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19. Magnetic force on currents or moving charged particles

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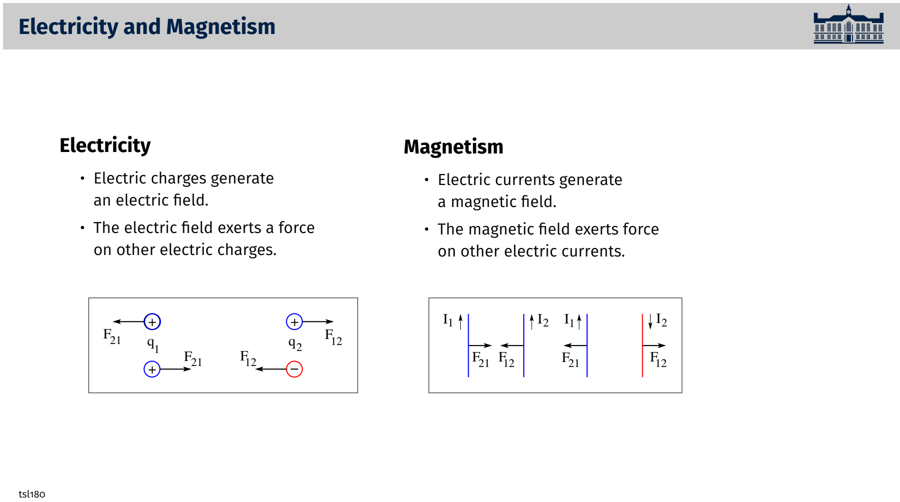
PHY204 Lecture 19 [rln19]



Today we begin studying the effects of a magnetic field. Electric and magnetic phenomena exhibit intriguing similarities and differences. We highlight a few up front.

We shall discover, as we move ahead, that electricity and magnetism are inextricable aspects of one and the same natural phenomenon named *electromagnetism*.

In the slide of this page, we see (on the left) the Coulomb force in action: like charges exert a repulsive force on each other and unlike charges an attractive force.

On the right, we see (and later analyze in detail) that electric currents flowing in the same direction exert an attractive force on each other whereas two currents flowing in opposite direction exert a repulsive force on each other.

As the text on the slide says, a more productive way of describing the situation introduces fields.

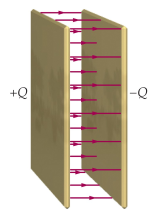
- One charge generates an electric field $\vec{E}$, which then exerts an electric force on a second charge.
- One current generates a magnetic field $\vec{B}$, which then exerts a magnetic force on a second current.

The roles of the two charges or currents are interchangeable. The forces between charges or currents are, effectively, an action-reaction pair.

Sources of Electric and Magnetic Fields

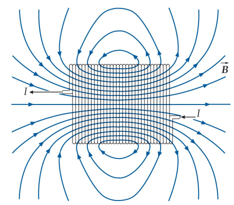
Capacitor

The parallel-plate capacitor generates a near uniform electric field provided the linear dimensions of the plates are large compared to the distance between them.



Solenoid

The solenoid (a tightly wound cylindrical coil) generates a near uniform magnetic field provided the length of the coil is large compared to its radius.



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The main purpose of this slide is to show how uniform fields can be generated.

We know from earlier that a uniform electric field $\vec{E}$ exists between oppositely charged conducting plates. The field is directed from the plate with charge $+Q$ toward the plate with charge $-Q$.

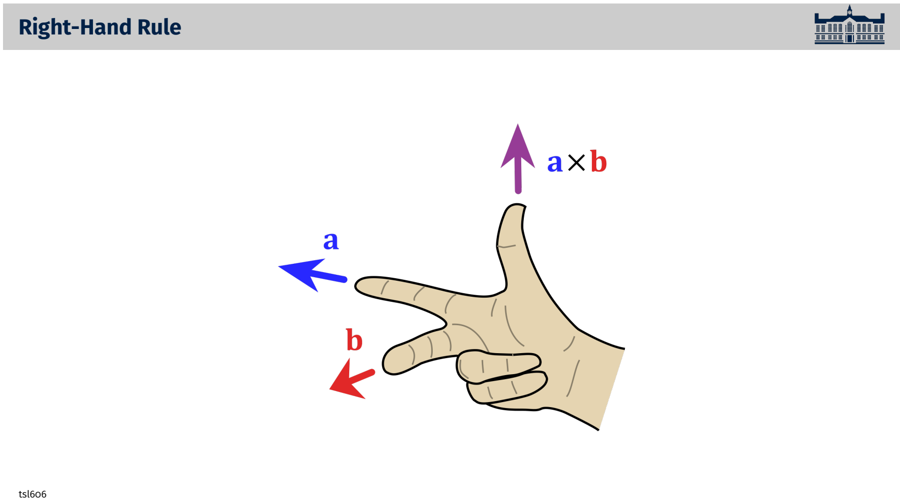
When a wire is tightly wound into a cylindrical coil and current I is sent through the wire it generates a magnetic field $\vec{B}$ with field lines as shown.

