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22. Applications of magnetic force and torque. Hall effect

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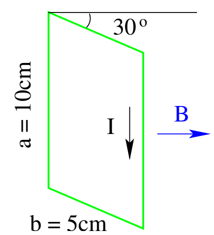
PHY204 Lecture 22 [rln22]

Magnetic Force Application (7)

The rectangular 20-turn loop of wire is 10cm high and 5cm wide.
It carries a current $I = 0.1$A and is hinged along one long side.
It is mounted with its plane at an angle of $30°$ to the direction of a uniform magnetic field of magnitude $B = 0.50$T.

- Calculate the magnetic moment μ of the loop.
- Calculate the torque τ acting on the loop about the hinge line.



tsl202

We begin this lecture with a few quantitative but simple applications of what we have learned in the previous lecture.

The magnetic dipole moment of the rectangular 20-turn loop has magnitude,

$$\mu = (20)(0.1\text{A})(0.1\text{m})(0.05\text{m}) = 0.01\text{Am}^2.$$

Its direction is perpendicular to the rectangular frame. Is it toward the front or back? Use your right hand, curl your fingers in the clockwise current direction, which makes the thumb point toward the back.

The magnitude of the torque acting on the current loop is,

$$\tau = \mu B \sin\theta = (0.01\text{Am}^2)(0.5\text{T})\sin 60° = 4.33 \times 10^{-3}\text{Nm}.$$

Note that the relevant angle is between $\vec{B}$ and $\vec{\mu}$.

Use your right hand to confirm that the direction of $\vec{\tau} = \vec{\mu} \times \vec{B}$ is $\downarrow$.

If the frame were a door half open, would the acting torque cause it to open further or do close it? Use your right hand with the thumb pointing in the direction of the torque. Then your finger curl in the way the door would rotate about the hinge line. It would open more.

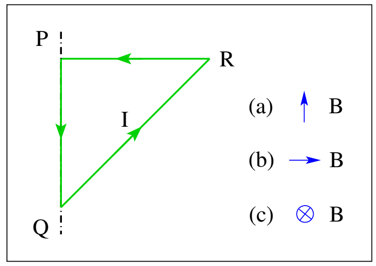
Magnetic Force Application (10)

A triangular current loop is free to rotate around the vertical axis PQ.

If a uniform magnetic field $\vec{B}$ is switched on, will the corner R of the triangle start to move out of the plane, into the plane, or will it not move at all?

Find the answer for a field $\vec{B}$ pointing

(a) up,
(b) to the right,
(c) into the plane.



tsl205

In this application, the focus is on directions exclusively. The purpose is to sharpen your sense of mental navigation in 3-dimensional space.

The triangular loop with a counterclockwise current can only rotate about the hinge line. Motion begins only under a torque, $\vec{\tau} = \vec{\mu} \times \vec{B}$, with a component parallel to the hinge line.

The vector $\vec{\mu}$ is directed out of the plane ($\odot$) as you quickly verify using your right hand.

Therefore, in the three cases the following happens:

(a) The torque is directed left ($\leftarrow$), implying that it has no component parallel to the hinge line. The triangle will not start moving. The torque causes extra stress on the hinges though.

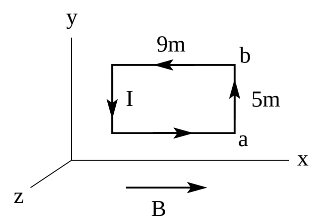
(b) The torque is directed up ($\uparrow$), implying that the corner R begins moving into the plane of the page.

(c) The torque in this case is the cross product of two vectors that are anti-parallel. Such a cross product vanishes. The triangle will not start moving.

Verify these conclusions using the two renditions of right-hand rule introduced previously.

Intermediate Exam III: Problem #1 (Spring '07)

Consider a rectangular conducting loop in the xy-plane with a counterclockwise current $I = 7\text{A}$ in a uniform magnetic field $\vec{B} = 3\text{T}\hat{\imath}$.
(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
(b) Find the force $\vec{F}$ (magnitude and direction) acting on the side ab of the rectangle.
(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



Solution:

(a) $\vec{\mu} = (7\text{A})(45\text{m}^2)\hat{k} = 315\text{Am}^2\hat{k}$.

(b) $\vec{F} = I\vec{L} \times \vec{B} = (7\text{A})(5\text{m}\hat{\jmath}) \times (3\text{T}\hat{\imath}) = -105\text{N}\hat{k}$.

