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14. Capacitors with dielectrics

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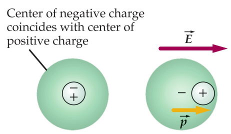
PHY204 Lecture 14 [rln14]

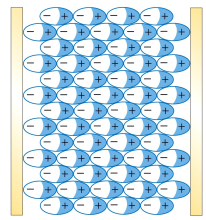
Capacitor with Dielectric

Most capacitors have a dielectric (insulating solid or liquid material) in the space between the conductors.
This has several advantages:

- Physical separation of the conductors.
- Prevention of dielectric breakdown.
- Enhancement of capacitance.

The dielectric is polarized by the electric field between the capacitor plates.





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Dielectric materials are electrically polarizable. When placed into an electric field, atomic electric dipole moments are induced or permanent molecular dipole moments are aligned as discussed earlier (lecture 3).

Filling the space between the two conductors of a capacitor with a solid dielectric material has three advantages as stated on the slide. The first two are mechanical and electrical aspects of the same thing: Prevent the two conductors from touching, which facilitates an electrical discharge across it.

In the following we focus on the third advantage, the enhancement of capacitance by the mere presence of a dielectric.

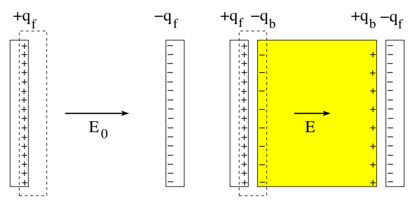
When a dielectric is placed between oppositely charged parallel plates, the electric field (directed to the right) pushes all positive atomic charge slightly to the right and all negative atomic charge to the left.

In the bulk of the dielectric, electric neutrality is conserved when averaged over an atomic length scale. However, at the interfaces between the dielectric and the conducting plates, there is a layer of bound (i.e. immobile) charge.

The bound charge near the positively charged plate (on the left) is negative and the bound charge near the negatively charged plate (on the right) is positive. What are the consequences for the capacitance and other quantities?

Parallel-Plate Capacitor with Dielectric (1)

The polarization produces a bound charge on the surface of the dielectric.



The bound surface charge has the effect of reducing the electric field between the plates from $\vec{E}_0$ to $\vec{E}$.

- A: area of plates
- d: separation between plates
- $\pm q_f$: free charge on plate
- $\pm q_b$: bound charge on surface of dielectric
- $\vec{E}_0$: electric field in vacuum
- $\vec{E}$: electric field in dielectric

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On the slide we see the same charged parallel-plate capacitor without dielectric (left) and with dielectric (right). All relevant specifications are listed.

The free charge on the conducting plates, $+q_f$ on the left and $-q_f$ on the right, are uniformly distributed on the inside surface.

The electric field polarizes the dielectric and generates bound charge on the surface next to the conductors, $-q_b$ on the left and $+q_b$ on the right.

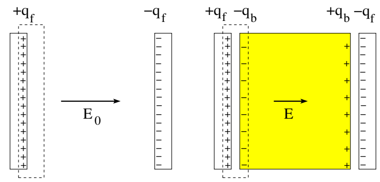
The bound charge cannot move from the dielectric to the conductor across the interface nor can the free charge move in the opposite direction.

The free charge is assumed to be the same on both capacitors, which is the case if the device is disconnected from any circuit while the dielectric is added or removed.

The electric field is assumed to be uniform between the plates, which is accurate except for fringe effects near the edges.

Parallel-Plate Capacitor with Dielectric (2)

Use Gauss' law to determine the electric fields $\vec{E}_0$ and $\vec{E}$.



- Field in vacuum: $E_0 A = \frac{q_f}{\epsilon_0} \quad \Rightarrow \quad E_0 = \frac{q_f}{\epsilon_0 A}$
- Field in dielectric: $EA = \frac{q_f - q_b}{\epsilon_0} \quad \Rightarrow \quad E = \frac{q_f - q_b}{\epsilon_0 A} < E_0$
- Voltage: $V_0 = E_0 d$ (vacuum), $V = Ed = \frac{V_0}{\kappa} < V_0$ (dielectric)

Dielectric constant: $\kappa \equiv \frac{E_0}{E} = \frac{q_f}{q_f - q_b} > 1$. Permittivity of dielectric: $\epsilon = \kappa\epsilon_0$.

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The electric field E_0 between the plates in the absence of a dielectric can be inferred from Gauss's law in the form of a pizza box (shown dashed in cross section) that envelops the inside surface of the plate with $+q_f$ on it. The result is shown as the first item on the slide.