When the coil is made longer and longer, the field inside becomes more and more uniform and the field outside weaker and weaker except at the ends of the coil. When the current inside the wire flows in the opposite direction, the magnetic field also switches direction.

Both physical objects shown are used as devices in electric circuits. The device on the left is a capacitor and the device on the right is an inductor.

Solenoid is the name of an inductor of cylindrical shape.

In what follows next we will take sources of magnetic field for granted and study their effects. Later we will return to exploring sources of magnetic field.



In preparation of what comes next, we introduce the cross product between two vectors and the right-hand rule.

We are already familiar with the *dot product* between two vectors $\vec{a}$ and $\vec{b}$. We can write,

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = ab \cos \phi,$$

where ϕ is the angle between the two vector. The dot product gives you a positive number if $\phi < 90°$ and a negative number if $\phi > 90°$. The dot product between two vectors that are perpendicular to each other vanishes. The sequencing of the factors makes no difference.

The *cross product* between the same two vectors produces a vector again and the sequencing of the factor does matter. We can write,

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}, \quad |\vec{a} \times \vec{b}| = ab \sin \phi.$$

The last equation states how to calculate the magnitude of the product vector. The vector product of two vectors that are parallel ($\phi = 0$) or antiparallel ($\phi = 180°$) vanishes.

The direction of the product vector is perpendicular to the plane spanned by the factor vectors. If the index of your right hand point in the direction of the first factor $\vec{a}$ and your middle finger in the direction of the second factor $\vec{b}$, then the thumb points in the direction of the product $\vec{a} \times \vec{b}$.

More about the math of vector products later.

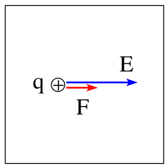
Electric and Magnetic Forces on Point Charge

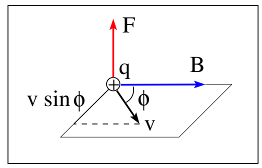
Electric Force

- $\vec{F} = q\vec{E}$
- electric force is parallel to electric field
- SI unit of E: 1N/C=1V/m

Magnetic Force

- $\vec{F} = q\vec{v} \times \vec{B}, \quad F = qvB\sin\phi$
- magnetic force is perpendicular to magnetic field
- SI unit of B: 1Ns/Cm=1T (Tesla)
- 1T=10^4G (Gauss)





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This slide shows familiar content on the left and new content on the right. We take the existence of a field for granted, an electric field $\vec{E}$ on the left and a magnetic field $\vec{B}$ on the right. What is the effect of the field on a particle carrying an electric charge q? The effect is a force in both cases.

We know from earlier that the electric force, $\vec{F} = q\vec{E}$, is in the same direction as the electric field $\vec{E}$ if the charge q is positive (as shown) or in the opposite direction if it is negative.

The magnetic force only comes into action when the charged particle moves. Then the force is perpendicular to the plane spanned by the velocity vector $\vec{v}$ of the charged particle and the magnetic field vector $\vec{B}$.

The magnetic force vector can be expressed as a cross product between these two vectors $\vec{v}$ (first factor) and $\vec{B}$ (second factor).

The product vector $\vec{v} \times \vec{B}$ is then to be multiplied by q, which is not a vector but can be positive or negative. Hence if the charge q is positive, then the magnetic force is in the direction of $\vec{v} \times \vec{B}$ as determined by the right-hand rule, but in the opposite direction if q is negative.

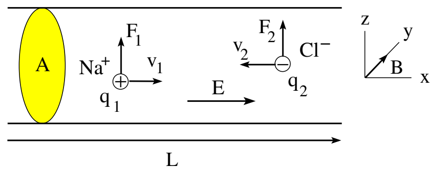
A charged particle moving in the direction of the magnetic field or in opposite direction experiences no magnetic force: the cross product vanishes.