(c) $\vec{\tau} = \vec{\mu} \times \vec{B} = (315\text{Am}^2\hat{k}) \times (3\text{T}\hat{\imath}) = 945\text{Nm}\hat{\jmath}$

tsl365 – p.1/3

This is an elementary and straightforward application of magnetic moment, force, and torque. It takes half a line or less to work out each part.

Most mistakes in solving such problems originate in misunderstandings from a hasty reading of the problem and the accompanying graph.

When you practice your skills by solving previous exam problems such as this one, only use resources that you also have available during the exam. Conceal the model solution until you have come up with your own solution. Then check you solution against the model solution. Resolve any discrepancies.

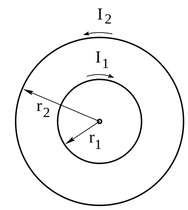
We all make mistakes. There is no shame in making mistakes. The key is to recognize and admit mistakes, and then learn from them.

Unit Exam III: Problem #1 (Spring '08)

Consider two circular currents $I_1 = 3\text{A}$ at radius $r_1 = 2\text{m}$ and $I_2 = 5\text{A}$ at radius $r_2 = 4\text{m}$ in the directions shown.

(a) Find magnitude B and direction $(\odot, \otimes)$ of the resultant magnetic field at the center.

(b) Find magnitude μ and direction $(\odot, \otimes)$ of the magnetic dipole moment generated by the two currents.



Solution:

(a) $B = \dfrac{\mu_0(3\text{A})}{2(2\text{m})} - \dfrac{\mu_0(5\text{A})}{2(4\text{m})} = (9.42 - 7.85) \times 10^{-7}\text{T}$

$\Rightarrow B = 1.57 \times 10^{-7}\text{T} \quad \otimes$

(b) $\mu = \pi(4\text{m})^2(5\text{A}) - \pi(2\text{m})^2(3\text{A}) = (251 - 38)\text{Am}^2$

$\Rightarrow \mu = 213\text{Am}^2 \quad \odot$

tsl381

Here is another previous exam problem. At this point we are only ready to do part (b). We will return to part (a) later.

We are dealing with two concentric current loops with currents flowing in opposite directions. Each loop independently produces a magnetic dipole moment $\vec{\mu}$. For each loop the relevant area is the entire area inside the loop irrespective of the presence of another loop.

The right-hand rule tells us that the magnetic moment of the outer loop is directed out of the plane of the page ($\odot$) and that of the inner loop into the page ($\otimes$).

On the slide, the first direction is (tacitly) declared to be the positive direction. Hence the minus sign for the result of the inner loop. A positive result then means "out of the page"

If we had decided that positive meant "into the page" then our result would have come out negative, meaning again " out of the page".

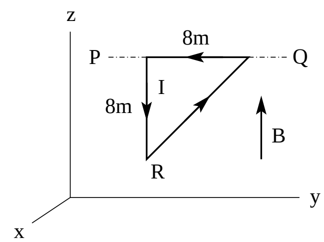
Unit Exam III: Problem #1 (Spring ’09)

A triangular conducting loop in the yz-plane with a counterclockwise current $I = 3\text{A}$ is free to rotate about the axis PQ. A uniform magnetic field $\vec{B} = 0.5\text{T}\hat{k}$ is present. (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the triangle.

(b) Find the magnetic torque $\vec{\tau}$ (magnitude and direction) acting on the triangle.

(c) Find the magnetic force $\vec{F}_H$ (magnitude and direction) acting on the long side (hypotenuse) of the triangle.

(d) Find the force $\vec{F}_R$ (magnitude and direction) that must be applied to the corner R to keep the triangle from rotating.



Solution:

(a) $\vec{\mu} = (3\text{A})(32\text{m}^2)\hat{i} = 96\text{Am}^2\hat{i}$.

(b) $\vec{\tau} = \vec{\mu} \times \vec{B} = (96\text{Am}^2\hat{i}) \times (0.5\text{T}\hat{k}) = -48\text{Nm}\hat{j}$.

(c) $F_H = (3\text{A})(8\sqrt{2}\text{m})(0.5\text{T})(\sin 45^\circ) = 12\text{N} \quad \odot$.

(d) $(-8\text{m}\hat{k}) \times \vec{F}_R = -\vec{\tau} = 48\text{Nm}\hat{j} \quad \Rightarrow \vec{F}_R = -6\text{N}\hat{i}$.

tsl395

Here we have another hinged triangle. Parts (a) and (b) are now a routine affair. With a coordinate system specified in the graph, the most practical way to indicate directions are unit vectors $\hat{\mathbf{i}}$ (in x-direction), $\hat{\mathbf{j}}$ (in y-direction), and $\hat{\mathbf{k}}$ (in z-direction).

