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19. LC and RLC Oscillators

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Abstract

Lecture slides 19 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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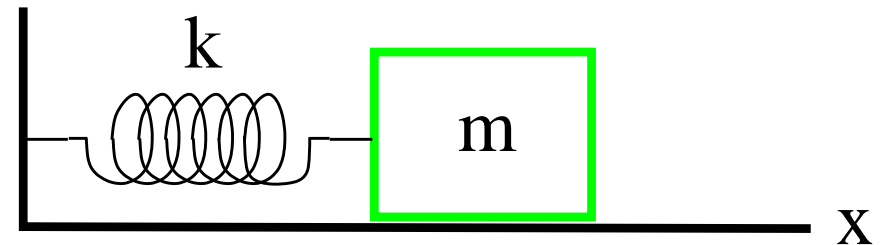
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Mechanical Oscillator



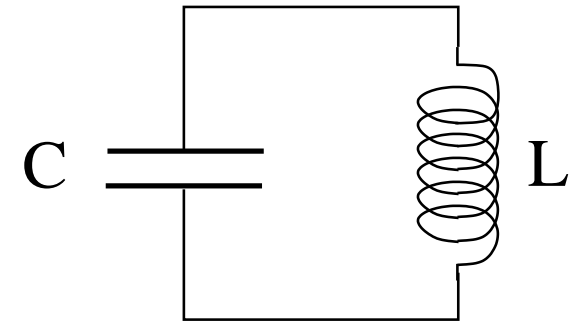
- law of motion: $F = ma$, $a = \frac{d^2x}{dt^2}$
- law of force: $F = -kx$
- equation of motion: $\frac{d^2x}{dt^2} = -\frac{k}{m}x$
- displacement: $x(t) = x_{max} \cos(\omega t)$
- velocity: $v(t) = -\omega x_{max} \sin(\omega t)$
- angular frequency: $\omega = \sqrt{\frac{k}{m}}$
- kinetic energy: $K = \frac{1}{2}mv^2$
- potential energy: $U = \frac{1}{2}kx^2$
- total energy: $E = K + U = \text{const.}$



Electromagnetic Oscillator (LC Circuit)



- loop rule: $\frac{Q}{C} + L \frac{dI}{dt} = 0, I = \frac{dQ}{dt}$
- equation of motion: $\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$
- charge on capacitor: $Q(t) = Q_{max} \cos(\omega t)$
- current through inductor: $I(t) = -\omega Q_{max} \sin(\omega t)$
- angular frequency: $\omega = \frac{1}{\sqrt{LC}}$
- magnetic energy: $U_B = \frac{1}{2}LI^2$ (stored on inductor)
- electric energy: $U_E = \frac{Q^2}{2C}$ (stored on capacitor)
- total energy: $E = U_B + U_E = \text{const.}$

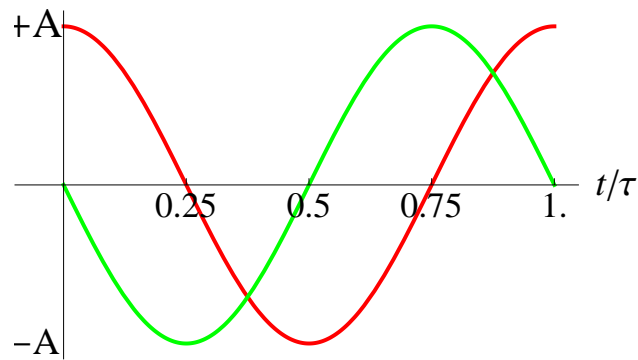


Mechanical vs Electromagnetic Oscillations

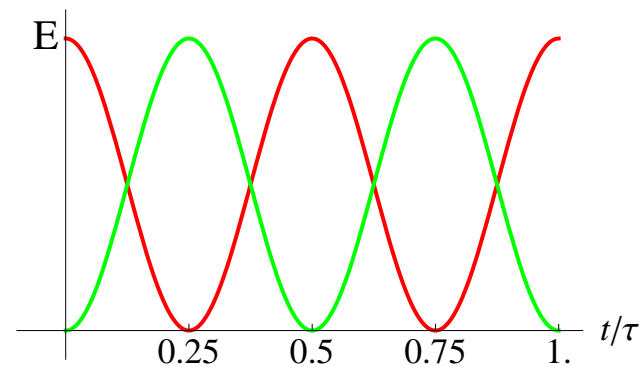


mechanical oscillations

- position: $x(t) = A \cos(\omega t)$ [red]
- velocity: $v(t) = -A \sin(\omega t)$ [green]
- period: $\tau = \frac{2\pi}{\omega}$, $\omega = \sqrt{\frac{k}{m}}$