Note the SI units of electric and magnetic field. "Gauss" is not an SI unit but commonly used by practitioners working with magnetism.

Magnetic Force on Current-Carrying Conductor

Consider drift of Na^+ and Cl^- ions in a plastic pipe filled with salt water.

- $v_{1x} > 0$, $v_{2x} < 0$: drift velocities; $q_1 > 0$, $q_2 < 0$: charge on ions
- n_1, n_2: number of charge carriers per unit volume



- Electric current through A: $I = A(n_1 q_1 v_{1x} + n_2 q_2 v_{2x})$
- Force on Na^+: $\vec{F}_1 = q_1 \vec{v}_1 \times \vec{B} \;\Rightarrow\; F_{1z} = q_1 v_{1x} B_y$
- Force on Cl^-: $\vec{F}_2 = q_2 \vec{v}_2 \times \vec{B} \;\Rightarrow\; F_{2z} = q_2 v_{2x} B_y$
- Force on current-carrying pipe: $F_z = (n_1 q_1 v_{1x} + n_2 q_2 v_{2x}) A L B_y = I L B_y$
- Vector relation: $\vec{F} = I\vec{L} \times \vec{B}$

tsl183

The magnetic force on moving charges translates into a magnetic force on current-carrying conductors as explained on this slide. I am using the example from earlier (lecture 15) with positive and negative mobile charge carriers.

Positive ions drifting to the right and negative ions drifting to the left both contribute to a positive current I toward right.

The electric field $\vec{E}$ inside the conductor (in x-direction) is what drives the current. It represents a voltage across the pipe with high potential on the left and low potential on the right.

The conductor is positioned into a magnetic field $\vec{B}$ directed toward the rear (y-direction). Both kinds of charge carriers experience a magnetic force in the same direction, namely up (z-direction).

The vector product $\vec{v}_2 \times \vec{B}$ is opposite to the vector product $\vec{v}_1 \times \vec{B}$, but when the former is multiplied by the negative q_2 the direction is switched. Hence $\vec{F}_1$ and $\vec{F}_2$ are both directed up.

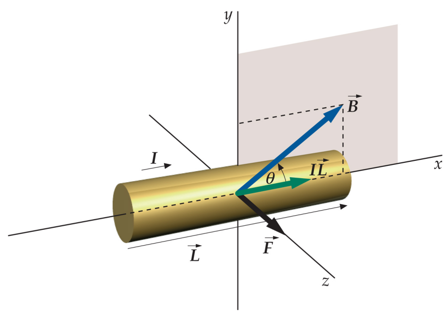
The important result is that when a straight segment of wire or pipe is positioned in a magnetic field $\vec{B}$, it experiences a magnetic force,

$$\vec{F} = I\vec{L} \times \vec{B},$$

where the vector $\vec{L}$ is in current direction and its magnitude is the length of the segment. The electric current is I.

Direction of Magnetic Force (1)

$\vec{F} = I\vec{L} \times \vec{B}$



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Here we look at an illustration of the last point made on the previous page.

A current I flows through a copper wire of which only a short segment is shown. Remember that I is not a vector.

A coordinate system is used with the x-axis in current direction. It must be right handed, meaning (according to page 2) index x cross middle finger y equals thumb z.

A vector $\vec{L}$ is constructed whose direction is the current direction and whose magnitude is the length of the segment. The magnetic field vector $\vec{B}$ at the position of the segment is given.

The magnetic force can thus be determined as $\vec{F} = I\vec{L} \times \vec{B}$.

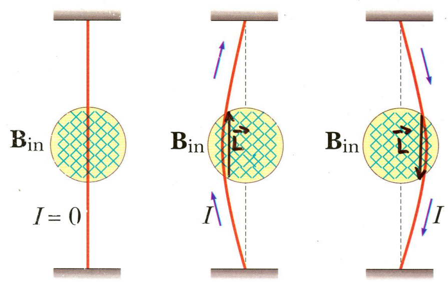
The right-hand rule dictates that $\vec{F}$ points in the positive z-direction.

The magnitude of the magnetic force is $F = ILB \sin\theta$, where L and B are the magnitudes of the vectors $\vec{L}$ and $\vec{B}$, respectively, and θ the angle between them.

The force on a current-carrying wire of arbitrary shape can be calculated by dividing the wire into short segments and determining the force on each segment, followed by a vector summation of all these forces.

