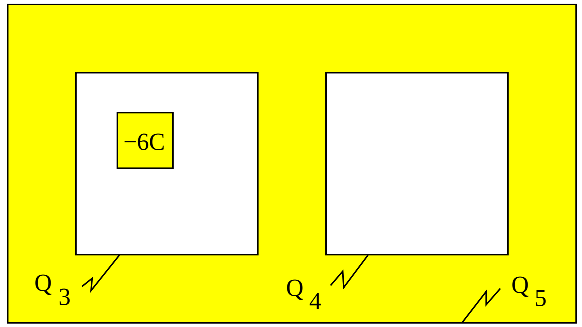
Unit Exam I: Problem #3 (Spring '09)

(a) Consider a conducting box with no net charge on it. Inside the box are two small charged conducting cubes. For the given charges on the surface of one cube and on the inside surface of the box find the charges Q_1 on the surface of the other cube and Q_2 on the outside surface of the box.

(b) Consider a conducting box with two compartments and no net charge on it. Inside one compartment is a small charged conducting cube. For the given charge on the surface of the cube find the charges Q_3, Q_4, and Q_5 on the three surfaces of the box.



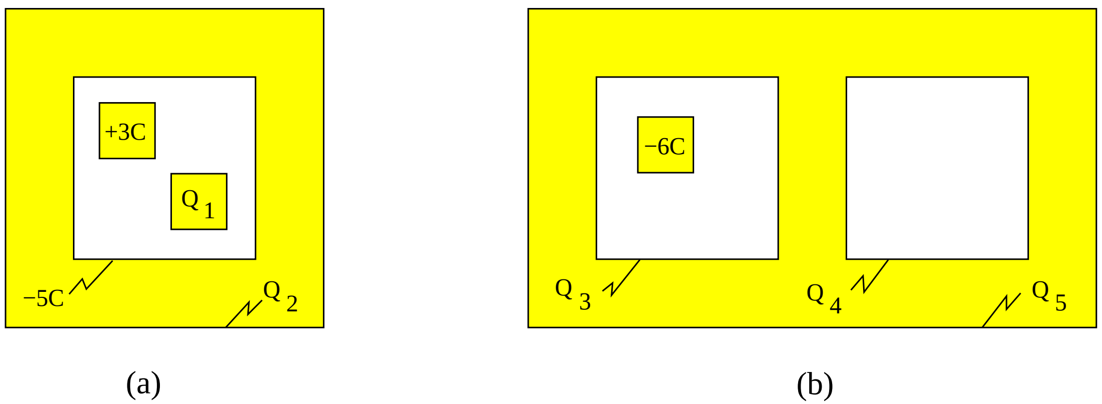
(a)



(b)

(a) Consider a conducting box with no net charge on it. Inside the box are two small charged conducting cubes. For the given charges on the surface of one cube and on the inside surface of the box find the charges Q_1 on the surface of the other cube and Q_2 on the outside surface of the box.

(b) Consider a conducting box with two compartments and no net charge on it. Inside one compartment is a small charged conducting cube. For the given charge on the surface of the cube find the charges Q_3, Q_4, and Q_5 on the three surfaces of the box.



Solution:

(a) Gauss’s law implies $Q_1 + 3\mathrm{C} + (-5\mathrm{C}) = 0 \quad \Rightarrow \quad Q_1 = +2\mathrm{C}$.
Net charge on the box: $Q_2 + (-5\mathrm{C}) = 0 \quad \Rightarrow \quad Q_2 = +5\mathrm{C}$.

(b) Gauss’s law implies $Q_3 + (-6\mathrm{C}) = 0 \quad \Rightarrow \quad Q_3 = +6\mathrm{C}$.
Gauss’s law implies $Q_4 = 0$.
Net charge on box: $Q_3 + Q_4 + Q_5 = 0 \quad \Rightarrow \quad Q_5 = -6\mathrm{C}$.

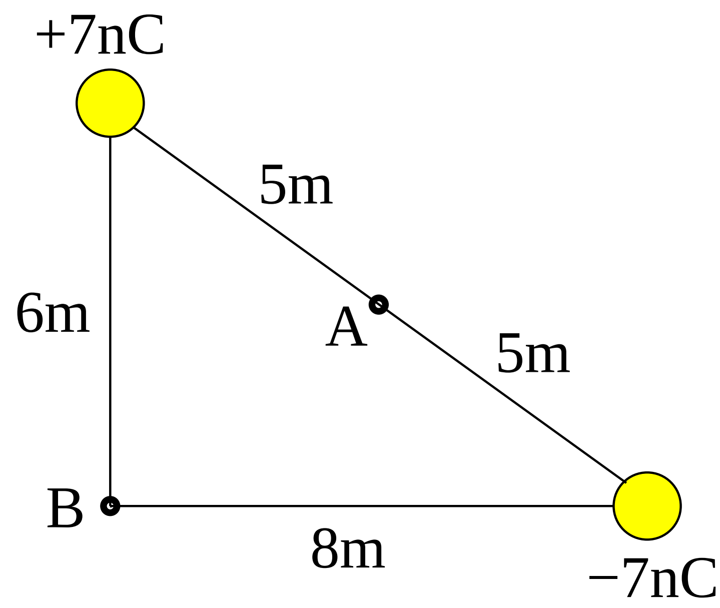
Consider two point charges positioned as shown.

(a) Find the magnitude of the electric field at point A.

(b) Find the electric potential at point A.

(c) Find the magnitude of the electric field at point B.

(d) Find the electric potential at point B.



Unit Exam I: Problem #1 (Fall '10)

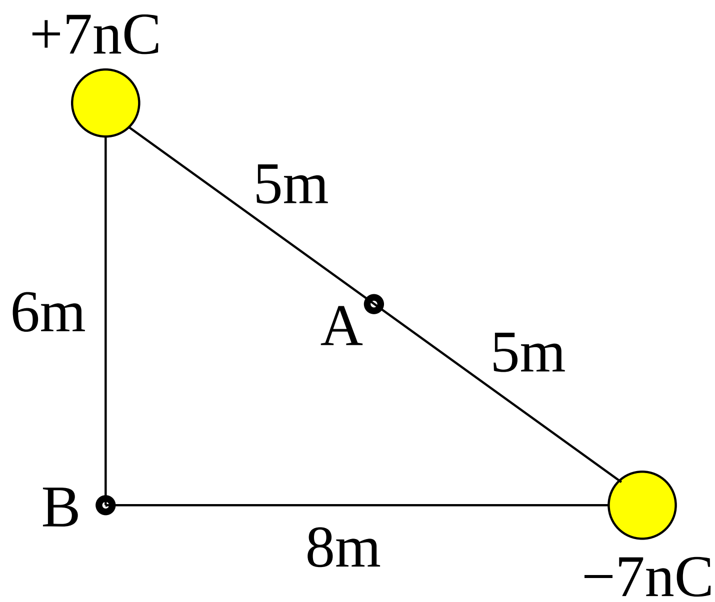
Consider two point charges positioned as shown.

(a) Find the magnitude of the electric field at point A.

(b) Find the electric potential at point A.

(c) Find the magnitude of the electric field at point B.

(d) Find the electric potential at point B.



Solution:

(a) $E_A = 2k\dfrac{|7\text{nC}|}{(5\text{m})^2} = 2(2.52\text{V/m}) = 5.04\text{V/m}.$

(b) $V_A = k\dfrac{(+7\text{nC})}{5\text{m}} + k\dfrac{(-7\text{nC})}{5\text{m}} = 12.6\text{V} - 12.6\text{V} = 0.$

(c) $E_B = \sqrt{\left(k\dfrac{|7\text{nC}|}{(6\text{m})^2}\right)^2 + \left(k\dfrac{|7\text{nC}|}{(8\text{m})^2}\right)^2} \Rightarrow E_B = \sqrt{(1.75\text{V/m})^2 + (0.98\text{V/m})^2} = 2.01\text{V/m}.$

(d) $V_B = k\dfrac{(+7\text{nC})}{6\text{m}} + k\dfrac{(-7\text{nC})}{8\text{m}} = 10.5\text{V} - 7.9\text{V} = 2.6\text{V}.$

Unit Exam I: Problem #2 (Fall ’10)

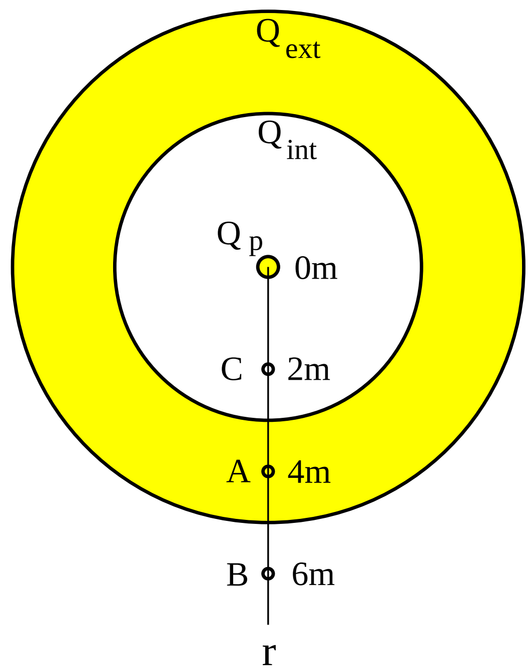
A point charge Q_p is positioned at the center of a conducting spherical shell of inner radius $r_{int} = 3$m and outer radius $r_{ext} = 5$m. The charge on the inner surface of the shell is $Q_{int} = -4\mathrm{nC}$ and the charge on the outer surface is $Q_{ext} = +3\mathrm{nC}$.

(a) Find the value of the point charge Q_p.

(b) Find direction (up/down/none) and magnitude of the electric field at point A.

(c) Find direction (up/down/none) and magnitude of the electric field at point B.

(d) Find direction (up/down/none) and magnitude of the electric field at point C. [not on exam]



Unit Exam I: Problem #2 (Fall '10)

A point charge Q_p is positioned at the center of a conducting spherical shell of inner radius $r_{int} = 3\text{m}$ and outer radius $r_{ext} = 5\text{m}$. The charge on the inner surface of the shell is $Q_{int} = -4\text{nC}$ and the charge on the outer surface is $Q_{ext} = +3\text{nC}$.

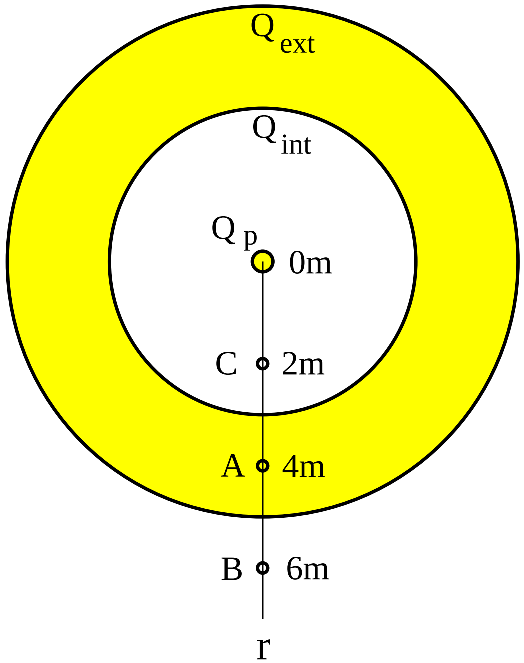
(a) Find the value of the point charge Q_p.

(b) Find direction (up/down/none) and magnitude of the electric field at point A.

(c) Find direction (up/down/none) and magnitude of the electric field at point B.

(d) Find direction (up/down/none) and magnitude of the electric field at point C. [not on exam]

Solution:



(a) $Q_p = -Q_{int} = +4\text{nC}.$

(b) $E_A = 0$ inside conductor (no direction).

(c) $E_B[4\pi(6\text{m})^2] = \dfrac{Q_p + Q_{int} + Q_{ext}}{\epsilon_0}$

$$\Rightarrow E_B = k\frac{3\text{nC}}{(6\text{m})^2} = 0.75\text{N/C} \quad \text{(down)}.$$

(d) $E_C[4\pi(2\text{m})^2] = \dfrac{Q_p}{\epsilon_0} \quad \Rightarrow E_C = k\dfrac{4\text{nC}}{(2\text{m})^2} = 9\text{N/C} \quad \text{(down)}.$

Unit Exam I: Problem #3 (Fall '10)

An electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) and a proton ($m = 1.67 \times 10^{-27}$kg, $q = +1.60 \times 10^{-19}$C) are released from rest midway between oppositely charged parallel plates. The plates are at the electric potentials shown.

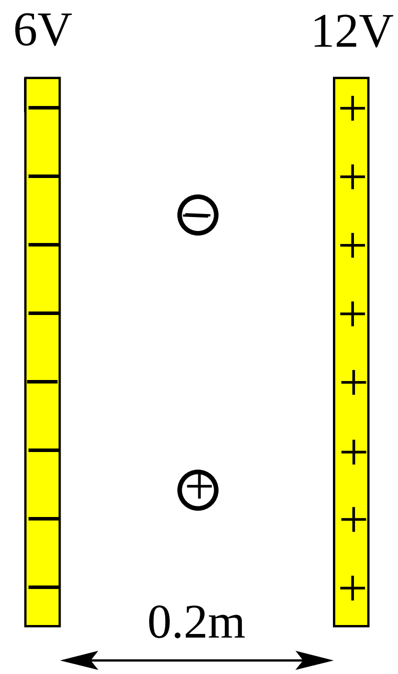
(a) Find the magnitude of the electric field between the plates.

(b) What direction (left/right) does the electric field have?

(c) Which particle (electron/proton/both) is accelerated to the left?

(d) Why does the electron reach the plate before the proton?

(e) Find the kinetic energy of the proton when it reaches the plate.



Unit Exam I: Problem #3 (Fall ’10)

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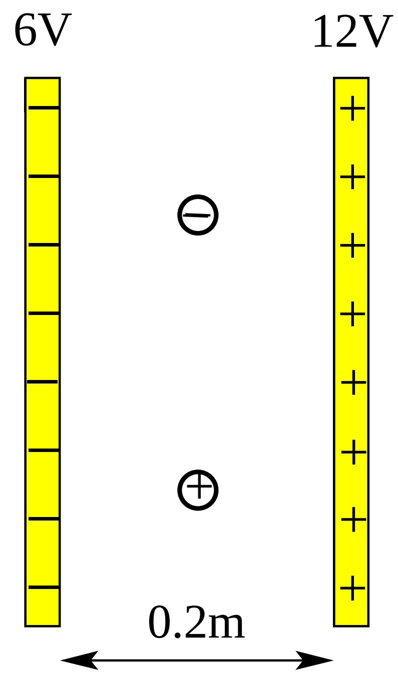
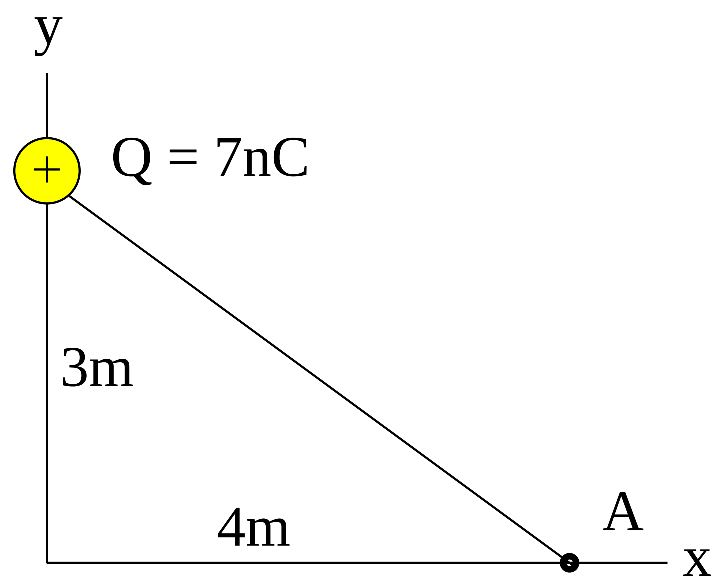


Solution:

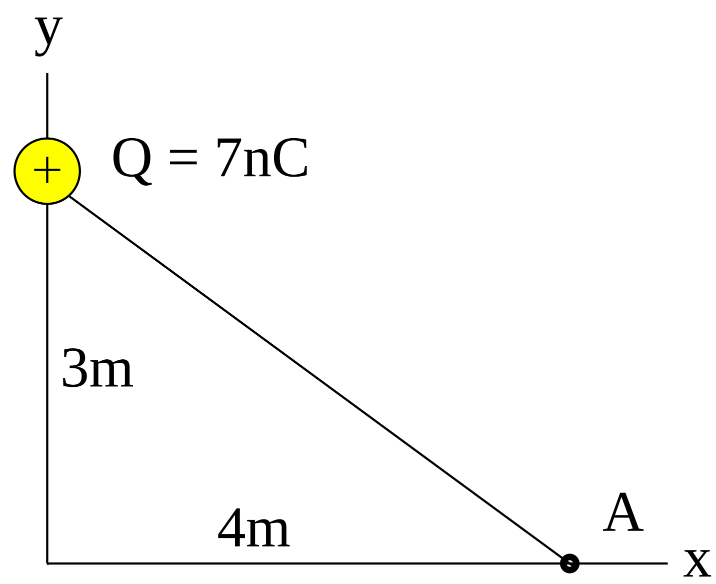
(a) $E = 6\mathrm{V}/0.2\mathrm{m} = 30\mathrm{V/m}$.
(b) left
(c) proton (positive charge)
(d) smaller m, equal $|q|$ $\Rightarrow$ larger $|q|E/m$
(e) $K = |q\Delta V| = (1.6 \times 10^{-19}\mathrm{C})(3\mathrm{V}) = 4.8 \times 10^{-19}\mathrm{J}$.

The point charge Q has a fixed position as shown.
(a) Find the components E_x and E_y of the electric field at point A.
(b) Find the electric potential V at point A.
Now place a proton $(m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C})$ at point A.
(c) Find the the electric force F (magnitude only) experienced by the proton.
(d) Find the electric potential energy U of the proton.



Unit Exam I: Problem #1 (Spring ’11)

The point charge Q has a fixed position as shown.
(a) Find the components E_x and E_y of the electric field at point A.
(b) Find the electric potential V at point A.
Now place a proton $(m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C})$ at point A.
(c) Find the the electric force F (magnitude only) experienced by the proton.
(d) Find the electric potential energy U of the proton.



Solution:

(a) $E = k\dfrac{|7\mathrm{nC}|}{(5\mathrm{m})^2} = 2.52\mathrm{N/C},$

$$E_x = \frac{4}{5}E = 2.02\mathrm{N/C}, \quad E_y = -\frac{3}{5}E = -1.51\mathrm{N/C}$$

(b) $V = k\dfrac{7\mathrm{nC}}{5\mathrm{m}} = 12.6\mathrm{V}.$

(c) $F = qE = 4.03 \times 10^{-19}\mathrm{N}.$

(d) $U = qV = 2.02 \times 10^{-18}\mathrm{J}.$

Unit Exam I: Problem #2 (Spring '11)

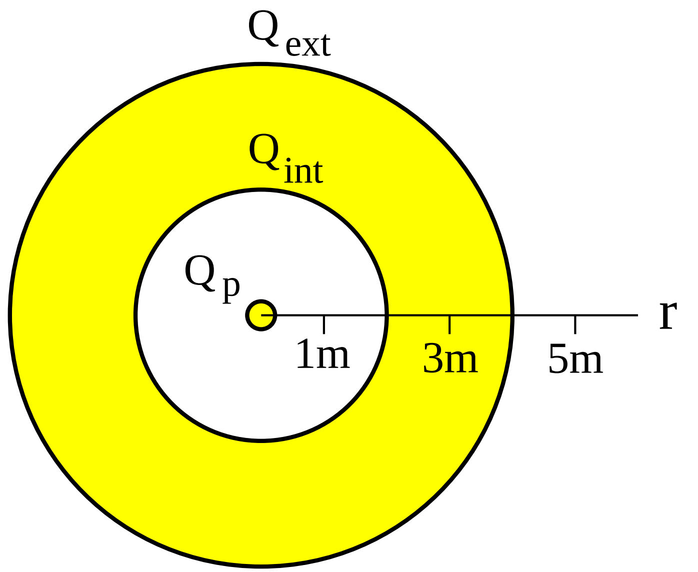
The charged conducting spherical shell has a 2m inner radius and a 4m outer radius. The charge on the outer surface is $Q_{\rm ext} = 8{\rm nC}$. There is a point charge $Q_{\rm p} = 3{\rm nC}$ at the center.

(a) Find the charge $Q_{\rm int}$ on the inner surface of the shell.

(b) Find the surface charge density $\sigma_{\rm ext}$ on the outer surface of the shell.

(c) Find the electric flux Φ_E through a Gaussian sphere of radius $r = 5$m.

(d) Find the magnitude of the electric field E at radius $r = 3$m.



The charged conducting spherical shell has a 2m inner radius and a 4m outer radius. The charge on the outer surface is $Q_{\rm ext} = 8{\rm nC}$. There is a point charge $Q_{\rm p} = 3{\rm nC}$ at the center.

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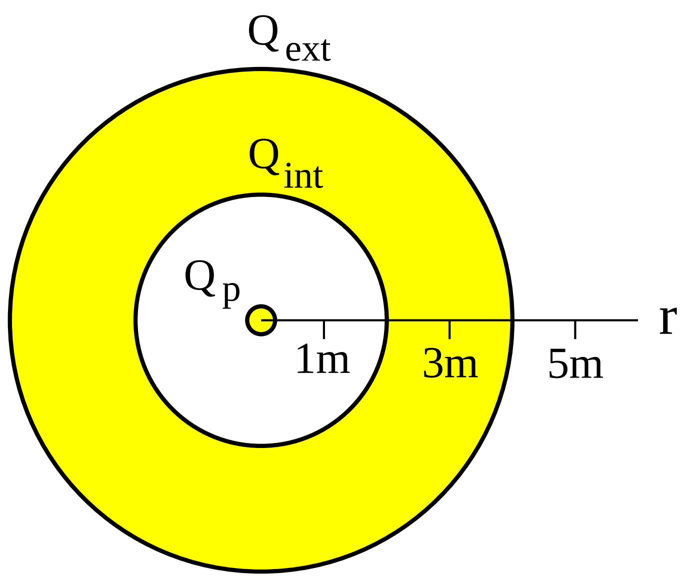


Solution:

(a) $Q_{\rm int} = -Q_{\rm p} = -3{\rm nC}$.

(b) $\sigma_{\rm ext} = \dfrac{Q_{\rm ext}}{4\pi(4{\rm m})^2} = 3.98 \times 10^{-11}{\rm C/m}^2$.

(c) $\Phi_E = \dfrac{Q_{\rm ext}}{\epsilon_0} = 904{\rm Nm}^2/{\rm C}$.

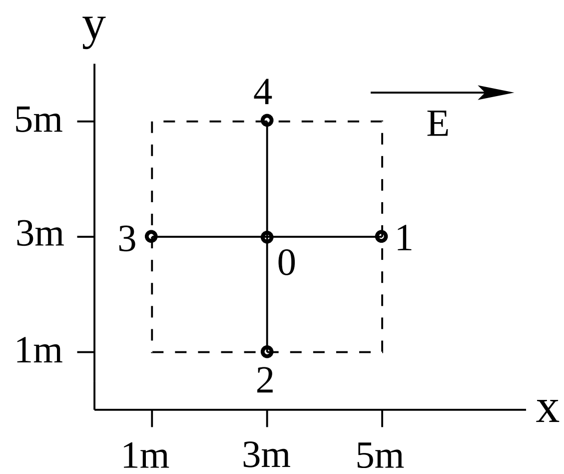
(d) $E = 0$ inside conductor.