The electric field E between the plates in the presence of a dielectric can be inferred similarly. The pizza box now encloses the free charge $+q_f$ on the inside surface of the plate and the bound charge $-q_b$ on the left surface of the dielectric. The result is shown as the second item on the slide.

We see that the dielectric weakens the electric field between the plates.

The third item calculates the voltage across the capacitor without and with dielectric. We see that the dielectric has the effect of decreasing the voltage across a capacitor of given free charge on it.

The ratio $E_0/E = \kappa$ of electric fields is determined by a dimensionless material specification of dielectrics, named dielectric constant.

The dielectric constant is often absorbed into the permittivity constant ϵ_0. The quantity $\epsilon = \kappa\epsilon_0$ is the (enhanced) permittivity of a dielectric.

The dielectric constant then also determines the ratio V_0/V of voltages and the ration C/C_0 of capacitances. The voltage decreases in the presence of the dielectric as does the electric field, whereas the capacitance increases.

Dielectric Materials

TABLE 24-1

Dielectric Constants and Dielectric Strengths of Various Materials

Material	Dielectric Constant κ	Dielectric Strength, kV/mm
Air	1.00059	3
Bakelite	4.9	24
Glass (Pyrex)	5.6	14
Mica	5.4	10–100
Neoprene	6.9	12
Paper	3.7	16
Paraffin	2.1–2.5	10
Plexiglas	3.4	40
Polystyrene	2.55	24
Porcelain	7	5.7
Transformer oil	2.24	12

- Dielectrics increase the capacitance: $C/C_0 = \kappa$.
- The capacitor is discharged spontaneously across the dielectric if the electric field exceeds the value quoted as dielectric strength.

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Empirical data exist for the dielectric constant of a large range of materials. A short list is compiled on the slide.

Included in the same list are data for the dielectric strength, which is the maximum voltage per mm thickness that can be applied without inadvertent electric discharge.

The data for dielectric constant and dielectric strength are useful for the design of capacitors built for specific applications.

Impact of Dielectric (1)

What happens when a dielectric is placed into a capacitor with the **charge on the capacitor** kept constant?

	vacuum	dielectric
charge	Q_0	$Q = Q_0$
electric field	E_0	$E = \frac{E_0}{\kappa} < E_0$
voltage	V_0	$V = \frac{V_0}{\kappa} < V_0$
capacitance	$C_0 = \frac{Q_0}{V_0}$	$C = \frac{Q}{V} = \kappa C_0 > C_0$
potential energy	$U_0 = \frac{Q_0^2}{2C_0}$	$U = \frac{Q^2}{2C} = \frac{U_0}{\kappa} < U_0$
energy density	$u_E^{(0)} = \frac{1}{2}\epsilon_0 E_0^2$	$u_E = \frac{1}{2}\epsilon E^2 = \frac{u_E^{(0)}}{\kappa} < u_E^{(0)}$

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The table gives a more complete list of what the impact of the dielectric in a (parallel-plate) capacitor is when it is inserted while the device is disconnected from a circuit and thus maintains the same charge on the plates.

We have already determined that the electric field and the voltage decrease when the dielectric is inserted. It follows directly that the capacitance increases, which, in turn, has the consequence that the potential energy decreases.

In lecture 12 we have derived an expression for the energy density contained in an electric field. That expression must be generialized in the presence of a dielectric as follows:

$$u_E = \frac{1}{2}\epsilon E^2, \quad \epsilon = \kappa\epsilon_0.$$

We simply replace the permittivity constant ϵ_0 by the permittivity ϵ of the dielectric.

The consequence is that when a dielectric is inserted into a capacitor with fixed charge, the energy density decreases, as stated on the last line of the table.

Impact of Dielectric (2)

What happens when a dielectric is placed into a capacitor with the **voltage across the capacitor** kept constant?

	vacuum	dielectric
voltage	V_0	$V = V_0$
electric field	E_0	$E = E_0$
capacitance	$C_0 = \frac{Q_0}{V_0}$	$C = \frac{Q}{V} = \kappa C_0 > C_0$
charge	Q_0	$Q = \kappa Q_0 > Q_0$
potential energy	$U_0 = \frac{1}{2}C_0V_0^2$	$U = \frac{1}{2}CV^2 = \kappa U_0 > U_0$
energy density	$u_E^{(0)} = \frac{1}{2}\epsilon_0 E_0^2$	$u_E = \frac{1}{2}\epsilon E^2 = \kappa u_E^{(0)} > u_E^{(0)}$

tsl128

The impact of a dielectric is different if it is inserted while the capacitor stays connected to a power source. In this case, the voltage across the device remains the same. In consequence, the electric field does not change either.

