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18. RL Circuits

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Abstract

Lecture slides 18 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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RL Circuit: Fundamentals

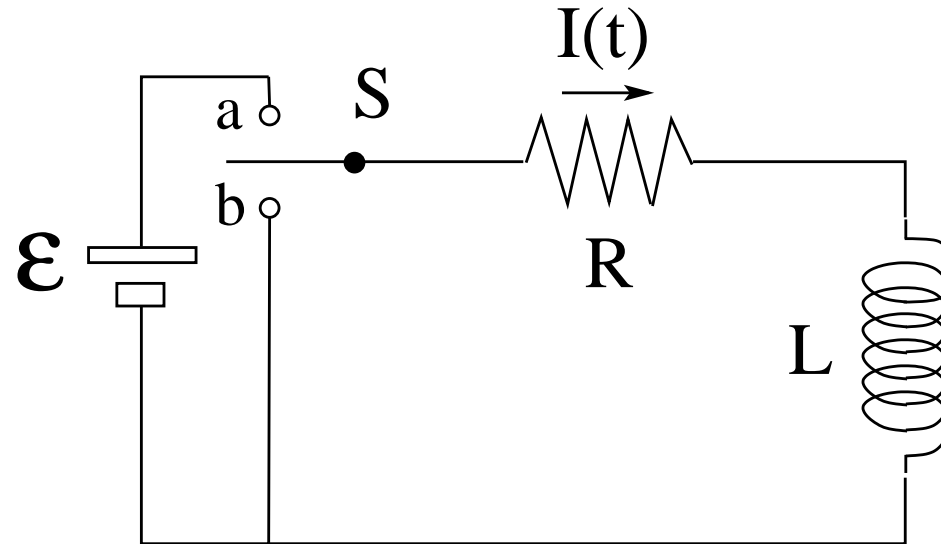


Specifications:

- \mathcal{E} (emf)
- R (resistance)
- L (inductance)

Switch S :

- a: current buildup
- b: current shutdown



Time-dependent quantities:

- $I(t)$: instantaneous current through inductor
- $\frac{dI}{dt}$: rate of change of instantaneous current
- $V_R(t) = I(t)R$: instantaneous voltage across resistor
- $V_L(t) = L \frac{dI}{dt}$: instantaneous voltage across inductor

RL Circuit: Current Buildup in Inductor



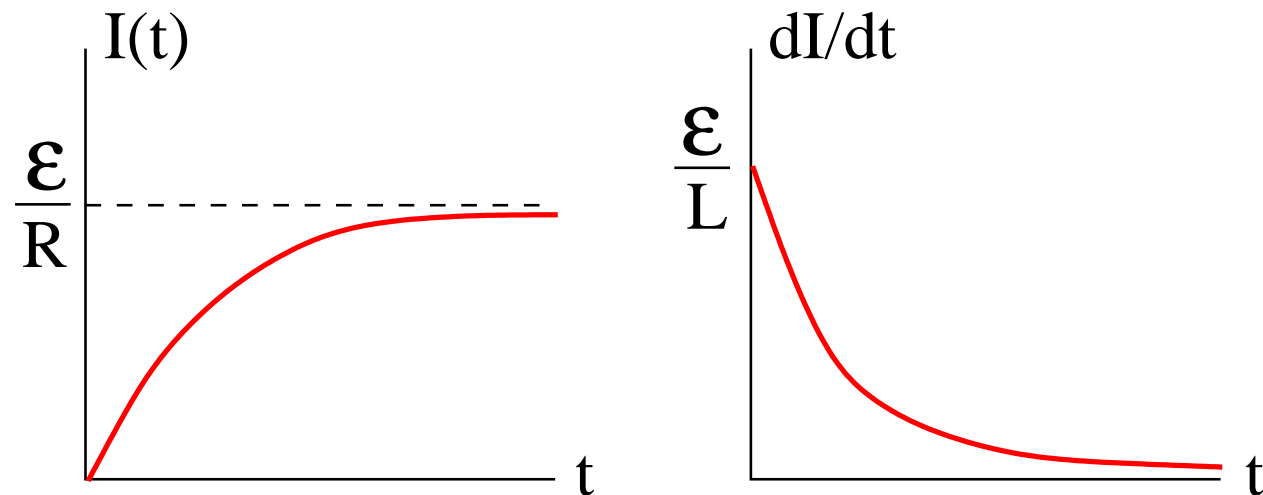
- Loop rule: $\mathcal{E} - IR - L \frac{dI}{dt} = 0$

- Differential equation: $L \frac{dI}{dt} = \mathcal{E} - IR \Rightarrow \frac{dI}{dt} = \frac{\mathcal{E}/R - I}{L/R}$

$$\int_0^I \frac{dI}{\mathcal{E}/R - I} = \int_0^t \frac{dt}{L/R} \Rightarrow -\ln\left(\frac{\mathcal{E}/R - I}{\mathcal{E}/R}\right) = \frac{t}{L/R} \Rightarrow \frac{\mathcal{E}/R - I}{\mathcal{E}/R} = e^{-Rt/L}$$