Consider a region of space with a uniform electric field $\mathbf{E} = 0.5\mathrm{V/m}\,\hat{\mathbf{i}}$. Ignore gravity.

(a) If the electric potential vanishes at point 0, what are the electric potentials at points 1 and 2?

(b) If an electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) is released from rest at point 0, toward which point will it start moving?

(c) What will be the speed of the electron when it gets there?



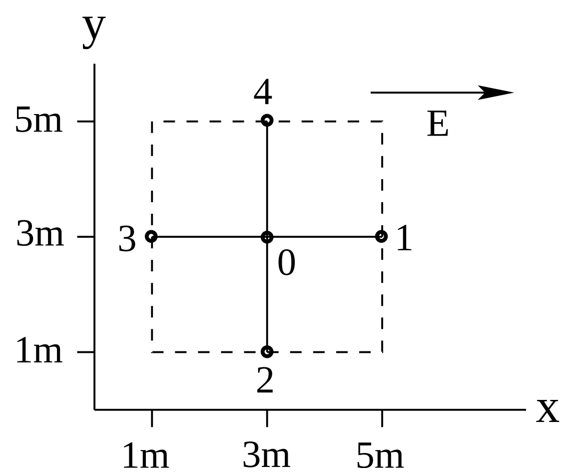
Unit Exam I: Problem #3 (Spring '11)

Consider a region of space with a uniform electric field $\mathbf{E} = 0.5\mathrm{V/m}\,\hat{\mathbf{i}}$. Ignore gravity.

(a) If the electric potential vanishes at point 0, what are the electric potentials at points 1 and 2?

(b) If an electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) is released from rest at point 0, toward which point will it start moving?

(c) What will be the speed of the electron when it gets there?



Solution:

(a) $V_1 = -(0.5\mathrm{V/m})(2\mathrm{m}) = -1\mathrm{V}, \quad V_2 = 0.$

(b) $\mathbf{F} = q\mathbf{E} = -|qE|\hat{\mathbf{i}}$ (toward point 3).

(c) $\Delta V = (V_3 - V_0) = 1\mathrm{V}, \quad \Delta U = q\Delta V = -1.60 \times 10^{-19}\mathrm{J},$

$K = -\Delta U = 1.60 \times 10^{-19}\mathrm{J}, \quad v = \sqrt{\frac{2K}{m}} = 5.93 \times 10^5\mathrm{m/s}.$

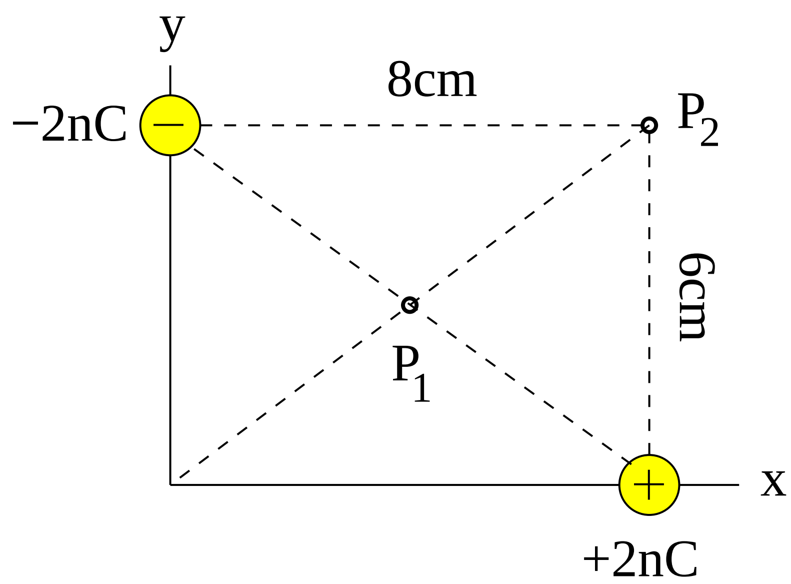
Alternatively:

$F = qE = 8.00 \times 10^{-20}\mathrm{N}, \quad a = \frac{F}{m} = 8.78 \times 10^{10}\mathrm{m/s^2},$

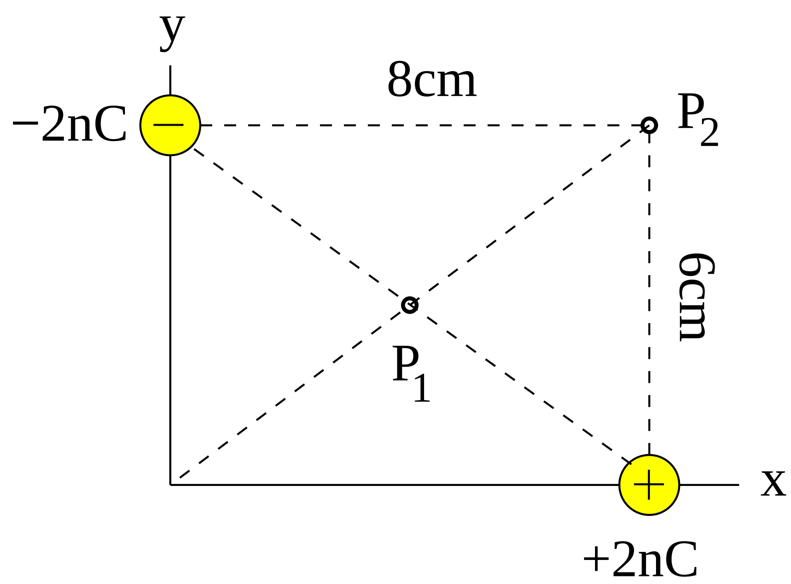
$|\Delta x| = 2\mathrm{m}, \quad v = \sqrt{2a|\Delta x|} = 5.93 \times 10^5\mathrm{m/s}.$

Unit Exam I: Problem #1 (Spring ’12)

Consider two point charges at the positions shown.
(a) Find the magnitude E of the electric field at point P_1.
(b) Find the components E_x and E_y of the electric field at point P_2.
(c) Draw the direction of the electric field at points P_1 and P_2 in the diagram.
(d) Calculate the potential difference $\Delta V = V_2 - V_1$ between point P_2 and P_1.



Consider two point charges at the positions shown.
(a) Find the magnitude E of the electric field at point P_1.
(b) Find the components E_x and E_y of the electric field at point P_2.
(c) Draw the direction of the electric field at points P_1 and P_2 in the diagram.
(d) Calculate the potential difference $\Delta V = V_2 - V_1$ between point P_2 and P_1.



Solution:

(a) $E = 2k\dfrac{2\mathrm{nC}}{(5\mathrm{cm})^2} = 1.44 \times 10^4\mathrm{N/C}.$

(b) $E_x = -k\dfrac{2\mathrm{nC}}{(8\mathrm{cm})^2} = -2.81 \times 10^3\mathrm{N/C}.$

$E_y = k\dfrac{2\mathrm{nC}}{(6\mathrm{cm})^2} = 5.00 \times 10^3\mathrm{N/C}.$

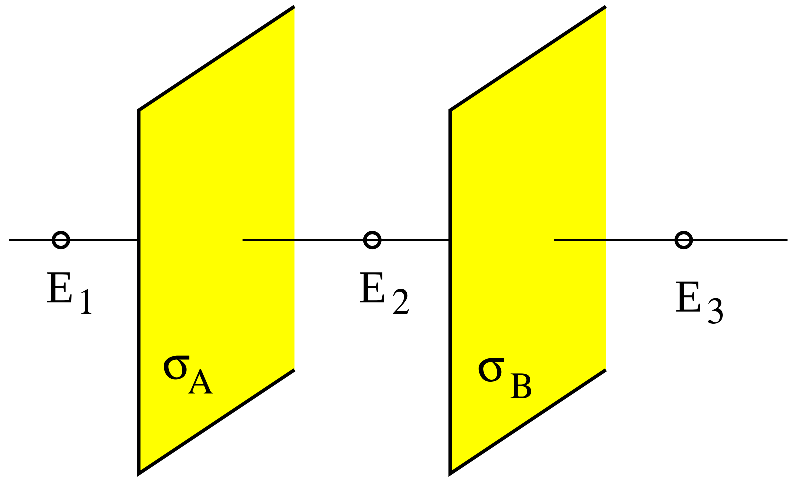
(c) $\mathbf{E}_1$ up and left toward negative charge; $\mathbf{E}_2$ more up and less left

(d) $\Delta V = V_2 - 0 = k\dfrac{2\mathrm{nC}}{6\mathrm{cm}} + k\dfrac{-2\mathrm{nC}}{8\mathrm{cm}} = 300\mathrm{V} - 225\mathrm{V} = 75\mathrm{V}.$

Unit Exam I: Problem #2 (Spring '12)

Two very large, thin, uniformly charged, parallel sheets are positioned as shown.
Find the values of the charge densities (charge per area), σ_A and σ_B, if you know the electric fields $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$.

Consider two situations.
(a) $E_1 = 2\mathrm{N/C}$ (directed left), $E_2 = 0$, $E_3 = 2\mathrm{N/C}$ (directed right).
(b) $E_1 = 0$, $E_2 = 2\mathrm{N/C}$ (directed right), $E_3 = 0$.



Two very large, thin, uniformly charged, parallel sheets are positioned as shown.
Find the values of the charge densities (charge per area), σ_A and σ_B, if you know the electric fields $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$.

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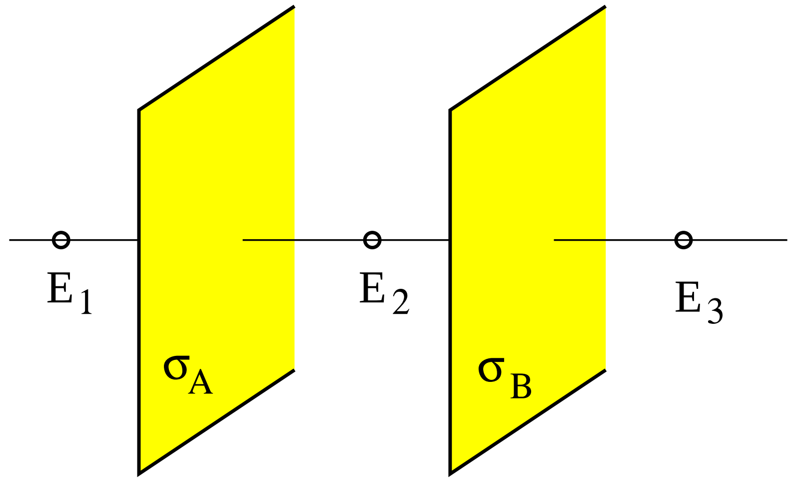
(a) $E_1 = 2\mathrm{N/C}$ (directed left), $E_2 = 0$, $E_3 = 2\mathrm{N/C}$ (directed right).
(b) $E_1 = 0$, $E_2 = 2\mathrm{N/C}$ (directed right), $E_3 = 0$.



Solution:

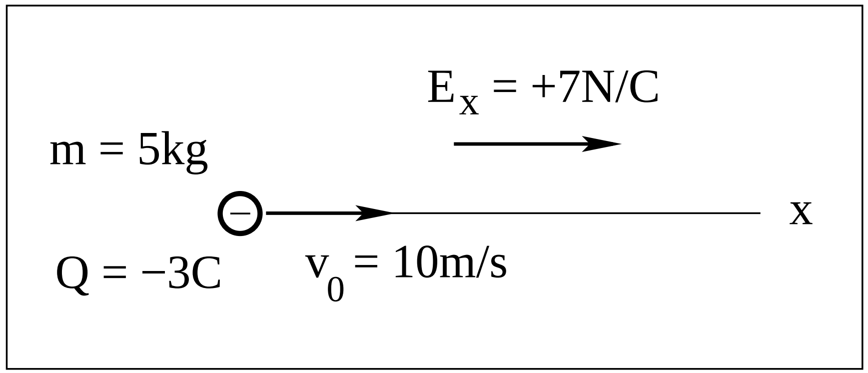
(a) The two sheets are equally charged:
$\sigma_A = \sigma_B = 2\epsilon_0(1\mathrm{N/C}) = 1.77 \times 10^{-11}\mathrm{C/m^2}$.

(b) The two sheets are oppositely charged:
$\sigma_A = -\sigma_B = 2\epsilon_0(1\mathrm{N/C}) = 1.77 \times 10^{-11}\mathrm{C/m^2}$.

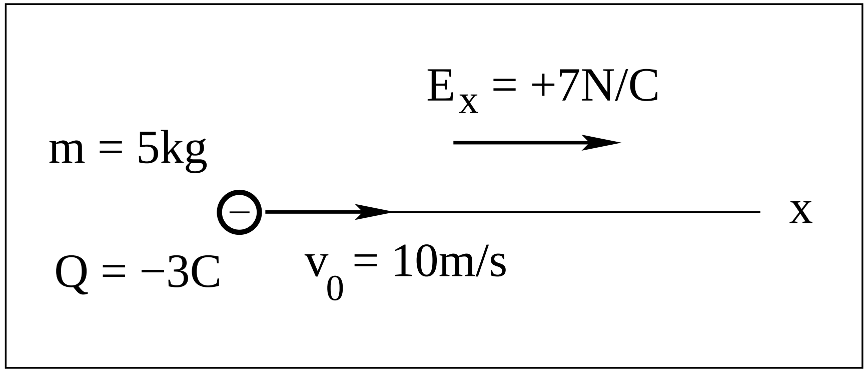
Unit Exam I: Problem #3 (Spring '12)

Consider a region of uniform electric field $E_x = +7\text{N/C}$. A charged particle (charge $Q = -3\text{C}$, mass $m = 5\text{kg}$) is launched at time $t = 0$ from initial position $x = 0$ with velocity $v_0 = 10\text{m/s}$ in the positive x-direction. Ignore gravity.

(a) Find the force F_x acting on the particle at time $t = 0$.
(b) Find the force F_x acting on the particle at time $t = 3\text{s}$.
(c) Find the kinetic energy of the particle at time $t = 0$.
(d) Find the kinetic energy of the particle at time $t = 3\text{s}$.
(e) Find the work done on the particle between $t = 0$ and $t = 3\text{s}$.



Consider a region of uniform electric field $E_x = +7$N/C. A charged particle (charge $Q = -3$C, mass $m = 5$kg) is launched at time $t = 0$ from initial position $x = 0$ with velocity $v_0 = 10$m/s in the positive x-direction. Ignore gravity.
(a) Find the force F_x acting on the particle at time $t = 0$.
(b) Find the force F_x acting on the particle at time $t = 3$s.
(c) Find the kinetic energy of the particle at time $t = 0$.
(d) Find the kinetic energy of the particle at time $t = 3$s.
(e) Find the work done on the particle between $t = 0$ and $t = 3$s.



Solution:

(a) $F_x = QE_x = (-3\mathrm{C})(7\mathrm{N/C}) = -21\mathrm{N}$.

(b) no change from (a).

(c) $K = \frac{1}{2}(5\mathrm{kg})(10\mathrm{m/s})^2 = 250\mathrm{J}$.

(d) $v_x = v_0 + a_x t = v_0 + (F_x/m)t = 10\mathrm{m/s} + (-21\mathrm{N}/5\mathrm{kg})(3\mathrm{s}) = -2.6\mathrm{m/s}$.
$K = \frac{1}{2}(5\mathrm{kg})(-2.6\mathrm{m/s})^2 = 16.9\mathrm{J}$.

(e) $W = \Delta K = 16.9\mathrm{J} - 250\mathrm{J} = -233\mathrm{J}$.

Unit Exam I: Problem #1 (Spring ’13)

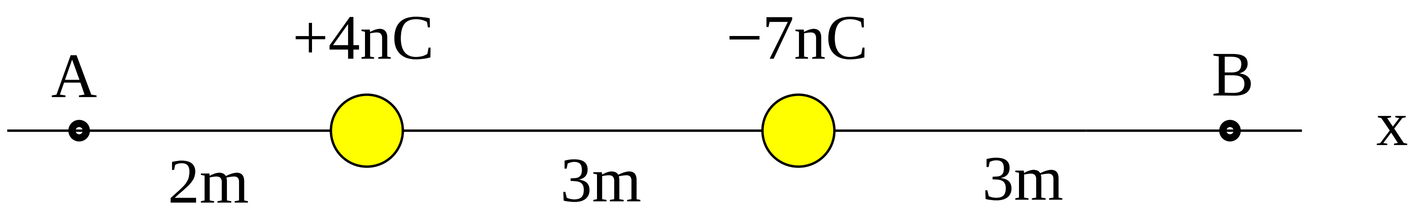
Consider two point charges positioned on the x-axis as shown.

(a) Find magnitude and direction of the electric field at points A and B.

(b) Find the electric potential at points A and B.

(c) Find the electric potential energy of a proton (mass $m = 1.67 \times 10^{-27}$kg, charge $q = 1.60 \times 10^{-19}$C) when placed at point A or point B.

(d) Find magnitude and direction of the acceleration the proton experiences when released at point A or point B.



Unit Exam I: Problem #1 (Spring '13)

Solution:

(a) $E_x = -k\dfrac{4\text{nC}}{(2\text{m})^2} - k\dfrac{(-7\text{nC})}{(5\text{m})^2} = -9.00\text{N/C} + 2.52\text{N/C} = -6.48\text{N/C}.$

$E_x = k\dfrac{4\text{nC}}{(6\text{m})^2} + k\dfrac{(-7\text{nC})}{(3\text{m})^2} = 1.00\text{N/C} - 7.00\text{N/C} = -6.00\text{N/C}.$

(b) $V = +k\dfrac{4\text{nC}}{2\text{m}} + k\dfrac{(-7\text{nC})}{5\text{m}} = 18.0\text{V} - 12.6\text{V} = 5.4\text{V}.$

$V = +k\dfrac{4\text{nC}}{6\text{m}} + k\dfrac{(-7\text{nC})}{3\text{m}} = 6.0\text{V} - 21.0\text{V} = -15.0\text{V}.$

(c) $U = qV = (5.4\text{V})(1.6 \times 10^{-19}\text{C}) = 8.64 \times 10^{-19}\text{J}.$

$U = qV = (-15.0\text{V})(1.6 \times 10^{-19}\text{C}) = -2.40 \times 10^{-18}\text{J}.$

(d) $a_x = \dfrac{qE_x}{m} = \dfrac{(1.6 \times 10^{-19}\text{C})(-6.48\text{N/C})}{1.67 \times 10^{-27}\text{kg}} = -6.21 \times 10^{8}\text{ms}^{-2}.$

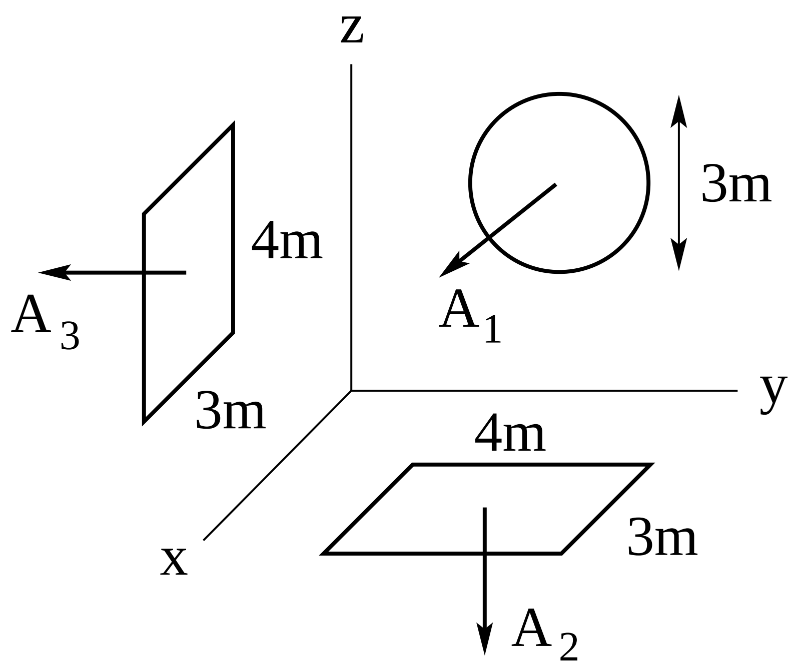
$a_x = \dfrac{qE_x}{m} = \dfrac{(1.6 \times 10^{-19}\text{C})(-6.00\text{N/C})}{1.67 \times 10^{-27}\text{kg}} = -5.75 \times 10^{8}\text{ms}^{-2}.$

Consider three plane surfaces (one circle and two rectangles) with area vectors $\vec{A}_1$ (pointing in positive x-direction), $\vec{A}_2$ (pointing in negative z-direction), and $\vec{A}_3$ (pointing in negative y-direction) as shown. The region is filled with a uniform electric field $\vec{E} = (-3\hat{i} + 9\hat{j} - 4\hat{k})$N/C or $\vec{E} = (2\hat{i} - 6\hat{j} + 5\hat{k})$N/C.

(a) Find the electric flux $\Phi_E^{(1)}$ through surface 1.

(b) Find the electric flux $\Phi_E^{(2)}$ through surface 2.

(c) Find the electric flux $\Phi_E^{(3)}$ through surface 3.



Solution:

(a) $\vec{A}_1 = \pi(1.5\text{m})^2\hat{i} = 7.07\text{m}^2\hat{i}, \quad \Phi_E^{(1)} = \vec{E}\cdot\vec{A}_1 = (-3\text{N/C})(7.07\text{m}^2) = -21.2\text{Nm}^2/\text{C}.$

$\vec{A}_1 = \pi(1.5\text{m})^2\hat{i} = 7.07\text{m}^2\hat{i}, \quad \Phi_E^{(1)} = \vec{E}\cdot\vec{A}_1 = (2\text{N/C})(7.07\text{m}^2) = 14.1\text{Nm}^2/\text{C}.$

(b) $\vec{A}_2 = (3\text{m})(4\text{m})(-\hat{k}) = -12\text{m}^2\hat{k}, \quad \Phi_E^{(2)} = \vec{E}\cdot\vec{A}_2 = (-4\text{N/C})(-12\text{m}^2) = 48\text{Nm}^2/\text{C}.$

$\vec{A}_2 = (3\text{m})(4\text{m})(-\hat{k}) = -12\text{m}^2\hat{k}, \quad \Phi_E^{(2)} = \vec{E}\cdot\vec{A}_2 = (5\text{N/C})(-12\text{m}^2) = -60\text{Nm}^2/\text{C}.$

(c) $\vec{A}_3 = (3\text{m})(4\text{m})(-\hat{j}) = -12\text{m}^2\hat{j}, \quad \Phi_E^{(3)} = \vec{E}\cdot\vec{A}_3 = (9\text{N/C})(-12\text{m}^2) = -108\text{Nm}^2/\text{C}.$

$\vec{A}_3 = (3\text{m})(4\text{m})(-\hat{j}) = -12\text{m}^2\hat{j}, \quad \Phi_E^{(3)} = \vec{E}\cdot\vec{A}_3 = (-6\text{N/C})(-12\text{m}^2) = 72\text{Nm}^2/\text{C}.$

Unit Exam I: Problem #3 (Spring '13)

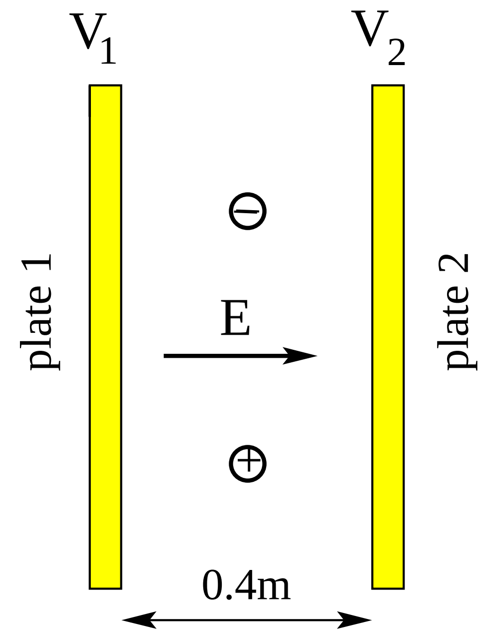
An electron ($m_e = 9.11 \times 10^{-31}$kg, $q_e = -1.60 \times 10^{-19}$C) and a proton ($m_p = 1.67 \times 10^{-27}$kg, $q_p = +1.60 \times 10^{-19}$C) are released from rest midway between oppositely charged parallel plates. The electric field between the plates is uniform and has strength $E = 40$V/m. Ignore gravity.