- potential energy: $U(t) = \frac{1}{2}kx^2(t)$ [r]
- kinetic energy: $K(t) = \frac{1}{2}mv^2(t)$ [g]
- total energy: $E = U(t) + K(t) = \text{const}$



electromagnetic oscillations

- charge: $Q(t) = A \cos(\omega t)$ [red]
- current: $I(t) = -A \sin(\omega t)$ [green]
- period: $\tau = \frac{2\pi}{\omega}$, $\omega = \frac{1}{\sqrt{LC}}$

- electric energy: $U_E(t) = \frac{1}{2C}Q^2(t)$ [r]
- magnetic energy: $U_B(t) = \frac{1}{2}LI^2(t)$ [g]
- total energy: $E = U_E(t) + U_B(t) = \text{const}$

Mechanical Oscillator with Damping



- law of motion: $F = ma$, $a = \frac{d^2x}{dt^2}$
- law of force: $F = -kx - bv$, $v = \frac{dx}{dt}$
- equation of motion: $\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$

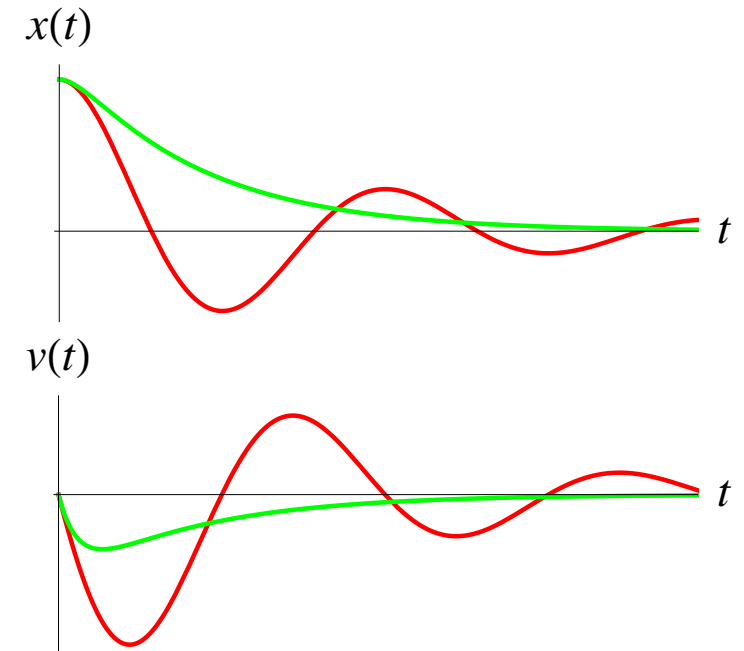
Solution for initial conditions $x(0) = A$, $v(0) = 0$:

(a) underdamped motion: $b^2 < 4km$

$$x(t) = Ae^{-bt/2m} \left[\cos(\omega't) + \frac{b}{2m\omega'} \sin(\omega't) \right] \quad \text{with} \quad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

(b) overdamped motion: $b^2 > 4km$

$$x(t) = Ae^{-bt/2m} \left[\cosh(\Omega't) + \frac{b}{2m\Omega'} \sinh(\Omega't) \right] \quad \text{with} \quad \Omega' = \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}$$



Damped Electromagnetic Oscillator (RLC Circuit)



- loop rule: $RI + L \frac{dI}{dt} + \frac{Q}{C} = 0, I = \frac{dQ}{dt}$
- equation of motion: $\frac{d^2Q}{dt^2} + \frac{R}{L} \frac{dQ}{dt} + \frac{1}{LC}Q = 0$

