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## 16. Faraday's Law

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### Abstract

Lecture slides 16 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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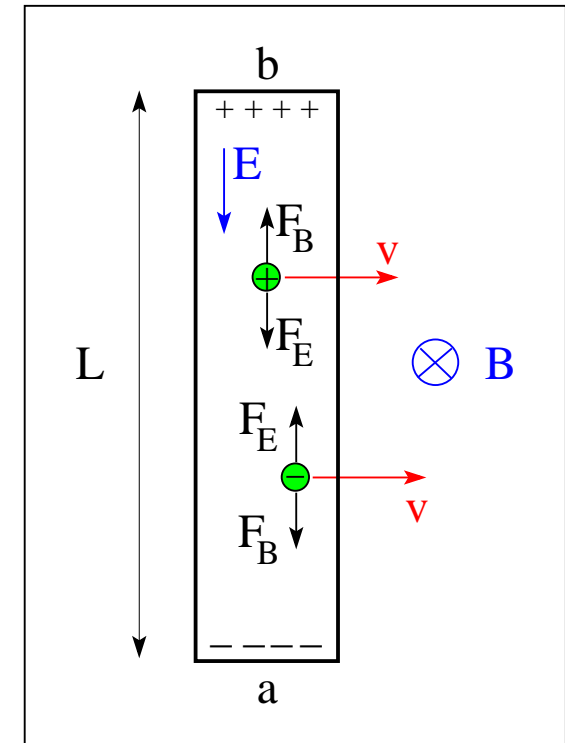
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# Motional EMF



Conducting rod moving across region of uniform magnetic field

- moving charge carriers
- magnetic force  $\vec{F}_B = q\vec{v} \times \vec{B}$
- charge separation
- electric field  $\vec{E}$
- electric force  $\vec{F}_E = q\vec{E}$



Equilibrium between electric and magnetic force:

$$F_E = F_B \Rightarrow qE = qvB \Rightarrow E = vB$$

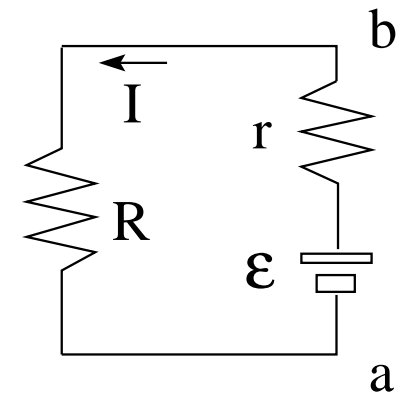
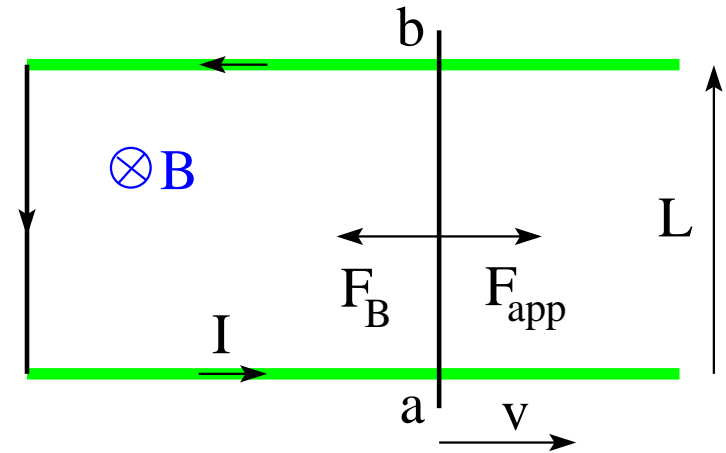
Potential difference induced between endpoints of rod:

$$V_{ab} \equiv V_b - V_a = EL \Rightarrow V_{ab} = vBL \quad (\text{motional EMF})$$

# Current Produced by Motional EMF



- Motional EMF:  $\mathcal{E} = vBL$
- Terminal voltage:  $V_{ab} = \mathcal{E} - Ir$
- Electric current:  $\mathcal{E} - Ir - IR = 0 \Rightarrow I = \frac{\mathcal{E}}{r + R}$
- Applied mechanical force:  $\vec{F}_{app}$
- Magnetic force:  $\vec{F}_B = I\vec{L} \times \vec{B}$
- Motion at constant velocity:  $\vec{F}_{app} = -\vec{F}_B$
- Electrical power generated:  $P_{gen} = \mathcal{E}I$
- Mechanical power input:  $P_{in} = Fv = (ILB)v = (vBL)I = \mathcal{E}I$
- Electrical power output:  $P_{out} = V_{ab}I = \mathcal{E}I - I^2r$

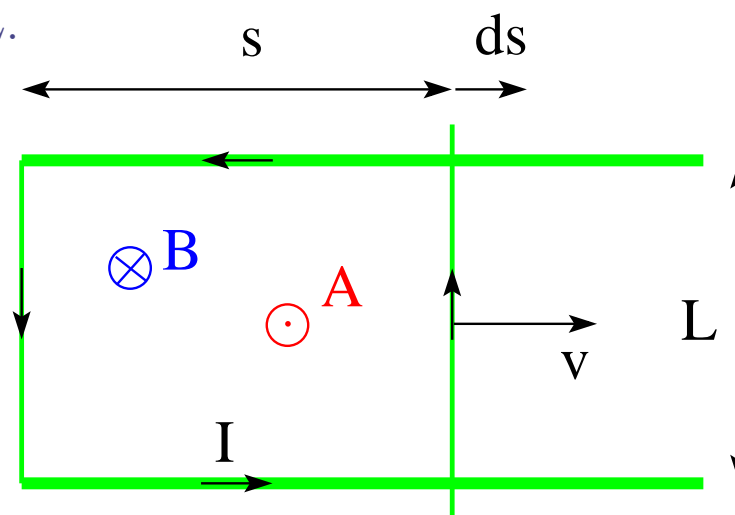


# Faraday's Law of Induction (1)



Prototype: motional EMF reformulated.

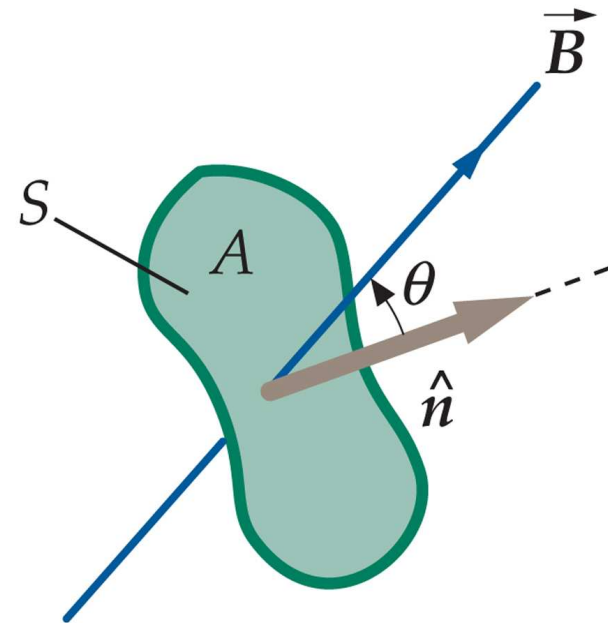
- Choose area vector  $\vec{A}$  for current loop:  $A = Ls \odot$ .
- Magnetic flux:  $\Phi_B = \int \vec{B} \cdot d\vec{A}$ . Here  $\Phi_B = -BLs$ .
- Motional EMF:  $\mathcal{E} = vBL$ .
- Change in area of loop:  $dA = Lds$ .
- Change in magnetic flux:  $d\Phi_B = -BdA = -BLds$ .
- SI unit of magnetic flux:  $1\text{Wb}=1\text{Tm}^2$  (Weber).
- Rate of change of flux:  $\frac{d\Phi_B}{dt} = -BL\frac{ds}{dt} = -vBL$ .
- Faraday's law:  $\mathcal{E} = -\frac{d\Phi_B}{dt}$ .