(a) Which plate is positively (negatively) charged?

(b) Find the electric forces $\vec{F}_p$ acting on the proton and $\vec{F}_e$ acting on the electron (magnitude and direction).

(c) Find the accelerations $\vec{a}_p$ of the proton and $\vec{a}_e$ of the electron (magnitude and direction).

(d) If plate 1 is at potential $V_1 = 1$V at what potential V_2 is plate 2?
If plate 2 is at potential $V_2 = 2$V at what potential V_1 is plate 1?



Unit Exam I: Problem #3 (Spring '13)

Solution:

(a) plate 1 (plate 2)

(b) $F_p = |q_p|E = 6.40 \times 10^{-18}\mathrm{N}.$ (directed right).

$F_e = |q_e|E = 6.40 \times 10^{-18}\mathrm{N}.$ (directed left).

(c) $a_p = F_p/m_p = 3.83 \times 10^{9}\mathrm{m/s^2}.$ (directed right).

$a_e = F_e/m_e = 7.03 \times 10^{12}\mathrm{m/s^2}.$ (directed left).

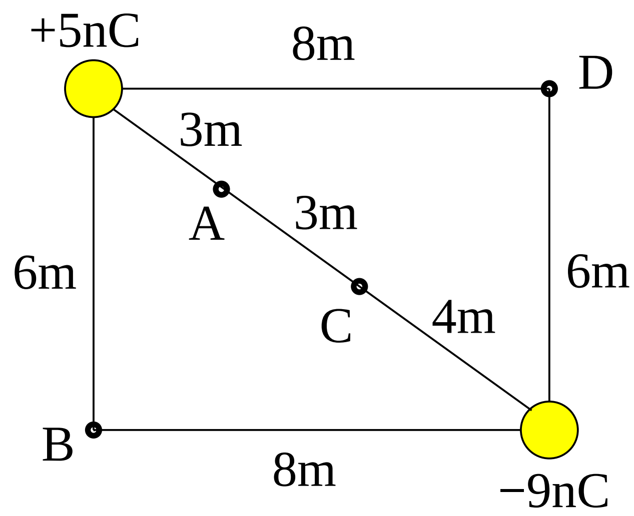
(d) $V_2 = 1\mathrm{V} - (40\mathrm{V/m})(0.4\mathrm{m}) = -15\mathrm{V}.$

$V_1 = 2\mathrm{V} + (40\mathrm{V/m})(0.4\mathrm{m}) = 18\mathrm{V}.$

Unit Exam I: Problem #1 (Spring '14)

Consider two point charges positioned as shown.

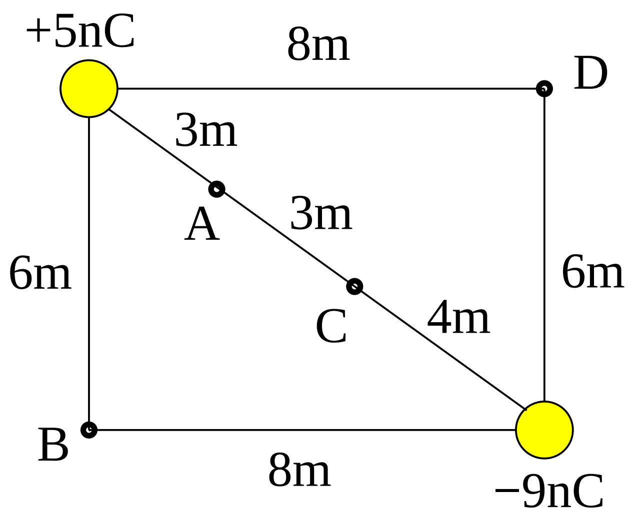
- Find the magnitude of the electric field at point A.
- Find the electric potential at point B.
- Find the magnitude of the electric field at point C.
- Find the electric potential at point D.



Unit Exam I: Problem #1 (Spring ’14)

Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point A.
- Find the electric potential at point B.
- Find the magnitude of the electric field at point C.
- Find the electric potential at point D.



Solution:

- $E_A = k\dfrac{|5\mathrm{nC}|}{(3\mathrm{m})^2} + k\dfrac{|-9\mathrm{nC}|}{(7\mathrm{m})^2} = 5.00\mathrm{V/m} + 1.65\mathrm{V/m} = 6.65\mathrm{V/m}.$
- $V_B = k\dfrac{(+5\mathrm{nC})}{6\mathrm{m}} + k\dfrac{(-9\mathrm{nC})}{8\mathrm{m}} = 7.50\mathrm{V} - 10.13\mathrm{V} = -2.63\mathrm{V}.$
- $E_C = k\dfrac{|5\mathrm{nC}|}{(6\mathrm{m})^2} + k\dfrac{|-9\mathrm{nC}|}{(4\mathrm{m})^2} = 1.25\mathrm{V/m} + 5.06\mathrm{V/m} = 6.31\mathrm{V/m}.$
- $V_D = k\dfrac{(+5\mathrm{nC})}{8\mathrm{m}} + k\dfrac{(-9\mathrm{nC})}{6\mathrm{m}} = 5.63\mathrm{V} - 13.5\mathrm{V} = -7.87\mathrm{V}.$

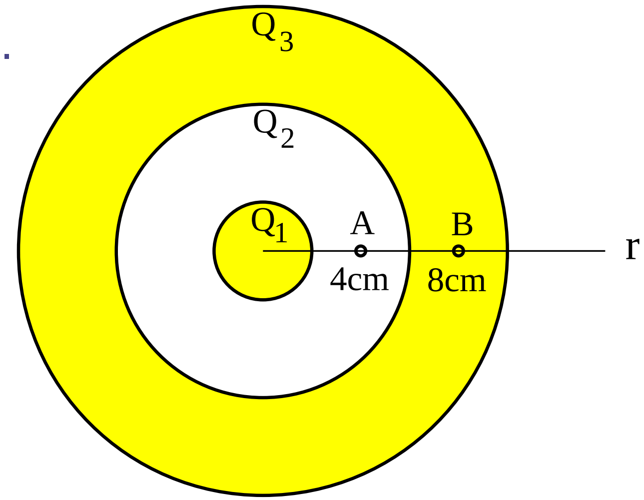
Unit Exam I: Problem #2 (Spring ’14)

Consider a conducting sphere of radius $r_1 = 2$cm and a conducting spherical shell of inner radius $r_2 = 6$cm and outer radius $r_3 = 10$cm. The charges on the two surfaces of the shell are $Q_2 = Q_3 = 1.3$nC [3.1nC].

(a) Find the charge Q_1 on the surface of the conducting sphere.

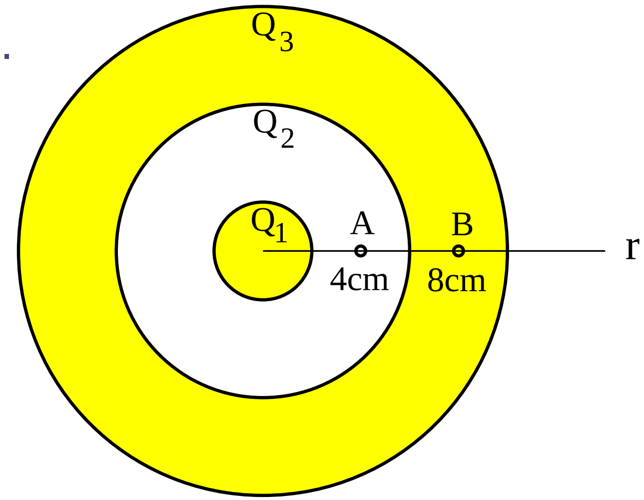
(b) Find the magnitude of the electric field at points A and B.

(c) Find the surface charge density σ_3 on the outermost surface.



Unit Exam I: Problem #2 (Spring '14)

Consider a conducting sphere of radius $r_1 = 2$cm and a conducting spherical shell of inner radius $r_2 = 6$cm and outer radius $r_3 = 10$cm. The charges on the two surfaces of the shell are $Q_2 = Q_3 = 1.3$nC [3.1nC].
(a) Find the charge Q_1 on the surface of the conducting sphere.
(b) Find the magnitude of the electric field at points A and B.
(c) Find the surface charge density σ_3 on the outermost surface.



Solution:

(a) Gauss' law implies that
$Q_1 = -Q_2 = -1.3$nC   [-3.1nC].

(b) $E_A = k\dfrac{1.3\text{nC}}{(4\text{cm})^2} = 7.31 \times 10^3\text{N/C}$

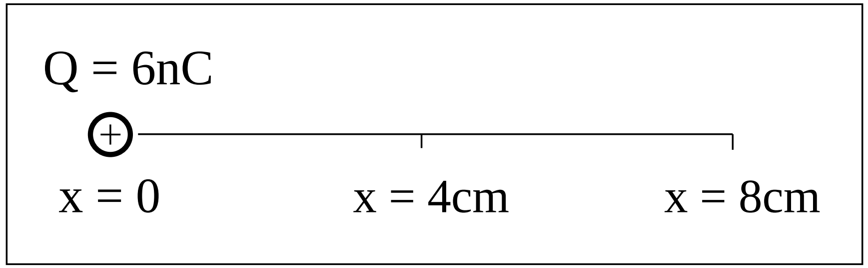
$$\left[k\frac{3.1\text{nC}}{(4\text{cm})^2} = 1.74 \times 10^4\text{N/C}\right].$$

$E_B = 0$ inside conductor.

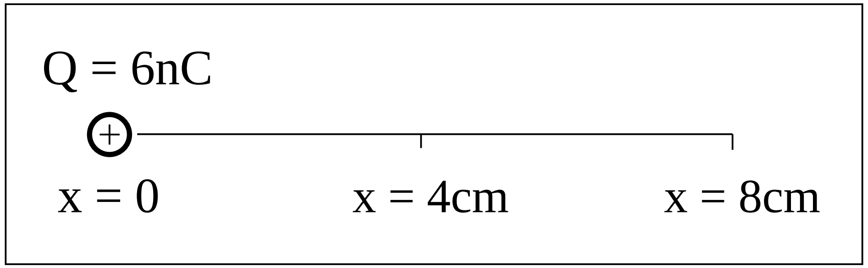
(c) $\sigma_3 = \dfrac{Q_3}{4\pi r_3^2} = \dfrac{1.3\text{nC}}{1257\text{cm}^2} = 1.03 \times 10^{-8}\text{C/m}^2 \quad \left[\dfrac{3.1\text{nC}}{1257\text{cm}^2} = 2.47 \times 10^{-8}\text{C/m}^2\right]$

Unit Exam I: Problem #3 (Spring ’14)

Consider a point charge $Q = 6$nC fixed at position $x = 0$.
(a) Find the electric potential energy U_4 of a charged particle with mass $m = 1$mg and charge $q = 2\mu$C placed at position $x = 4$cm.
(b) Find the electric potential energy U_8 of a charged particle with mass $m = 2$mg and charge $q = -1\mu$C placed at position $x = 8$cm.
(c) Find the kinetic energy K_8 of that particle, released from rest at $x = 4$cm, when it has reached position $x = 8$cm.
(d) Find the kinetic energy K_4 of that particle, released from rest at $x = 8$cm, when it has reached position $x = 4$cm.
(e) Find the velocity v_8 of that particle at $x = 8$cm.
(f) Find the velocity v_4 of that particle at $x = 4$cm.



Consider a point charge $Q = 6\text{nC}$ fixed at position $x = 0$.
(a) Find the electric potential energy U_4 of a charged particle with mass $m = 1\text{mg}$ and charge $q = 2\mu\text{C}$ placed at position $x = 4\text{cm}$.
(b) Find the electric potential energy U_8 of a charged particle with mass $m = 2\text{mg}$ and charge $q = -1\mu\text{C}$ placed at position $x = 8\text{cm}$.
(c) Find the kinetic energy K_8 of that particle, released from rest at $x = 4\text{cm}$, when it has reached position $x = 8\text{cm}$.
(d) Find the kinetic energy K_4 of that particle, released from rest at $x = 8\text{cm}$, when it has reached position $x = 4\text{cm}$.
(e) Find the velocity v_8 of that particle at $x = 8\text{cm}$.
(f) Find the velocity v_4 of that particle at $x = 4\text{cm}$.



Solution:

(a) $U_4 = k\dfrac{qQ}{4\text{cm}} = 2.7\text{mJ}.$

(b) $U_8 = k\dfrac{qQ}{8\text{cm}} = -0.675\text{mJ}.$

(c) $K_8 = (2.7 - 1.35)\text{mJ} = 1.35\text{mJ}.$

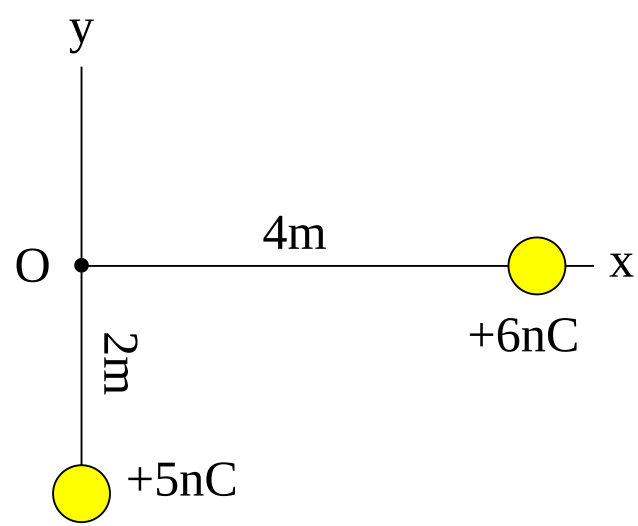
(d) $K_4 = (1.35 - 0.675)\text{mJ} = 0.675\text{mJ}.$

(e) $v_8 = \sqrt{\dfrac{2K_8}{m}} = 52.0\text{m/s}.$

(f) $v_4 = \sqrt{\dfrac{2K_4}{m}} = 26.0\text{m/s}.$

Unit Exam I: Problem #1 (Fall '14)

Two point charges are placed in the xy-plane as shown.
(a) Find the components E_x and E_y of the electric field at point O.
(b) Draw an arrow indicating the direction of $\vec{E}$ at point O.
(c) Find the electric potential V at point O.
(d) Find the magnitude F of the electric force between the two charges.



Unit Exam I: Problem #1 (Fall '14)

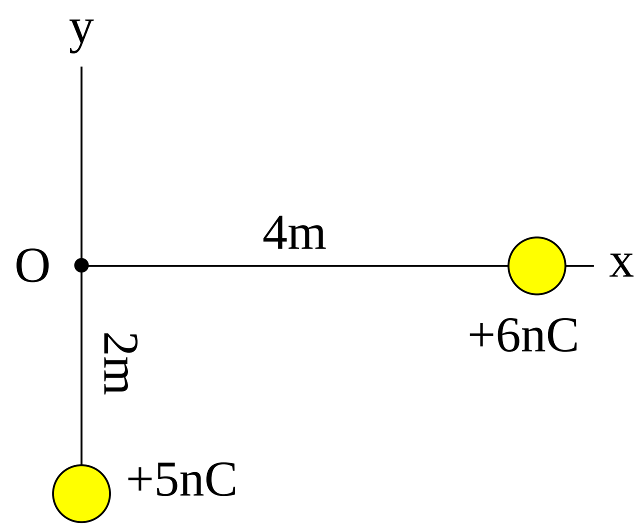
Two point charges are placed in the xy-plane as shown.

(a) Find the components E_x and E_y of the electric field at point O.

(b) Draw an arrow indicating the direction of $\vec{E}$ at point O.

(c) Find the electric potential V at point O.

(d) Find the magnitude F of the electric force between the two charges.



Solution:

(a) $$E_x = -k\frac{|6\text{nC}|}{(4\text{m})^2} = -3.38\text{N/C}$$

$$E_y = +k\frac{|5\text{nC}|}{(2\text{m})^2} = 11.25\text{N/C}.$$

(b) Up and left.

(c) $$V = k\frac{6\text{nC}}{4\text{m}} + k\frac{5\text{nC}}{2\text{m}} = 13.5\text{V} + 22.5\text{V} = 36\text{V}.$$

(d) $$F = k\frac{|6\text{nC}||5\text{nC}|}{20\text{m}^2} = 13.5\text{nN}.$$

Unit Exam I: Problem #2 (Fall ’14)

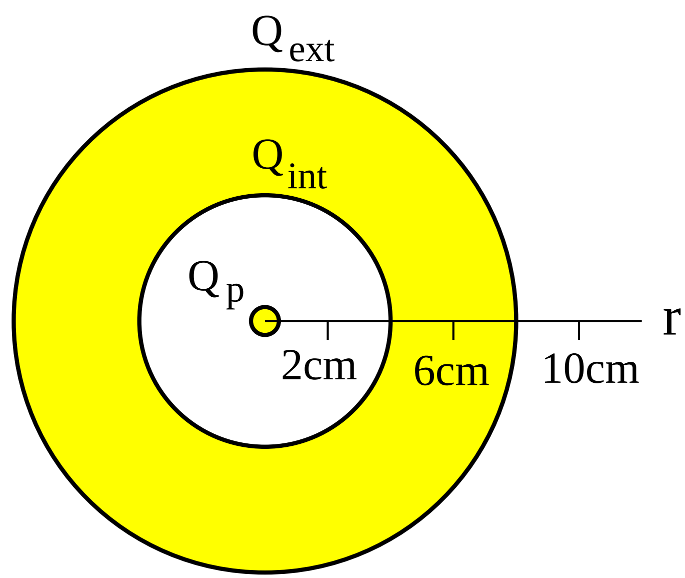
The conducting spherical shell shown in cross section has a 4cm inner radius and an 8cm outer radius. A point charge Q_p is placed at the center. The charges on the inner and outer surfaces of the shell are $Q_\mathrm{int} = 5\mathrm{nC}$ and $Q_\mathrm{ext} = 7\mathrm{nC}$, respectively.

(a) Find the charge Q_p.

(b) Find the magnitude of the electric field E at radius $r = 10$cm.

(c) Find the surface charge density σ_int on the inner surface of the shell.

(d) Find the electric flux Φ_E through a Gaussian sphere of radius $r = 6$cm.



Unit Exam I: Problem #2 (Fall ’14)

The conducting spherical shell shown in cross section has a 4cm inner radius and an 8cm outer radius. A point charge Q_p is placed at the center. The charges on the inner and outer surfaces of the shell are $Q_\mathrm{int} = 5\mathrm{nC}$ and $Q_\mathrm{ext} = 7\mathrm{nC}$, respectively.

(a) Find the charge Q_p.

(b) Find the magnitude of the electric field E at radius $r = 10$cm.

(c) Find the surface charge density σ_int on the inner surface of the shell.

(d) Find the electric flux Φ_E through a Gaussian sphere of radius $r = 6$cm.

Solution:

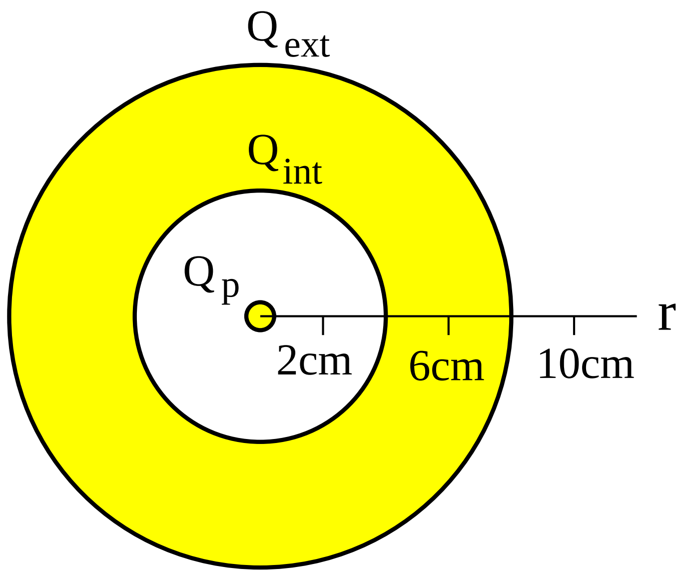
(a) $Q_\mathrm{p} = -Q_\mathrm{int} = -5\mathrm{nC}.$

(b) $E[4\pi(10\mathrm{cm})^2] = \dfrac{Q_\mathrm{p} + Q_\mathrm{int} + Q_\mathrm{ext}}{\epsilon_0} = \dfrac{Q_\mathrm{ext}}{\epsilon_0}$

$\Rightarrow\ E = 6300\mathrm{N/C}.$

(c) $\sigma_\mathrm{int} = \dfrac{Q_\mathrm{int}}{4\pi(4\mathrm{cm})^2} = 2.49 \times 10^{-7}\mathrm{C/m^2}.$

(d) $\Phi_E = 0$ inside conducting material.



Unit Exam I: Problem #3 (Fall ’14)

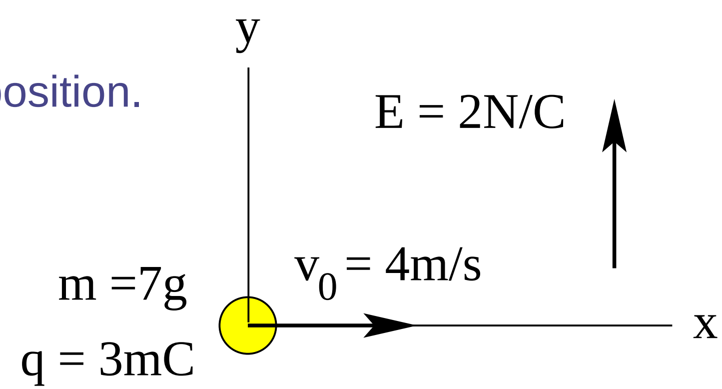
Consider a region of uniform electric field as shown. A charged particle is projected at time $t = 0$ with initial velocity as shown.

(a) Find the components a_x and a_y of the acceleration at time $t = 0$.

(b) Find the components v_x and v_y of the velocity at time $t = 2$s.

(c) Find the kinetic energy at time $t = 2$s.

