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18. RL Circuits

Gerhard Müller
University of Rhode Island, gmuller@uri.edu

Abstract
Lecture slides 18 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.
Some of the slides contain figures from the textbook, Paul A. Tipler and Gene Mosca. *Physics for Scientists and Engineers*, 5th/6th editions. The copyright to these figures is owned by W.H. Freeman. We acknowledge permission from W.H. Freeman to use them on this course web page. The textbook figures are not to be used or copied for any purpose outside this class without direct permission from W.H. Freeman.

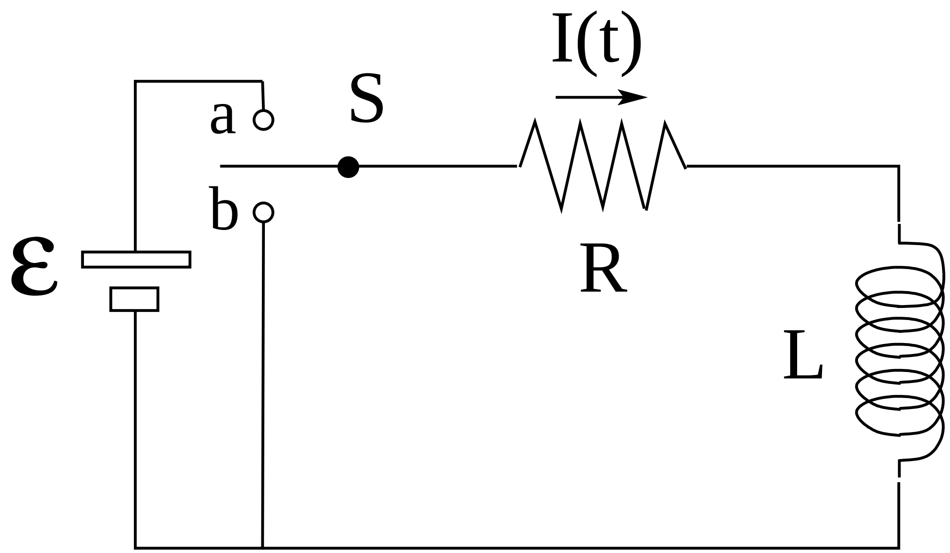
RL Circuit: Fundamentals

Specifications:

- $\mathcal{E}$ (emf)
- R (resistance)
- L (inductance)

Switch S:

- a: current buildup
- b: current shutdown



Time-dependent quantities:

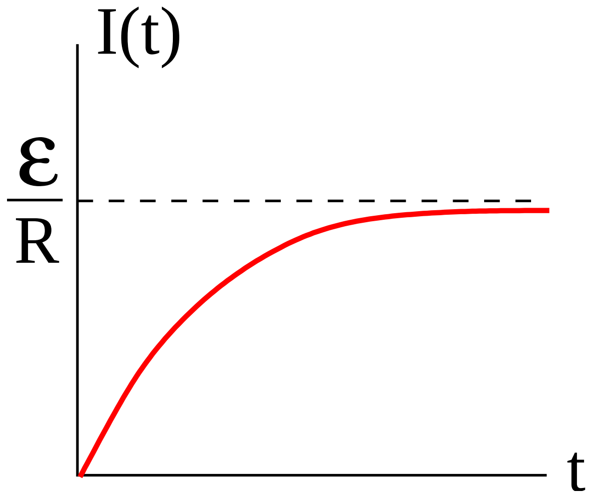
- $I(t)$: instantaneous current through inductor
- $\dfrac{dI}{dt}$: rate of change of instantaneous current
- $V_R(t) = I(t)R$: instantaneous voltage across resistor
- $V_L(t) = L\,\dfrac{dI}{dt}$: instantaneous voltage across inductor

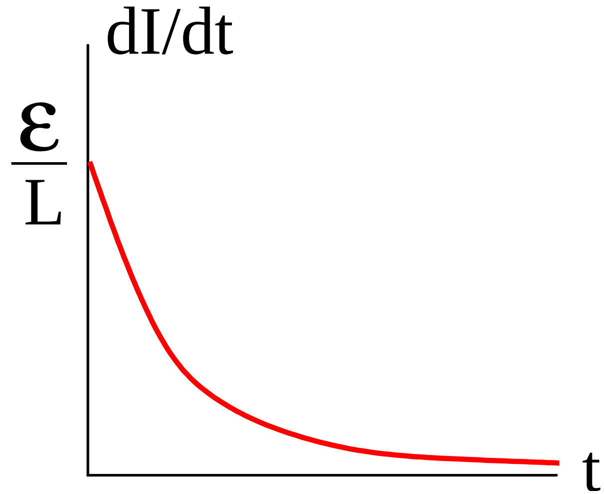
RL Circuit: Current Buildup in Inductor

- Loop rule: $\mathcal{E} - IR - L\frac{dI}{dt} = 0$
- Differential equation: $L\frac{dI}{dt} = \mathcal{E} - IR \quad \Rightarrow \frac{dI}{dt} = \frac{\mathcal{E}/R - I}{L/R}$

$$\int_0^I \frac{dI}{\mathcal{E}/R - I} = \int_0^t \frac{dt}{L/R} \quad \Rightarrow \; -\ln\left(\frac{\mathcal{E}/R - I}{\mathcal{E}/R}\right) = \frac{t}{L/R} \quad \Rightarrow \frac{\mathcal{E}/R - I}{\mathcal{E}/R} = e^{-Rt/L}$$