- Current through inductor: $I(t) = \frac{\mathcal{E}}{R} [1 - e^{-Rt/L}]$

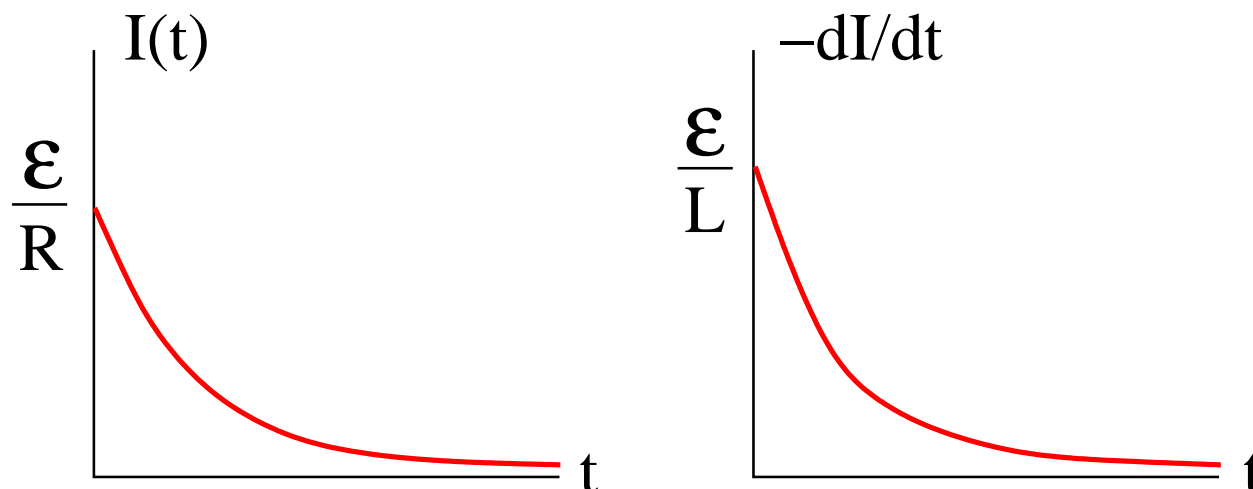
- Rate of current change: $\frac{dI}{dt} = \frac{\mathcal{E}}{L} e^{-Rt/L}$



RL Circuit: Current Shutdown in Inductor



- Loop rule: $-IR - L \frac{dI}{dt} = 0$
- Differential equation: $L \frac{dI}{dt} + IR = 0 \Rightarrow \frac{dI}{dt} = -\frac{R}{L} I$
 $\Rightarrow \int_{\mathcal{E}/R}^I \frac{dI}{I} = -\frac{R}{L} \int_0^t dt \Rightarrow \ln \frac{I}{\mathcal{E}/R} = -\frac{R}{L} t \Rightarrow \frac{I}{\mathcal{E}/R} = e^{-Rt/L}$
- Current: $I(t) = \frac{\mathcal{E}}{R} e^{-Rt/L}$
- Rate of current change: $\frac{dI}{dt} = -\frac{\mathcal{E}}{L} e^{-Rt/L}$



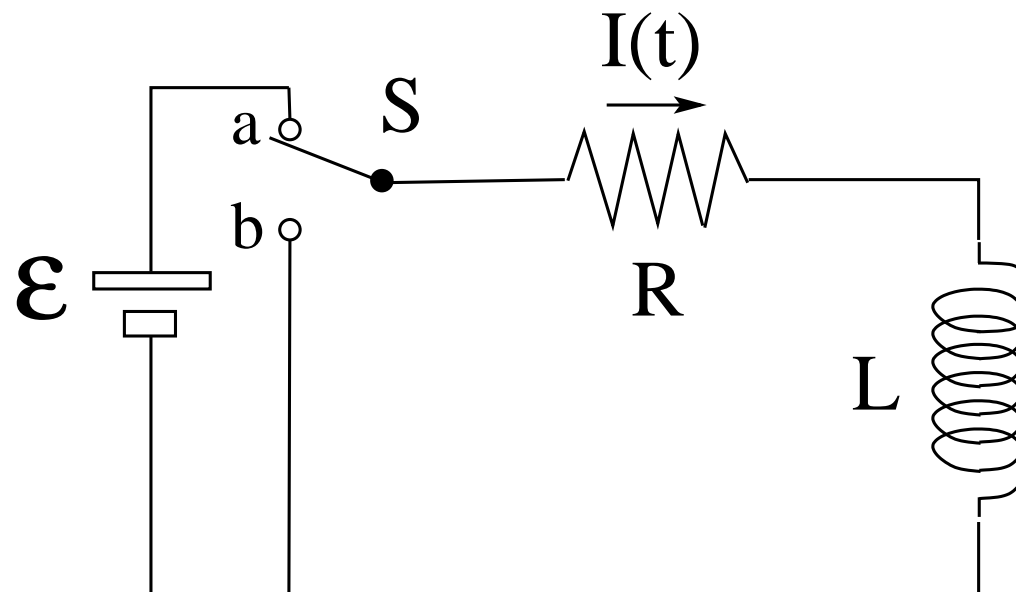
RL Circuit: Energy Transfer During Current Buildup



Loop rule: $IR + L \frac{dI}{dt} = \mathcal{E}$ ($I > 0$, $\frac{dI}{dt} > 0$)

- $I\mathcal{E}$: rate at which EMF source delivers energy
- $IV_R = I^2R$: rate at which energy is dissipated in resistor
- $IV_L = LI \frac{dI}{dt}$: rate at which energy is stored in inductor