(d) Sketch the path of the particle as it moves from the initial position.



Unit Exam I: Problem #3 (Fall '14)

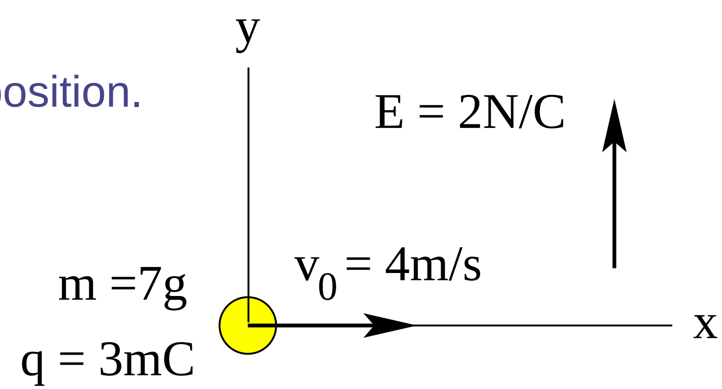
Consider a region of uniform electric field as shown. A charged particle is projected at time $t = 0$ with initial velocity as shown.

(a) Find the components a_x and a_y of the acceleration at time $t = 0$.

(b) Find the components v_x and v_y of the velocity at time $t = 2$s.

(c) Find the kinetic energy at time $t = 2$s.

(d) Sketch the path of the particle as it moves from the initial position.



Solution:

(a) $a_x = 0, \quad a_y = \dfrac{q}{m}E = \dfrac{3 \times 10^{-3}\mathrm{C}}{7 \times 10^{-3}\mathrm{kg}}(2\mathrm{N/C}) = 0.857\mathrm{m/s^2}.$

(b) $v_x = v_0 = 4\mathrm{m/s}, \quad v_y = a_y t = (0.857\mathrm{m/s^2})(2\mathrm{s}) = 1.71\mathrm{m/s}.$

(c) $E = \dfrac{1}{2}(7 \times 10^{-3}\mathrm{kg})[(4\mathrm{m/s})^2 + (1.71\mathrm{m/s})^2] = 6.62 \times 10^{-2}\mathrm{J}.$

(d) Upright parabolic path.

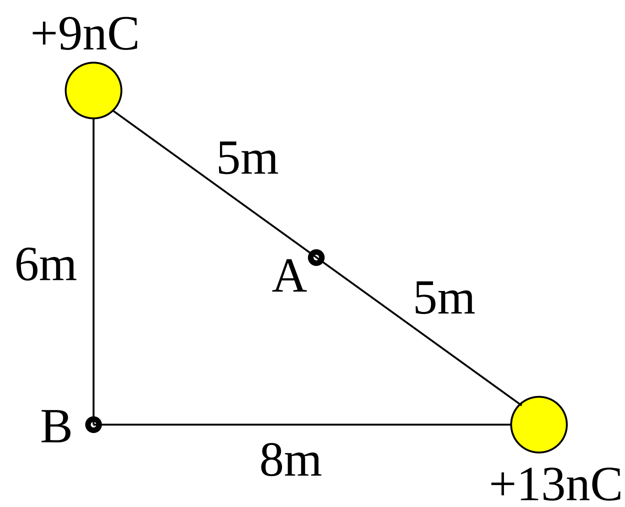
Unit Exam I: Problem #1 (Spring ’15)

Consider two point charges positioned as shown.

(a) Find the magnitude of the electric force acting between the two charges.

(b) Find the electric potential at point B.

(c) Find the magnitude and direction of the electric field at point A.

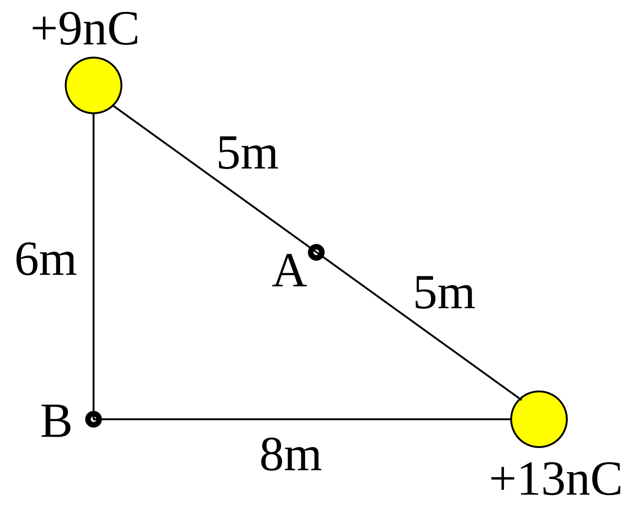


Consider two point charges positioned as shown.

(a) Find the magnitude of the electric force acting between the two charges.

(b) Find the electric potential at point B.

(c) Find the magnitude and direction of the electric field at point A.



Solution:

(a) $F = k\dfrac{|(9\text{nC})(13\text{nC}|)}{(10\text{m})^2} = 10.53\text{nN}.$

(b) $V_B = k\dfrac{(9\text{nC})}{6\text{m}} + k\dfrac{(13\text{nC})}{8\text{m}} = 13.5\text{V} + 14.6\text{V} = 28.1\text{V}.$

(c) $E_A = \left|k\dfrac{9\text{nC}}{(5\text{m})^2} - k\dfrac{13\text{nC}}{(5\text{m})^2}\right| = |3.24\text{N/C} - 4.68\text{N/C}| = 1.44\text{N/C}.$

Direction along hypotenuse toward upper left.

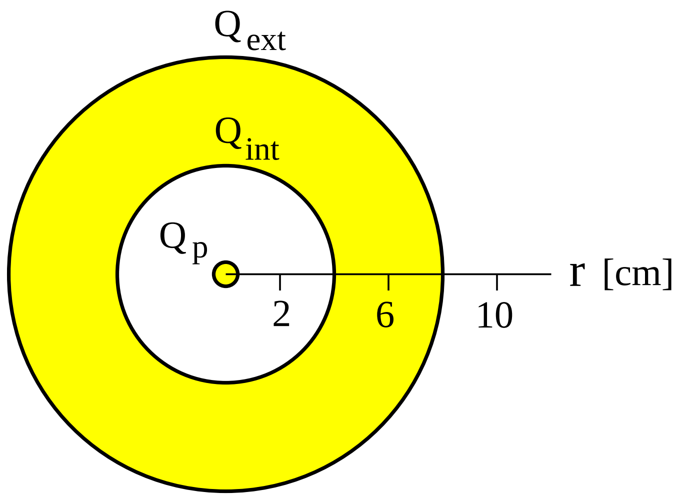
Unit Exam I: Problem #2 (Spring '15)

The conducting spherical shell shown in cross section has a 4cm inner radius and an 8cm outer radius. The excess charges on its inner and outer surfaces are $Q_{\rm int} = +7{\rm nC}$ and $Q_{\rm ext} = +11{\rm nC}$, respectively. There is a point charge $Q_{\rm p}$ at the center of the cavity.

(a) Find the point charge $Q_{\rm p}$.

(b) Find the surface charge density $\sigma_{\rm int}$ on the inner surface of the shell.

(c) Find the magnitude E of the electric field at radius $r = 10$cm.



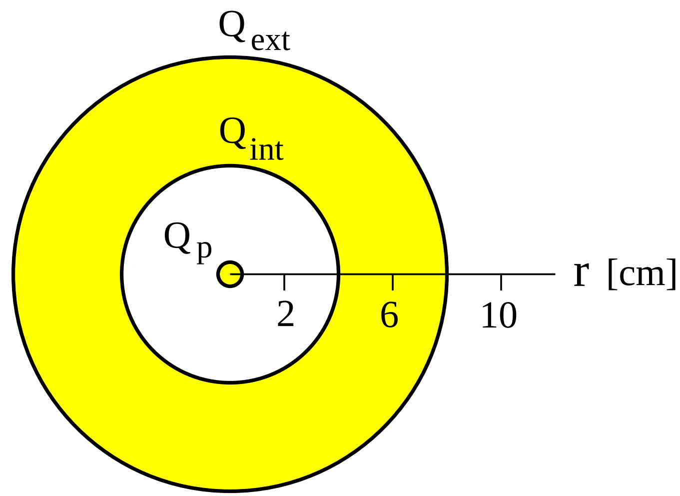
Unit Exam I: Problem #2 (Spring ’15)

The conducting spherical shell shown in cross section has a 4cm inner radius and an 8cm outer radius. The excess charges on its inner and outer surfaces are $Q_{\mathrm{int}} = +7\mathrm{nC}$ and $Q_{\mathrm{ext}} = +11\mathrm{nC}$, respectively. There is a point charge Q_{p} at the center of the cavity.

(a) Find the point charge Q_{p}.

(b) Find the surface charge density σ_{int} on the inner surface of the shell.

(c) Find the magnitude E of the electric field at radius $r = 10\mathrm{cm}$.



Solution:

(a) $Q_{\mathrm{p}} = -Q_{\mathrm{int}} = -7\mathrm{nC}$.

(b) $\sigma_{\mathrm{int}} = \dfrac{Q_{\mathrm{int}}}{4\pi(4\mathrm{cm})^2} = 3.48 \times 10^{-7}\mathrm{C/m^2}$.

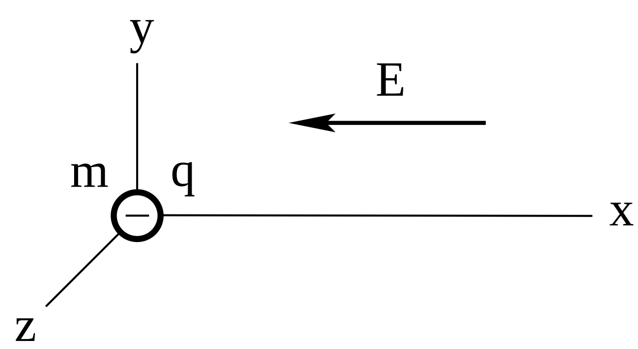
(c) $E = \dfrac{k(11\mathrm{nC})}{(10\mathrm{cm})^2} = 9900\mathrm{N/C}$.

Consider a region of uniform electric field $\mathbf{E} = -7\hat{\mathbf{i}}$ N/C. At time $t = 0$ a charged particle (charge $q = -5$nC, mass $m = 4 \times 10^{-6}$kg) is released from rest at the origin of the coordinate system as shown.

(a) Find the acceleration, the velocity, and the position of the particle $t = 0$.

(b) Find the acceleration, the velocity, and the position of the particle at $t = 3$s.

(c) Find the work W done by the electric field on the particle between $t = 0$ and $t = 3$s.



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(a) Find the acceleration, the velocity, and the position of the particle $t = 0$.

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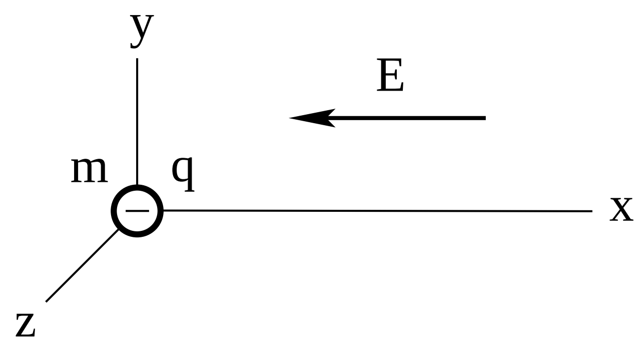


Solution:

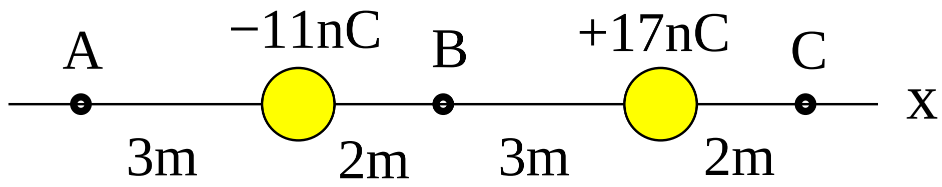
(a) $a_x = \dfrac{(-5\text{nC})}{4 \times 10^{-6}\text{kg}}(-7\text{N/C}) = 8.75 \times 10^{-3}\text{m/s}^2$,
$v_x = 0, \quad x = 0.$

(b) $a_x = 8.75 \times 10^{-3}\text{m/s}^2$,
$v_x = a_x t = (8.75 \times 10^{-3}\text{m/s}^2)(3\text{s}) = 2.63 \times 10^{-2}\text{m/s}$,
$x = \dfrac{1}{2}a_x t^2 = (0.5)(8.75 \times 10^{-3}\text{m/s}^2)(3\text{s})^2 = 3.94 \times 10^{-2}\text{m}.$

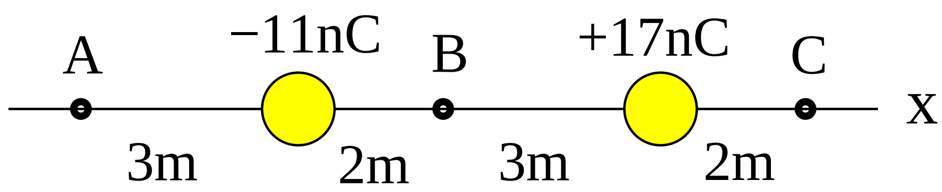
(c) $W = F\Delta x = (-5\text{nC})(-7\text{N/C})(3.94 \times 10^{-2}\text{m}) = 1.38\text{nJ}.$
$W = \Delta K = \dfrac{1}{2}(4 \times 10^{-6}\text{kg})(2.63 \times 10^{-2}\text{m/s})^2 = 1.38\text{nJ}.$

Unit Exam I: Problem #1 (Fall ’15)

Consider two point charges positioned on the x-axis as shown.
(1a) Find magnitude and direction of the electric field at point C.
(1b) Find the electric potential at point B.
(2a) Find magnitude and direction of the electric field at point B.
(2b) Find the electric potential at point A.



Consider two point charges positioned on the x-axis as shown.
(1a) Find magnitude and direction of the electric field at point C.
(1b) Find the electric potential at point B.
(2a) Find magnitude and direction of the electric field at point B.
(2b) Find the electric potential at point A.



Solution:

(1a) $E_x = -k\frac{|-11\mathrm{nC}|}{(7\mathrm{m})^2} + k\frac{|17\mathrm{nC}|}{(2\mathrm{m})^2} = -2.02\mathrm{N/C} + 38.25\mathrm{N/C} = +36.23\mathrm{N/C}.$

(1b) $V = k\frac{(-11\mathrm{nC})}{2\mathrm{m}} + k\frac{(17\mathrm{nC})}{3\mathrm{m}} = -49.5\mathrm{V} + 51.0\mathrm{V} = 1.5\mathrm{V}.$

(2a) $E_x = -k\frac{|-11\mathrm{nC}|}{(2\mathrm{m})^2} - k\frac{|17\mathrm{nC}|}{(3\mathrm{m})^2} = -24.75\mathrm{N/C} - 17.00\mathrm{N/C} = -41.75\mathrm{N/C}.$

(2b) $V = k\frac{(-11\mathrm{nC})}{3\mathrm{m}} + k\frac{17\mathrm{nC}}{8\mathrm{m}} = -33.0\mathrm{V} + 19.1\mathrm{V} = -13.9\mathrm{V}.$

Consider two plane surfaces (of rectangular and a circular shape) with area vectors $\vec{A}_1$ pointing in positive z-direction) and $\vec{A}_2$ pointing in positive x-direction.

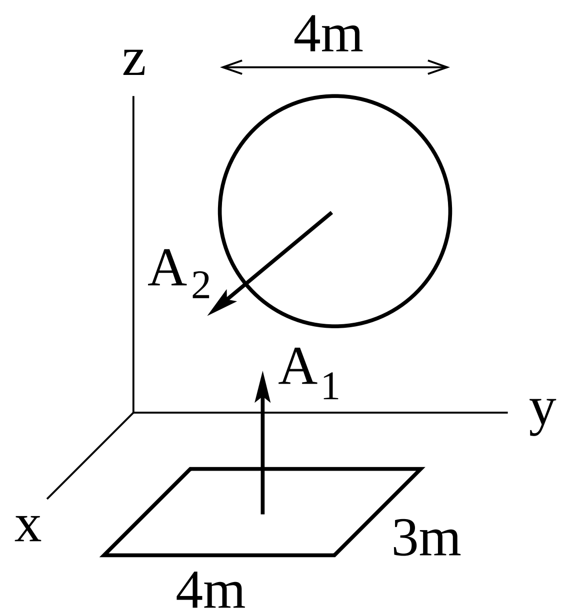
The region is filled with a uniform electric field

(1) $\vec{E} = (4\hat{i} + 5\hat{j} - 7\hat{k})$N/C,

(2) $\vec{E} = (-6\hat{i} + 4\hat{j} + 5\hat{k})$N/C.

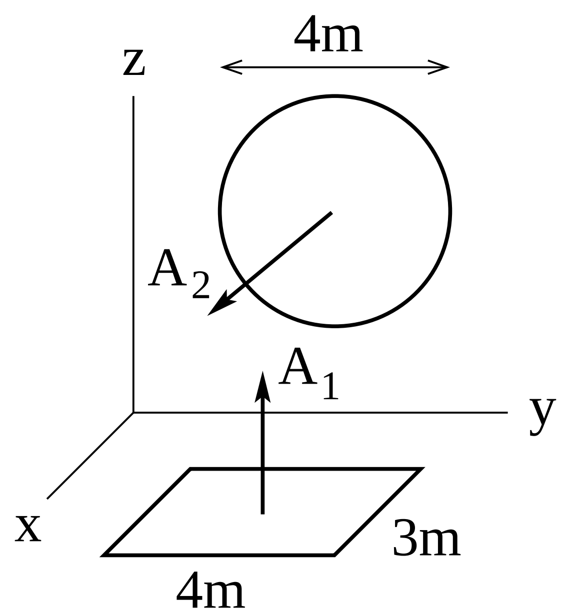
(a) Find the electric flux $\Phi_E^{(1)}$ through area A_1.

(b) Find the electric flux $\Phi_E^{(2)}$ through area A_2.



Unit Exam I: Problem #2 (Fall ’15)

Consider two plane surfaces (of rectangular and a circular shape) with area vectors $\vec{A}_1$ pointing in positive z-direction) and $\vec{A}_2$ pointing in positive x-direction.
The region is filled with a uniform electric field
(1) $\vec{E} = (4\hat{\imath} + 5\hat{\jmath} - 7\hat{k})$N/C,
(2) $\vec{E} = (-6\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$N/C.
(a) Find the electric flux $\Phi_E^{(1)}$ through area A_1.
(b) Find the electric flux $\Phi_E^{(2)}$ through area A_2.



Solution:

(1a) $\Phi_E^{(1)} = \vec{E} \cdot \vec{A}_1 = (-7\mathrm{N/C})(12.0\mathrm{m}^2) = -84.0\mathrm{Nm}^2/\mathrm{C}.$

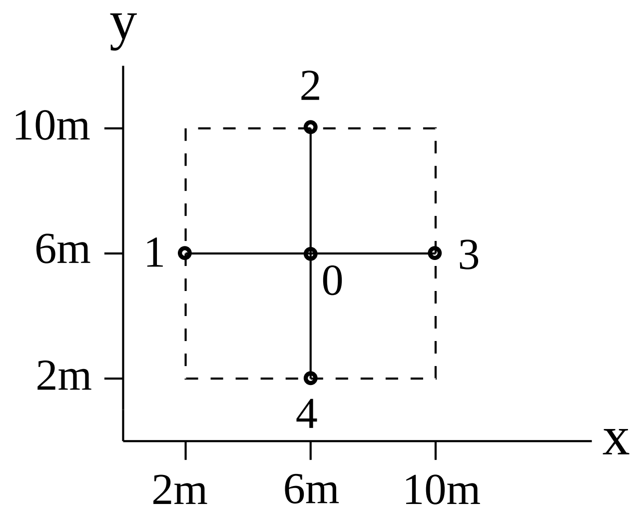
(1b) $\Phi_E^{(2)} = \vec{E} \cdot \vec{A}_2 = (4\mathrm{N/C})(12.6\mathrm{m}^2) = 50.4\mathrm{Nm}^2/\mathrm{C}.$

(2a) $\Phi_E^{(1)} = \vec{E} \cdot \vec{A}_1 = (5\mathrm{N/C})(12.0\mathrm{m}^2) = 60.0\mathrm{Nm}^2/\mathrm{C}.$

(2b) $\Phi_E^{(2)} = \vec{E} \cdot \vec{A}_2 = (-6\mathrm{N/C})(12.6\mathrm{m}^2) = -75.6\mathrm{Nm}^2/\mathrm{C}.$

Unit Exam I: Problem #3 (Fall ’15)

Consider a region of space with a uniform electric field
(1) $\mathbf{E} = 1.2\mathrm{V/m}\,\hat{\mathbf{j}}$, (2) $\mathbf{E} = 0.6\mathrm{V/m}\,\hat{\mathbf{i}}$. Ignore gravity.
(a) If the electric potential vanishes at point 0, what are the electric potentials at points 1, 2, 3, 4?
(b) If a proton ($m = 1.67 \times 10^{-27}$kg, $q = 1.60 \times 10^{-19}$C) is released from rest at point 0, toward which point will it start moving?
(c) What will be the kinetic energy of the proton when it gets there?



Unit Exam I: Problem #3 (Fall ’15)

Consider a region of space with a uniform electric field
(1) $\mathbf{E} = 1.2\mathrm{V/m}\,\hat{\mathbf{j}}$, (2) $\mathbf{E} = 0.6\mathrm{V/m}\,\hat{\mathbf{i}}$. Ignore gravity.
(a) If the electric potential vanishes at point 0, what are the electric potentials at points 1, 2, 3, 4?
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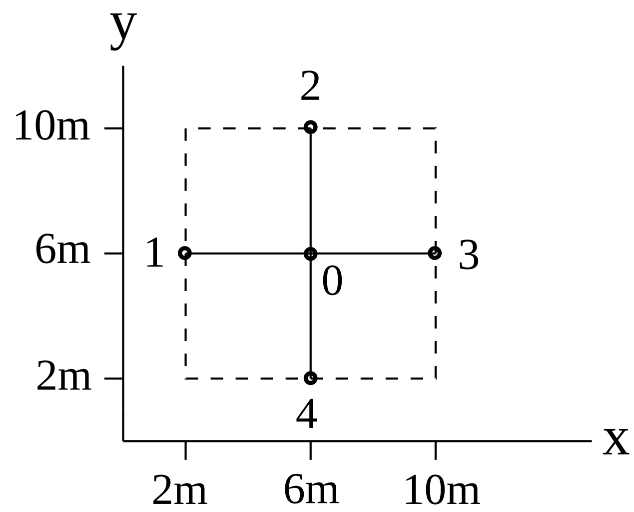


Solution:

(1a) $V_1 = 0,\ V_2 = -4.8\mathrm{V},\ V_3 = 0,\ V_4 = +4.8\mathrm{V}.$

(1b) $\mathbf{F} = q\mathbf{E}$ (toward point 2).