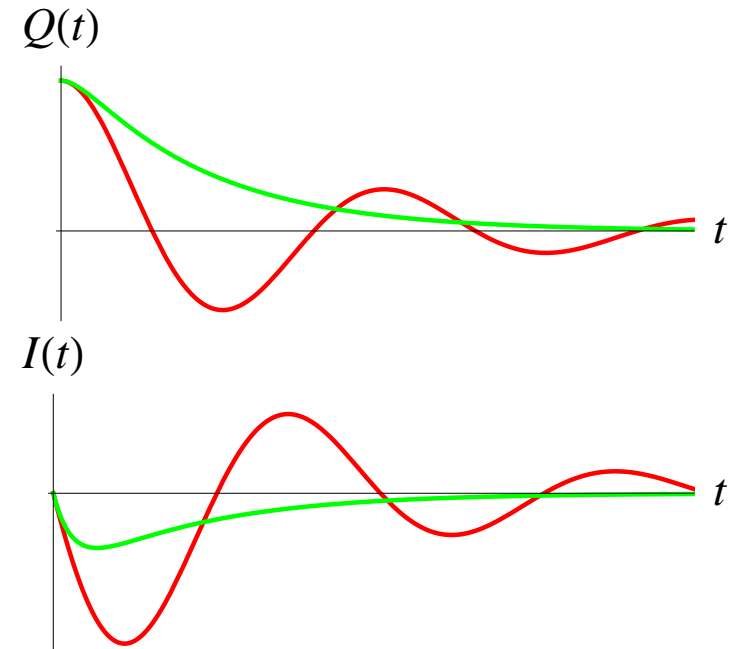
Solution for initial conditions $Q(0) = Q_{max}, I(0) = 0$:

(a) underdamped motion: $R^2 < \frac{4L}{C}$

$$Q(t) = Q_{max} e^{-Rt/2L} \left[\cos(\omega' t) + \frac{R}{2L\omega'} \sin(\omega' t) \right] \quad \text{with} \quad \omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

(b) overdamped motion: $R^2 > \frac{4L}{C}$

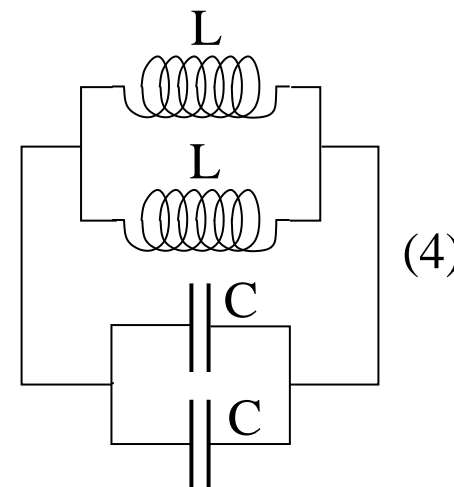
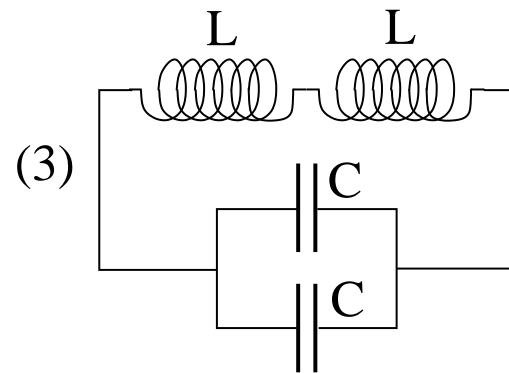
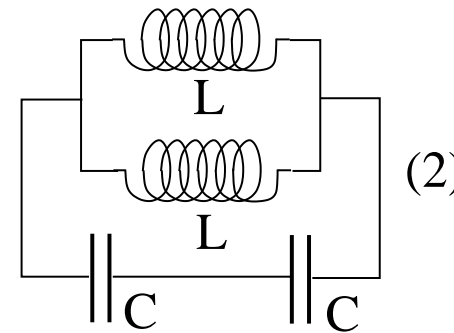
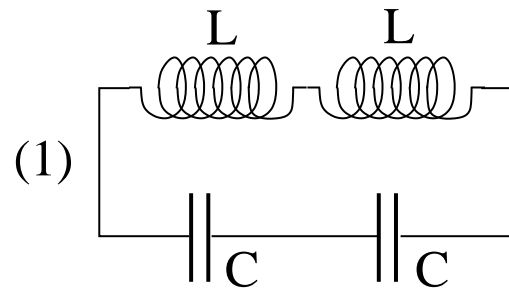
$$Q(t) = Q_{max} e^{-Rt/2L} \left[\cosh(\Omega' t) + \frac{R}{2L\Omega'} \sinh(\Omega' t) \right] \quad \text{with} \quad \Omega' = \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$



LC Circuit: Application (1)



Name the LC circuit with the highest and the lowest angular frequency of oscillation.