Balance of energy transfer: $I^2R + LI \frac{dI}{dt} = I\mathcal{E}$



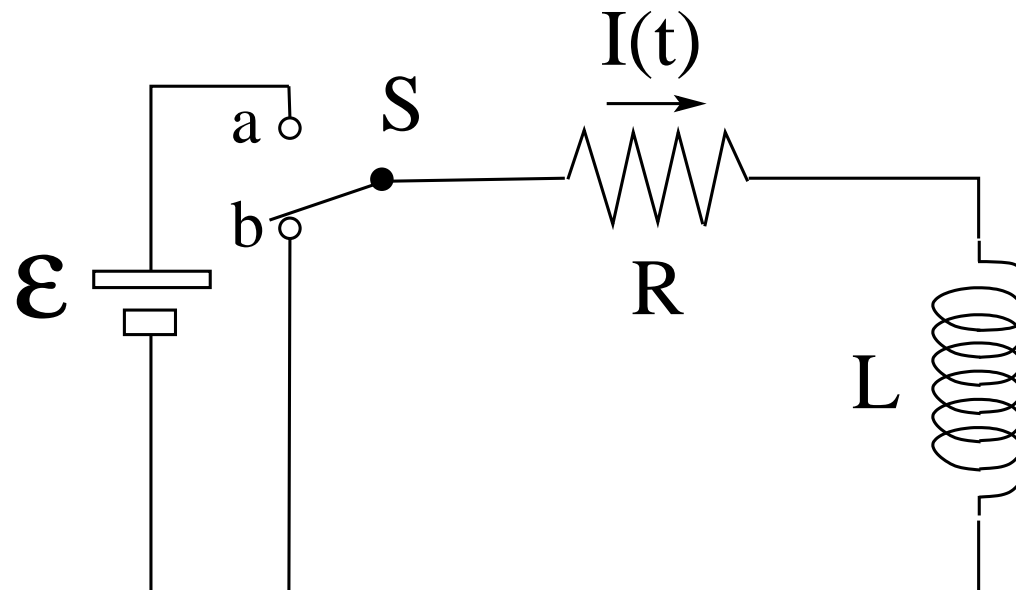
RL Circuit: Energy Transfer During Current Shutdown



Loop rule: $IR + L \frac{dI}{dt} = 0$ ($I > 0$, $\frac{dI}{dt} < 0$)

- $IV_L = LI \frac{dI}{dt}$: rate at which inductor releases energy
- $IV_R = I^2 R$: rate at which energy is dissipated in resistor

Balance of energy transfer: $I^2 R + LI \frac{dI}{dt} = 0$

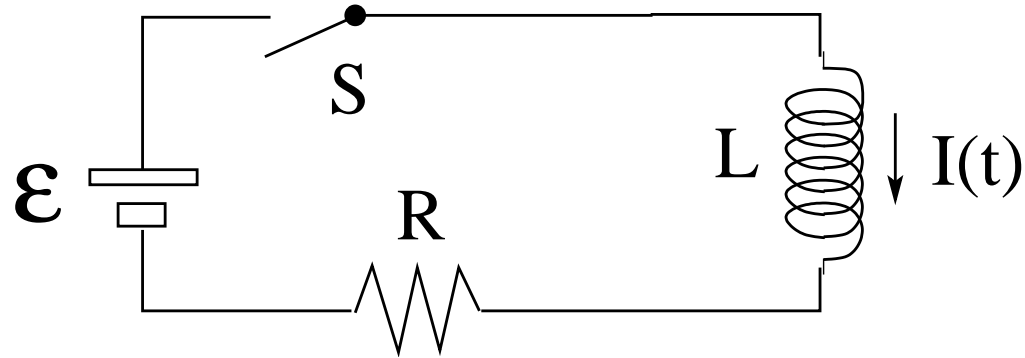


RL Circuit: Some Physical Properties



Specification of RL circuit
by 3 device properties:

- \mathcal{E} [V] (emf)
- R [Ω] (resistance)
- L [H] (inductance)



Physical properties of RL circuit during current buildup determined by 3 combinations of the device properties:

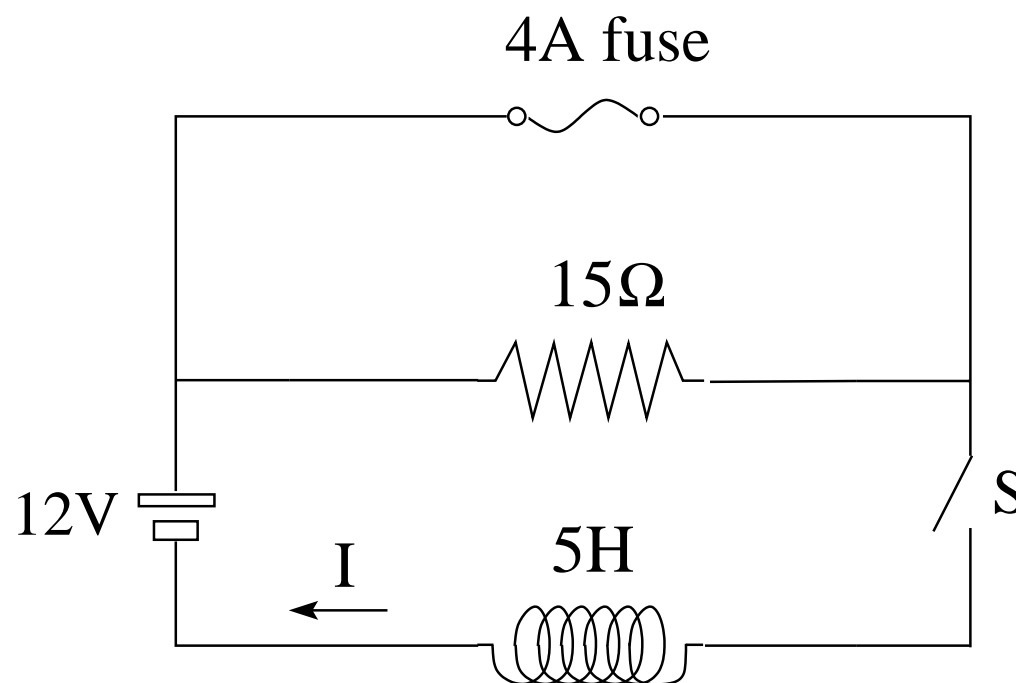
- $\frac{\mathcal{E}}{L} = \left. \frac{dI}{dt} \right|_{t=0}$: initial rate at which current increases
- $\frac{\mathcal{E}}{R} = I(t = \infty)$: final value of current
- $L/R = \tau$: time it takes to build up 63% of the current through the circuit
[$1 - e^{-1} = 0.632 \dots$]

RL Circuit: Application (7)



In the circuit shown the switch S is closed at time $t = 0$.

- (a) Find the current I as a function of time for $0 < t < t_F$, where t_F marks the instant the fuse breaks.
- (b) Find the current I as a function of time for $t > t_F$.

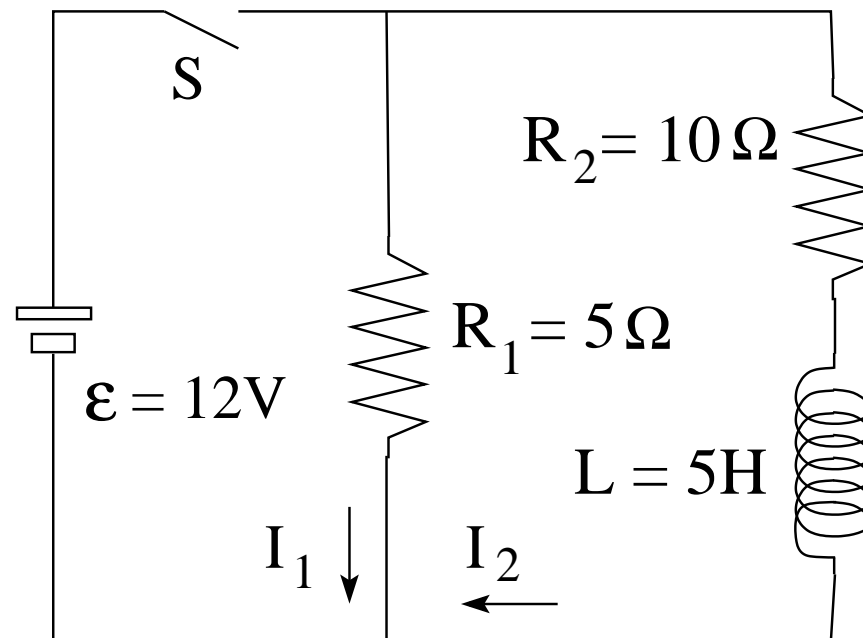


RL Circuit: Application (8)



In the circuit shown the switch has been open for a long time.
Find the currents I_1 and I_2

- just after the switch has been closed,
- a long time later,
- as functions of time for $0 < t < \infty$.

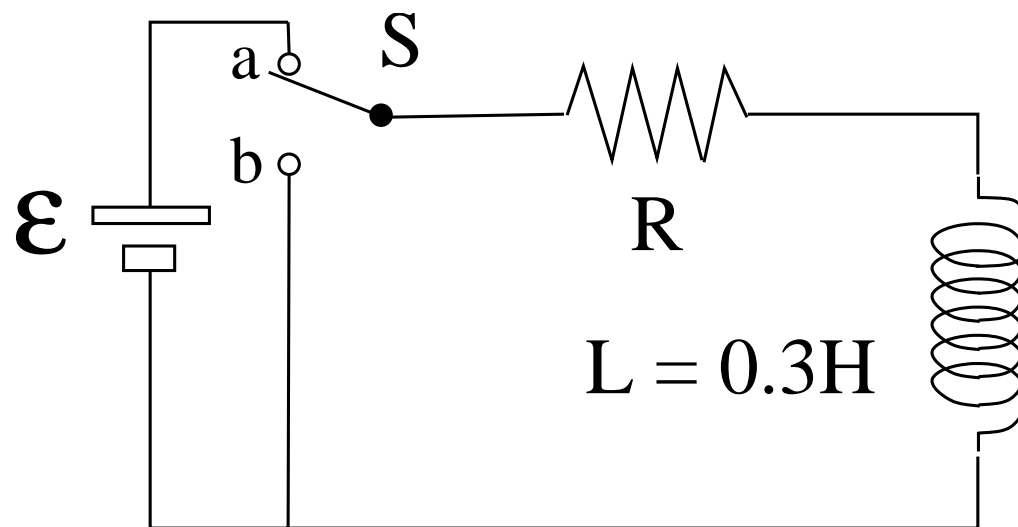


RL Circuit: Application (6)



In the RL circuit shown the switch has been at position a for a long time and is thrown to position b at time $t = 0$. At that instant the current has the value $I_0 = 0.7\text{A}$ and decreases at the rate $dI/dt = -360\text{A/s}$.