(1c) $\Delta V = (V_2 - V_0) = -4.8\mathrm{V},$
$\Delta U = q\Delta V = -7.68 \times 10^{-19}\mathrm{J},$
$K = -\Delta U = +7.68 \times 10^{-19}\mathrm{J}.$

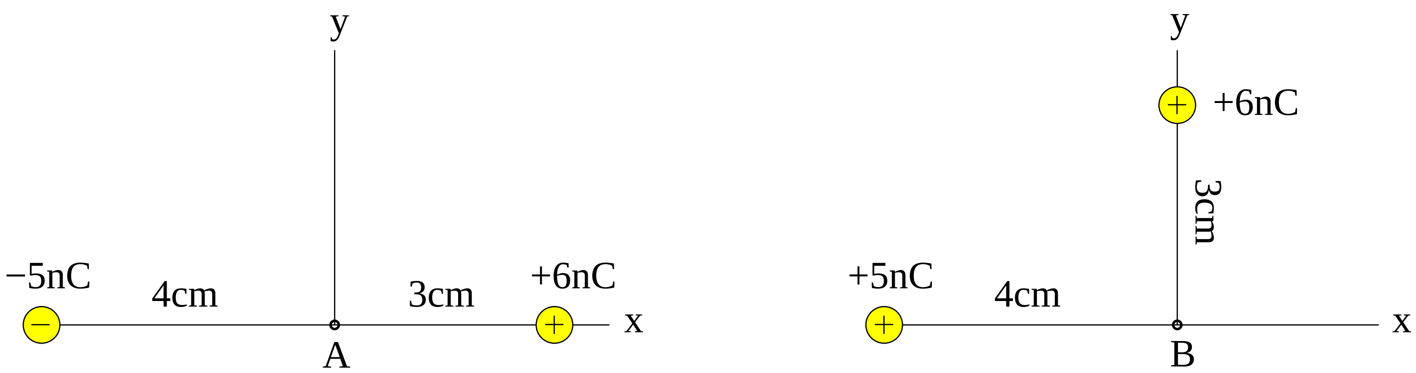
(2a) $V_1 = 2.4\mathrm{V},\ V_2 = 0,\ V_3 = -2.4\mathrm{V},\ V_4 = 0.$

(2b) $\mathbf{F} = q\mathbf{E}$ (toward point 3).

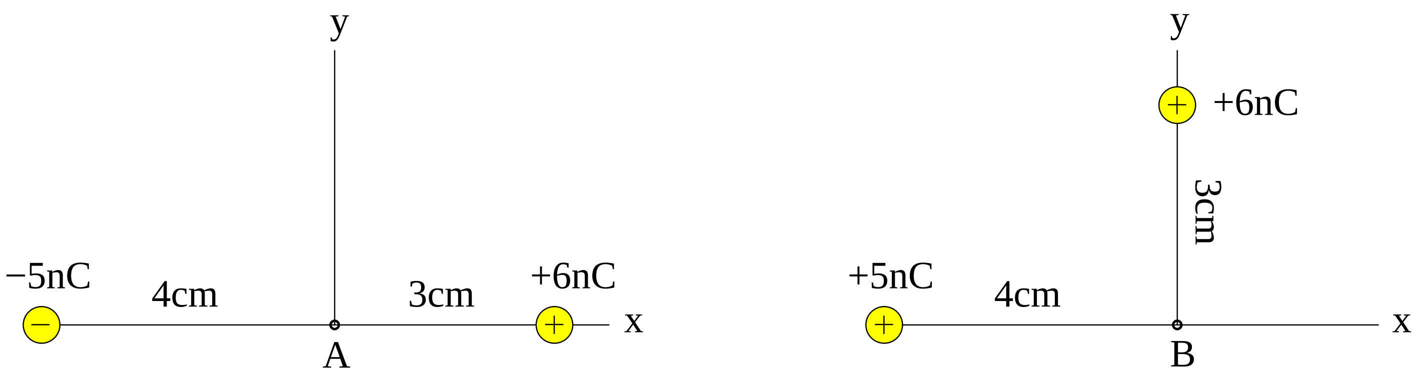
(2c) $\Delta V = (V_3 - V_0) = -2.4\mathrm{V},$
$\Delta U = q\Delta V = -3.84 \times 10^{-19}\mathrm{J},$
$K = -\Delta U = +3.84 \times 10^{-19}\mathrm{J}.$

Unit Exam I: Problem #1 (Spring ’16)

Consider a pair of point charges in two different configurations. Find the electric potential V and the components E_x and E_y of the electric field at point A and at point B.



Consider a pair of point charges in two different configurations. Find the electric potential V and the components E_x and E_y of the electric field at point A and at point B.



Solution:

- $V^{(A)} = k\dfrac{6\text{nC}}{3\text{cm}} + k\dfrac{(-5\text{nC})}{4\text{cm}} = 1800\text{V} - 1125\text{V} = 675\text{V}.$
- $E_x^{(A)} = -k\dfrac{|6\text{nC}|}{(3\text{cm})^2} - k\dfrac{|-5\text{nC}|}{(4\text{cm})^2} = -88\,125\text{V/m}, \qquad E_y^{(A)} = 0.$
- $V^{(B)} = k\dfrac{6\text{nC}}{3\text{cm}} + k\dfrac{5\text{nC}}{4\text{cm}} = 1800\text{V} + 1125\text{V} = 2925\text{V}.$
- $E_x^{(B)} = k\dfrac{|5\text{nC}|}{(4\text{cm})^2} = 28\,125\text{V/m}, \qquad E_y^{(B)} = -k\dfrac{|6\text{nC}|}{(3\text{cm})^2} = -60\,000\text{V/m}.$

A charged conducting spherical shell has a 4m inner radius and an 8m outer radius. The charge on the outer surface is $Q_{\text{ext}} = -7\text{nC}$.

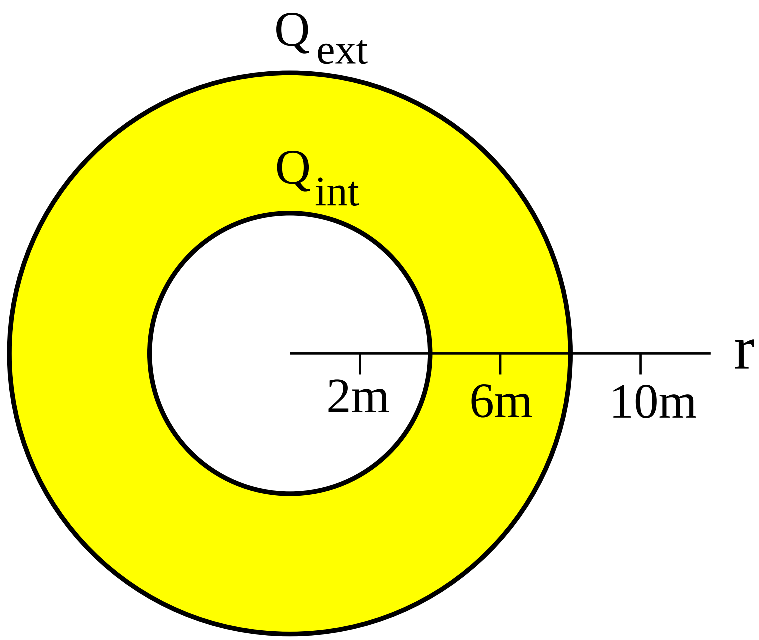
(a) Find the charge Q_{int} on the inner surface of the shell.

(b) Find the surface charge density σ_{ext} on the outer surface of the shell.

(c) Find the magnitude of the electric field E at radius $r = 6\text{m}$.

(d) Find the electric flux Φ_E through a Gaussian sphere of radius $r = 10\text{m}$.

(e) Find the magnitude of the electric field E at radius $r = 10\text{m}$.



Unit Exam I: Problem #2 (Spring ’16)

A charged conducting spherical shell has a 4m inner radius and an 8m outer radius. The charge on the outer surface is $Q_{\rm ext} = -7{\rm nC}$.

(a) Find the charge $Q_{\rm int}$ on the inner surface of the shell.

(b) Find the surface charge density $\sigma_{\rm ext}$ on the outer surface of the shell.

(c) Find the magnitude of the electric field E at radius $r = 6{\rm m}$.

(d) Find the electric flux Φ_E through a Gaussian sphere of radius $r = 10{\rm m}$.

(e) Find the magnitude of the electric field E at radius $r = 10{\rm m}$.

Solution:

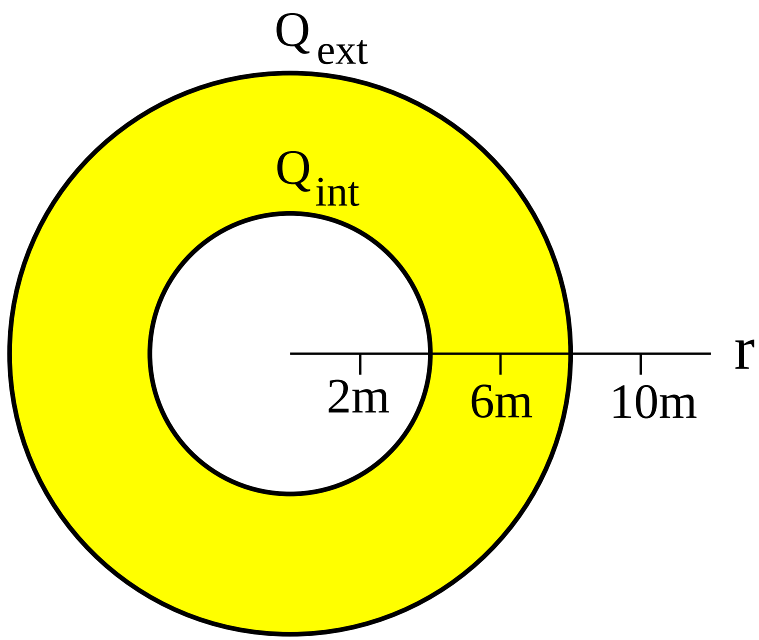
(a) $Q_{\rm int} = 0$ (inferred from Gauss’ law.)

(b) $\sigma_{\rm ext} = \dfrac{-7{\rm nC}}{4\pi(8{\rm m})^2} = -8.70 \times 10^{-12}{\rm C/m^2}$.

(c) $E = 0$ (inside conducting material.)

(d) $\Phi_E = \dfrac{-7{\rm nC}}{\epsilon_0} = -791{\rm Nm^2/C}$.

(e) $E = k\dfrac{|-7{\rm nC}|}{(10{\rm m})^2} = 0.63{\rm V/m}$.



Unit Exam I: Problem #3 (Spring '15)

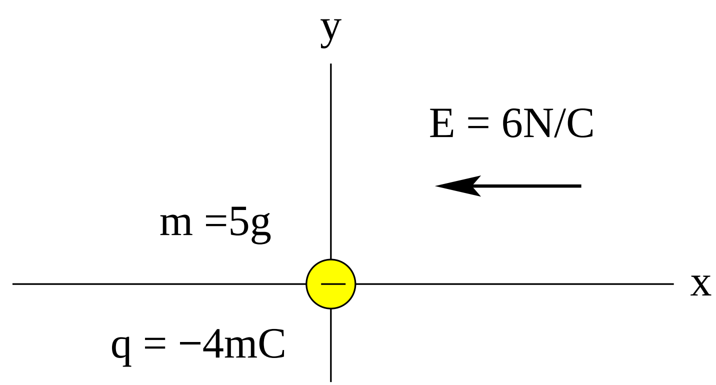
Consider a region of uniform electric field as shown. A charged particle is released from rest at time $t = 0$ at the origin of the coordinate system.

(a) Find the acceleration a_x of the particle at time $t = 3$s.

(b) Find the velocity v_x of the particle at time $t = 3$s.

(c) Find the position x of the particle at time $t = 3$s.

(d) In what time Δt does the particle move from $x = 10$m to $x = 20$m?



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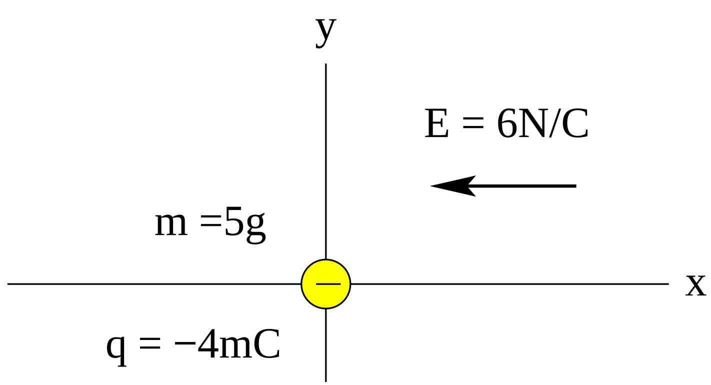


Solution:

(a) $a_x = \dfrac{q}{m}E = \dfrac{-4\times 10^{-3}\mathrm{C}}{5\times 10^{-3}\mathrm{kg}}(-6\mathrm{N/C}) = 4.8\mathrm{m/s}^2.$

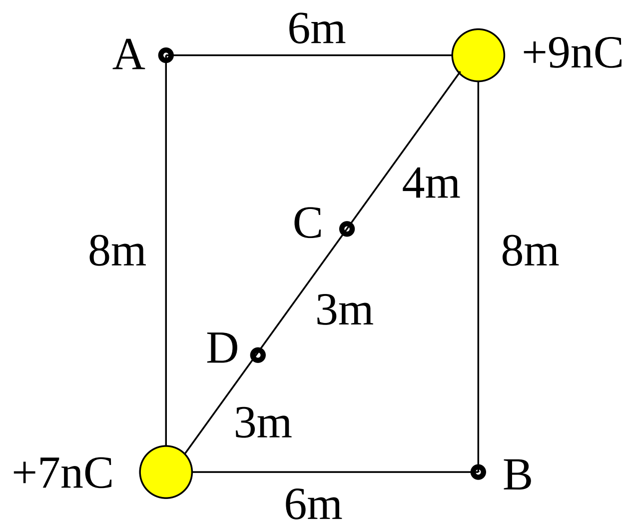
(b) $v_x = a_x t = (4.8\mathrm{m/s}^2)(3\mathrm{s}) = 14.4\mathrm{m/s}.$

(c) $x = \dfrac{1}{2}a_x t^2 = 0.5(4.8\mathrm{m/s}^2)(3\mathrm{s})^2 = 21.6\mathrm{m}.$

(d) $\Delta t = \sqrt{\dfrac{2(20\mathrm{m})}{4.8\mathrm{m/s}^2}} - \sqrt{\dfrac{2(10\mathrm{m})}{4.8\mathrm{m/s}^2}} = 2.89\mathrm{s} - 2.04\mathrm{s} = 0.85\mathrm{s}.$

Unit Exam I: Problem #1 (Fall '16)

Consider two point charges positioned as shown.
(a) Find the magnitude of the electric field at point C $[D]$.
(b) Draw the field direction at point C $[D]$ by an arrow.
(c) Find the electric potential at point A $[B]$.



Unit Exam I: Problem #1 (Fall '16)

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(a) Find the magnitude of the electric field at point C $[D]$.
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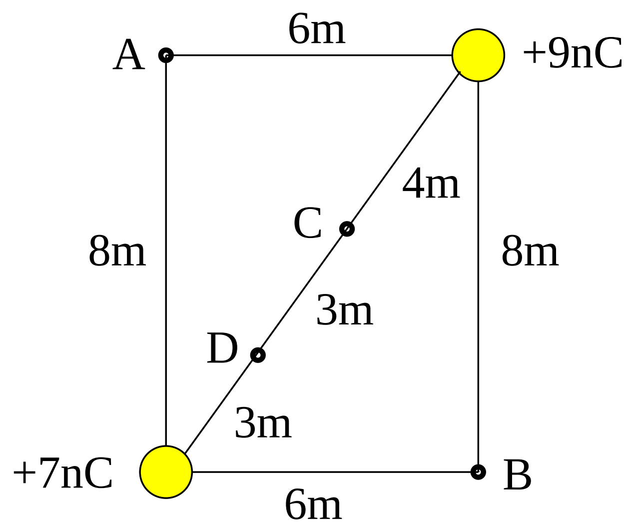
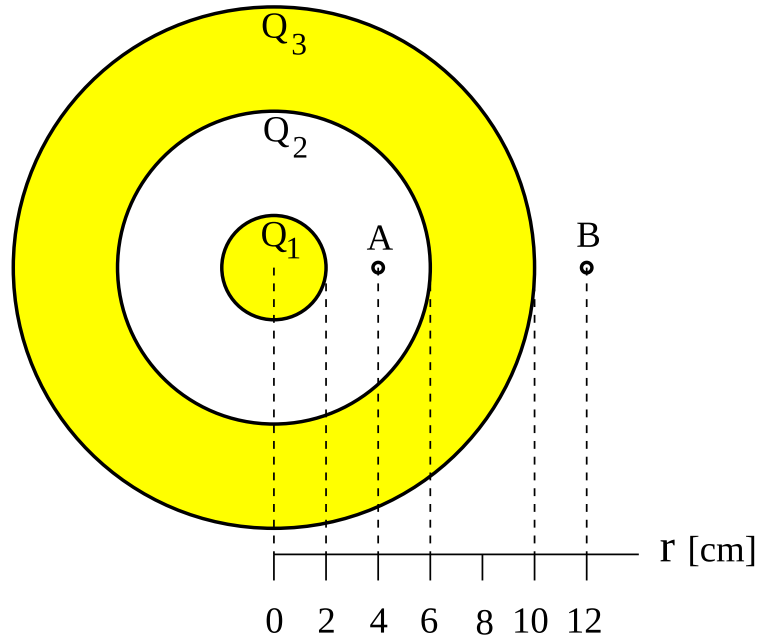


Solution:

- $E_C = k\dfrac{9\text{nC}}{(4\text{m})^2} - k\dfrac{7\text{nC}}{(6\text{m})^2} = 5.06\text{V/m} - 1.75\text{V/m} = 3.31\text{V/m}.$
  $[E_D = k\dfrac{7\text{nC}}{(3\text{m})^2} - k\dfrac{9\text{nC}}{(7\text{m})^2} = 7.00\text{V/m} - 1.65\text{V/m} = 5.35\text{V/m}].$
- Down/left along diagonal [Up/right along diagonal].
- $V_A = k\dfrac{9\text{nC}}{6\text{m}} + k\dfrac{7\text{nC}}{8\text{m}} = 13.50\text{V} + 7.88\text{V} = 21.4\text{V}.$
  $[V_B = k\dfrac{9\text{nC}}{8\text{m}} + k\dfrac{7\text{nC}}{6\text{m}} = 10.1\text{V} + 10.5\text{V} = 20.6\text{V}].$

Unit Exam I: Problem #2 (Fall '16)

Consider a conducting sphere and a conducting spherical shell as shown in cross section. The charges on the two surfaces of the shell are
$Q_2 = -5$nC and $Q_3 = +2$nC [$Q_2 = +4$nC and $Q_3 = -3$nC].
(a) Find the charge Q_1 on the surface of the conducting sphere.
(b) Find magnitude and direction of the electric field at point A.
(c) Find magnitude and direction of the electric field at point B.



Unit Exam I: Problem #2 (Fall '16)

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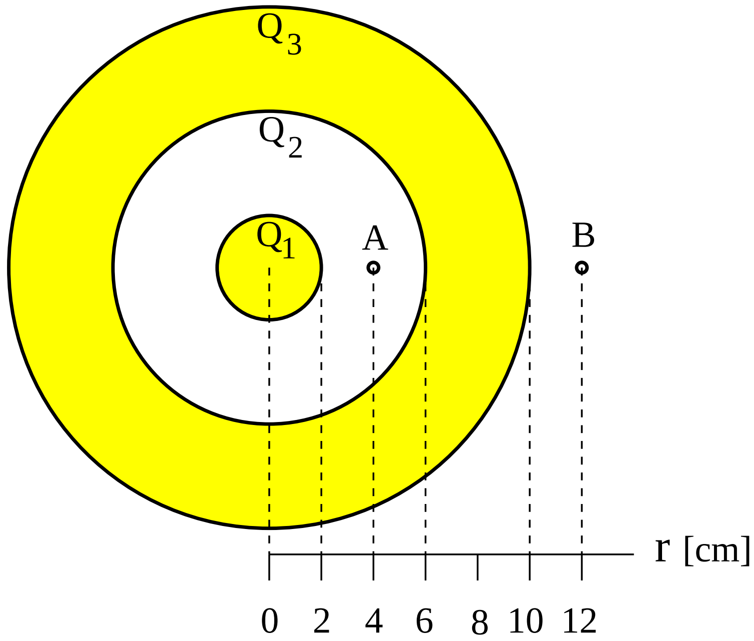
$Q_2 = -5$nC and $Q_3 = +2$nC [$Q_2 = +4$nC and $Q_3 = -3$nC].
(a) Find the charge Q_1 on the surface of the conducting sphere.
(b) Find magnitude and direction of the electric field at point A.
(c) Find magnitude and direction of the electric field at point B.



Solution:

(a) Gauss' law implies that
$Q_1 = -Q_2 = +5$nC [$Q_1 = -Q_2 = -4$nC].

(b) $E_A = k\dfrac{5\text{nC}}{(4\text{cm})^2} = 28.1 \times 10^3\text{N/C}$ (right)

$[E_A = k\dfrac{4\text{nC}}{(4\text{cm})^2} = 22.5 \times 10^3\text{N/C}$ (left)].

(c) $E_B = k\dfrac{2\text{nC}}{(12\text{cm})^2} = 1.25 \times 10^3\text{N/C}$ (right)

$[E_B = k\dfrac{3\text{nC}}{(12\text{cm})^2} = 1.88 \times 10^3\text{N/C}$ (left)].

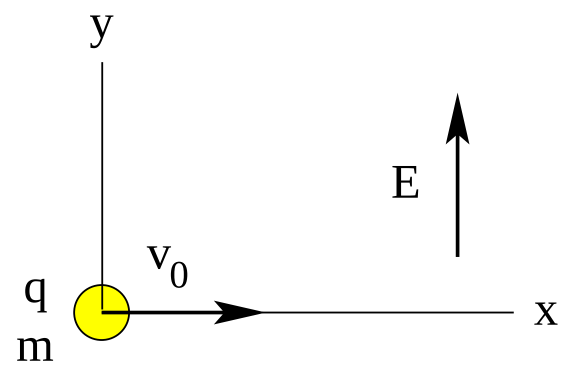
Unit Exam I: Problem #3 (Fall '16)

Consider a region of uniform electric field E. A particle with charge q and mass m is projected at time $t = 0$ with initial velocity v_0. The specifications are $m = 3$g, $q = 2$mC, $v_0 = 4$m/s, $E = 5$N/C. [$m = 2$g, $q = 3$mC, $v_0 = 5$m/s, $E = 4$N/C]. Ignore gravity.

(a) Find the components F_x and F_y of the electric force acting on the particle at time $t = 1.5$s.

(b) Find the components v_x and v_y of the velocity at time $t = 1.5$s.

(c) Find the kinetic energy at time $t = 1.5$s.

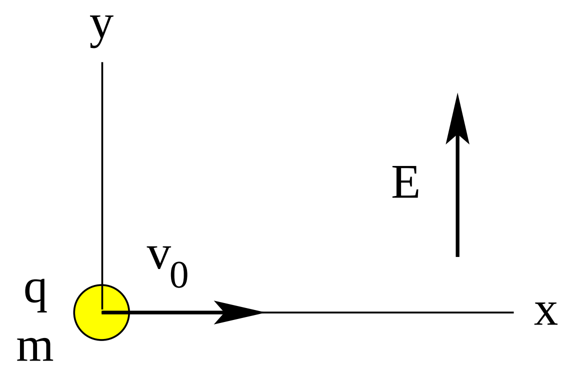


Consider a region of uniform electric field E. A particle with charge q and mass m is projected at time $t = 0$ with initial velocity v_0. The specifications are $m = 3$g, $q = 2$mC, $v_0 = 4$m/s, $E = 5$N/C. [$m = 2$g, $q = 3$mC, $v_0 = 5$m/s, $E = 4$N/C]. Ignore gravity.