# Magnetic flux and Faraday's law



- Magnetic field  $\vec{B}$  (given)
- Surface  $S$  with perimeter loop (given)
- Surface area  $A$  (given)
- Area vector  $\vec{A} = A\hat{n}$  (my choice)
- Positive direction around perimeter: ccw (consequence of my choice)
- Magnetic flux:  $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int \vec{B} \cdot \hat{n}dA$
- Consider situation with  $\frac{d\vec{B}}{dt} \neq 0$
- Induced electric field:  $\vec{E}$
- Induced EMF:  $\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell}$   
(integral ccw around perimeter)
- Faraday's law:  $\mathcal{E} = -\frac{d\Phi_B}{dt}$

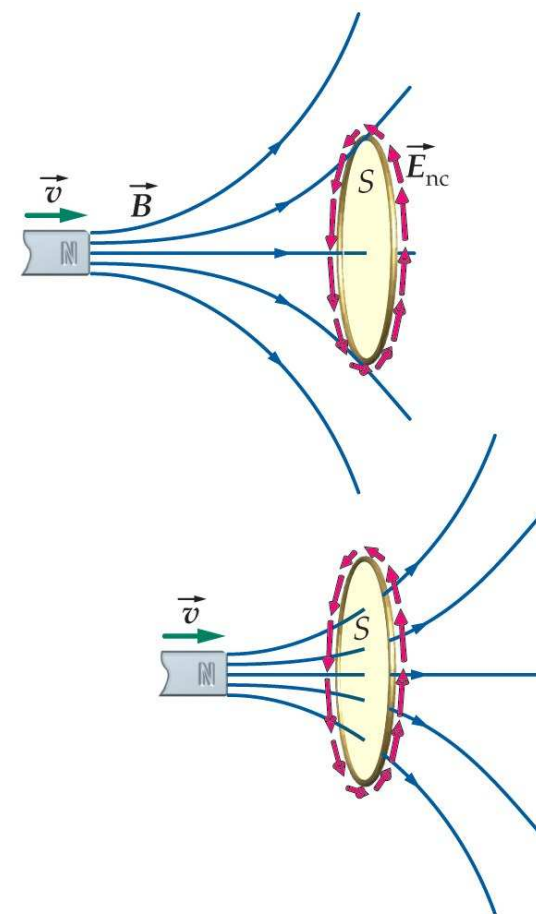


# Faraday's Law of Induction (2)



Here the change in magnetic flux  $\Phi_B$  is caused by a moving bar magnet.

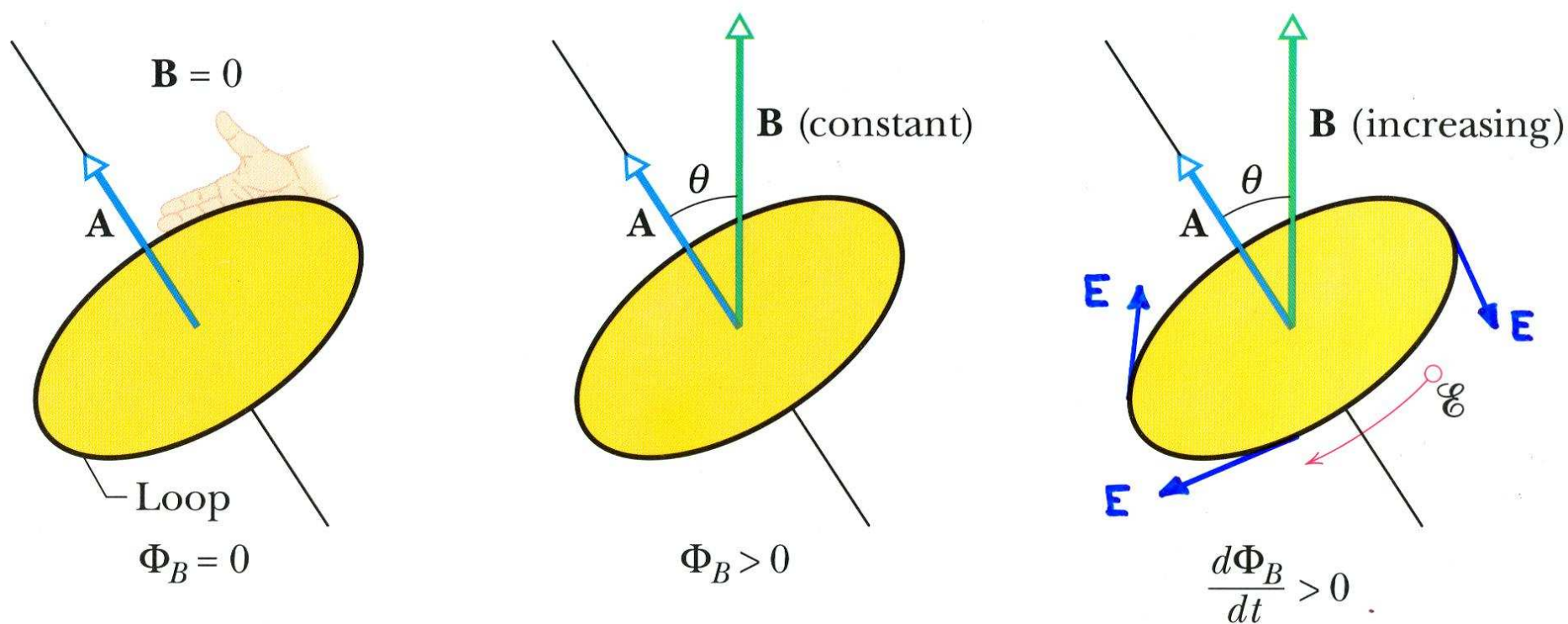
- Assume area vector  $\vec{A}$  of loop pointing right. Hence positive direction around loop is clockwise.
- Motion of bar magnet causes  $\frac{d\Phi_B}{dt} > 0$ .
- Faraday's law:  $\mathcal{E} = -\frac{d\Phi_B}{dt}$ .
- Induced EMF is in negative direction,  $\mathcal{E} < 0$ , which is counterclockwise.
- Induced EMF reflects induced electric field:  $\mathcal{E} = \oint_C \vec{E} \cdot d\vec{\ell}$ .
- Field lines of induced electric field are closed.
- Faraday's law is a dynamics relation between electric and magnetic fields:  $\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A}$ .



# Area – Field – Flux – EMF (1)



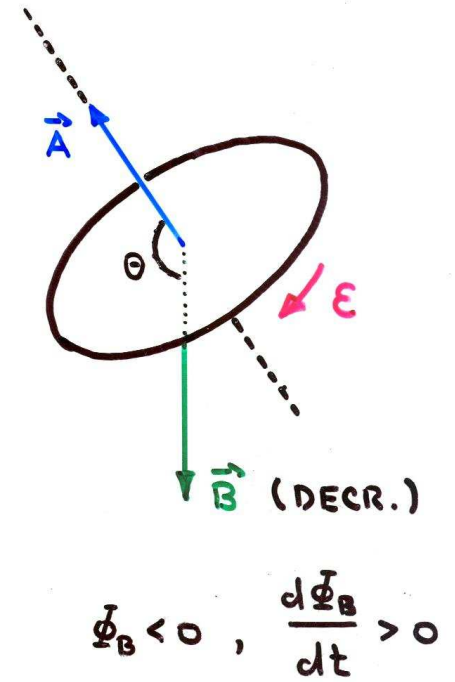
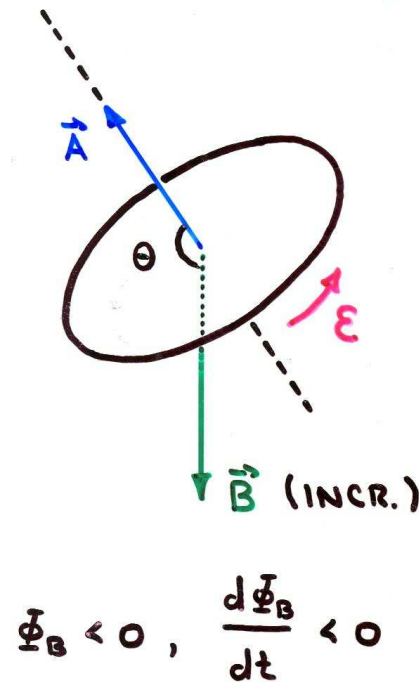
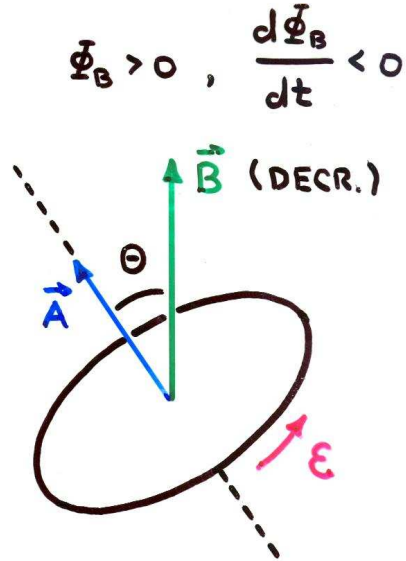
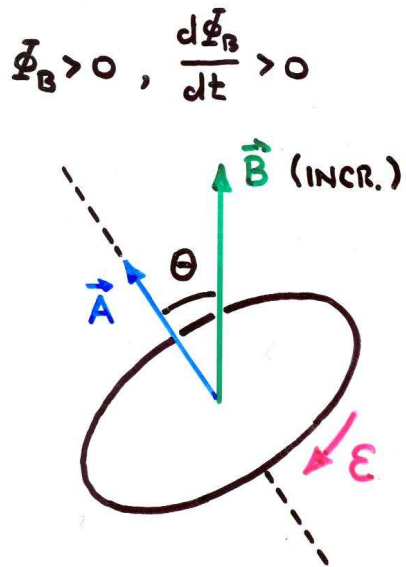
$$\Phi_B = \int \vec{B} \cdot d\vec{A}, \quad \mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$