- Current through inductor: $I(t) = \frac{\mathcal{E}}{R}\left[1 - e^{-Rt/L}\right]$
- Rate of current change: $\frac{dI}{dt} = \frac{\mathcal{E}}{L}\, e^{-Rt/L}$



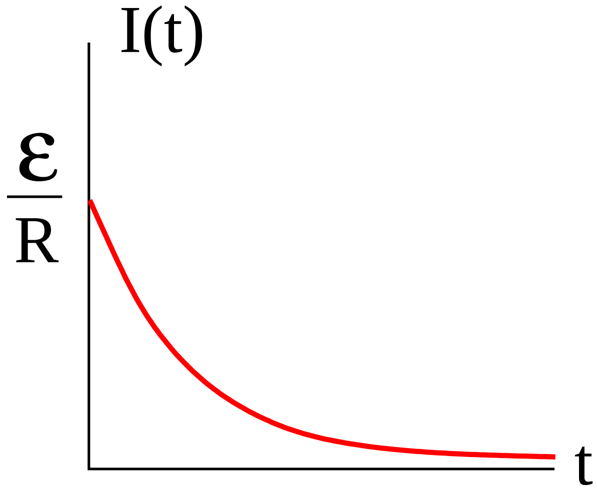


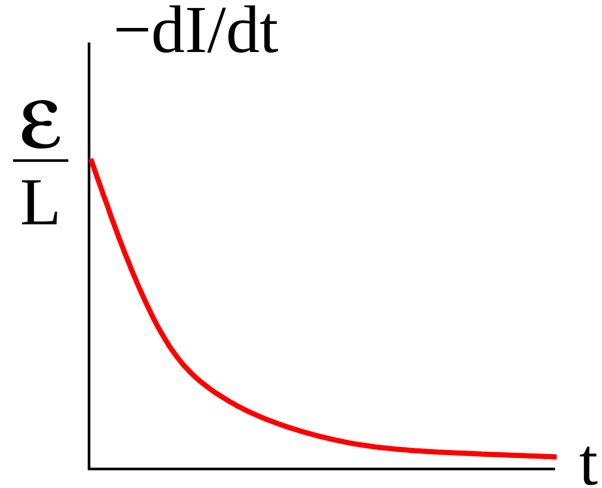
RL Circuit: Current Shutdown in Inductor

- Loop rule: $-IR - L\dfrac{dI}{dt} = 0$
- Differential equation: $L\dfrac{dI}{dt} + IR = 0 \quad \Rightarrow \quad \dfrac{dI}{dt} = -\dfrac{R}{L}I$

$$\Rightarrow \int_{\mathcal{E}/R}^{I} \frac{dI}{I} = -\frac{R}{L}\int_0^t dt \quad \Rightarrow \quad \ln\frac{I}{\mathcal{E}/R} = -\frac{R}{L}t \quad \Rightarrow \quad \frac{I}{\mathcal{E}/R} = e^{-Rt/L}$$

- Current: $I(t) = \dfrac{\mathcal{E}}{R}\, e^{-Rt/L}$
- Rate of current change: $\dfrac{dI}{dt} = -\dfrac{\mathcal{E}}{L}\, e^{-Rt/L}$



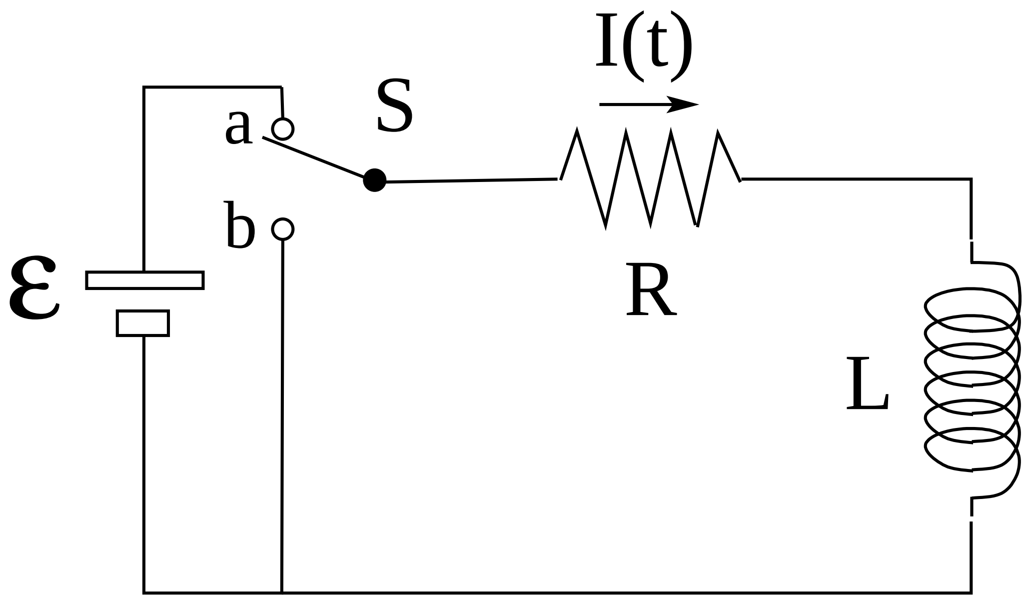


RL Circuit: Energy Transfer During Current Buildup

Loop rule: $IR + L\dfrac{dI}{dt} = \mathcal{E} \quad \left(I > 0,\ \dfrac{dI}{dt} > 0\right)$