Direction of Magnetic Force (2)

$$\vec{F} = I\vec{L} \times \vec{B}$$



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Here is another illustration intended to help you predict the direction of the magnetic force acting on a current-carrying conductor in a magnetic field.

Vectors are graphically represented by arrows like →, ↑, ↓, or ↙, as done on the previous page. If the vector is perpendicular to the plane of the page the arrow reduces to a point. It then matter whether the arrow is into or out of the page. The commonly used graphical conventions are as follows:

- ⊗: vector directed into the page (tail of arrow),
- ⊙: vector directed out of the page (tip of arrow).

In this illustration, the magnetic field $\vec{B}$, seemingly generated by a solenoid, is directed into the plane, as indicated by the crosses.

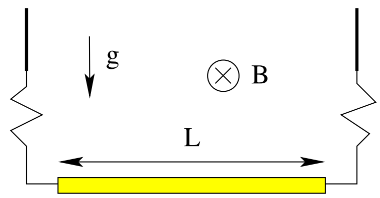
The wire on the left does not carry any current. Hence there is no magnetic force even if a magnetic field is present. Currents flow in opposite direction through the other two wires, producing magnetic forces in opposite directions.

Employ the right-hand rule to confirm that the force directions are as indicated by the bent wires: to the left (←) when the current is flowing up (↑) and to the right (→) when the current is flowing down (↓).

Magnetic Force Application (1)

A wire of length $L = 62$cm and mass $m = 13$g is suspended by a pair of flexible leads in a uniform magnetic field $B = 0.440$T pointing in to the plane.

- What are the magnitude and direction of the current required to remove the tension in the supporting leads?



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When we read the problem statement, our first thought is that this is an application of $\vec{F} = I\vec{L} \times \vec{B}$. This relation contains four quantities of which three are vectors.

The direction and magnitude of the vector $\vec{B}$ is given. The magnitude but not the direction of $\vec{L}$ is also given. However, we do know that $\vec{L}$ is directed either $\leftarrow$ or $\rightarrow$, in either case perpendicular to $\vec{B}$.

The magnetic force $\vec{F}$ must balance the downward gravitational force $m\vec{g}$. Hence it is directed $\uparrow$ and its magnitude is mg.

Given the directions $\uparrow$ of $\vec{F}$ and $\otimes$ of $\vec{B}$, the relation $\vec{F} = I\vec{L} \times \vec{B}$ can only be satisfied if the direction $\vec{L}$ is $\rightarrow$.

Now we use the fact that the angle between the vectors $\vec{L}$ and $\vec{B}$ is 90°, which allows us to write $F = ILB$ for the magnitude of the magnetic force.

This equation is readily solved for the unknown current I. Make sure to use SI units for all ingredients to produce the SI unit A:

$$I = \frac{mg}{LB} = \frac{(0.013\text{kg})(9.8\text{m/s}^2)}{(0.62\text{m})(0.44\text{T})} = 0.467\text{A}.$$

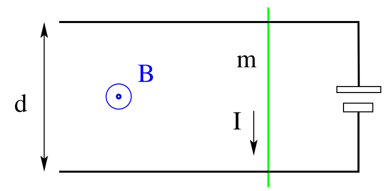
Magnetic Force Application (2)

A metal wire of mass $m = 1.5\text{kg}$ slides without friction on two horizontal rails spaced a distance $d = 3\text{m}$ apart.

The track lies in a vertical uniform magnetic field of magnitude $B = 24\text{mT}$ pointing out of the plane.

A constant current $I = 12\text{A}$ flows from a battery along one rail, across the wire, and back down the other rail. The wire starts moving from rest at $t = 0$.

- Find the direction and magnitude of the velocity of the wire at time $t = 5\text{s}$.



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In this application of $\vec{F} = I\vec{L} \times \vec{B}$, the given current I as driven by the battery shown is flowing counterclockwise through the loop.

The relevant segment is the portion of the wire between the rails. We infer that the vector $\vec{L}$ is directed $\downarrow$ and has magnitude d.

Direction and magnitude of the magnetic field $\vec{B}$ are given.

Use the right-hand rule to show that the magnetic force is directed $\leftarrow$.

We use the fact that $\vec{L}$ and $\vec{B}$ are perpendicular to each other to infer that that the magnitude of the magnetic force is $F = IdB$, which we calculate, paying attention to SI units:

$$F = IdB = (12\text{A})(3\text{m})(2.4 \times 10^{-2}\text{T}) = 0.864\text{N}.$$

The (constant) magnetic force is the cause of a constant acceleration of the wire as dictated by Newtons second law, $\vec{F} = m\vec{a}$.