# Area – Field – Flux – EMF (2)



$$\Phi_B = \int \vec{B} \cdot d\vec{A}, \quad \mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$



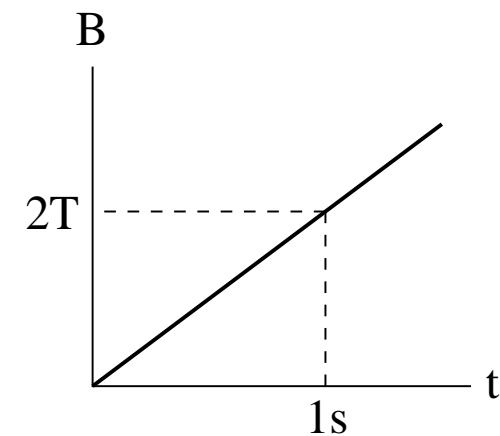
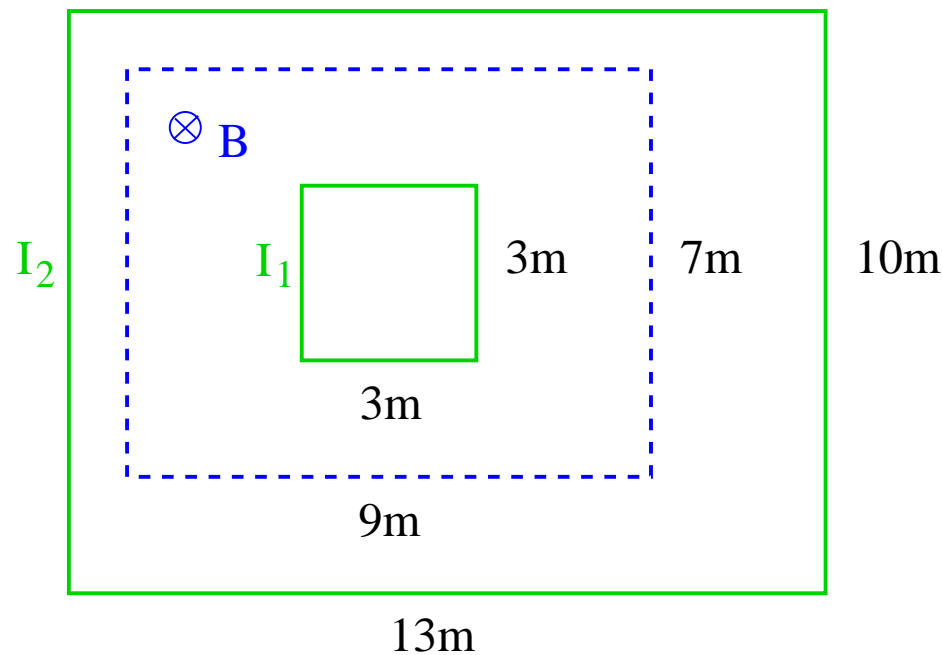


# Magnetic Induction: Application (3)



A uniform magnetic field  $\vec{B}$  pointing into the plane and increasing in magnitude as shown in the graph exists inside the dashed rectangle.

- Find the magnitude (in amps) and the direction (cw/ccw) of the currents  $I_1$ ,  $I_2$  induced in the small conducting square and in the big conducting rectangle, respectively. Each conducting loop has a resistance  $R = 9\Omega$

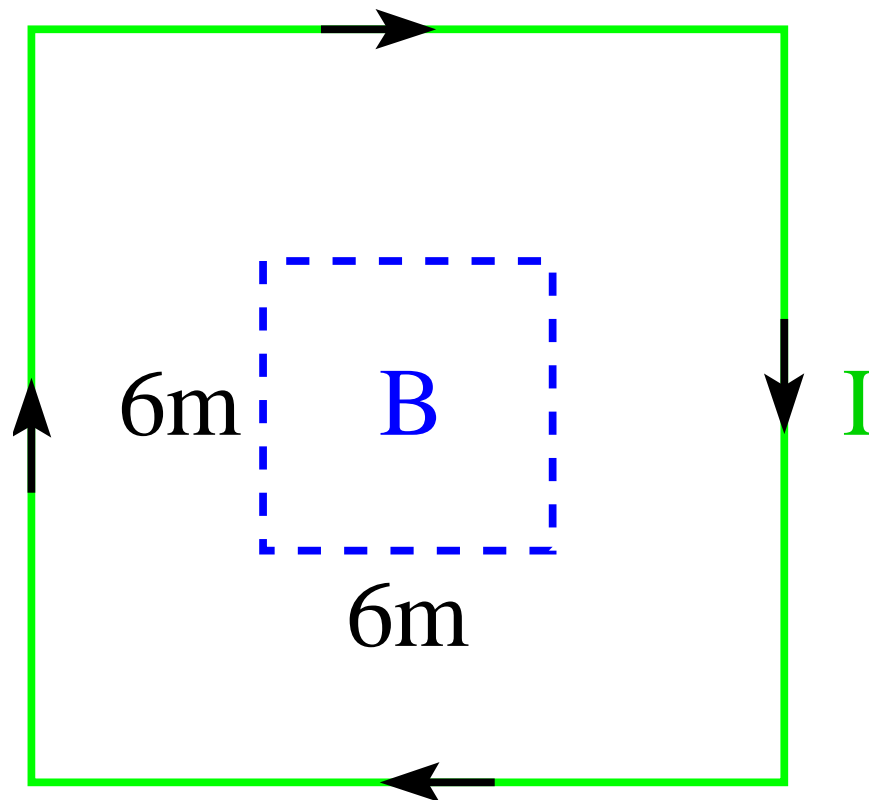


## Magnetic Induction: Application (4)



A magnetic field  $\vec{B}$  of increasing strength and directed perpendicular to the plane exists inside the dashed square. It induces a constant clockwise current  $I = 8\text{A}$  in the large conducting square with resistance  $R = 9\Omega$ .

- If  $\vec{B} = 0$  at time  $t = 0$ , find the direction ( $\odot, \otimes$ ) and magnitude of  $\vec{B}$  at time  $t = 5\text{s}$ .