(a) Find the components F_x and F_y of the electric force acting on the particle at time $t = 1.5$s.

(b) Find the components v_x and v_y of the velocity at time $t = 1.5$s.

(c) Find the kinetic energy at time $t = 1.5$s.



Solution:

(a) $F_x = 0, \quad F_y = qE = 10\text{mN}$
$[F_x = 0, \quad F_y = qE = 12\text{mN}]$.

(b) $v_x = v_0 = 4\text{m/s}, \quad v_y = \dfrac{F_y}{m}t = 5\text{m/s}$
$[v_x = v_0 = 5\text{m/s}, \quad v_y = \dfrac{F_y}{m}t = 9\text{m/s}]$.

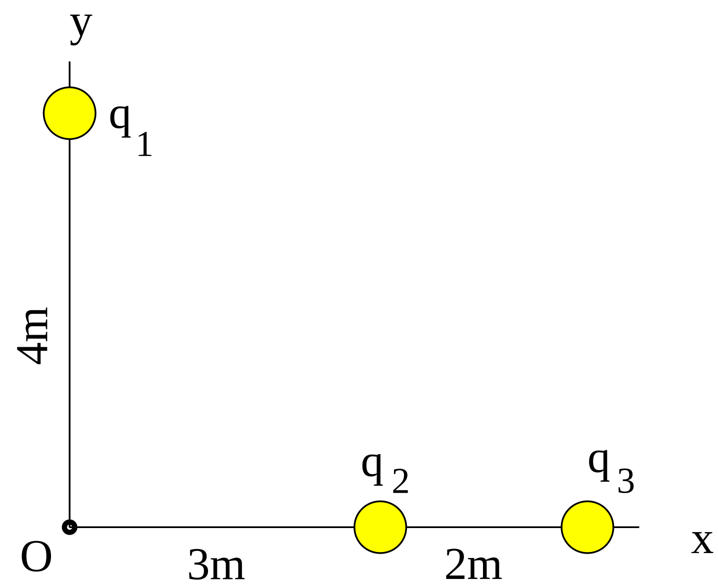
(c) $K = \dfrac{1}{2}(3 \times 10^{-3}\text{kg})[(4\text{m/s})^2 + (5\text{m/s})^2] = 61.5\text{mJ}$
$[K = \dfrac{1}{2}(2 \times 10^{-3}\text{kg})[(5\text{m/s})^2 + (9\text{m/s})^2] = 106\text{mJ}]$.

Point charges $q_1 = +1$nC, $q_2 = +2$nC, $q_3 = -3$nC [$q_1 = -1$nC, $q_2 = +2$nC, $q_3 = +3$nC] are positioned as shown.

(a) Find the components E_x and E_y of the electric field at point O.

(b) Find the electric potential V at point O.

(c) Find the direction $(\uparrow, \nearrow, \rightarrow, \searrow, \downarrow, \swarrow, \leftarrow, \nwarrow)$ of the resultant Coulomb force on charge q_2.



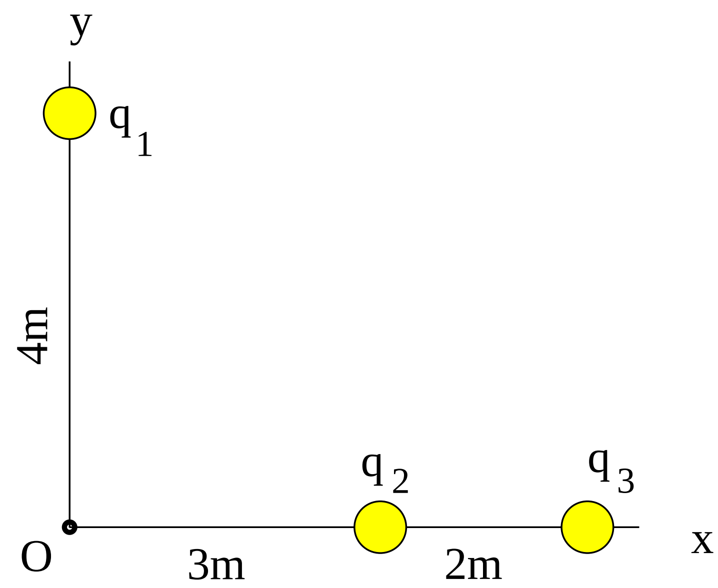
Unit Exam I: Problem #1 (Spring '17)

Point charges $q_1 = +1\text{nC}$, $q_2 = +2\text{nC}$, $q_3 = -3\text{nC}$ [$q_1 = -1\text{nC}$, $q_2 = +2\text{nC}$, $q_3 = +3\text{nC}$] are positioned as shown.

(a) Find the components E_x and E_y of the electric field at point O.

(b) Find the electric potential V at point O.

(c) Find the direction $(\uparrow, \nearrow, \rightarrow, \searrow, \downarrow, \swarrow, \leftarrow, \nwarrow)$ of the resultant Coulomb force on charge q_2.



Solution:

(a) $E_x = -k\dfrac{|q_2|}{(3\text{m})^2} + k\dfrac{|q_3|}{(5\text{m})^2} = -0.92\,\text{N/C}$

$\left[E_x = -k\dfrac{|q_2|}{(3\text{m})^2} - k\dfrac{|q_3|}{(5\text{m})^2} = -3.08\,\text{N/C}\right]$

$E_y = -k\dfrac{|q_1|}{(4\text{m})^2} = -0.56\,\text{N/C}$

$\left[E_y = +k\dfrac{|q_1|}{(4\text{m})^2} = +0.56\,\text{N/C}\right]$

(b) $V = k\dfrac{q_1}{4\text{m}} + k\dfrac{q_2}{3\text{m}} + k\dfrac{q_3}{5\text{m}} = 2.85\text{V}$ $\left[V = k\dfrac{q_1}{4\text{m}} + k\dfrac{q_2}{3\text{m}} + k\dfrac{q_3}{5\text{m}} = 9.15\text{V}\right]$

(c) $\searrow$ [$\nwarrow$]

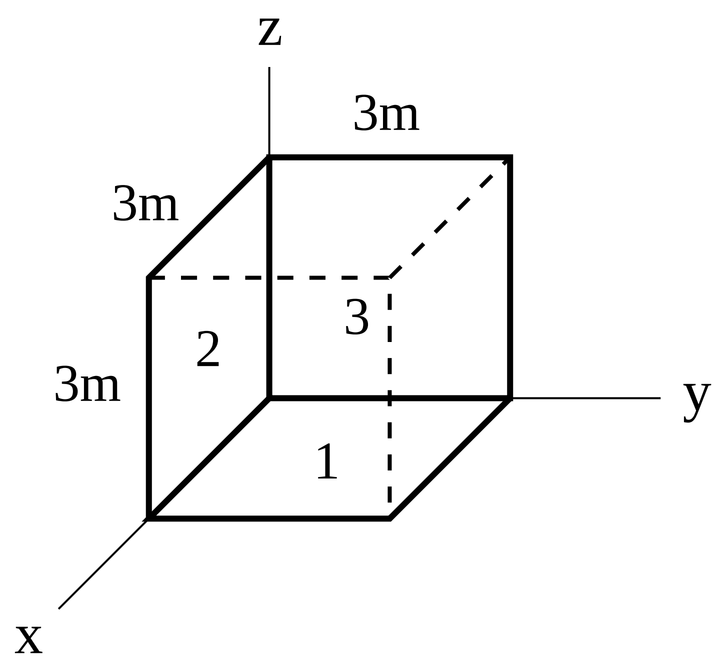
Consider a Gaussian surface in the form of a cube with edges of length 3m placed into a region of uniform electric field $\mathbf{E} = (5\hat{i} - 4\hat{j} + 6\hat{k})$N/C [$\mathbf{E} = (8\hat{i} + 7\hat{j} - 9\hat{k})$N/C].

(a) Find the electric flux $\Phi_E^{(1)}$ through face 1 (in xy plane).

(b) Find the electric flux $\Phi_E^{(2)}$ through face 2 (in xz plane).

(c) Find the electric flux $\Phi_E^{(3)}$ through face 3 (in yz plane).

(d) Find the electric flux $\Phi_E^{(tot)}$ through all six faces added up.



Consider a Gaussian surface in the form of a cube with edges of length 3m placed into a region of uniform electric field $\mathbf{E} = (5\hat{i} - 4\hat{j} + 6\hat{k})$N/C [$\mathbf{E} = (8\hat{i} + 7\hat{j} - 9\hat{k})$N/C].

(a) Find the electric flux $\Phi_E^{(1)}$ through face 1 (in xy plane).

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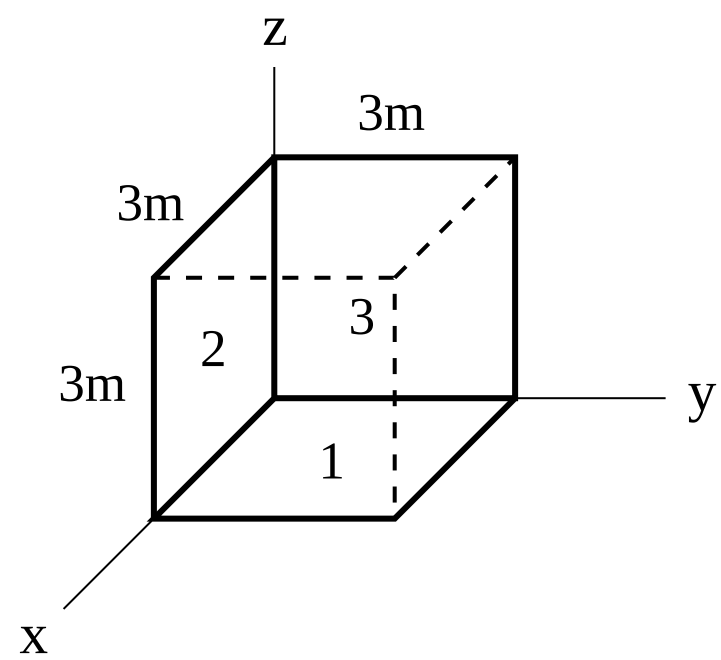
(d) Find the electric flux $\Phi_E^{(tot)}$ through all six faces added up.



Solution:

(a) $\Phi_E^{(1)} = \vec{E} \cdot \vec{A}_1 = (6\mathrm{N/C})\hat{k} \cdot (-9\mathrm{m}^2)\hat{k} = -54\mathrm{Nm}^2/\mathrm{C}$

$[\Phi_E^{(1)} = \vec{E} \cdot \vec{A}_1 = (-9\mathrm{N/C})\hat{k} \cdot (-9\mathrm{m}^2)\hat{k} = 81\mathrm{Nm}^2/\mathrm{C}]$

(b) $\Phi_E^{(2)} = \vec{E} \cdot \vec{A}_2 = (-4\mathrm{N/C})\hat{j} \cdot (-9\mathrm{m}^2)\hat{j} = +36\mathrm{Nm}^2/\mathrm{C}$

$[\Phi_E^{(2)} = \vec{E} \cdot \vec{A}_2 = (7\mathrm{N/C})\hat{j} \cdot (-9\mathrm{m}^2)\hat{j} = -63\mathrm{Nm}^2/\mathrm{C}]$

(c) $\Phi_E^{(3)} = \vec{E} \cdot \vec{A}_3 = (5\mathrm{N/C})\hat{i} \cdot (-9\mathrm{m}^2)\hat{i} = -45\mathrm{Nm}^2/\mathrm{C}$

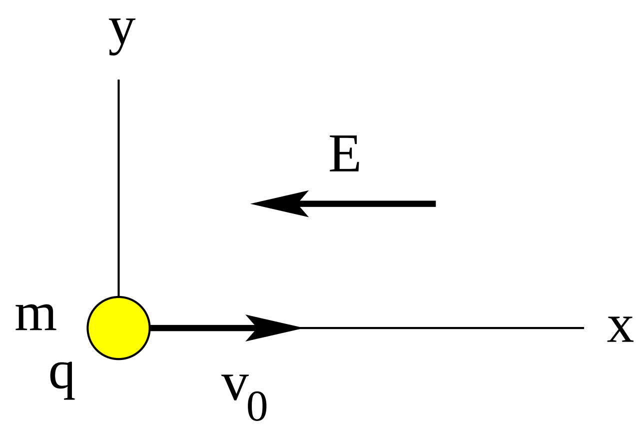
$[\Phi_E^{(3)} = \vec{E} \cdot \vec{A}_3 = (8\mathrm{N/C})\hat{i} \cdot (-9\mathrm{m}^2)\hat{i} = -72\mathrm{Nm}^2/\mathrm{C}]$

(d) $\Phi_E^{(tot)} = \dfrac{Q_{in}}{\epsilon_0} = 0 \quad \left[\Phi_E^{(tot)} = \dfrac{Q_{in}}{\epsilon_0} = 0\right]$

Unit Exam I: Problem #3 (Spring ’17)

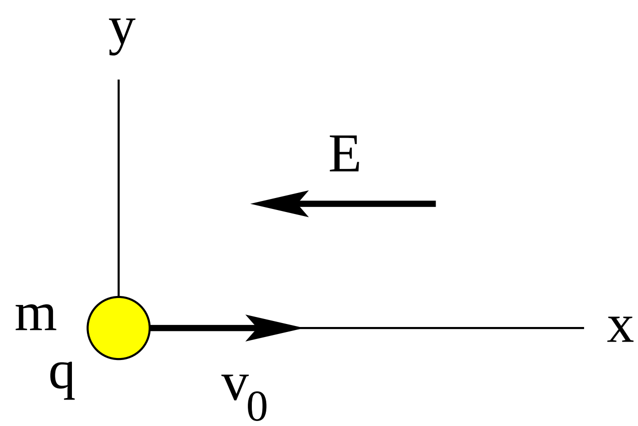
Consider a region of uniform electric field $\mathbf{E} = -2\mathrm{N/C}\hat{\mathbf{i}}$ [$\mathbf{E} = -3\mathrm{N/C}\hat{\mathbf{i}}$]. A charged particle $(m = 0.04\mathrm{kg},\ q = 6\mathrm{mC})$ [$(m = 0.05\mathrm{kg},\ q = 7\mathrm{mC})$] is projected at time $t = 0$ with initial velocity $\mathbf{v}_0 = 8\mathrm{m/s}\hat{\mathbf{i}}$ [$\mathbf{v}_0 = 9\mathrm{m/s}\hat{\mathbf{i}}$] from the origin of the coordinate system as shown.

(a) Find the the acceleration a_x of the particle at time $t = 2.5$s.
(b) Find its velocity v_x at time $t = 2.5$s.
(c) Find its position x at time $t = 2.5$s.



Unit Exam I: Problem #3 (Spring '17)

Consider a region of uniform electric field $\mathbf{E} = -2\mathrm{N/C}\hat{\mathbf{i}}$ [$\mathbf{E} = -3\mathrm{N/C}\hat{\mathbf{i}}$]. A charged particle $(m = 0.04\mathrm{kg},\ q = 6\mathrm{mC})$ [$(m = 0.05\mathrm{kg},\ q = 7\mathrm{mC})$] is projected at time $t = 0$ with initial velocity $\mathbf{v}_0 = 8\mathrm{m/s}\hat{\mathbf{i}}$ [$\mathbf{v}_0 = 9\mathrm{m/s}\hat{\mathbf{i}}$] from the origin of the coordinate system as shown.
(a) Find the the acceleration a_x of the particle at time $t = 2.5$s.
(b) Find its velocity v_x at time $t = 2.5$s.
(c) Find its position x at time $t = 2.5$s.



Solution:

(a) $a_x = -\frac{q}{m}E = -\frac{6\times10^{-3}\mathrm{C}}{4\times10^{-2}\mathrm{kg}}(2\mathrm{N/C}) = -0.3\mathrm{m/s}^2$

$\left[a_x = -\frac{q}{m}E = -\frac{7\times10^{-3}\mathrm{C}}{5\times10^{-2}\mathrm{kg}}(3\mathrm{N/C}) = -0.42\mathrm{m/s}^2\right]$

(b) $v_x = v_0 + a_x t = 8\mathrm{m/s} - (0.3\mathrm{m/s}^2)(2.5\mathrm{s}) = 7.25\mathrm{m/s}$

$[v_x = v_0 + a_x t = 9\mathrm{m/s} - (0.42\mathrm{m/s}^2)(2.5\mathrm{s}) = 7.95\mathrm{m/s}]$

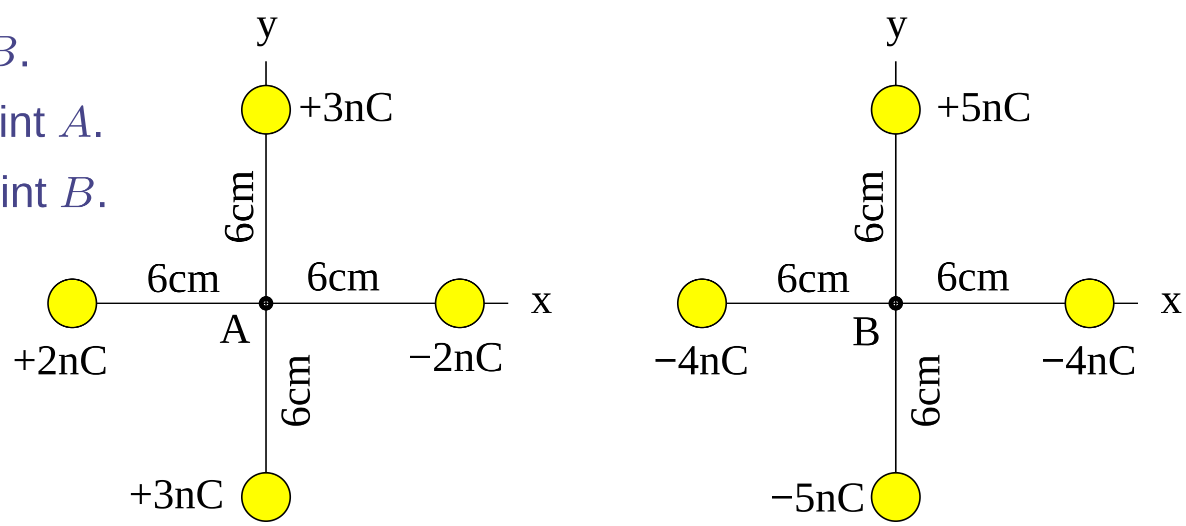
(c) $x = v_0 t + \frac{1}{2}a_x t^2 = (8\mathrm{m/s})(2.5\mathrm{s}) - 0.5(0.3\mathrm{m/s}^2)(2.5\mathrm{s})^2 = 19.1\mathrm{m}$

$\left[x = v_0 t + \frac{1}{2}a_x t^2 = (9\mathrm{m/s})(2.5\mathrm{s}) - 0.5(0.42\mathrm{m/s}^2)(2.5\mathrm{s})^2 = 21.2\mathrm{m}\right]$

Unit Exam I: Problem #1 (Fall ’17)

Consider point charges positioned in two coordinate systems as shown.

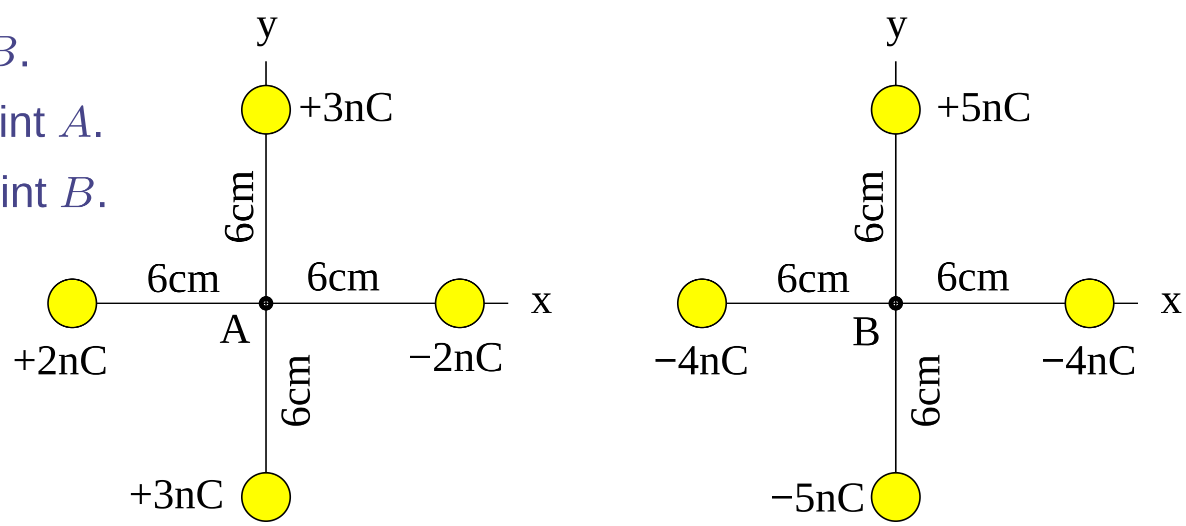
- Find the electric field $\mathbf{E}_A$ at point A.
- Find the electric field $\mathbf{E}_B$ at point B.
- Find the electric potential V_A at point A.
- Find the electric potential V_B at point B.



Unit Exam I: Problem #1 (Fall ’17)

Consider point charges positioned in two coordinate systems as shown.

- Find the electric field $\mathbf{E}_A$ at point A.
- Find the electric field $\mathbf{E}_B$ at point B.
- Find the electric potential V_A at point A.
- Find the electric potential V_B at point B.



Solution:

$$\mathbf{E}_A = 2k\frac{|2\text{nC}|}{(6\text{cm})^2}\,\hat{\mathbf{i}} = 1.00 \times 10^4\ \text{N/C}\,\hat{\mathbf{i}}$$

$$\mathbf{E}_B = -2k\frac{|5\text{nC}|}{(6\text{cm})^2}\,\hat{\mathbf{j}} = -2.50 \times 10^4\ \text{N/C}\,\hat{\mathbf{j}}$$

$$V_A = 2k\frac{3\text{nC}}{6\text{cm}} = 9.00 \times 10^2\text{V}$$

$$V_B = 2k\frac{(-4\text{nC})}{6\text{cm}} = -12.0 \times 10^2\text{V}$$

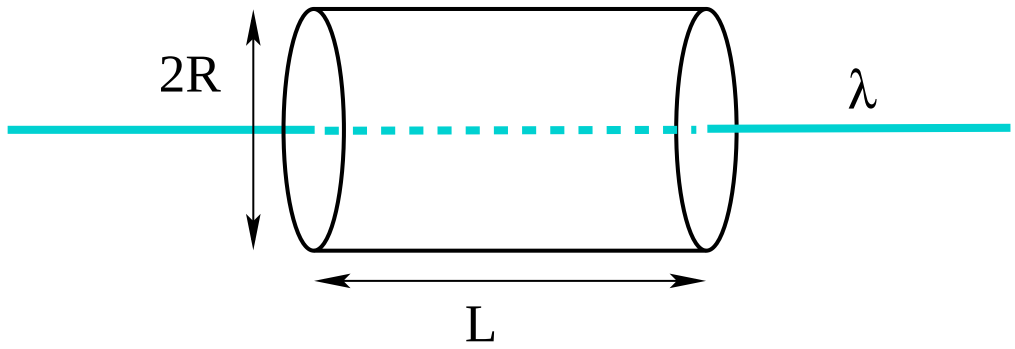
Unit Exam I: Problem #2 (Fall '17)

Consider a long charged rod with charge per unit length $\lambda = 3\mu\mathrm{C/m}$ [$\lambda = 2\mu\mathrm{C/m}$]. A Gaussian cylinder of radius $R = 4\mathrm{cm}$ [$R = 5\mathrm{cm}$] and length $L = 12\mathrm{cm}$ [$L = 15\mathrm{cm}$] is placed with its axis along the rod as shown.