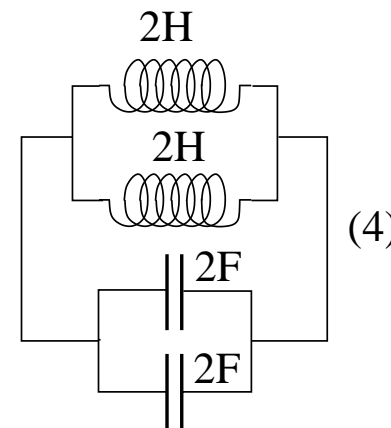
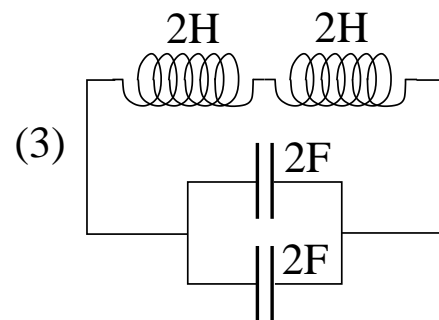
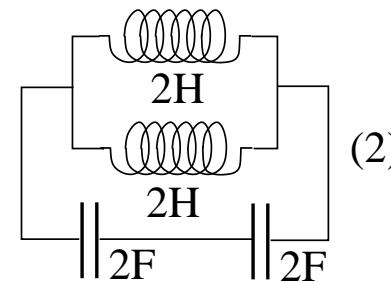
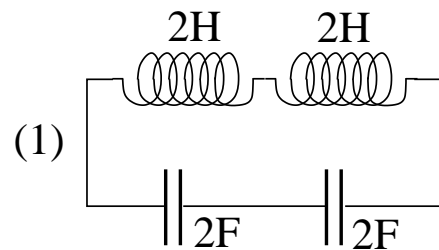


LC Circuit: Application (2)



At time $t = 0$ a charge $Q = 2C$ is on each capacitor and all currents are zero.

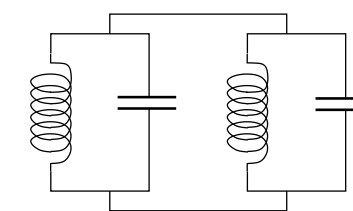
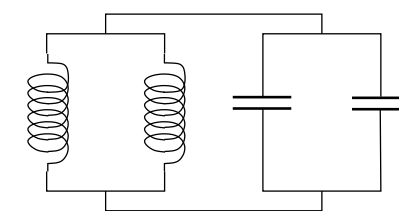
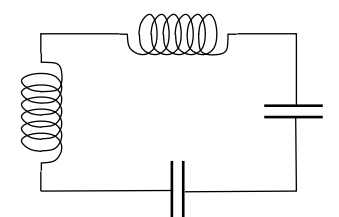
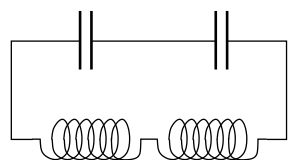
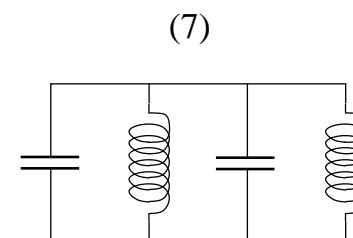
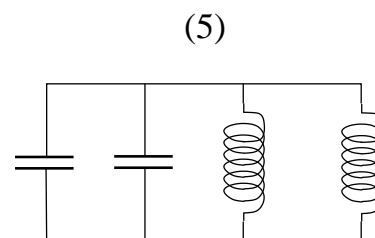
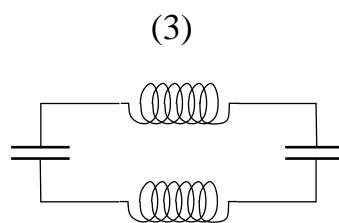
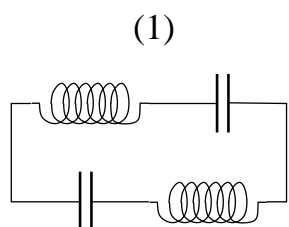
- (a) What is the energy stored in the circuit?
- (b) At what time t_1 are the capacitors discharged for the first time?
- (c) What is the current through each inductor at time t_1 ?