- $I\mathcal{E}$: rate at which EMF source delivers energy
- $IV_R = I^2R$: rate at which energy is dissipated in resistor
- $IV_L = LI\dfrac{dI}{dt}$: rate at which energy is stored in inductor

Balance of energy transfer: $I^2R + LI\dfrac{dI}{dt} = I\mathcal{E}$

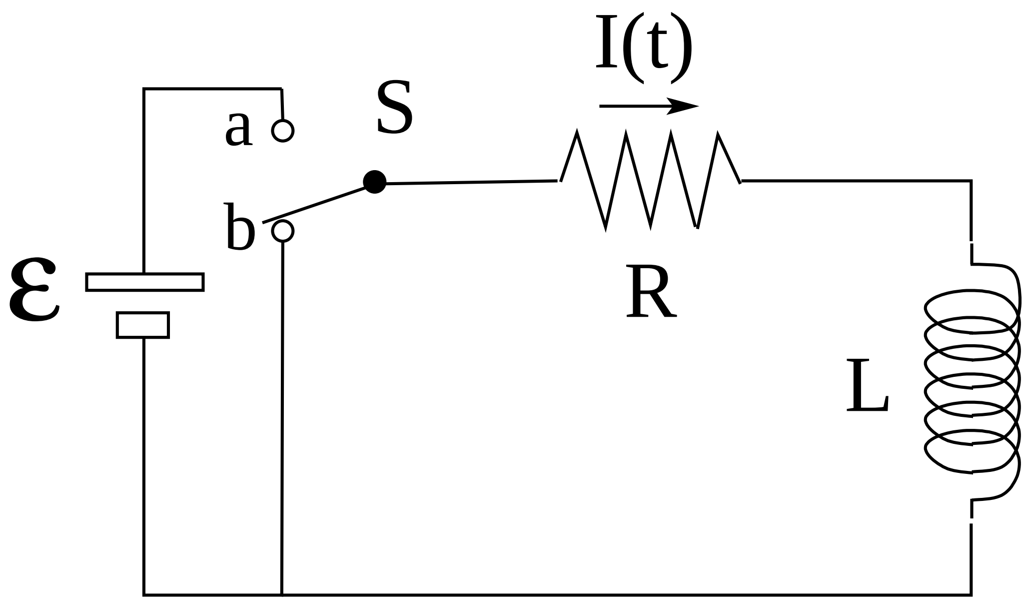


RL Circuit: Energy Transfer During Current Shutdown

Loop rule: $IR + L\frac{dI}{dt} = 0 \quad \left(I > 0,\ \frac{dI}{dt} < 0\right)$

- $IV_L = LI\frac{dI}{dt}$: rate at which inductor releases energy
- $IV_R = I^2R$: rate at which energy is dissipated in resistor

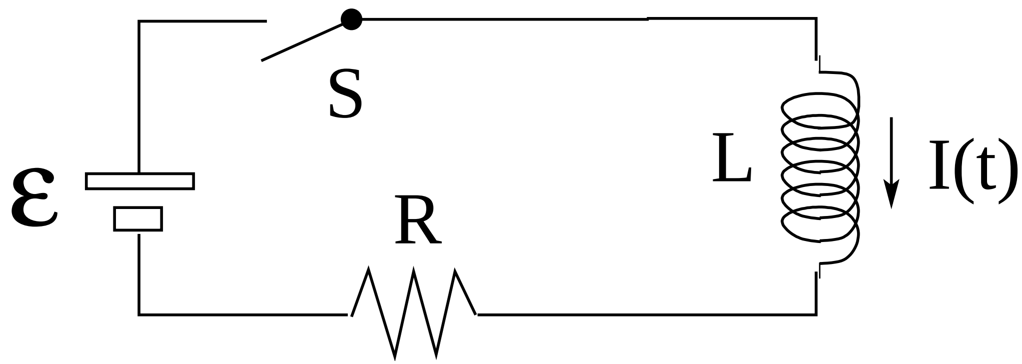
Balance of energy transfer: $I^2R + LI\frac{dI}{dt} = 0$



RL Circuit: Some Physical Properties

Specification of RL circuit
by 3 device properties:

- $\mathcal{E}$ [V] (emf)
- R $[\Omega]$ (resistance)
- L [H] (inductance)



Physical properties of RL circuit during current buildup determined by 3 combinations of the device properties:

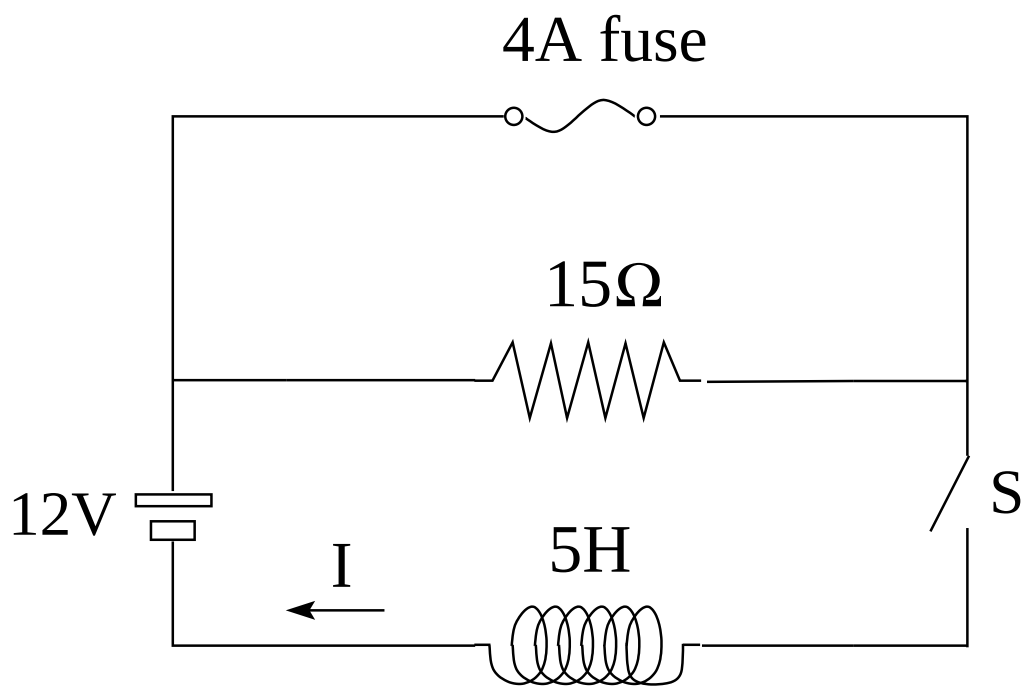
- $\dfrac{\mathcal{E}}{L} = \left.\dfrac{dI}{dt}\right|_{t=0}$: initial rate at which current increases
- $\dfrac{\mathcal{E}}{R} = I(t = \infty)$: final value of current
- $L/R = \tau$: time it takes to build up 63% of the current through the circuit
  $[1 - e^{-1} = 0.632\ldots]$

RL Circuit: Application (7)

In the circuit shown the switch S is closed at time $t = 0$.

(a) Find the current I as a function of time for $0 < t < t_F$, where t_F marks the instant the fuse breaks.

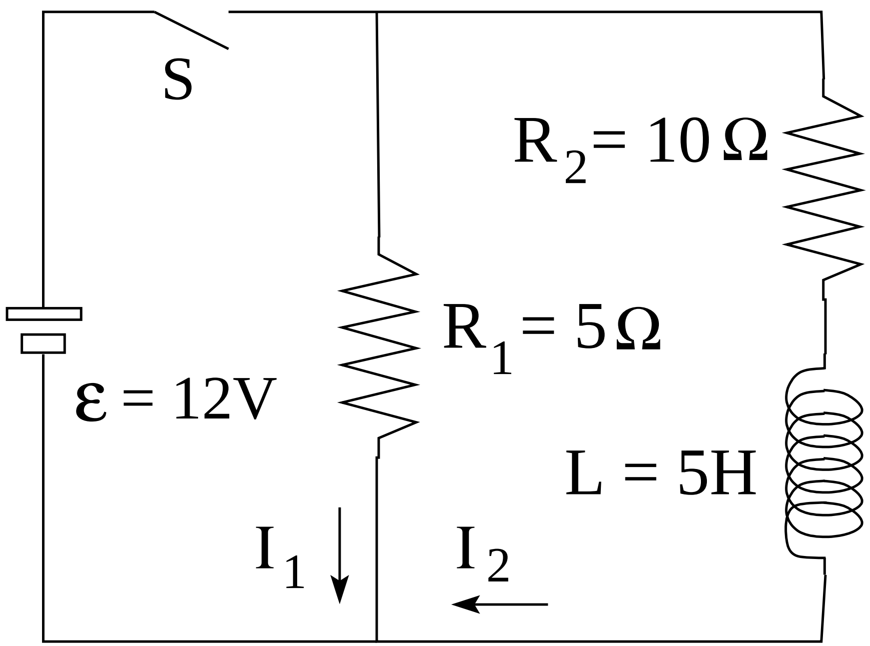
(b) Find the current I as a function of time for $t > t_F$.



RL Circuit: Application (8)

In the circuit shown the switch has been open for a long time.
Find the currents I_1 and I_2

- just after the switch has been closed,
- a long time later,
- as functions of time for $0 < t < \infty$.



RL Circuit: Application (6)

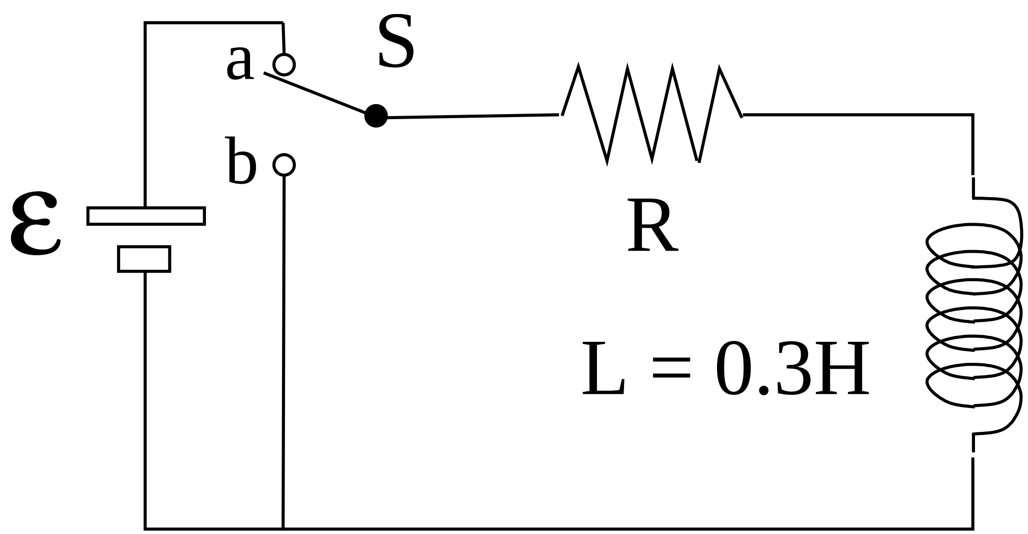
In the RL circuit shown the switch has been at position a for a long time and is thrown to position b at time $t = 0$. At that instant the current has the value $I_0 = 0.7$A and decreases at the rate $dI/dt = -360$A/s.