- (a) Find the EMF \mathcal{E} of the battery.
- (b) Find the resistance R of the resistor.
- (c) At what time t_1 has the current decreased to the value $I_1 = 0.2\text{A}$?
- (d) Find the voltage across the inductor at time t_1 .

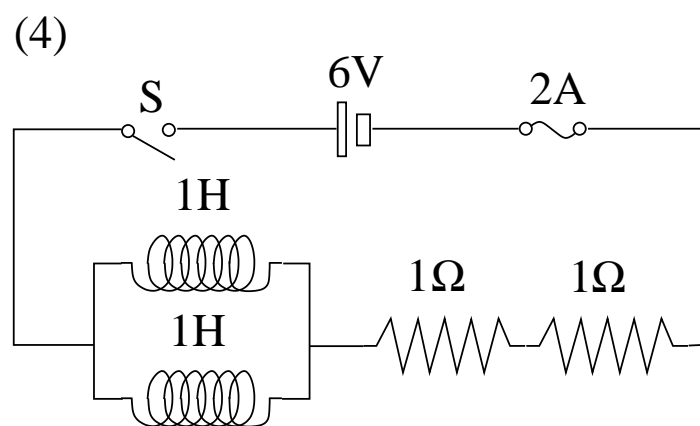
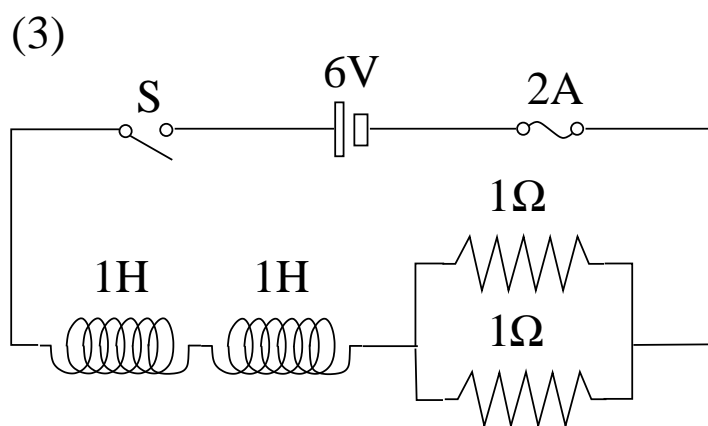
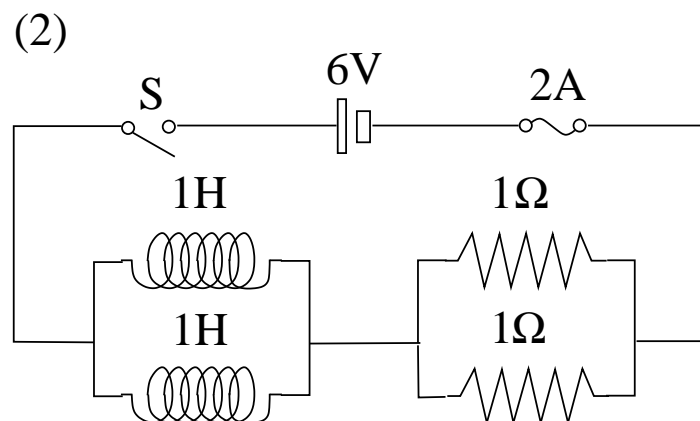
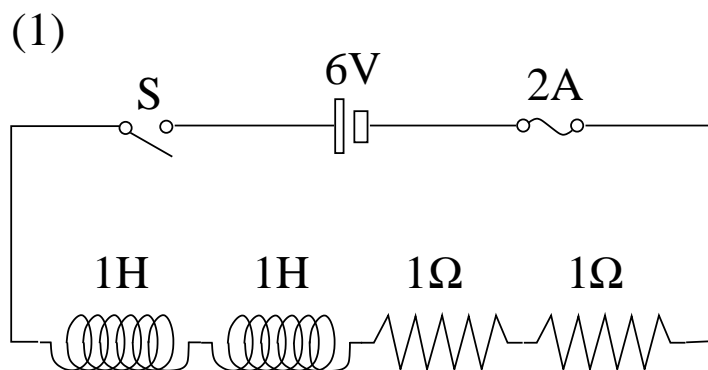


RL Circuit: Application (5)



Each RL circuit contains a 2A fuse. The switches are closed at $t = 0$.

- In what sequence are the fuses blown?