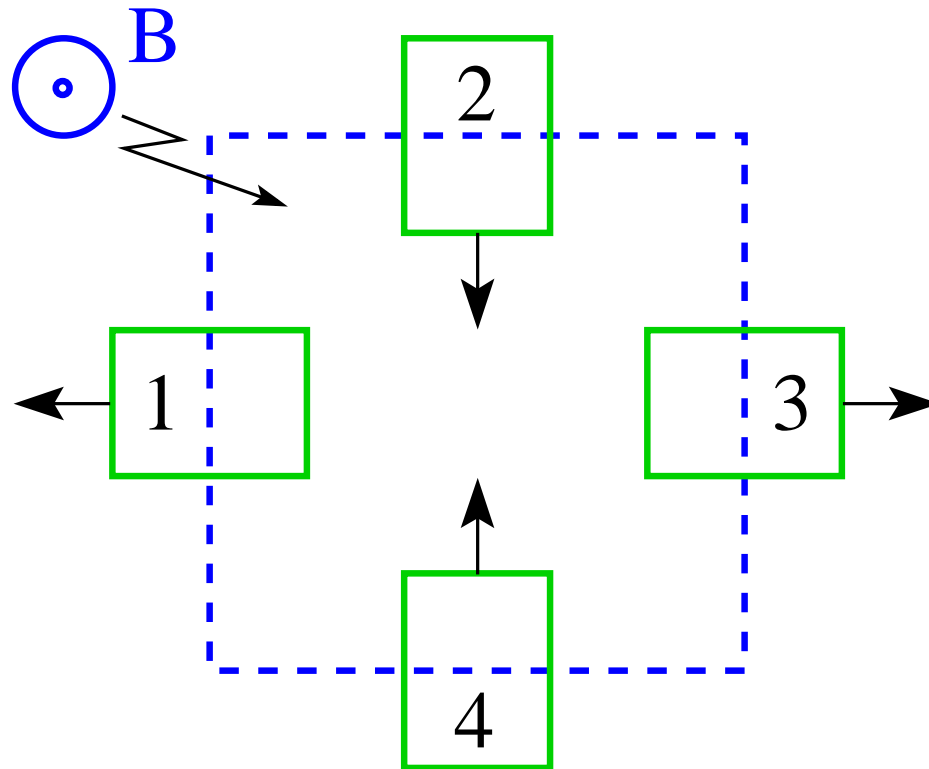


# Magnetic Induction: Application (5)



A uniform magnetic field  $\vec{B}$  pointing out of the plane exists inside the dashed square. Four conducting rectangles 1,2,3,4 move in the directions indicated.

- Find the direction (cw,ccw) of the current induced in each rectangle.



# Magnetic Induction: Application (9)

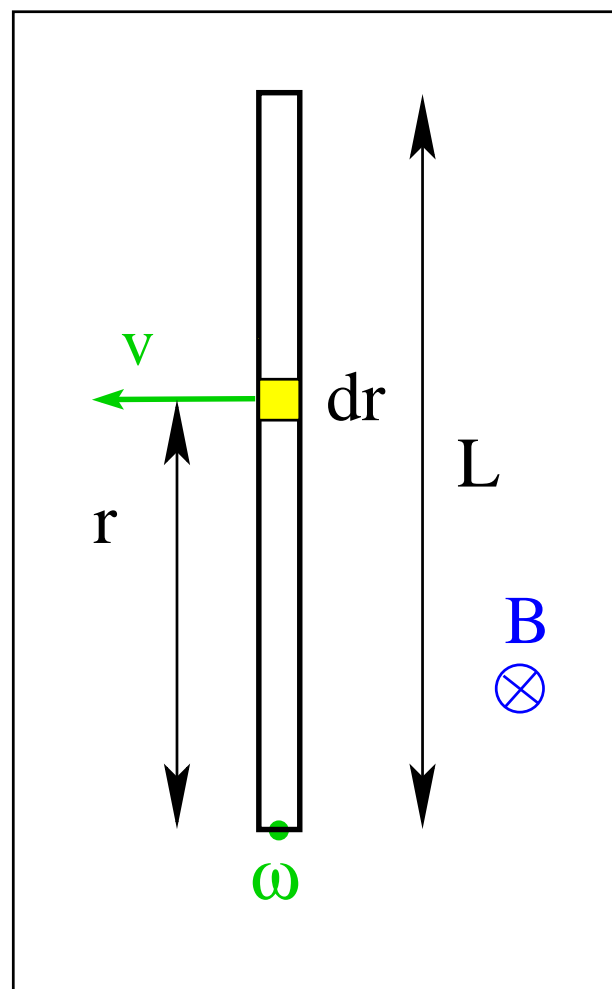


Consider a conducting rod of length  $L$  rotating with angular velocity  $\omega$  in a plane perpendicular to a uniform magnetic field  $\vec{B}$ .

- Angular velocity of slice:  $\omega$
- Linear velocity of slice:  $v = \omega r$
- EMF induced in slice:  $d\mathcal{E} = Bvdr$
- Slices are connected in series.
- EMF induced in rod:

$$\mathcal{E} = \int_0^L Bv dr = B\omega \int_0^L r dr$$

$$\Rightarrow \mathcal{E} = \frac{1}{2}B\omega L^2 = \frac{1}{2}BvL$$





**The induced emf and induced current are in such a direction as to oppose the cause that produces them.**

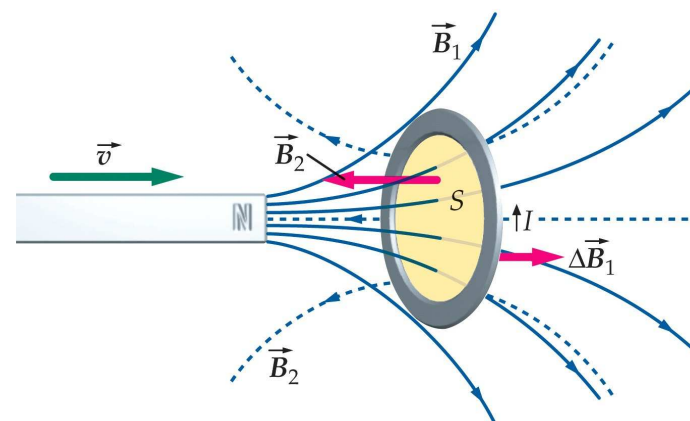
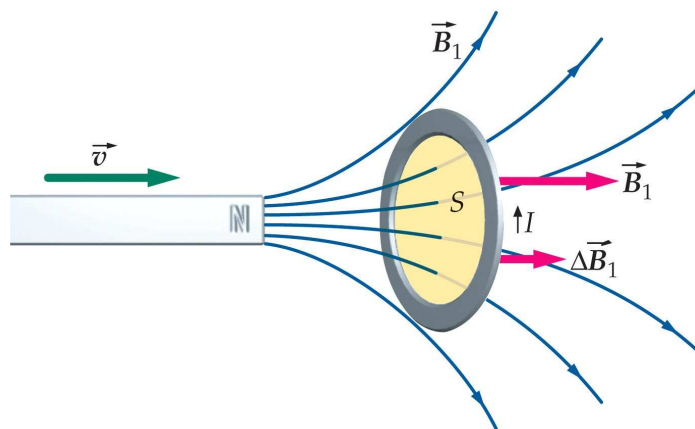
- Lenz's rule is a statement of negative feedback.
- The cause is a change in magnetic flux through some loop.
- The loop can be real or fictitious.
- What opposes the cause is a magnetic field generated by the induced emf.
  - If the loop is a conductor the opposing magnetic field is generated by the induced current as stated in the law of Biot and Savart or in the restricted version of Ampère's law.
  - If the loop is not a conductor the opposing magnetic field is generated by the induced electric field as stated by the extended version of Ampère's law (to be discussed later).

## Lenz's Rule (2)

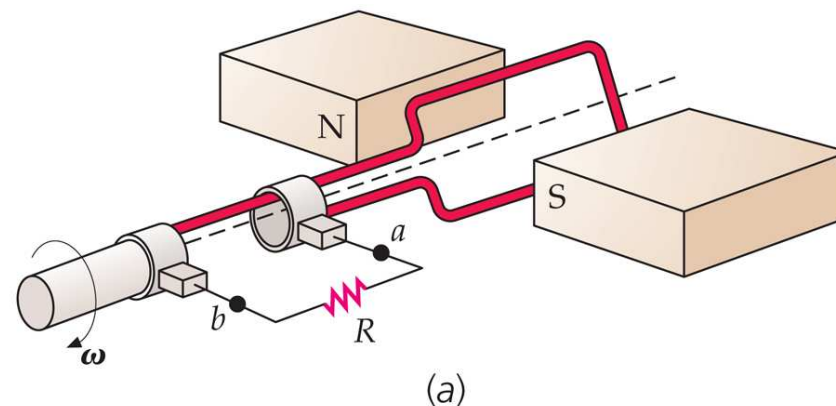


In the situation shown below the current induced in the conducting ring generates a magnetic field whose flux counteracts the change in magnetic flux caused by the bar magnet.