(a) Find the EMF $\mathcal{E}$ of the battery.

(b) Find the resistance R of the resistor.

(c) At what time t_1 has the current decreased to the value $I_1 = 0.2$A?

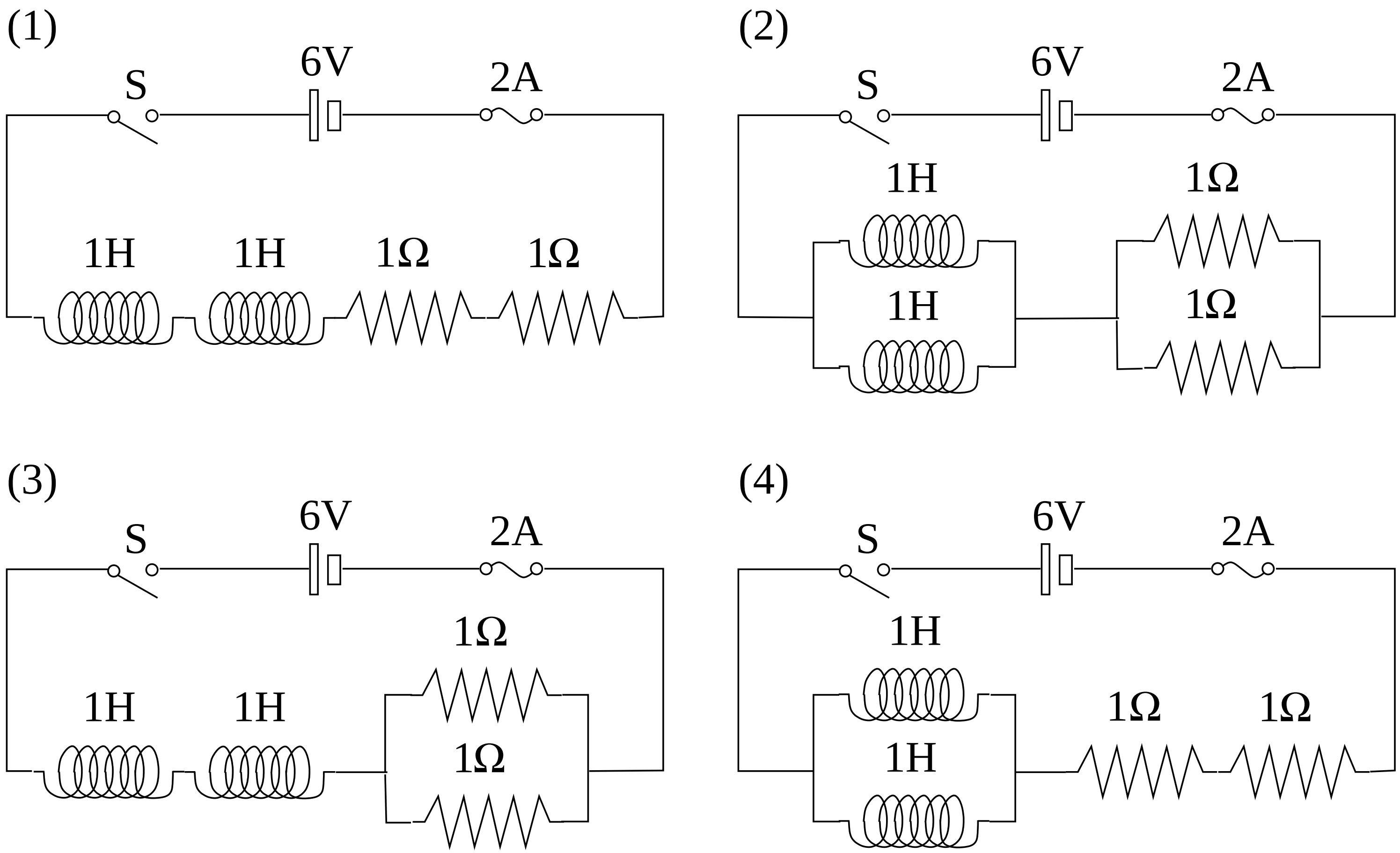
(d) Find the voltage across the inductor at time t_1.



RL Circuit: Application (5)

Each RL circuit contains a 2A fuse. The switches are closed at $t = 0$.

- In what sequence are the fuses blown?