For part (c) we must heed that fact that the angle between the two vectors $\vec{L}$ and $\vec{B}$ in the cross product of $\vec{F}_H = I\vec{L} \times \vec{B}$, is 45°. For the direction of the result, it would have been simpler to write $12\text{N}\hat{\mathbf{i}}$.

One way to solve part (d) is to say that the applied torque must counteract the torque from part (b). Given the length of the lever arm, the result shown follows directly.

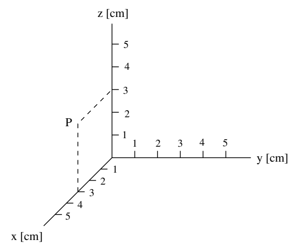
Unit Exam III: Problem #1 (Spring '13)

In a region of uniform magnetic field $\mathbf{B}$ a proton ($m = 1.67 \times 10^{-27}$kg, $q = 1.60 \times 10^{-19}$C) experiences a force $\mathbf{F} = 8.0 \times 10^{-19}\text{N}\,\hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 2000\text{m/s}\,\hat{\mathbf{k}}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).

(b) Calculate the radius r of the circular path.

(c) Locate the center C of the circular path in the coordinate system on the page.



Solution:

(a) $B = \dfrac{F}{qv_0} = 2.50 \times 10^{-3}\text{T}, \quad \hat{\mathbf{i}} = \hat{\mathbf{k}} \times (-\hat{\mathbf{j}})$

$\Rightarrow \mathbf{B} = -2.50 \times 10^{-3}\text{T}\,\hat{\mathbf{j}}.$

(b) $F = \dfrac{mv_0^2}{r} = qv_0B$

$\Rightarrow r = \dfrac{mv_0^2}{F} = \dfrac{mv_0}{qB} = 0.835\text{cm}.$

(c) $C = 3.84\text{cm}\,\hat{\mathbf{i}} + 3.00\text{cm}\,\hat{\mathbf{k}}.$

tsl462

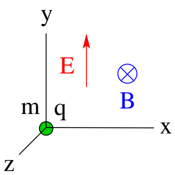
What makes this exercise a bit more challenging is that we have to reason backward in a certain sense. We are given the first factor of a cross product and the result. In part (a) we are asked to find magnitude and direction of the second factor.

For part (b) we must recall that the magnetic force is a centripetal force in this application.

Part (c) test our skills of visualization. We know that point P is on the circle. The particle at P moves up. That gives us a tangent. The (centripetal) force is toward the front. That gives us the direction from P toward the center C of the circle. The radius r gives us the distance between P and C.

Charged Particle in Crossed Electric and Magnetic Fields (1)

- Release particle from rest.
- Force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
- (1) $F_x = m\dfrac{dv_x}{dt} = -qv_yB \quad \Rightarrow \quad \dfrac{dv_x}{dt} = -\dfrac{qB}{m}v_y$
- (2) $F_y = m\dfrac{dv_y}{dt} = qv_xB + qE \;\Rightarrow\; \dfrac{dv_y}{dt} = \dfrac{qB}{m}v_x + \dfrac{qE}{m}$
- Ansatz: $v_x(t) = w_x\cos(\omega_0 t) + u_x,\ v_y(t) = w_y\sin(\omega_0 t) + u_y$
- Substitute ansatz into (1) and (2) to find $w_x, w_y, u_x, u_y, \omega_0$.
- (1) $-\omega_0 w_x \sin(\omega_0 t) = -\dfrac{qB}{m}w_y\sin(\omega_0 t) - \dfrac{qB}{m}u_y$
- (2) $\omega_0 w_y \cos(\omega_0 t) = \dfrac{qB}{m}w_x\cos(\omega_0 t) + \dfrac{qB}{m}u_x + \dfrac{qE}{m}$
- $\Rightarrow\ u_y = 0, \quad u_x = -\dfrac{E}{B}, \quad \omega_0 = \dfrac{qB}{m}, \quad w_x = w_y \equiv w$
- Initial condition: $v_x(0) = v_y(0) = 0 \Rightarrow\ w = \dfrac{E}{B}$



tsl208

On this page and the next, we discuss an advanced application of a charged particle moving in a combination of uniform electric and magnetic fields.

He have encountered crossed electric and magnetic fields previously in the velocity selector. We found that particles of one special velocity experience electric and magnetic forces that balance each other and thus have zero net effect on the path of the particle.