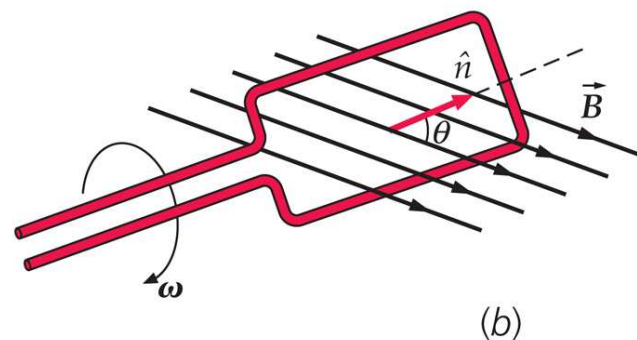
- Moving the bar magnet closer to the ring increases the magnetic field  $\vec{B}_1$  (solid field lines) through the ring by the amount  $\Delta\vec{B}_1$ .
- The resultant change in magnetic flux through the ring induces a current  $I$  in the direction shown.
- The induced current  $I$ , in turn, generates a magnetic field  $\vec{B}_2$  (dashed field lines) in a direction that opposes the change of flux caused by the moving bar magnet.

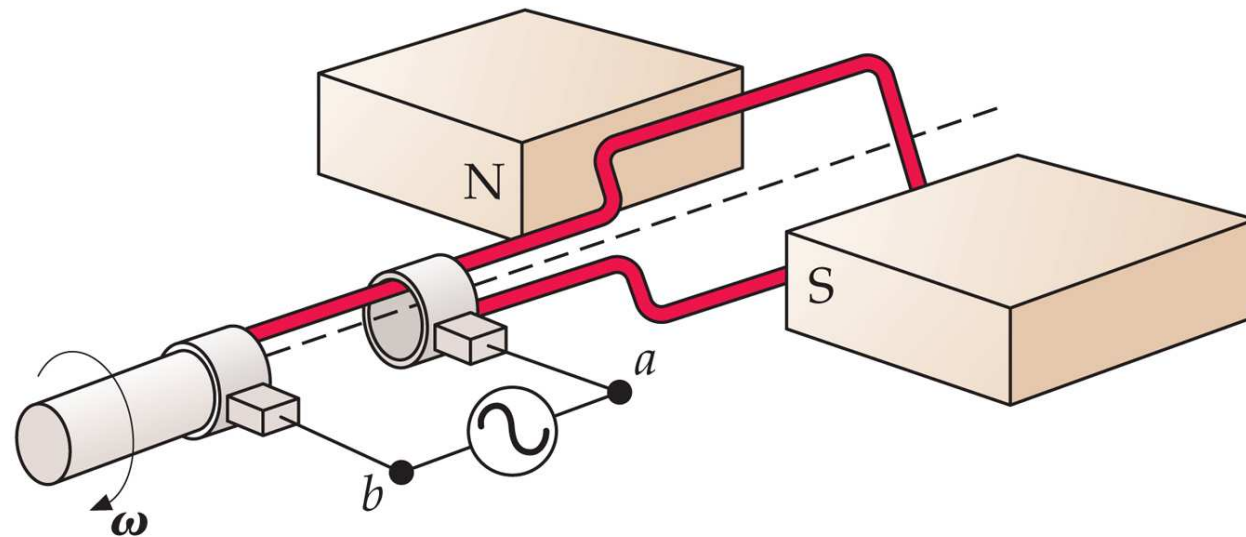


# AC Generator



- Area of conducting loop:  $A$
- Number of loops:  $N$
- Area vector:  $\vec{A} = A\hat{n}$
- Magnetic field:  $\vec{B}$
- Angle between vectors  $\vec{A}$  and  $\vec{B}$ :  $\theta = \omega t$
- Magnetic flux:  $\Phi_B = N\vec{A} \cdot \vec{B} = NAB \cos(\omega t)$
- Induced EMF:  $\mathcal{E} = -\frac{d\Phi_B}{dt} = \underbrace{NAB\omega}_{\mathcal{E}_{max}} \sin(\omega t)$





- Power source produces alternating current (ac) in loop.
- Current exerts torque on loop.
- Torque initiates and maintains rotation.
- Rotating loop induces back emf (counteracting source emf).

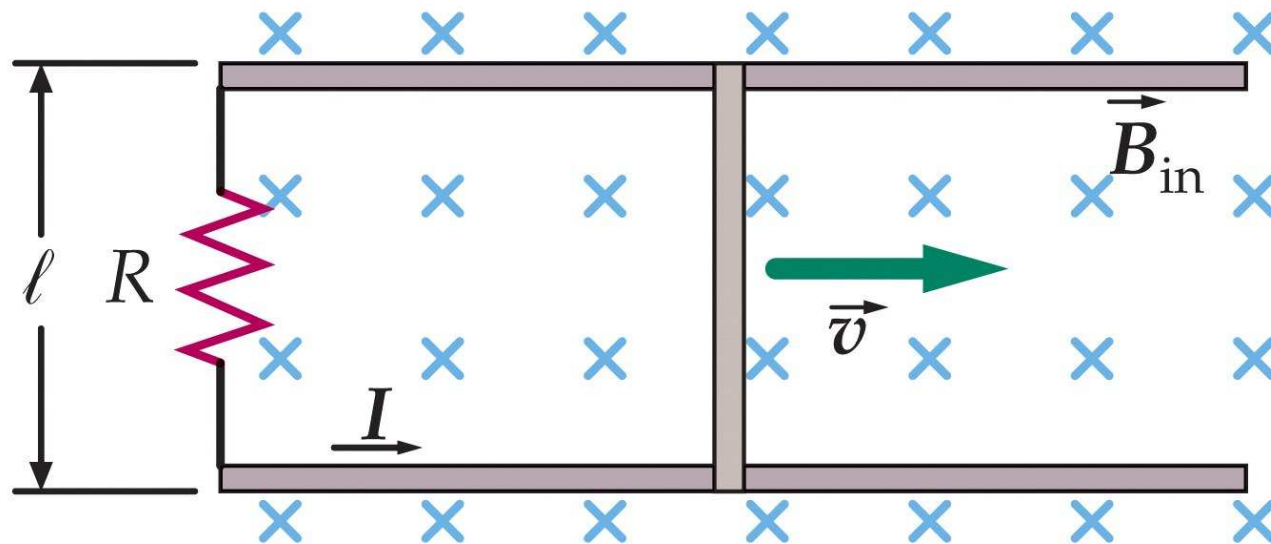


## Magnetic Induction: Application (8)



Consider a rectangular loop of width  $\ell$  in a uniform magnetic field  $\vec{B}$  directed into the plane. A slide wire of mass  $m$  is given an initial velocity  $\vec{v}_0$  to the right. There is no friction between the slide wire and the loop. The resistance  $R$  of the loop is constant.

- Find the magnetic force on the slide wire as a function of its velocity.
- Find the velocity of the slide wire as a function of time.
- Find the total distance traveled by the slide wire.