RL Circuit: Application (3)

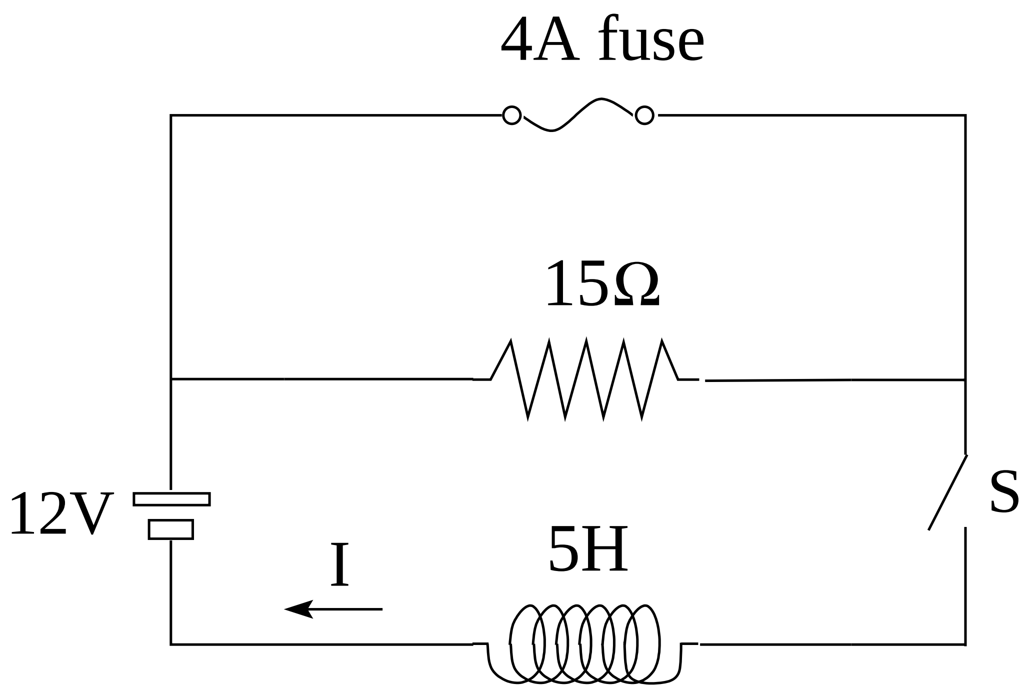
The switch is closed at $t = 0$. Find the current I

(a) immediately after the switch has been closed,

(b) immediately before the fuse breaks,

(c) immediately after the fuse has broken,

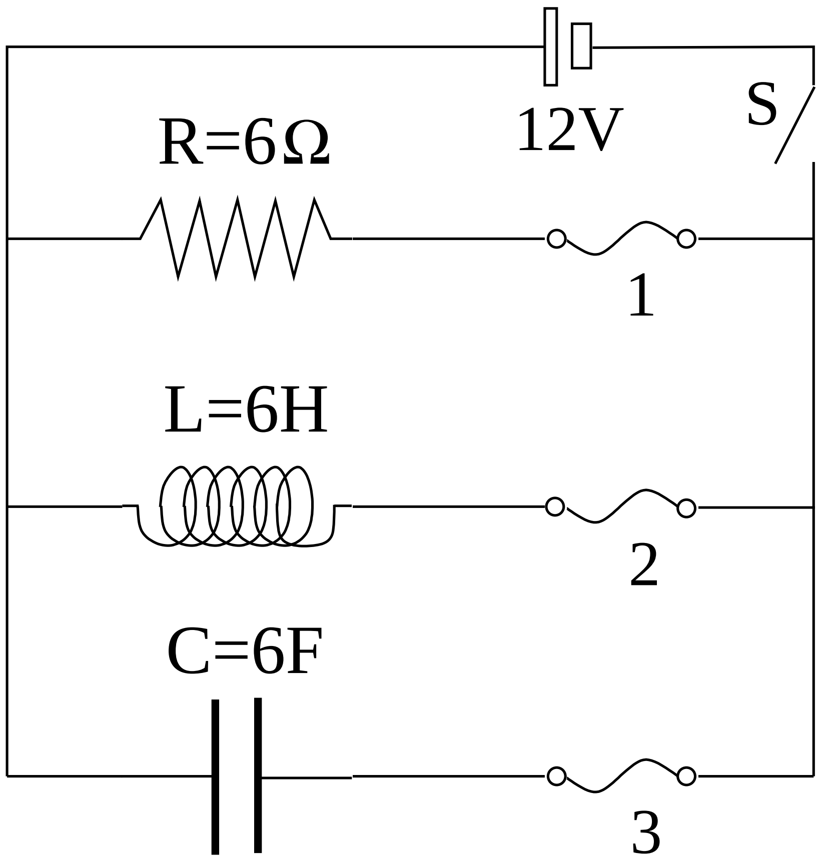
(d) a very long time later.



Each branch in the circuit shown contains a 3A fuse. The switch is closed at time $t = 0$.

(a) Which fuse is blown in the shortest time?

(b) Which fuse lasts the longest time?