Here we release a charged particle from rest with electric and magnetic fields in the directions as shown and find out what happens. Initially, only the electric force acts. It accelerates the particle straight up. As the particle gains speed, the magnetic force increases from zero and bends the path toward the left.

The solution presented on the slide splits Newton's second law, $\vec{F} = m(d\vec{v}/dt)$, into x- and y-components and then uses a trial function (ansatz) with undetermined parameters.

Substitution of the ansatz into the the equations of motion (1) and (2) determines what all parameters are except one. The last parameter is determined by imposing the appropriate initial conditions, which in the case of our interest means zero initial velocity.

The ansatz with all parameters determined represents a unique solution for the two components of the velocity of the particle as functions of time (see next page).

Charged Particle in Crossed Electric and Magnetic Fields (2)

- Solution for velocity of particle:

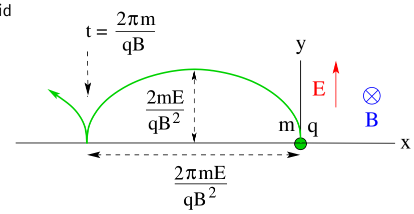
$$v_x(t) = \frac{E}{B}\left[\cos\left(\frac{qBt}{m}\right) - 1\right], \quad v_y(t) = \frac{E}{B}\sin\left(\frac{qBt}{m}\right)$$

- Solution for position of particle:

$$x(t) = \frac{E}{B}\int_0^t \left[\cos\left(\frac{qBt}{m}\right) - 1\right] dt = \frac{Em}{qB^2}\sin\left(\frac{qBt}{m}\right) - \frac{Et}{B}$$

$$y(t) = \frac{E}{B}\int_0^t \sin\left(\frac{qBt}{m}\right) dt = \frac{Em}{qB^2}\left[1 - \cos\left(\frac{qBt}{m}\right)\right]$$

- Path of particle in (x, y)-plane: cycloid



tsl209

We see that both $v_x(t)$ and $v_y(t)$ are periodic functions. The velocity in horizontal direction is always negative or zero. The velocity in vertical direction is alternatingly positive and negative. We thus conclude that the particle moves up and down and always toward the left with periodic stops.

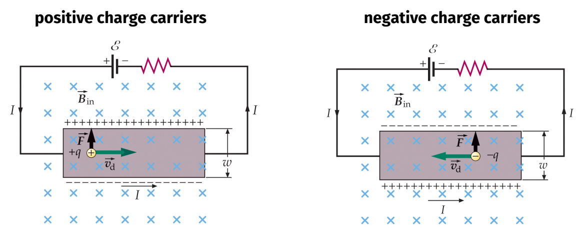
In order to find out the exact path of the particle, we need to know its position as a function of time. We get that by integration as done on the slide. We see that the particle traces a path as if it were hopping along the negative x-axis as shown. The hopping distance and the time between bounces are indicated in the graph.

This shape of path is called a cycloid. You also get a cycloid when you roll a cylinder with a pen attached to one point on the perimeter next to a wall.

Hall Effect

Method for dermining whether charge carriers are positively or negatively charged.

- Magnetic field $\vec{B}$ pulls charge carriers to one side of conducting strip.
- Accumulation of charge carriers on that side and depletion on opposite side produce transverse electric field $\vec{E}$.
- Transverse forces on charge carrier: $F_E = qE$ and $F_B = qv_dB$.
- In steady state forces are balanced: $\vec{F}_E = -\vec{F}_B$.
- Hall voltage in steady state: $V_H = Ew = v_dBw$.



tsl201

When we first introduced electric currents, we noted that a positive current from left to right in a conductor can be produced by positive charge carriers drifting from left to right or by negative charge carriers drifting from right to left or by a combination of both.

Later we noted that the magnetic force on a current depends on the direction of positive current irrespective of the direction in which the actual charge carriers drift.

That raises the question of how we can find out whether a positive current directed from left to right is produced by positive or negative charge carriers. Both conducting strips on the slide carry a positive current from left to right. Both experience an upward magnetic force. How can we determine the drift direction of the charge carriers?

The Hall effect gives the answer. Even though the shaded and zoomed-in part of the conductor experiences an upward magnetic force in both cases, it's positive charge carriers that do the pushing on the left and the negative charge carriers that do it on the right.