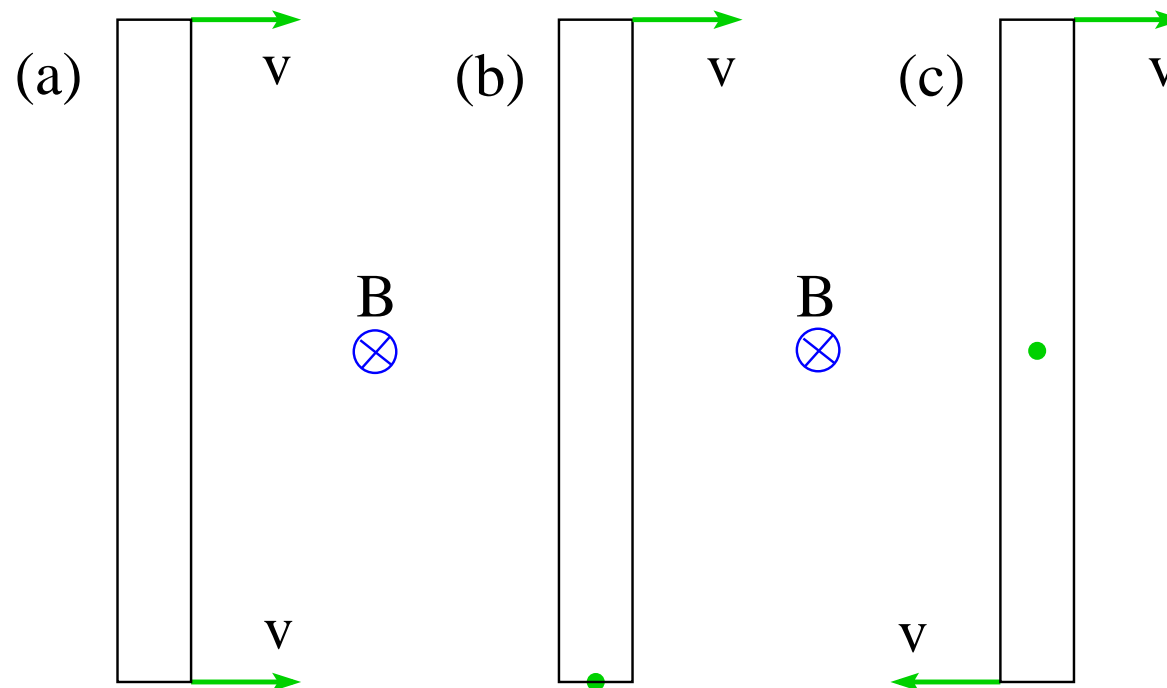


# Magnetic Induction: Application (1)



Consider three metal rods of length  $L = 2\text{m}$  moving translationally or rotationally across a uniform magnetic field  $B = 1\text{T}$  directed into the plane. All velocity vectors have magnitude  $v = 2\text{m/s}$ .

- Find the induced EMF  $\mathcal{E}$  between the ends of each rod.

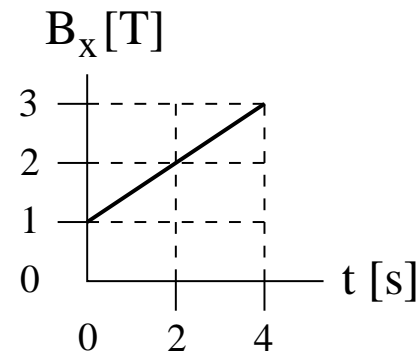
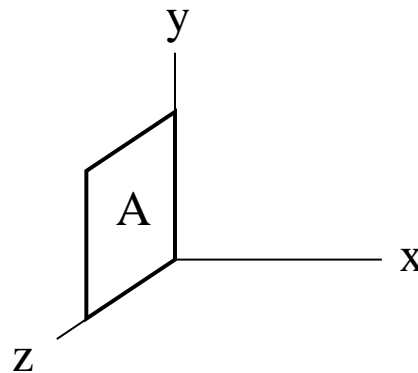


## Intermediate Exam III: Problem #2 (Spring '06)



A conducting loop in the shape of a square with area  $A = 4\text{m}^2$  and resistance  $R = 5\Omega$  is placed in the  $yz$ -plane as shown. A time-dependent magnetic field  $\mathbf{B} = B_x \hat{\mathbf{i}}$  is present. The dependence of  $B_x$  on time is shown graphically.

- Find the magnetic flux  $\Phi_B$  through the loop at time  $t = 0$ .
- Find magnitude and direction (cw/ccw) of the induced current  $I$  at time  $t = 2\text{s}$ .

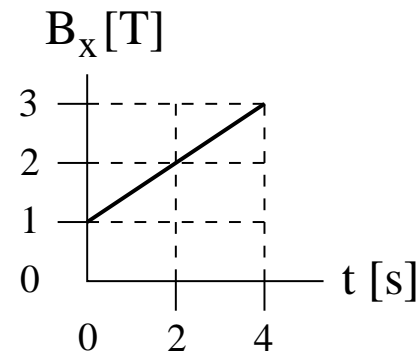
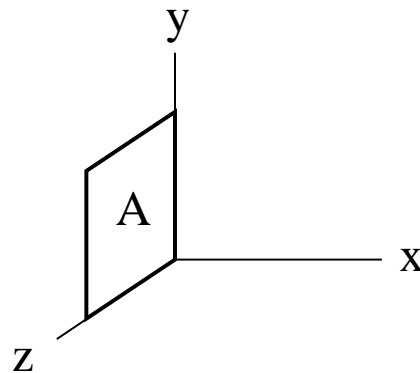


## Intermediate Exam III: Problem #2 (Spring '06)



A conducting loop in the shape of a square with area  $A = 4\text{m}^2$  and resistance  $R = 5\Omega$  is placed in the  $yz$ -plane as shown. A time-dependent magnetic field  $\mathbf{B} = B_x \hat{\mathbf{i}}$  is present. The dependence of  $B_x$  on time is shown graphically.

- (a) Find the magnetic flux  $\Phi_B$  through the loop at time  $t = 0$ .  
(b) Find magnitude and direction (cw/ccw) of the induced current  $I$  at time  $t = 2\text{s}$ .



Choice of area vector:  $\odot/\otimes \Rightarrow$  positive direction = ccw/cw.

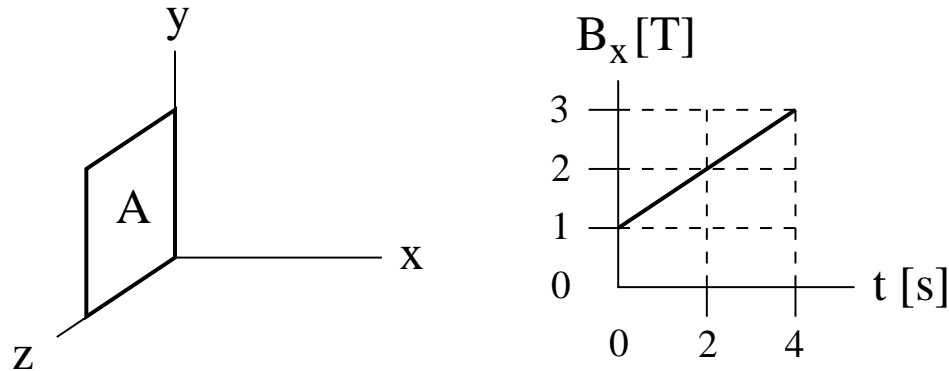
(a)  $\Phi_B = \pm(1\text{T})(4\text{m}^2) = \pm 4\text{Tm}^2$ .

## Intermediate Exam III: Problem #2 (Spring '06)



A conducting loop in the shape of a square with area  $A = 4\text{m}^2$  and resistance  $R = 5\Omega$  is placed in the  $yz$ -plane as shown. A time-dependent magnetic field  $\mathbf{B} = B_x \hat{\mathbf{i}}$  is present. The dependence of  $B_x$  on time is shown graphically.

- (a) Find the magnetic flux  $\Phi_B$  through the loop at time  $t = 0$ .  
(b) Find magnitude and direction (cw/ccw) of the induced current  $I$  at time  $t = 2\text{s}$ .



Choice of area vector:  $\odot/\otimes \Rightarrow$  positive direction = ccw/cw.

(a)  $\Phi_B = \pm(1\text{T})(4\text{m}^2) = \pm 4\text{Tm}^2$ .

(b)  $\frac{d\Phi_B}{dt} = \pm(0.5\text{T/s})(4\text{m}^2) = \pm 2\text{V} \quad \Rightarrow \quad \mathcal{E} = -\frac{d\Phi_B}{dt} = \mp 2\text{V}.$

$\Rightarrow I = \frac{\mathcal{E}}{R} = \mp \frac{2\text{V}}{5\Omega} = \mp 0.4\text{A} \quad (\text{cw}).$

## Intermediate Exam III: Problem #3 (Spring '07)

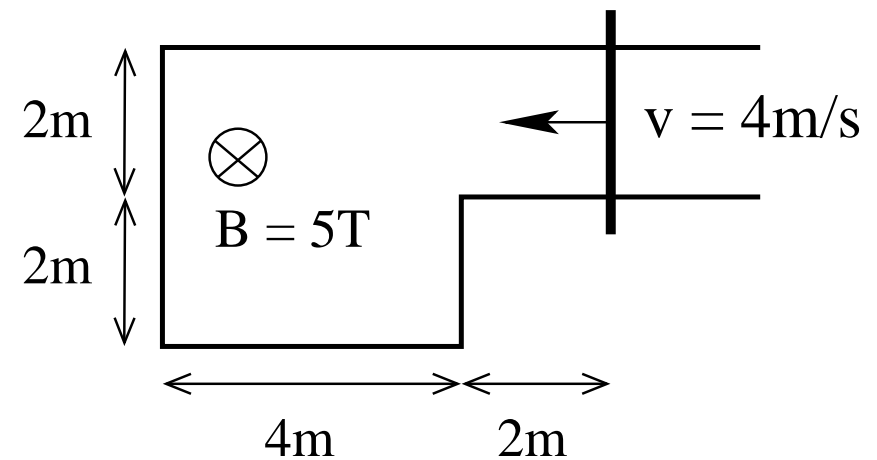


A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown.