LC Circuit: Application (3)



In these LC circuits all capacitors have equal capacitance C and all inductors have equal inductance L . Sort the circuits into groups that are equivalent.



Oscillator with Two Modes



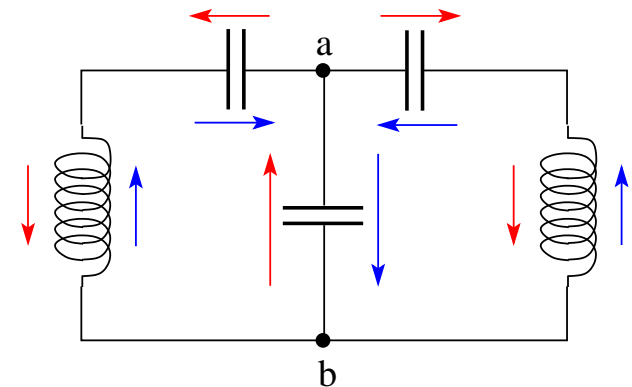
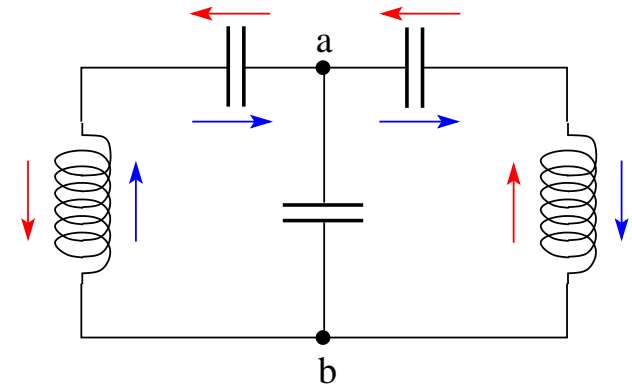
Electromagnetic:

$$\text{mode \#1: } L \frac{dI}{dt} + \frac{Q}{C} + \frac{Q}{C} + L \frac{dI}{dt} = 0, \quad I = \frac{dQ}{dt}$$

$$\Rightarrow \frac{dI}{dt} = -\frac{Q}{LC} \Rightarrow \frac{d^2Q}{dt^2} = -\omega^2 Q, \quad \omega = \frac{1}{\sqrt{LC}}$$

$$\text{mode \#2: } L \frac{dI}{dt} + \frac{Q}{C} + \frac{2Q}{C} = 0, \quad I = \frac{dQ}{dt}$$

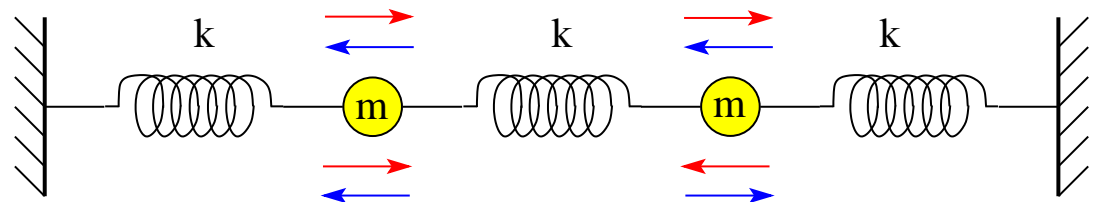
$$\Rightarrow \frac{dI}{dt} = -\frac{3Q}{LC} \Rightarrow \frac{d^2Q}{dt^2} = -\omega^2 Q, \quad \omega = \sqrt{\frac{3}{LC}}$$



Mechanical:

$$\text{mode \#1: } \omega = \sqrt{\frac{k}{m}}$$

$$\text{mode \#2: } \omega = \sqrt{\frac{3k}{m}}$$



RLC Circuit: Application (1)



In the circuit shown the capacitor is without charge.

When the switch is closed to position *a*...