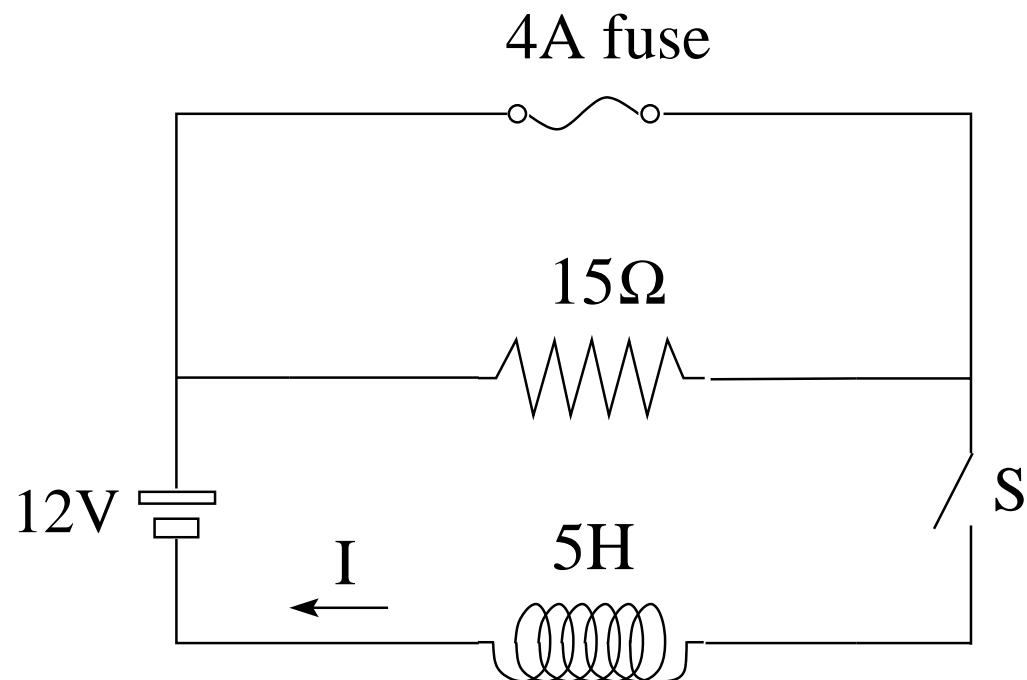


RL Circuit: Application (3)



The switch is closed at $t = 0$. Find the current I

- (a) immediately after the switch has been closed,
- (b) immediately before the fuse breaks,
- (c) immediately after the fuse has broken,
- (d) a very long time later.

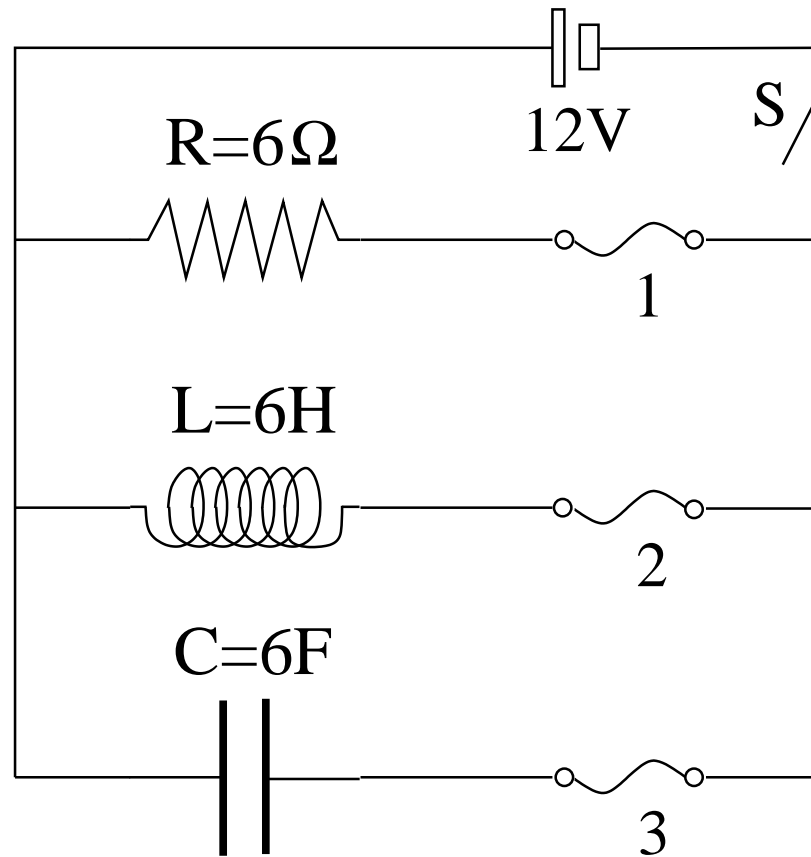


RL Circuit: Application (1)



Each branch in the circuit shown contains a 3A fuse. The switch is closed at time $t = 0$.

- (a) Which fuse is blown in the shortest time?
- (b) Which fuse lasts the longest time?