(a) Find the magnetic flux  $\Phi_B$  through the frame at the instant shown.

(b) Find the induced emf  $\mathcal{E}$  at the instant shown.

(c) Find the direction (cw/ccw) of the induced current.



## Intermediate Exam III: Problem #3 (Spring '07)

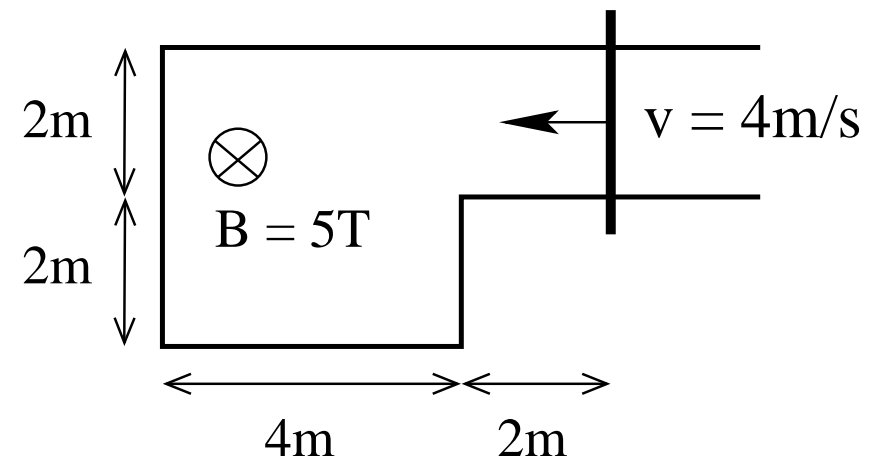


A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown.

(a) Find the magnetic flux  $\Phi_B$  through the frame at the instant shown.

(b) Find the induced emf  $\mathcal{E}$  at the instant shown.

(c) Find the direction (cw/ccw) of the induced current.



**Solution:**

(a)  $\Phi_B = \vec{A} \cdot \vec{B} = \pm(20\text{m}^2)(5\text{T}) = \pm 100\text{Wb}$ .

## Intermediate Exam III: Problem #3 (Spring '07)

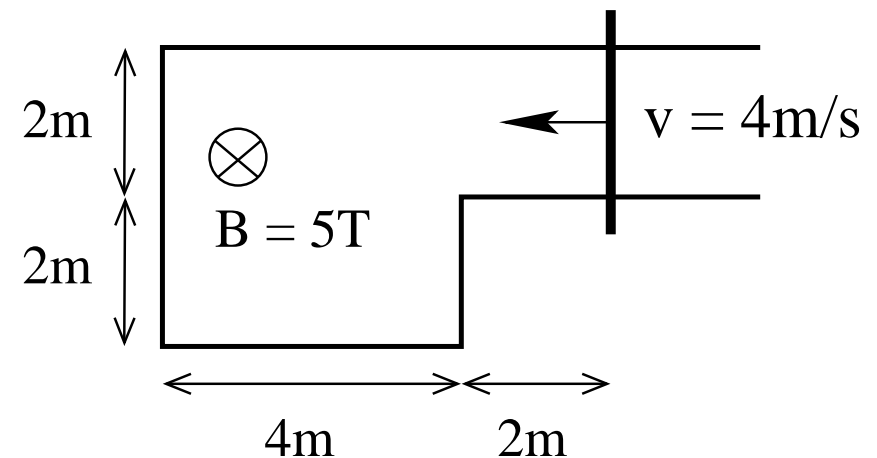


A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown.

(a) Find the magnetic flux  $\Phi_B$  through the frame at the instant shown.

(b) Find the induced emf  $\mathcal{E}$  at the instant shown.

(c) Find the direction (cw/ccw) of the induced current.



**Solution:**

$$(a) \Phi_B = \vec{A} \cdot \vec{B} = \pm(20\text{m}^2)(5\text{T}) = \pm 100\text{Wb}.$$

$$(b) \mathcal{E} = -\frac{d\Phi_B}{dt} = \pm(5\text{T})(2\text{m})(4\text{m/s}) = \pm 40\text{V}.$$



## Intermediate Exam III: Problem #3 (Spring '07)

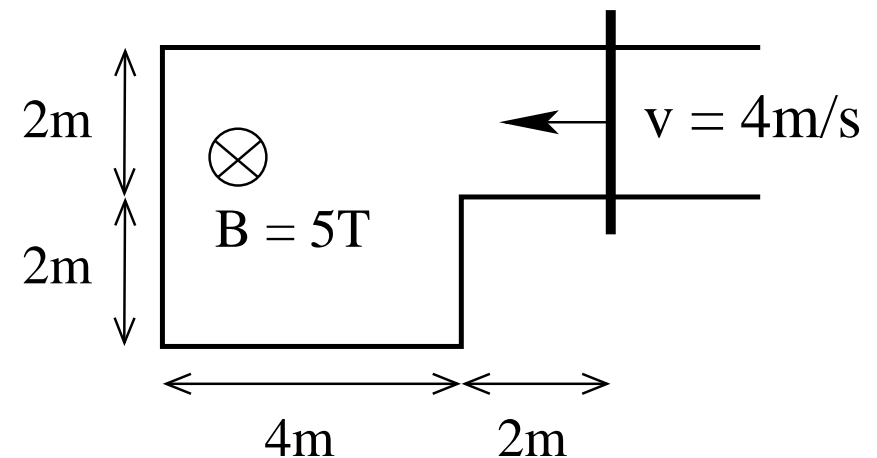


A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown.

(a) Find the magnetic flux  $\Phi_B$  through the frame at the instant shown.

(b) Find the induced emf  $\mathcal{E}$  at the instant shown.

(c) Find the direction (cw/ccw) of the induced current.



**Solution:**

(a)  $\Phi_B = \vec{A} \cdot \vec{B} = \pm(20\text{m}^2)(5\text{T}) = \pm 100\text{Wb}.$

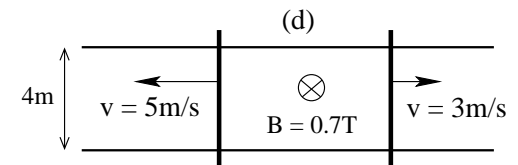
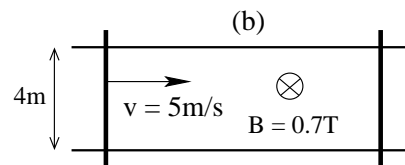
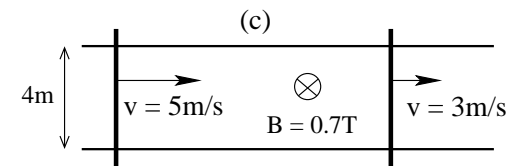
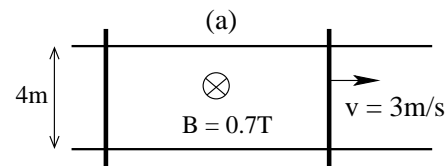
(b)  $\mathcal{E} = -\frac{d\Phi_B}{dt} = \pm(5\text{T})(2\text{m})(4\text{m/s}) = \pm 40\text{V}.$

(c) clockwise.

# Unit Exam III: Problem #3 (Spring '09)



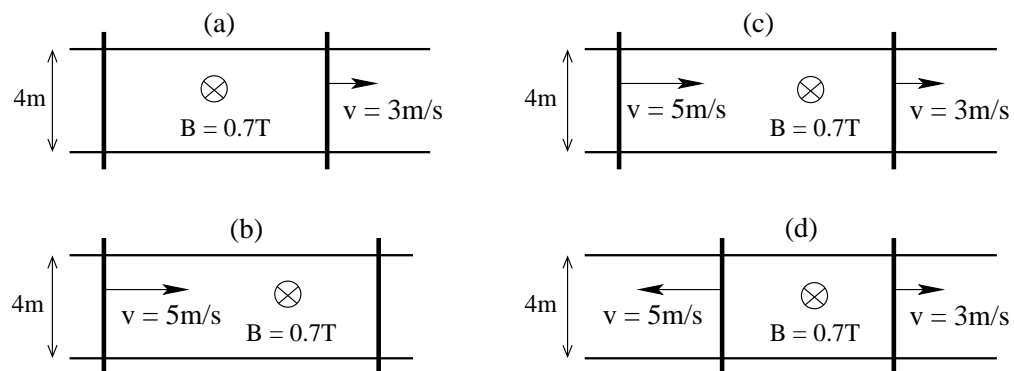
A pair of rails are connected by two mobile rods. A uniform magnetic field  $B$  directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is  $R = 0.2\Omega$  in each case. Find magnitude  $I$  and direction (cw/ccw) of the induced current in each case.