(a) Find the area A of the Gaussian cylinder.

(b) Find the electric charge Q_{in} inside the cylinder.

(c) Find the electric flux Φ_E through the Gaussian cylinder.



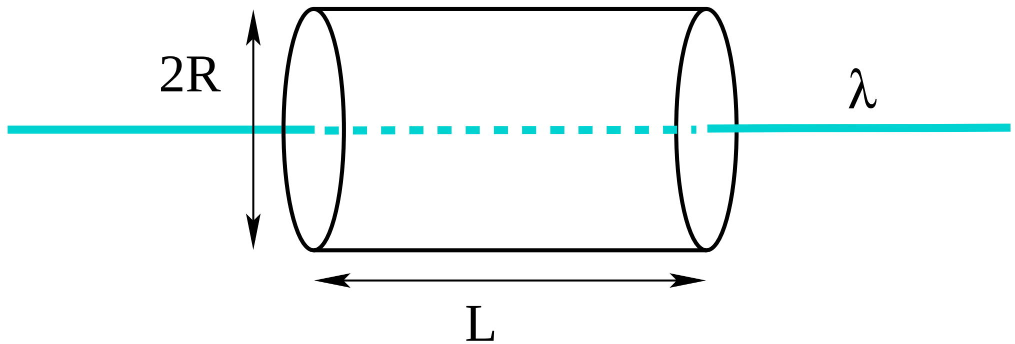
Unit Exam I: Problem #2 (Fall ’17)

Consider a long charged rod with charge per unit length $\lambda = 3\mu\mathrm{C/m}$ [$\lambda = 2\mu\mathrm{C/m}$]. A Gaussian cylinder of radius $R = 4\mathrm{cm}$ [$R = 5\mathrm{cm}$] and length $L = 12\mathrm{cm}$ [$L = 15\mathrm{cm}$] is placed with its axis along the rod as shown.

(a) Find the area A of the Gaussian cylinder.

(b) Find the electric charge Q_{in} inside the cylinder.

(c) Find the electric flux Φ_E through the Gaussian cylinder.



Solution:

(a) $A = 2 \times \pi(4\mathrm{cm})^2 + 2\pi(4\mathrm{cm})(12\mathrm{cm}) = 4.03 \times 10^{-2}\mathrm{m}^2$

$\left[A = 2 \times \pi(5\mathrm{cm})^2 + 2\pi(5\mathrm{cm})(15\mathrm{cm} = 6.28 \times 10^{-2}\mathrm{m}^2\right]$

(b) $Q_{in} = \lambda L = (3\mu\mathrm{C/m})(12\mathrm{cm}) = 0.36\mu\mathrm{C}$ $[Q_{in} = \lambda L = (2\mu\mathrm{C/m})(15\mathrm{cm}) = 0.30\mu\mathrm{C}]$

(c) $\Phi_E = \dfrac{Q_{in}}{\epsilon_0} = 4.07 \times 10^4\mathrm{Nm^2/C}$ $[\Phi_E = \dfrac{Q_{in}}{\epsilon_0} = 3.39 \times 10^4\mathrm{Nm^2/C}]$

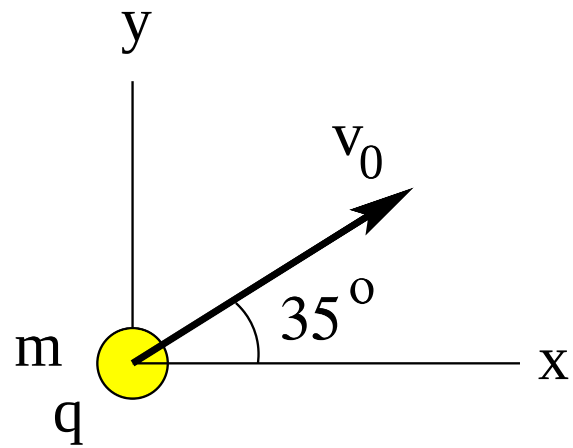
Unit Exam I: Problem #3 (Fall '17)

In a region of uniform electric field $\mathbf{E} = 9\mathrm{N/C}\,\hat{\mathbf{i}} + 7\mathrm{N/C}\,\hat{\mathbf{j}}$, a charged particle $(m = 0.02\mathrm{kg},\ q = 4\mathrm{mC})$ is projected at time $t = 0$ with initial speed $v_0 = 6\mathrm{m/s}$ in the direction shown. If we write $\mathbf{a} = a_x\,\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}}$ for the acceleration and $\mathbf{v}(t) = v_x(t)\,\hat{\mathbf{i}} + v_y(t)\,\hat{\mathbf{j}}$ for the velocity of the particle ...

(a) find a_x and a_y,

(b) find $v_x(0)$ and $v_y(0)$,

(c) find $v_x(6\mathrm{s})$ and $v_y(6\mathrm{s})$.



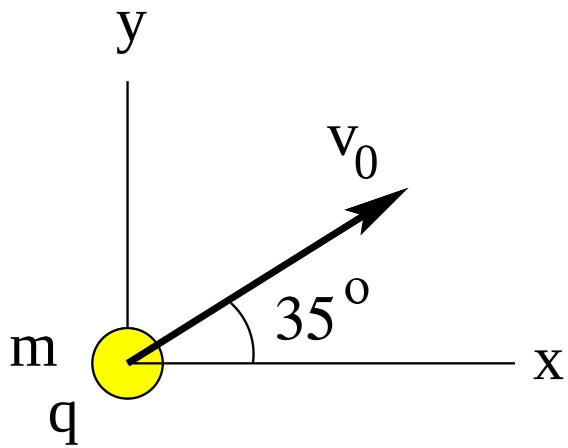
Unit Exam I: Problem #3 (Fall ’17)

In a region of uniform electric field $\mathbf{E} = 9\mathrm{N/C}\,\hat{\mathbf{i}} + 7\mathrm{N/C}\,\hat{\mathbf{j}}$, a charged particle ($m = 0.02\mathrm{kg},\ q = 4\mathrm{mC}$) is projected at time $t = 0$ with initial speed $v_0 = 6\mathrm{m/s}$ in the direction shown. If we write $\mathbf{a} = a_x\,\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}}$ for the acceleration and $\mathbf{v}(t) = v_x(t)\,\hat{\mathbf{i}} + v_y(t)\,\hat{\mathbf{j}}$ for the velocity of the particle ...

(a) find a_x and a_y,

(b) find $v_x(0)$ and $v_y(0)$,

(c) find $v_x(6\mathrm{s})$ and $v_y(6\mathrm{s})$.



Solution:

(a) $a_x = \dfrac{4\times10^{-3}\mathrm{C}}{2\times10^{-2}\mathrm{kg}}(9\mathrm{N/C}) = 1.80\mathrm{m/s^2}.$

$a_y = \dfrac{4\times10^{-3}\mathrm{C}}{2\times10^{-2}\mathrm{kg}}(7\mathrm{N/C}) = 1.40\mathrm{m/s^2}.$

(b) $v_x(0) = v_0\cos35^\circ = (6\mathrm{m/s})(0.819) = 4.91\mathrm{m/s}.$

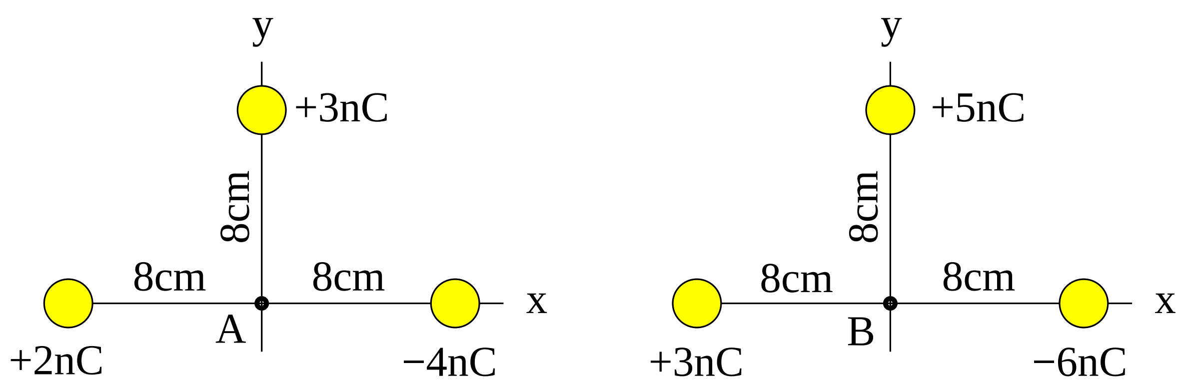
$v_y(0) = v_0\sin35^\circ = (6\mathrm{m/s})(0.574) = 3.44\mathrm{m/s}.$

(c) $v_x(6\mathrm{s}) = 4.91\mathrm{m/s} + (1.80\mathrm{m/s^2})(6\mathrm{s}) = 15.7\mathrm{m/s}.$

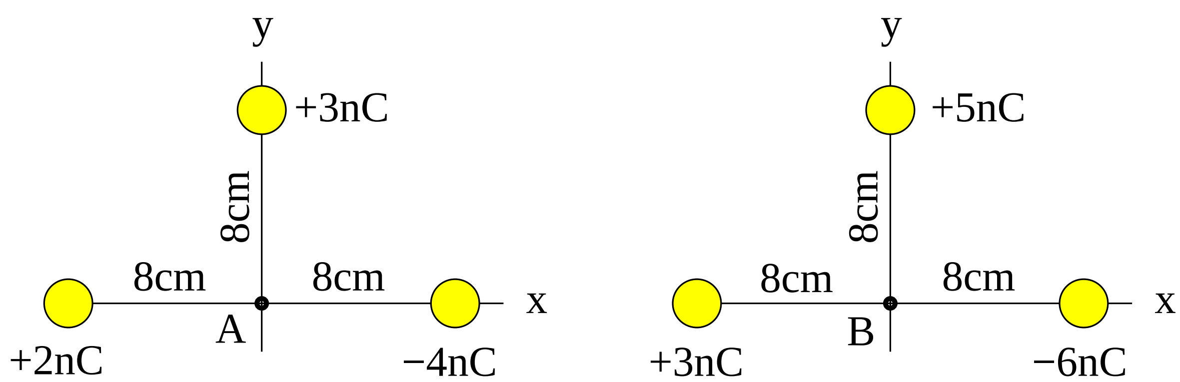
$v_y(6\mathrm{s}) = 3.44\mathrm{m/s} + (1.40\mathrm{m/s^2})(6\mathrm{s}) = 11.8\mathrm{m/s}.$

Unit Exam I: Problem #1 (Spring '18)

Consider the three point charges surrounding point A or point B.
Find the electric field $\mathbf{E}_A$ at point A and $\mathbf{E}_B$ at point B.
Find the electric potential V_A at point A and V_B at point B.
Find the magnitude F_{23} between the two positive charges on the left and F_{35} between the two positive charges on the right.



Consider the three point charges surrounding point A or point B.
Find the electric field $\mathbf{E}_A$ at point A and $\mathbf{E}_B$ at point B.
Find the electric potential V_A at point A and V_B at point B.
Find the magnitude F_{23} between the two positive charges on the left and F_{35} between the two positive charges on the right.



Solution:

- $\mathbf{E}_A = k\dfrac{|2\mathrm{nC}|}{(8\mathrm{cm})^2}\,\hat{\mathbf{i}} + k\dfrac{|4\mathrm{nC}|}{(8\mathrm{cm})^2}\,\hat{\mathbf{i}} - k\dfrac{|3\mathrm{nC}|}{(8\mathrm{cm})^2}\,\hat{\mathbf{j}} = 8.44\times 10^3\,\mathrm{N/C}\,\hat{\mathbf{i}} - 4.22\times 10^3\,\mathrm{N/C}\,\hat{\mathbf{j}}$

  $\mathbf{E}_B = k\dfrac{|3\mathrm{nC}|}{(8\mathrm{cm})^2}\,\hat{\mathbf{i}} + k\dfrac{|6\mathrm{nC}|}{(8\mathrm{cm})^2}\,\hat{\mathbf{i}} - k\dfrac{|5\mathrm{nC}|}{(8\mathrm{cm})^2}\,\hat{\mathbf{j}} = 12.7\times 10^3\,\mathrm{N/C}\,\hat{\mathbf{i}} - 7.03\times 10^3\,\mathrm{N/C}\,\hat{\mathbf{j}}$

- $V_A = k\dfrac{2\mathrm{nC}}{8\mathrm{cm}} + k\dfrac{3\mathrm{nC}}{8\mathrm{cm}} - k\dfrac{4\mathrm{nC}}{8\mathrm{cm}} = 113\mathrm{V}, \quad V_B = k\dfrac{3\mathrm{nC}}{8\mathrm{cm}} + k\dfrac{5\mathrm{nC}}{8\mathrm{cm}} - k\dfrac{6\mathrm{nC}}{8\mathrm{cm}} = 225\mathrm{V}$

- $F_{23} = k\dfrac{|(2\mathrm{nC})(3\mathrm{nC})|}{(8\mathrm{cm})^2+(8\mathrm{cm})^2} = 4.22\times 10^{-6}\,\mathrm{N}, \quad F_{35} = k\dfrac{|(3\mathrm{nC})(5\mathrm{nC})|}{(8\mathrm{cm})^2+(8\mathrm{cm})^2} = 10.5\times 10^{-6}\,\mathrm{N}$

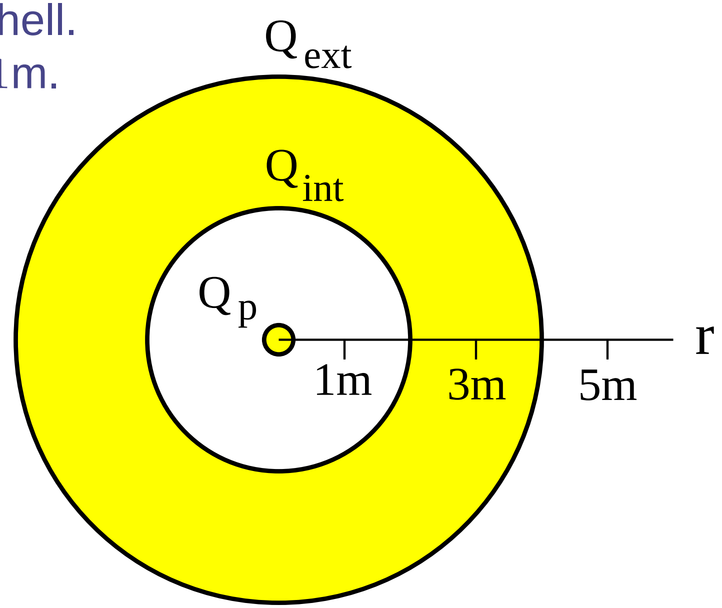
Unit Exam I: Problem #2 (Spring '18)

The conducting spherical shell with no net charge on it has a 2m inner radius and a 4m outer radius. There is a point charge $Q_{\rm p} = -4{\rm nC}$ [$Q_{\rm p} = 5{\rm nC}$] at the center.

(a) Find the charges $Q_{\rm int}$ and $Q_{\rm ext}$ on the two surfaces of the shell.

(b) Find the electric flux Φ_E through a Gaussian sphere of $r = 1$m.

(c) Find magnitude and direction of the electric field at $r = 5$m.



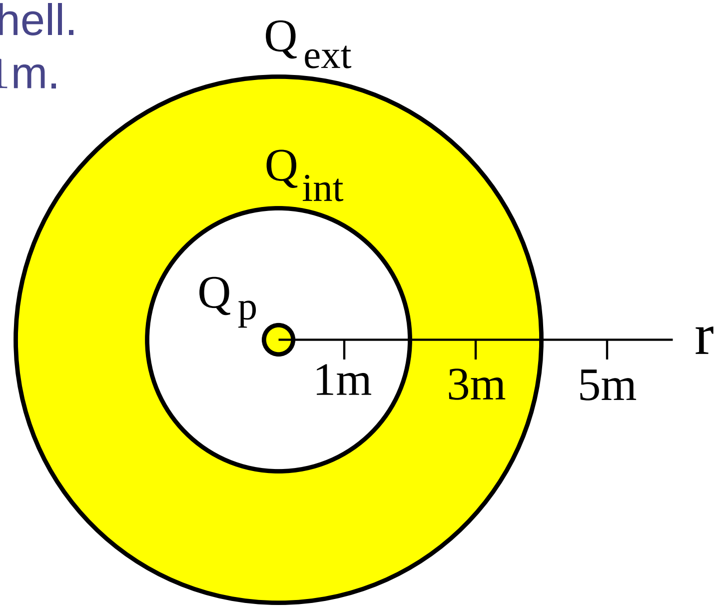
Unit Exam I: Problem #2 (Spring ’18)

The conducting spherical shell with no net charge on it has a 2m inner radius and a 4m outer radius. There is a point charge $Q_\mathrm{p} = -4\mathrm{nC}$ [$Q_\mathrm{p} = 5\mathrm{nC}$] at the center.

(a) Find the charges Q_int and Q_ext on the two surfaces of the shell.

(b) Find the electric flux Φ_E through a Gaussian sphere of $r = 1\mathrm{m}$.

(c) Find magnitude and direction of the electric field at $r = 5\mathrm{m}$.



Solution:

(a) $Q_\mathrm{int} = +4\mathrm{nC}, \quad Q_\mathrm{ext} = -4\mathrm{nC},$
$[Q_\mathrm{int} = -5\mathrm{nC}, \quad Q_\mathrm{ext} = +5\mathrm{nC}]\,.$

(b) $$\Phi_E = \frac{Q_\mathrm{p}}{\epsilon_0} = -452\mathrm{Nm}^2/\mathrm{C},$$
$$\left[\Phi_E = \frac{Q_\mathrm{p}}{\epsilon_0} = +565\mathrm{Nm}^2/\mathrm{C}\right].$$

(c) $$4\pi(5m)^2 E = \frac{(Q_\mathrm{p} + Q_\mathrm{int} + Q_\mathrm{ext})}{\epsilon_0} < 0 \quad \Rightarrow \; E = -1.44\mathrm{N/C} \quad (\mathrm{inward}),$$
$$\left[4\pi(5m)^2 E = \frac{(Q_\mathrm{p} + Q_\mathrm{int} + Q_\mathrm{ext})}{\epsilon_0} > 0 \quad \Rightarrow \; E = +1.80\mathrm{N/C} \quad (\mathrm{outward})\right].$$

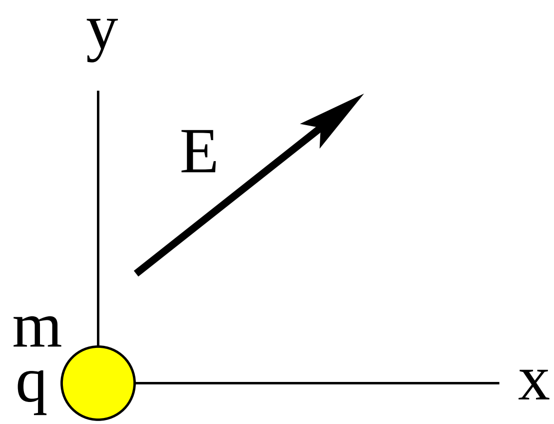
Unit Exam I: Problem #3 (Spring '18)

In a region of uniform electric field, $\mathbf{E} = 5\mathrm{N/C}\,\hat{\mathbf{i}} + 4\mathrm{N/C}\,\hat{\mathbf{j}}$, a charged particle $(m = 0.03\mathrm{kg},\ q = 2\mathrm{mC})$ [$(m = 0.02\mathrm{kg},\ q = 3\mathrm{mC})$] is released from rest at time $t = 0$ at the origin of the coordinate system.

(a) Find the electric force $\mathbf{F} = F_x\,\hat{\mathbf{i}} + F_y\,\hat{\mathbf{j}}$ acting on the particle.

(b) Find the position $\mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$ of the particle at time $t = 7$s.

(c) Draw the shape of the path into the diagram.



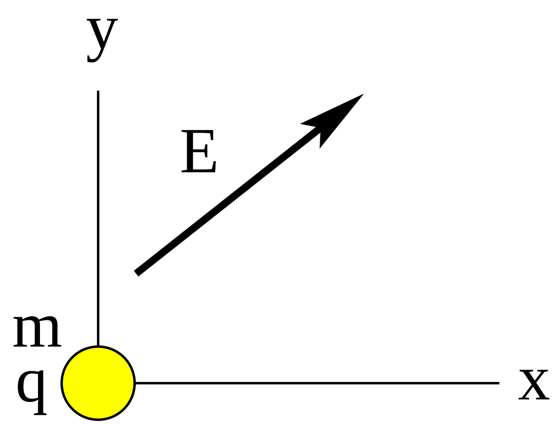
Unit Exam I: Problem #3 (Spring '18)

In a region of uniform electric field, $\mathbf{E} = 5\mathrm{N/C}\,\hat{\mathbf{i}} + 4\mathrm{N/C}\,\hat{\mathbf{j}}$, a charged particle $(m = 0.03\mathrm{kg},\ q = 2\mathrm{mC})$ [$(m = 0.02\mathrm{kg},\ q = 3\mathrm{mC})$] is released from rest at time $t = 0$ at the origin of the coordinate system.

(a) Find the electric force $\mathbf{F} = F_x\,\hat{\mathbf{i}} + F_y\,\hat{\mathbf{j}}$ acting on the particle.

(b) Find the position $\mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$ of the particle at time $t = 7$s.

(c) Draw the shape of the path into the diagram.