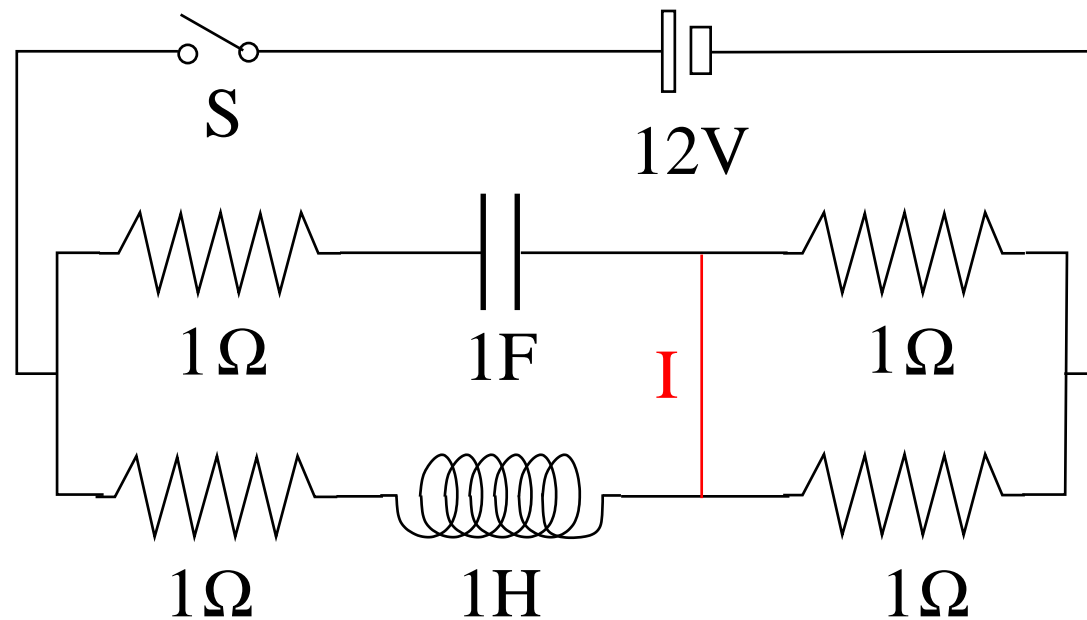


RL Circuit: Application (4)



Find the magnitude (in amps) and the direction (\uparrow , \downarrow) of the current I

- (a) right after the switch has been closed,
- (b) a very long time later.

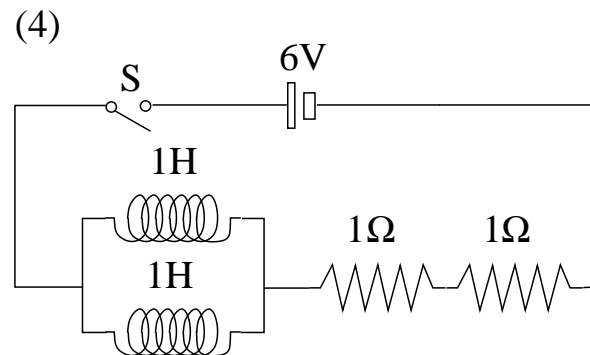
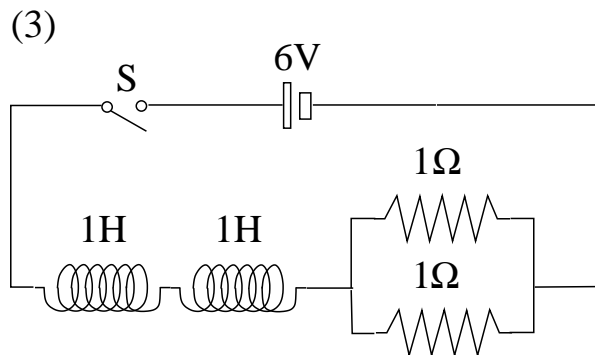
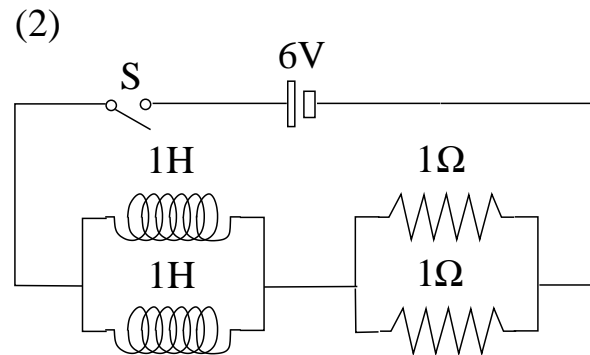
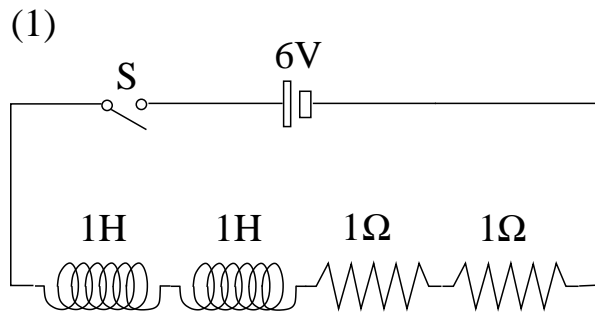


RL Circuit: Application (2)



The switch in each RL circuit is closed at $t = 0$.
Rank the circuits according to three criteria:

- (a) magnitude of current at $t = 1\text{ms}$,
- (b) magnitude of current at $t = \infty$,
- (c) time it takes I to reach 63% of its ultimate value.