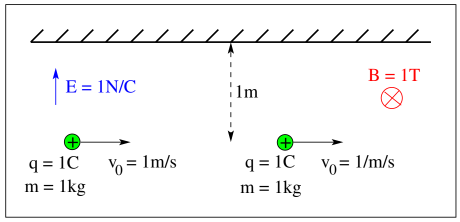
The result of the pushing is an accumulation of mobile charge carriers on the upper surface and a deficiency on the lower surface. This imbalance causes an electric field between the two surfaces that is pointing down on the left and up on the right. This electric field, in turn, is associated with a potential difference between the two surfaces, called *Hall voltage*.

Positive charge carriers produce a Hall voltage with the higher potential on the upper surface. Negative charge carriers produce a Hall voltage with the higher potential on the lower surface.

Two charged particles are released in different uniform fields. Ignore gravity.

(a) Find the the horizontal velocity components v_{Ex}, v_{Bx} and the vertical velocity components v_{Ey}, v_{By} at the instant each particle hits the wall.

(b) Find the times t_E, t_B it takes each particle to reach the wall.



tsl204

The charged particle in the electric field is launched to the right and accelerated up by the electric field. Its path is parabolic. It will hit the wall at an angle while going up and to the right.

The horizontal velocity will not change because there is no horizontal force acting. Hence we have $v_{Ex} = 1\text{m/s}$.

The (constant) vertical acceleration is $a_{Ey} = F/m = qE/m = 1\text{m/s}^2$. The time of flight is the solution of $1\text{m} = \frac{1}{2} m a_{Ey} t_E^2$. Hence we have $t_E = \sqrt{2}\,\text{s}$.

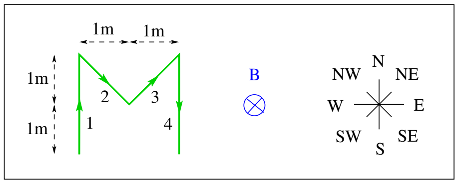
The vertical velocity component of the particle when it hits the wall is $v_{Ey} = a_{Ey} t_E = \sqrt{2}\,\text{m/s}$.

The magnetic field bends the particle on the right into a circular path of radius $r = mv_0/qB = 1\text{m}$. Hence the particle hits the wall after a quarter circle, heading straight up, implying that $v_{Bx} = 0$ and $v_{By} = v_0 = 1\text{m/s}$.

The time it takes is a quarter of the period $T = 2\pi m/qB = 2\pi\text{s}$, implying that $t_B = T/4 = (\pi/2)\text{s}$.

An electric current $I = 1\text{A}$ flows through the M-shaped wire in the direction indicated. The wire is placed in a magnetic field $B = 1\text{T}$ pointing into the plane.

(a) Find the magnitude of the magnetic forces F_1, F_2, F_3, F_4 acting on each part of the wire.
(b) Find the direction of the resultant force $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$ acting on the wire.



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All magnetic forces are perpendicular to the direction of the magnetic field. Hence they can only have a (horizontal) x-component and/or a (vertical) y-component.

The magnitude of the magnetic force on each segment is $F_i = IL_iB$, with $L_1 = L_4 = 2\text{m}$ and $L_2 = L_3 = \sqrt{2}\,\text{m}$. Hence we get $F_1 = F_4 = 2\text{N}$ and $F_2 = F_3 = \sqrt{2}\,\text{N}$.

The right-hand rule tells us that the direction of $\vec{F}_i = I\vec{L}_i \times \vec{B}$ is W for segment 1, NE for segment 2, NW for segment 3, and E for segment 4.

The forces on segments 1 and 4 cancel each other. The horizontal components of the forces on segments 2 and 3 are E and W, respectively, and cancel each other as well. The vertical components are both positive and add up. The net force is 1N pointing N.

Magnetic Force Application (4)

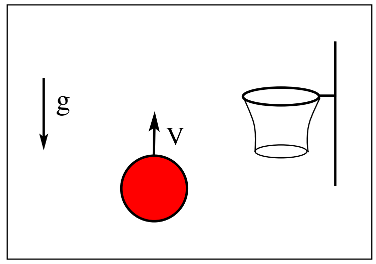
A negatively charged basketball is thrown vertically up against the gravitational field $\vec{g}$.

Which direction of

(a) a uniform electric field $\vec{E}$,

(b) a uniform magnetic field $\vec{B}$

will give the ball a chance to find its way into the basket? (up/down/left/right/back/front)



tsl191

This is the quiz for lecture 22.

The negatively charged basketball must experience an electric force $\vec{F}_E = q\vec{E}$ or a magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$ toward the right for it to make it into the basket.

The six possible direction for (a) $\vec{E}$ and (b) $\vec{B}$ are $\uparrow$, $\downarrow$, $\leftarrow$, $\rightarrow$, $\otimes$, $\odot$.