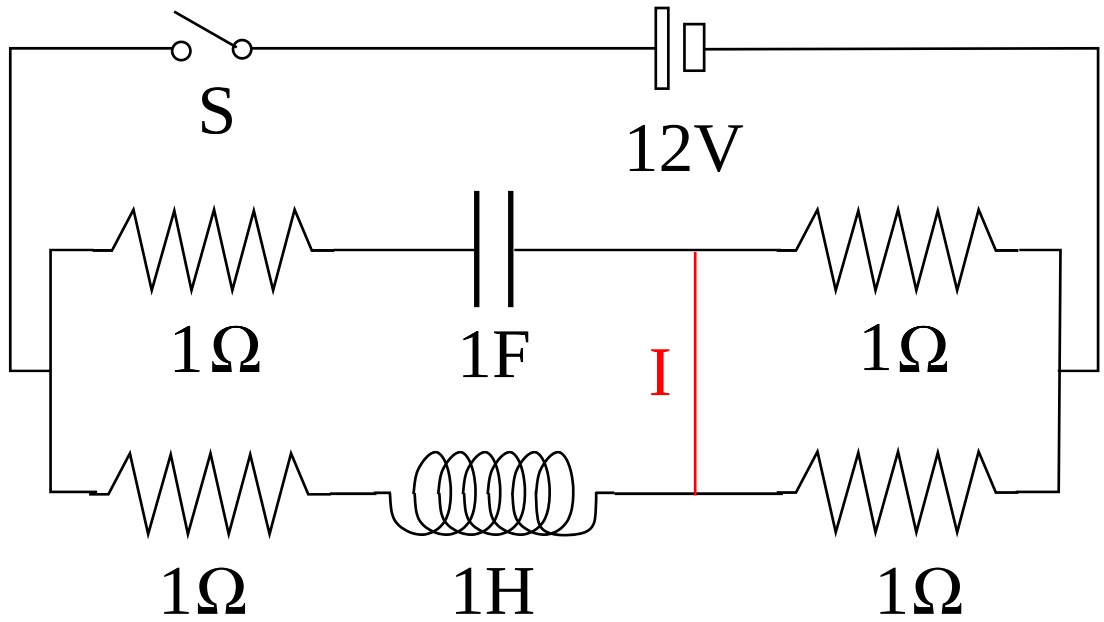


RL Circuit: Application (4)

Find the magnitude (in amps) and the direction $(\uparrow, \downarrow)$ of the current I

(a) right after the switch has been closed,

(b) a very long time later.



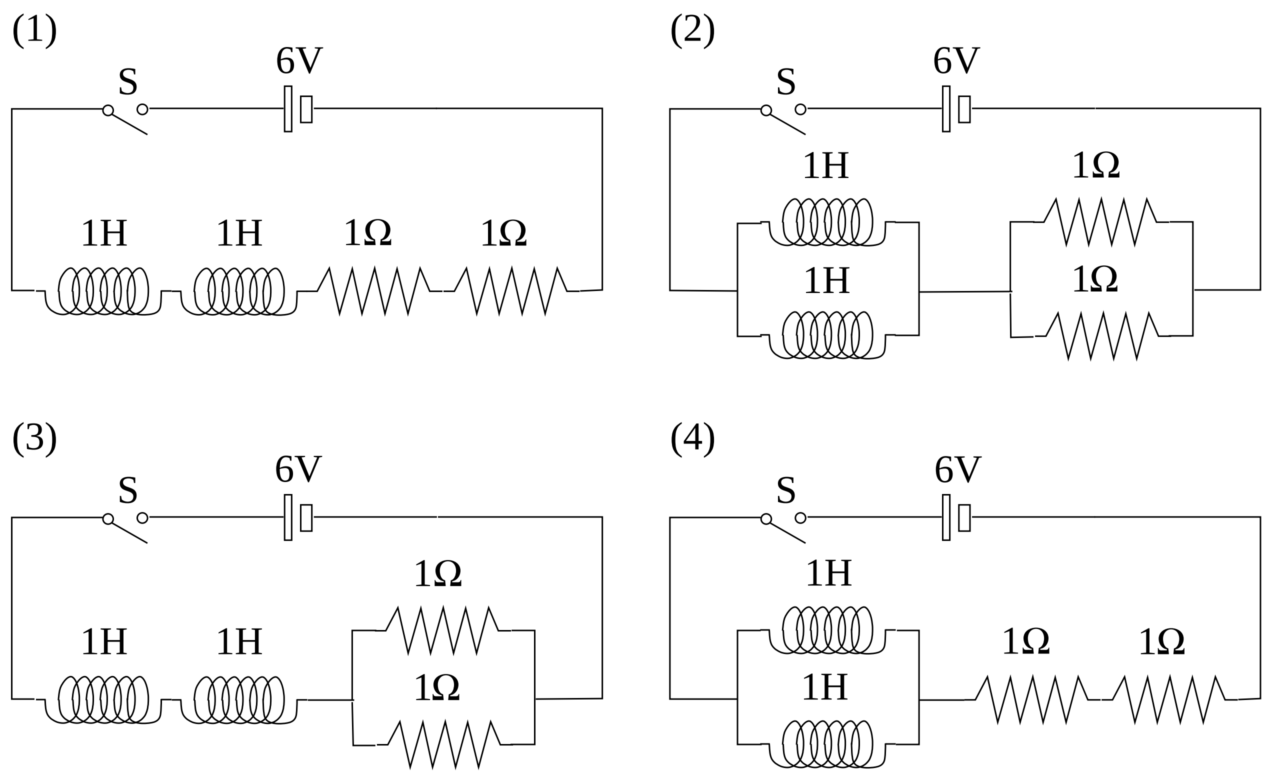
RL Circuit: Application (2)

The switch in each RL circuit is closed at $t = 0$.
Rank the circuits according to three criteria:

(a) magnitude of current at $t =$1ms,

(b) magnitude of current at $t = \infty$,

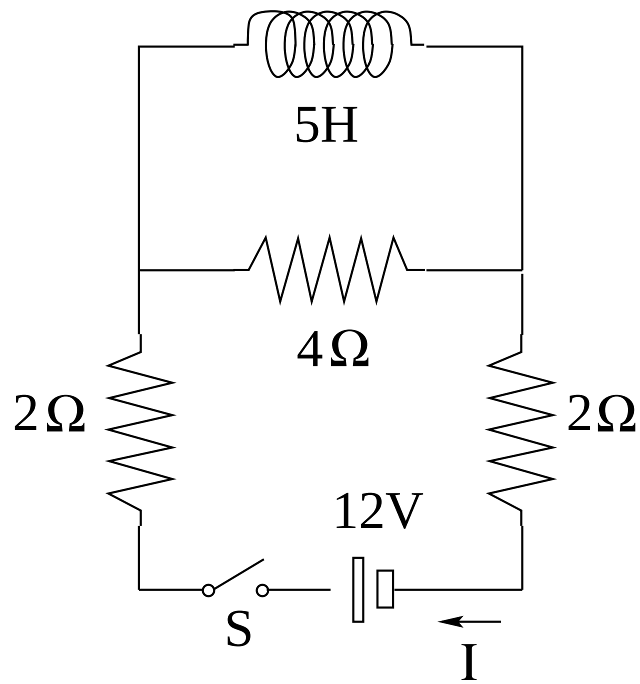
(c) time it takes I to reach 63% of its ultimate value.



In the circuit shown we close the switch S at time $t = 0$. Find the current I through the battery and the voltage V_L across the inductor

(a) immediately after the switch has been closed,

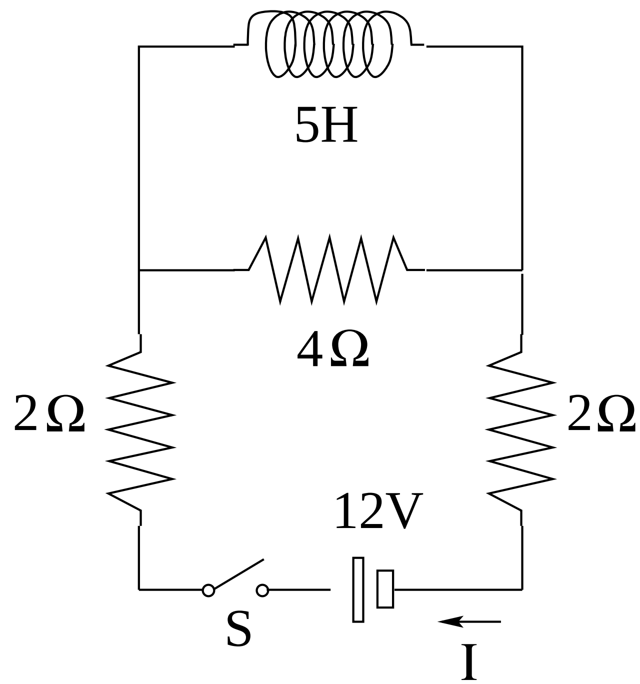
(b) a very long time later.



In the circuit shown we close the switch S at time $t = 0$. Find the current I through the battery and the voltage V_L across the inductor

(a) immediately after the switch has been closed,

(b) a very long time later.



Solution:

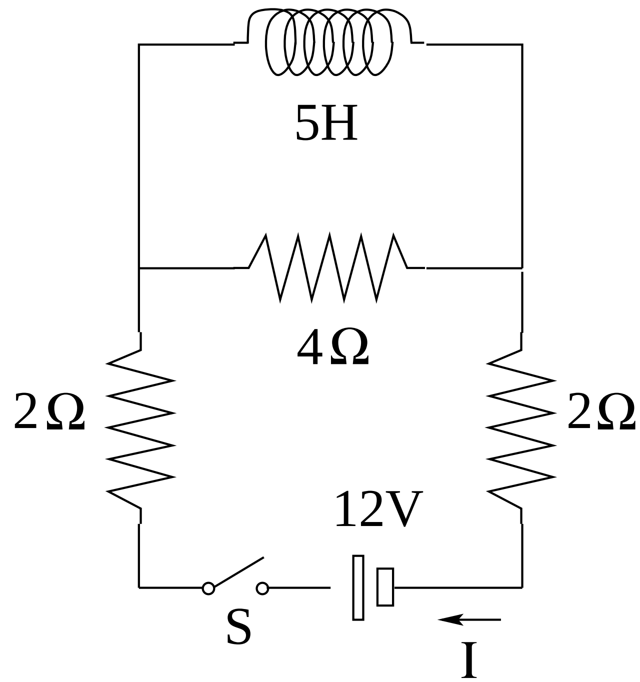
(a) $I = \dfrac{12\text{V}}{2\Omega + 4\Omega + 2\Omega} = 1.5\text{A}, \qquad V_L = (4\Omega)(1.5\text{A}) = 6\text{V}.$

Intermediate Exam III: Problem #2 (Spring ’05)

In the circuit shown we close the switch S at time $t = 0$. Find the current I through the battery and the voltage V_L across the inductor

(a) immediately after the switch has been closed,

(b) a very long time later.



Solution:

(a) $I = \dfrac{12\text{V}}{2\Omega + 4\Omega + 2\Omega} = 1.5\text{A}, \qquad V_L = (4\Omega)(1.5\text{A}) = 6\text{V}.$

(b) $I = \dfrac{12\text{V}}{2\Omega + 2\Omega} = 3\text{A}, \qquad V_L = 0.$