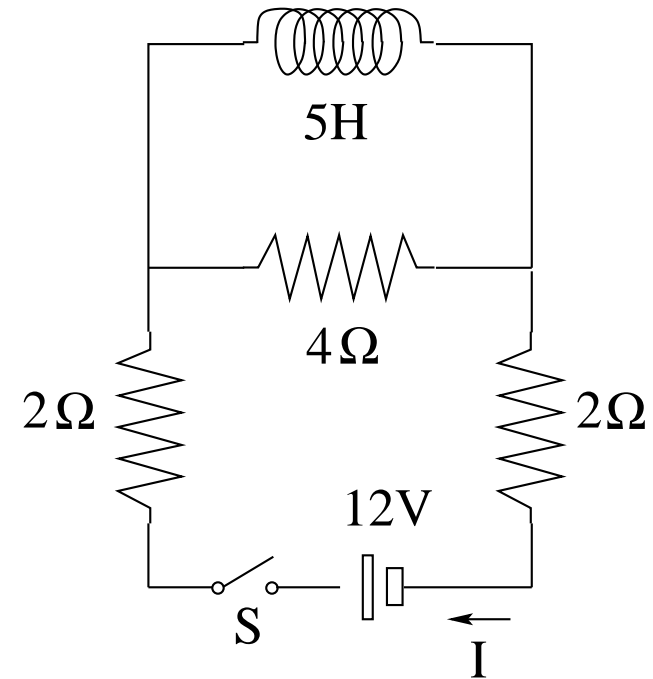


Intermediate Exam III: Problem #2 (Spring '05)



In the circuit shown we close the switch S at time $t = 0$. Find the current I through the battery and the voltage V_L across the inductor

- (a) immediately after the switch has been closed,
- (b) a very long time later.

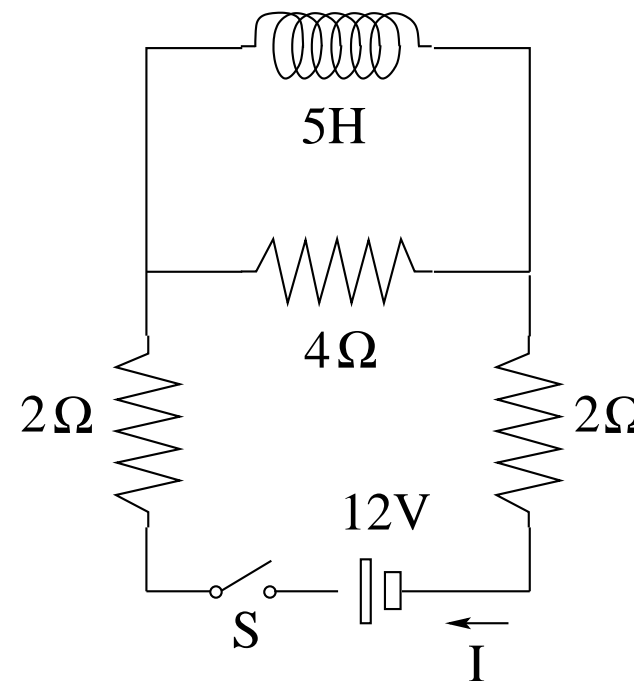


Intermediate Exam III: Problem #2 (Spring '05)



In the circuit shown we close the switch S at time $t = 0$. Find the current I through the battery and the voltage V_L across the inductor

- (a) immediately after the switch has been closed,
- (b) a very long time later.



Solution:

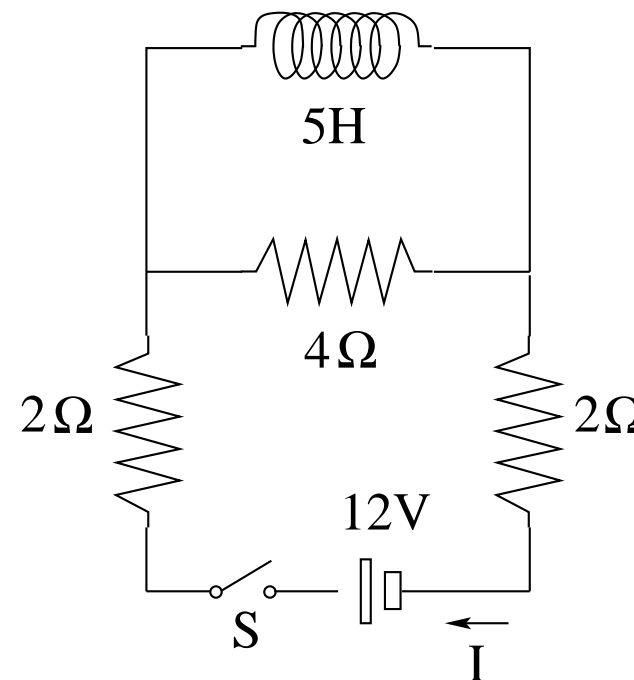
$$(a) \quad I = \frac{12V}{2\Omega + 4\Omega + 2\Omega} = 1.5A, \quad V_L = (4\Omega)(1.5A) = 6V.$$

Intermediate Exam III: Problem #2 (Spring '05)



In the circuit shown we close the switch S at time $t = 0$. Find the current I through the battery and the voltage V_L across the inductor

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Solution:

$$(a) \quad I = \frac{12V}{2\Omega + 4\Omega + 2\Omega} = 1.5A, \quad V_L = (4\Omega)(1.5A) = 6V.$$

$$(b) \quad I = \frac{12V}{2\Omega + 2\Omega} = 3A, \quad V_L = 0.$$