The acceleration is in the same direction as the force, namely $\leftarrow$. The wire is initially at rest. Hence it will pick up velocity $\vec{v} = \vec{a}t$ directed $\leftarrow$.

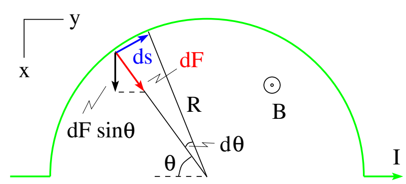
The speed at the given time is

$$v = at = \frac{F}{m}t = \frac{0.864\text{N}}{1.5\text{kg}}(5\text{s}) = 2.88\text{m/s}.$$

Magnetic Force on Semicircular Current (1)

Fancy solution:

- Uniform magnetic field $\vec{B}$ points out of the plane.
- Magnetic force on segment ds: $dF = IBds = IBRd\theta$.
- Integrate $dF_x = dF \sin\theta$ and $dF_y = dF \cos\theta$ along semicircle.
- $F_x = IBR \int_0^\pi \sin\theta d\theta = 2IBR, \quad F_y = IBR \int_0^\pi \cos\theta d\theta = 0.$



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On this page and the next we calculate the magnetic force on a clockwise semicircular current I of radius R in a uniform magnetic field $\vec{B}$ directed out of the plane.

The method used on this page divides the semicircle into infinitesimal segments $d\vec{s}$. We calculate the force for each segment, using $d\vec{F} = Id\vec{s} \times \vec{B}$, and then sum up the contributions from successive segments via integration.

We account for the fact that the force $d\vec{F}$ changes direction between segments by splitting it into two components: $d\vec{F} = dF_x\,\hat{\mathbf{i}} + dF_y\,\hat{\mathbf{j}}$ using the coordinate system shown.

The (definite) integrals to be evaluated are elementary ones:

$$\int_0^\pi \sin\theta\, d\theta = \Big[-\cos\theta\Big]_0^\pi = 2, \quad \int_0^\pi \cos\theta\, d\theta = \Big[\sin\theta\Big]_0^\pi = 0.$$

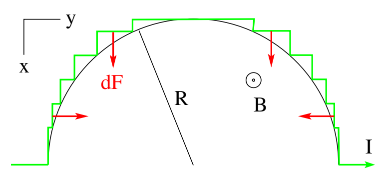
The result is a force directed ↓:

$$\vec{F} = 2IBR\,\hat{\mathbf{i}}.$$

Magnetic Force on Semicircular Current (2)

Clever solution:

- Replace the semicircle by symmetric staircase of tiny wire segments.
- Half the vertical segments experience a force to the left, the other half a force to the right. The resultant horizontal force is zero.
- All horizontal segments experience a downward force. The total length is $2R$.
  The total downward force is $2IBR$.
- Making the segments infinitesimally small does not change the result.



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We can reduce the math in the analysis of the semicircular current substantially if we replace each infinitesimal segment from the previous page, which are tangential to the semicircle, by two segments, one vertical and one horizontal.

The magnetic force on each horizontal segment is directed $\downarrow$. On each vertical segment it is directed $\rightarrow$ or $\leftarrow$, always toward the inside of the semicircle.

Summing up the forces is straightforward. The horizontal forces directed left and right cancel each other. The vertical forces are equivalent to the force on a straight horizontal current of length $2R$.

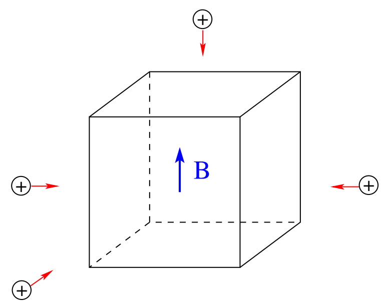
The results is the downward force found on the previous page.

Magnetic Force Application (5)

Inside the cube there is a magnetic field $\vec{B}$ directed vertically up.

Find the direction of the magnetic force experienced by a proton entering the cube

(a) from the left,
(b) from the front,
(c) from the right,
(d) from the top.



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This is the quiz for lecture 19. It is an application of the vector equation $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$.

Possible answers are either no force or one of the six directions,
$\rightarrow$ (right), $\leftarrow$ (left), $\uparrow$ (up), $\downarrow$ (down), $\otimes$ (in), $\odot$ (out).