# Unit Exam III: Problem #3 (Spring '09)



A pair of rails are connected by two mobile rods. A uniform magnetic field  $B$  directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is  $R = 0.2\Omega$  in each case. Find magnitude  $I$  and direction (cw/ccw) of the induced current in each case.



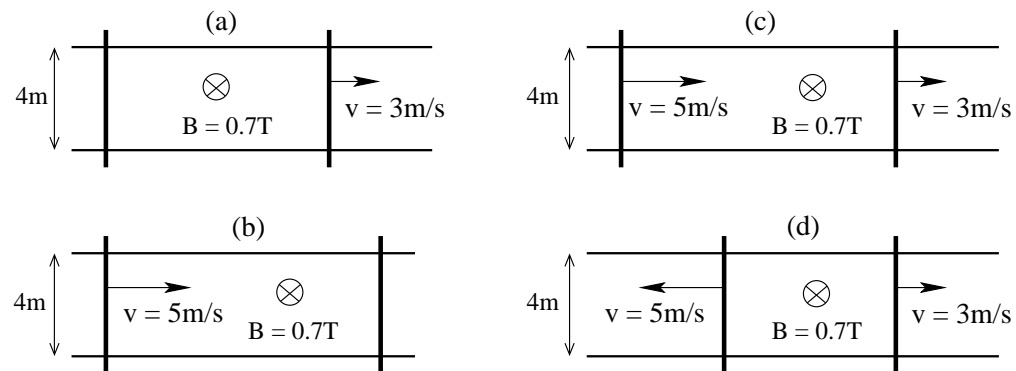
**Solution:**

(a)  $|\mathcal{E}| = (3\text{m/s})(0.7\text{T})(4\text{m}) = 8.4\text{V}, \quad I = \frac{8.4\text{V}}{0.2\Omega} = 42\text{A} \quad \text{ccw}$

# Unit Exam III: Problem #3 (Spring '09)



A pair of rails are connected by two mobile rods. A uniform magnetic field  $B$  directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is  $R = 0.2\Omega$  in each case. Find magnitude  $I$  and direction (cw/ccw) of the induced current in each case.



**Solution:**

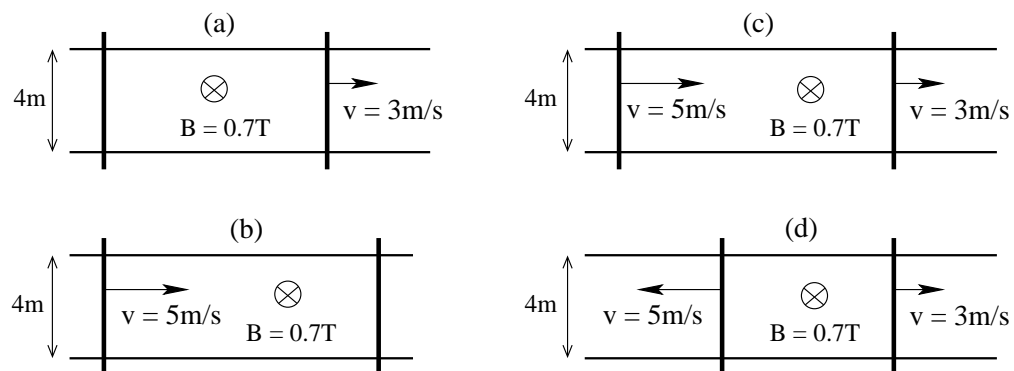
(a)  $|\mathcal{E}| = (3\text{m/s})(0.7\text{T})(4\text{m}) = 8.4\text{V}, \quad I = \frac{8.4\text{V}}{0.2\Omega} = 42\text{A} \quad \text{CCW}$

(b)  $|\mathcal{E}| = (5\text{m/s})(0.7\text{T})(4\text{m}) = 14\text{V}, \quad I = \frac{14\text{V}}{0.2\Omega} = 70\text{A} \quad \text{CW}$

# Unit Exam III: Problem #3 (Spring '09)



A pair of rails are connected by two mobile rods. A uniform magnetic field  $B$  directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is  $R = 0.2\Omega$  in each case. Find magnitude  $I$  and direction (cw/ccw) of the induced current in each case.



**Solution:**

$$(a) \quad |\mathcal{E}| = (3\text{m/s})(0.7\text{T})(4\text{m}) = 8.4\text{V}, \quad I = \frac{8.4\text{V}}{0.2\Omega} = 42\text{A} \quad \text{CCW}$$

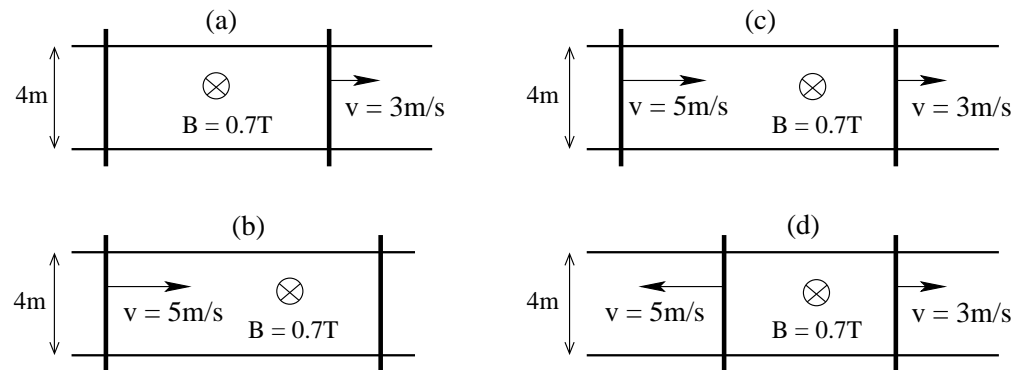
$$(b) \quad |\mathcal{E}| = (5\text{m/s})(0.7\text{T})(4\text{m}) = 14\text{V}, \quad I = \frac{14\text{V}}{0.2\Omega} = 70\text{A} \quad \text{CW}$$

$$(c) \quad |\mathcal{E}| = (5\text{m/s} - 3\text{m/s})(0.7\text{T})(4\text{m}) = 5.6\text{V}, \quad I = \frac{5.6\text{V}}{0.2\Omega} = 28\text{A} \quad \text{CW}$$

# Unit Exam III: Problem #3 (Spring '09)



A pair of rails are connected by two mobile rods. A uniform magnetic field  $B$  directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is  $R = 0.2\Omega$  in each case. Find magnitude  $I$  and direction (cw/ccw) of the induced current in each case.



**Solution:**

$$\begin{aligned}
 \text{(a)} \quad |\mathcal{E}| &= (3\text{m/s})(0.7\text{T})(4\text{m}) = 8.4\text{V}, & I &= \frac{8.4\text{V}}{0.2\Omega} = 42\text{A} & \text{CCW} \\
 \text{(b)} \quad |\mathcal{E}| &= (5\text{m/s})(0.7\text{T})(4\text{m}) = 14\text{V}, & I &= \frac{14\text{V}}{0.2\Omega} = 70\text{A} & \text{CW} \\
 \text{(c)} \quad |\mathcal{E}| &= (5\text{m/s} - 3\text{m/s})(0.7\text{T})(4\text{m}) = 5.6\text{V}, & I &= \frac{5.6\text{V}}{0.2\Omega} = 28\text{A} & \text{CW} \\
 \text{(d)} \quad |\mathcal{E}| &= (5\text{m/s} + 3\text{m/s})(0.7\text{T})(4\text{m}) = 22.4\text{V}, & I &= \frac{22.4\text{V}}{0.2\Omega} = 112\text{A} & \text{CCW}
 \end{aligned}$$