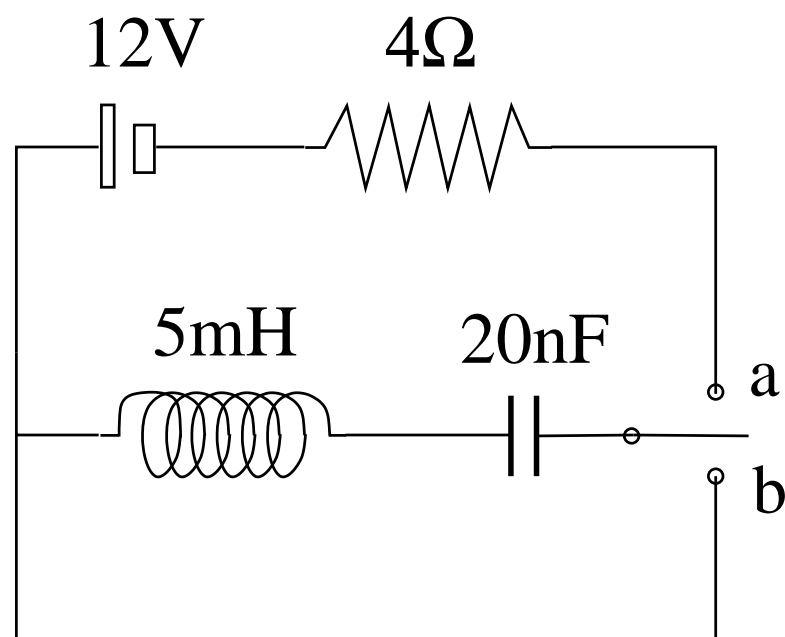
(a) find the initial rate dI/dt at which the current increases from zero,

(b) find the charge Q on the capacitor after a long time.

Then, when the switch is thrown from *a* to *b*...

(c) find the time t_1 it takes the capacitor to fully discharge,

(d) find the maximum current I_{max} in the process of discharging.



RLC Circuit: Application (2)



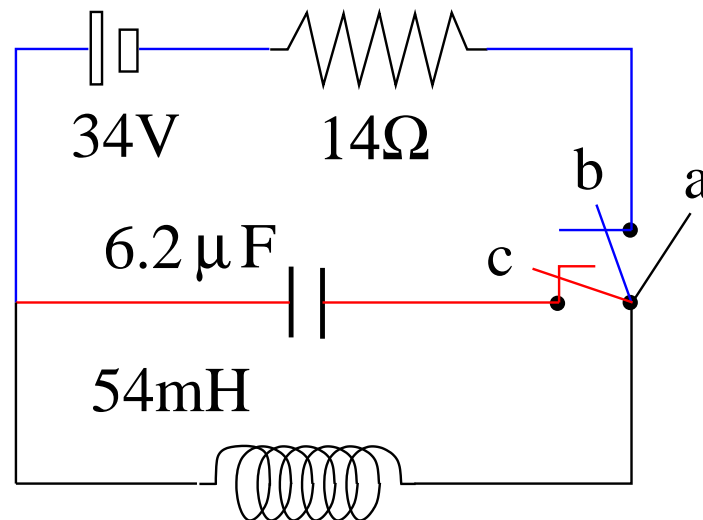
In the circuit shown the capacitor is without charge and the switch is in position a .

(i) When the switch is moved to position b we have an RL circuit with the current building up gradually: $I(t) = (\mathcal{E}/R)[1 - e^{-t/\tau}]$.

Find the time constant τ and the current I_{max} after a long time.

(ii) Then we reset the clock and move the switch from b to c with no interruption of the current through the inductor. We now have a an LC circuit: $I(t) = I_{max} \cos(\omega t)$.

Find the angular frequency of oscillation ω and the maximum charge Q_{max} that goes onto the capacitor periodically.



RLC Circuit: Application (3)



In the circuit shown the capacitor is without charge and the switch is in position *a*.

(i) When the switch is moved to position *b* we have an *RC* circuit with the capacitor being charged up gradually: $Q(t) = \mathcal{E}C[1 - e^{-t/\tau}]$.

Find the time constant τ and the charge Q_{max} after a long time.

(ii) Then we reset the clock and move the switch from *b* to *c*.

We now have a an *LC* circuit: $Q(t) = Q_{max} \cos(\omega t)$.

Find the angular frequency of oscillation ω and the maximum current I_{max} that flows through the inductor periodically.