Solution:

(a) $F_x = (2\times10^{-3}\mathrm{C})(5\mathrm{N/C}) = 10\times10^{-3}\mathrm{N}, \quad F_y = (2\times10^{-3}\mathrm{C})(4\mathrm{N/C}) = 8\times10^{-3}\mathrm{N}.$

$\left[F_x = (3\times10^{-3}\mathrm{C})(5\mathrm{N/C}) = 15\times10^{-3}\mathrm{N}, \quad F_y = (3\times10^{-3}\mathrm{C})(4\mathrm{N/C}) = 12\times10^{-3}\mathrm{N}.\right]$

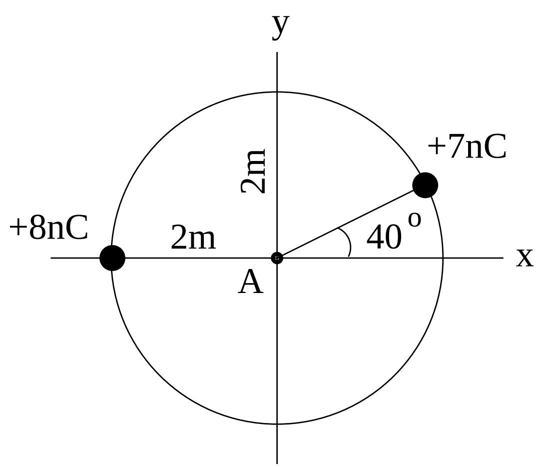
(b) $x = \frac{1}{2}\left(\frac{10\times10^{-3}\mathrm{N}}{3\times10^{-2}\mathrm{kg}}\right)(7s)^2 = 8.17\mathrm{m}, \quad y = \frac{1}{2}\left(\frac{8\times10^{-3}\mathrm{N}}{3\times10^{-2}\mathrm{kg}}\right)(7s)^2 = 6.53\mathrm{m}.$

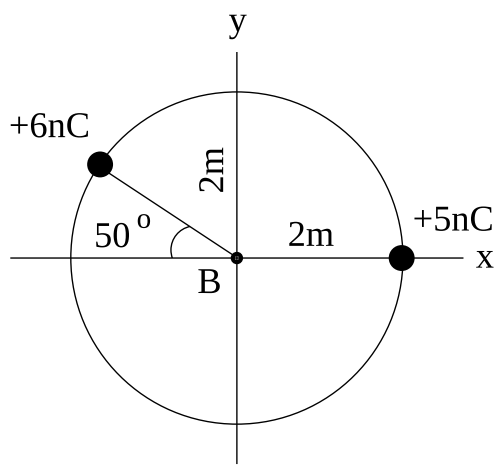
$\left[x = \frac{1}{2}\left(\frac{15\times10^{-3}\mathrm{N}}{2\times10^{-2}\mathrm{kg}}\right)(7s)^2 = 18.4\mathrm{m}, \quad y = \frac{1}{2}\left(\frac{12\times10^{-3}\mathrm{N}}{2\times10^{-2}\mathrm{kg}}\right)(7s)^2 = 14.7\mathrm{m}.\right]$

(c) Straight line through origin parallel to direction of electric field.

Unit Exam I: Problem #1 (Fall ’18)

Consider two point charges positioned on a circle as shown left and right.
(a) Find the horizontal component E_x of the electric field at points A and B.
(b) Find the vertical component E_y of the electric field at points A and B.
(c) Find the electric potential V at points A and B.





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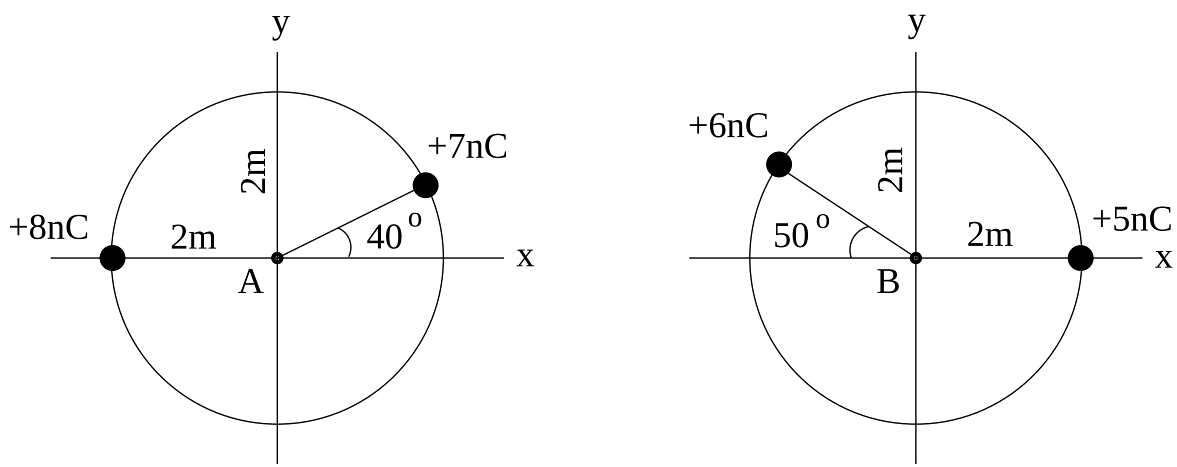


Solution:

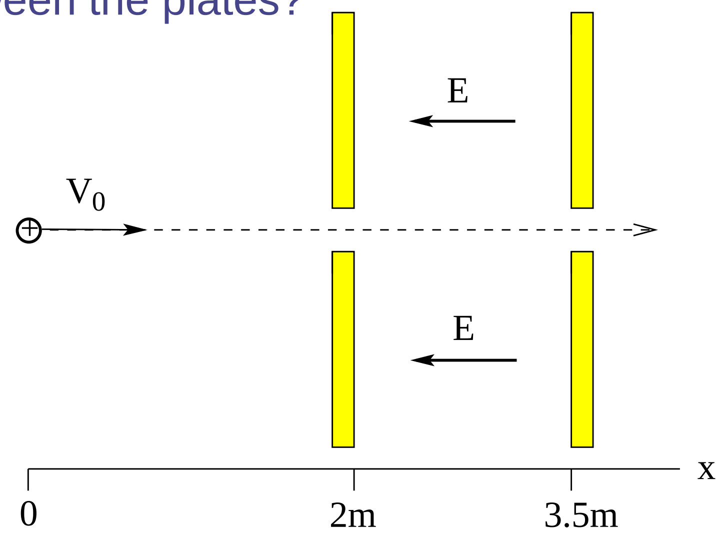
(a) $E_x = k\dfrac{8\text{nC}}{(2\text{m})^2} - k\dfrac{7\text{nC}}{(2\text{m})^2}\cos 40^\circ = 5.9\text{N/C}$ $E_x = k\dfrac{6\text{nC}}{(2\text{m})^2}\cos 50^\circ - k\dfrac{5\text{nC}}{(2\text{m})^2} = -2.57\text{N/C}$

(b) $E_y = -k\dfrac{7\text{nC}}{(2\text{m})^2}\sin 40^\circ = -10.1\text{N/C}$ $E_y = -k\dfrac{6\text{nC}}{(2\text{m})^2}\sin 50^\circ = -10.4\text{N/C}$

(c) $V = k\dfrac{8\text{nC}}{2\text{m}} + k\dfrac{7\text{nC}}{2\text{m}} = 67.5\text{V}.$ $V = k\dfrac{6\text{nC}}{2\text{m}} + k\dfrac{5\text{nC}}{2\text{m}} = 49.5\text{V}.$

Two oppositely charged plates positioned as shown produce between them a uniform electric field $E = 1.4$N/C [$E = 2.3$N/C] in the direction shown. A proton $(m = 1.67 \times 10^{-27}\mathrm{kg}$, $q = 1.60 \times 10^{-19}\mathrm{C})$ is launched at $x = 0$ with initial velocity $v_0 = 3.5 \times 10^4$m/s [$v_0 = 4.2 \times 10^4$m/s] as shown. The proton enters and exits the region of electric field through holes in the plates.

(a) At what time after launch does the proton reach the first plate?
(b) What is the acceleration of the proton between the plates?
(c) What is the potential difference between the plates?
(d) Does the proton gain or lose kinetic energy as it travels between the plates?
(e) What is the amount ΔK of gain or loss?



Unit Exam I: Problem #2 (Fall '18)

Two oppositely charged plates positioned as shown produce between them a uniform electric field $E = 1.4$N/C [$E = 2.3$N/C] in the direction shown. A proton $(m = 1.67 \times 10^{-27}\text{kg},$ $q = 1.60 \times 10^{-19}\text{C})$ is launched at $x = 0$ with initial velocity $v_0 = 3.5 \times 10^4$m/s [$v_0 = 4.2 \times 10^4$m/s] as shown. The proton enters and exits the region of electric field through holes in the plates.

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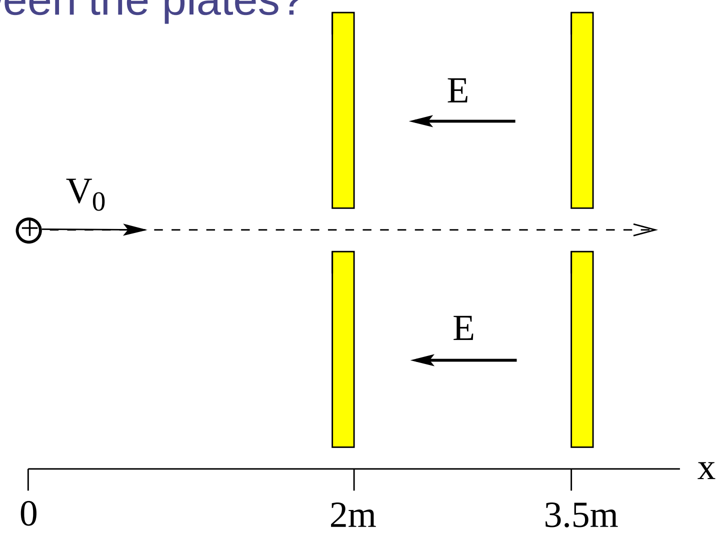


Solution:

(a) $t = \dfrac{(2\text{m})}{v_0} = 5.71 \times 10^{-5}\text{s} \quad [4.76 \times 10^{-5}\text{s}]$.

(b) $a = -\dfrac{qE}{m} = -1.34 \times 10^{8}\text{m/s}^2 \quad [-2.20 \times 10^{8}\text{m/s}^2]$.

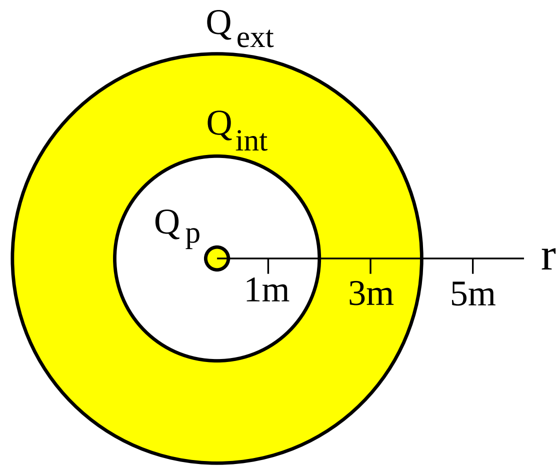
(c) $|\Delta V| = E(1.5\text{m}) = 2.1\text{V} \quad [3.45\text{V}]$.

(d) loss

(e) $\Delta K = -q|\Delta V| = -3.36 \times 10^{-19}\text{J} \quad [-5.52 \times 10^{-19}\text{J}]$.

Unit Exam I: Problem #3 (Fall '18)

A point charge $Q_\mathrm{p} = 7\mathrm{nC}$ [$Q_\mathrm{p} = 8\mathrm{nC}$] is surrounded by a conducting spherical shell with a 2m inner radius and a 4m outer radius. There is zero net charge on the shell.
(a) What is the magnitude of the electric field E at radius $r = 1\mathrm{m}$?
(b) What is the charge Q_int on the inner surface of the shell?
(c) What is the magnitude of the electric field E at radius $r = 3\mathrm{m}$?
(d) What is the charge Q_ext on the outer surface of the shell?
(e) What is the electric flux Φ_E through a Gaussian sphere of radius $r = 5\mathrm{m}$.



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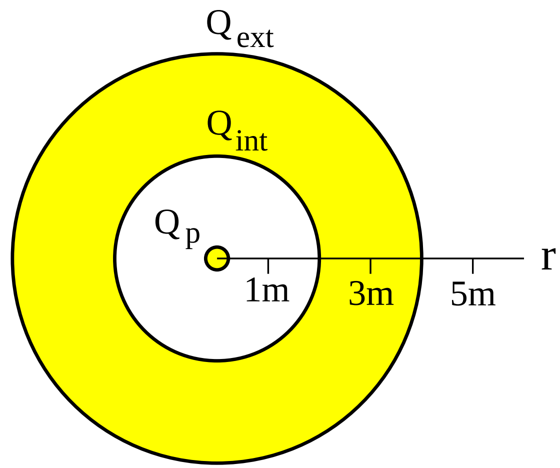


Solution:

(a) $E = \dfrac{kQ_\mathrm{p}}{(1\mathrm{m})^2} = 63\mathrm{N/C} \quad [72\mathrm{N/C}]$.

(b) $Q_\mathrm{int} = -Q_\mathrm{p} = -7\mathrm{nC} \quad [-8\mathrm{nC}]$.

(c) $E = 0$.

(d) $Q_\mathrm{ext} = -Q_\mathrm{int} = +7\mathrm{nC} \quad [+8\mathrm{nC}]$.

(e) $\Phi_E = \dfrac{Q_\mathrm{p}}{\epsilon_0} = 791\mathrm{Nm}^2/\mathrm{C} \quad [904\mathrm{Nm}^2/\mathrm{C}]$.

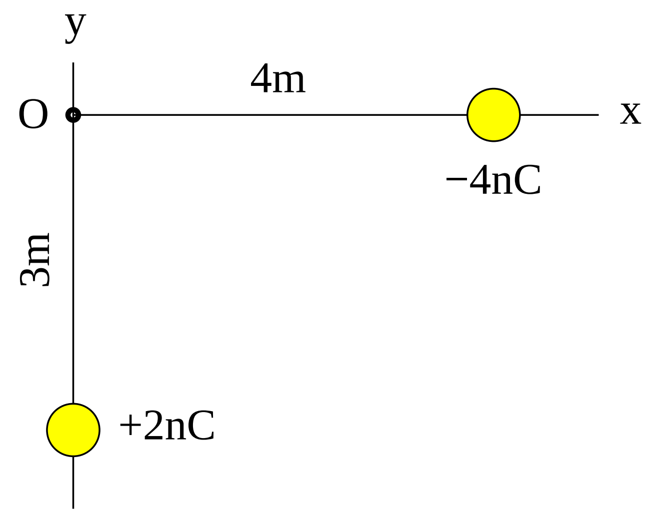
Unit Exam I: Problem #1 (Spring '19)

Consider two point charges positioned as shown. Use $k = 9 \times 10^9 \, \mathrm{Nm^2/C^2}$.

(a) Find the electric field $\mathbf{E} = E_x \, \hat{\mathbf{i}} + E_y \, \hat{\mathbf{j}}$ at point O.

(b) Find the electric potential V at point O.

(c) Find the magnitude F of the force between the two charges.



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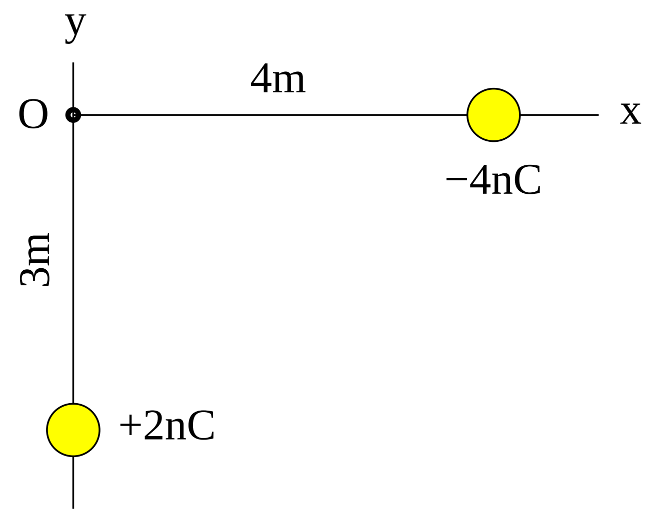


Solution:

(a) $E_x = k\dfrac{|-4\mathrm{nC}|}{(4\mathrm{m})^2} = \dfrac{9}{4}\,\mathrm{N/C} = 2.25\,\mathrm{N/C}, \quad E_y = k\dfrac{|2\mathrm{nC}|}{(3\mathrm{m})^2} = 2\,\mathrm{N/C}.$

(b) $V = k\dfrac{(-4\mathrm{nC})}{4\mathrm{m}} + k\dfrac{2\mathrm{nC}}{3\mathrm{m}} = -9\mathrm{V} + 6\mathrm{V} = -3\mathrm{V}.$

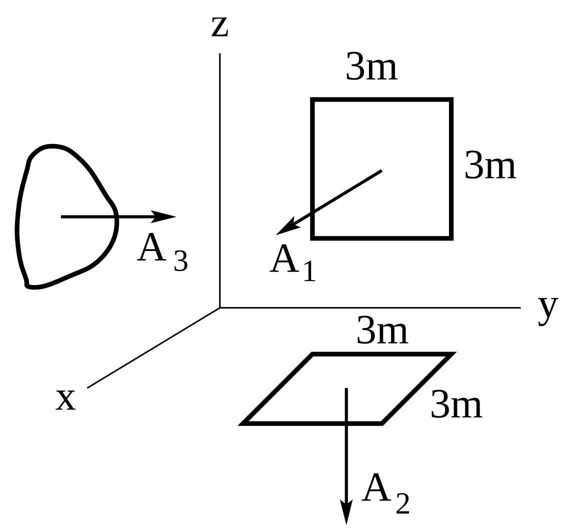
(c) $F = k\dfrac{|(-4\mathrm{nC})(2\mathrm{nC})|}{(5\mathrm{m})^2} = \dfrac{72}{25}\mathrm{nN} = 2.88\mathrm{nN}$

Consider three plane surfaces (two squares, one odd shape) with area vectors $\mathbf{A}_1$ (in positive x-direction), $\mathbf{A}_2$ (in negative z-direction), and $\mathbf{A}_3$ (in positive y-direction). The region is filled with a uniform electric field $\mathbf{E} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$N/C. The electric flux through surface 3 is $\Phi_E^{(3)} = 21\text{Nm}^2/\text{C}$.

(a) Find the electric flux $\Phi_E^{(1)}$ through surface 1.

(b) Find the electric flux $\Phi_E^{(2)}$ through surface 2.

(c) Find the area vector $\mathbf{A}_3$ of surface 3.



Unit Exam I: Problem #2 (Spring '19)

Consider three plane surfaces (two squares, one odd shape) with area vectors $\mathbf{A}_1$ (in positive x-direction), $\mathbf{A}_2$ (in negative z-direction), and $\mathbf{A}_3$ (in positive y-direction). The region is filled with a uniform electric field $\mathbf{E} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$N/C. The electric flux through surface 3 is $\Phi_E^{(3)} = 21\text{Nm}^2/\text{C}$.

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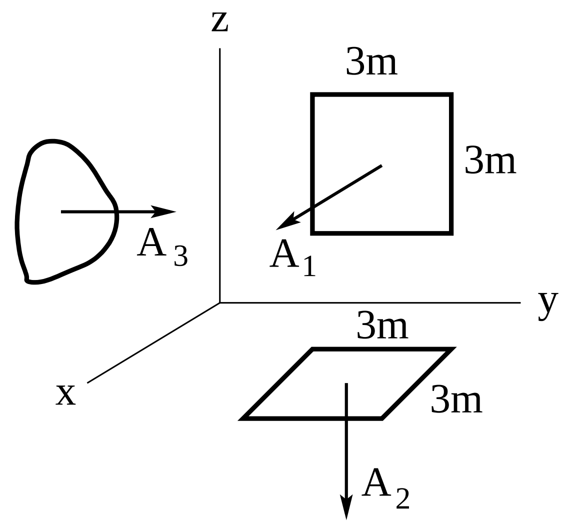


Solution:

(a) $\mathbf{A}_1 = 9\text{m}^2\,\hat{\mathbf{i}}, \quad \Phi_E^{(1)} = \mathbf{E}\cdot\mathbf{A}_1 = (2\text{N/C})(9\text{m}^2) = 18\text{Nm}^2/\text{C}.$

(b) $\mathbf{A}_2 = -9\text{m}^2\,\hat{\mathbf{k}}, \quad \Phi_E^{(2)} = \mathbf{E}\cdot\mathbf{A}_2 = -(4\text{N/C})(9\text{m}^2) = -36\text{Nm}^2/\text{C}.$

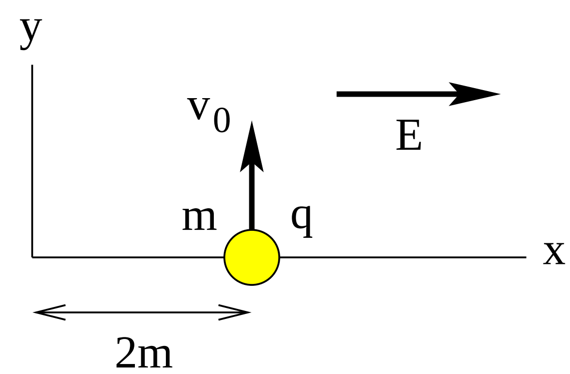
(c) $\mathbf{A}_3 = A_3\,\hat{\mathbf{j}}, \quad \Phi_E^{(3)} = A_3(3\text{N/C}) = 21\text{Nm}^2/\text{C} \quad \Rightarrow \; A_3 = 7\text{m}^2.$

Consider a region of uniform electric field $\mathbf{E} = 3\mathrm{N/C}\,\hat{\mathbf{i}}$. A charged particle $(m = 2\mathrm{kg},\ q = 4\mathrm{C})$ is projected at time $t = 0$ with initial velocity $\mathbf{v}_0 = 5\mathrm{m/s}\,\hat{\mathbf{j}}$ from the position shown.

(a) Find the acceleration $\mathbf{a} = a_x\,\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}}$ of the particle at time $t = 3\mathrm{s}$.

(b) Find its velocity $\mathbf{v} = v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}}$ at time $t = 3\mathrm{s}$.

(c) Find its position $\mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$ at time $t = 3\mathrm{s}$.



Consider a region of uniform electric field $\mathbf{E} = 3\mathrm{N/C}\,\hat{\mathbf{i}}$. A charged particle $(m = 2\mathrm{kg},\ q = 4\mathrm{C})$ is projected at time $t = 0$ with initial velocity $\mathbf{v}_0 = 5\mathrm{m/s}\,\hat{\mathbf{j}}$ from the position shown.

(a) Find the acceleration $\mathbf{a} = a_x\,\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}}$ of the particle at time $t = 3$s.

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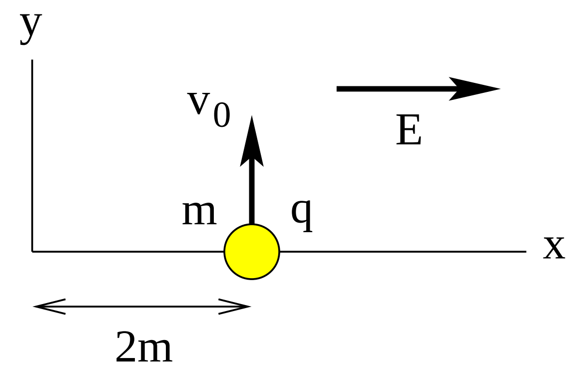

Solution:

(a) $a_x = \dfrac{q}{m}E = \dfrac{4\mathrm{C}}{2\mathrm{kg}}(3\mathrm{N/C}) = 6\mathrm{m/s}^2, \quad a_y = 0.$

(b) $v_x = a_x t = (6\mathrm{m/s}^2)(3\mathrm{s}) = 18\mathrm{m/s}, \quad v_y = v_0 = 5\mathrm{m/s}.$

(c) $x = x_0 + \dfrac{1}{2}a_x t^2 = 2\mathrm{m} + 0.5(6\mathrm{m/s}^2)(3\mathrm{s})^2 = 29\mathrm{m}, \quad y = v_0 t = (5\mathrm{m/s})(3\mathrm{s}) = 15\mathrm{m}.$