The capacitance, a device property, must change in the same way, whether or not it is connected to the power source, when the dielectric is inserted. It increases by a factor κ as on the previous page.

This has the consequence that the charge on the capacitor must also increase by a factor κ factor. The same conclusion holds for the energy stored on the capacitor. It goes up by a factor κ.

For the energy density we again use the generalized expression,

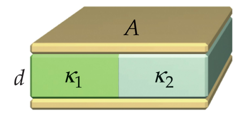
$$u_E = \frac{1}{2}\epsilon E^2, \quad \epsilon = \kappa\epsilon_0.$$

In this case, the insertion of the dielectric has the effect of increasing the energy density.

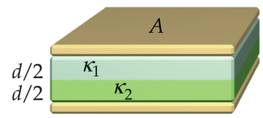
Stacked Dielectrics

Consider a parallel-plate capacitor with area A of each plate and spacing d.

- Capacitance without dielectric: $C_0 = \frac{\epsilon_0 A}{d}$.
- Dielectrics stacked in parallel: $C = C_1 + C_2$
  with $C_1 = \kappa_1 \epsilon_0 \frac{A/2}{d}$, $C_2 = \kappa_2 \epsilon_0 \frac{A/2}{d}$.
  $\Rightarrow C = \frac{1}{2}(\kappa_1 + \kappa_2) C_0$.



- Dielectrics stacked in series: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$
  with $C_1 = \kappa_1 \epsilon_0 \frac{A}{d/2}$, $C_2 = \kappa_2 \epsilon_0 \frac{A}{d/2}$
  $\Rightarrow C = \frac{2\kappa_1\kappa_2}{\kappa_1 + \kappa_2} C_0$.



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When we analyze stacked dielectrics, as represented by the two cases shown on the slide, we can take advantage of the insights gained on the previous two pages.

When the dielectrics are stacked in parallel, the voltage is the same left and right, which implies that the electric field is the same as well. It follows that the densities of bound and free charge must add up to the same values left and right.

Since different dielectrics produce different amounts of bound charge, the free charge is different left and right as well. When we separate the two halves left and right, we have two capacitors in parallel with different amounts of charge on them. They have different capacitances.

When the dielectrics are stacked in series, the electric field is not the same inside the two dielectrics. What is the same on both sides of the interface between the two dielectrics is the quantity $D = \epsilon E$, named displacement field. We are not quite ready to prove this here.

If we insert a pair of conducting plates between the dielectrics, we have two capacitors in series. They have the same charge on them but different voltages across. Their capacitances are different

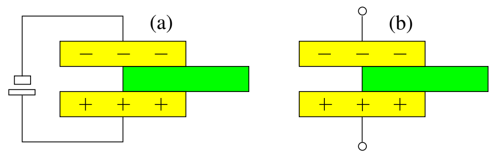
Lateral Force on Dielectric

Consider two charged capacitors with dielectrics only halfway between the plates.

In configuration (a) any lateral motion of the dielectric takes place at **constant voltage** across the plates.

In configuration (b) any lateral motion of the dielectric takes place at **constant charge** on the plates.

Determine in each case the direction (left/zero/right) of the lateral force experienced by the dielectric.



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This exercise can be solved at different levels of sophistication. Here we are only asked to predict whether the dielectric is pulled in (force to the left) or pushed out (force to the right). We can answer this question by a simple energetic argument.

When the potential energy of the total system goes down during insertion of the dielectric slab, then the slab is being pulled in. If the potential energy goes up, the slab is being pushed out.

Notice the clause “total system”. It is not the same for parts (a) and (b).

(b) We begin with the analysis of the simpler case. The system here contains only the conducting plates with fixed charge Q_0 and the dielectric. The relevant information is found in the table on page 5. There we read that the potential energy decreases:

$$\Delta U = U - U_0 = U_0 \left(\frac{1}{\kappa} - 1 \right) < 0.$$

We conclude that the force acting on the dielectric is toward the left. The dielectric is being pulled into the space between the plates.

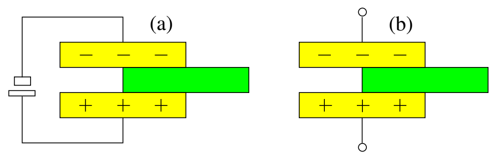
Lateral Force on Dielectric

Consider two charged capacitors with dielectrics only halfway between the plates.

In configuration (a) any lateral motion of the dielectric takes place at **constant voltage** across the plates.

In configuration (b) any lateral motion of the dielectric takes place at **constant charge** on the plates.

Determine in each case the direction (left/zero/right) of the lateral force experienced by the dielectric.



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(a) Here the system consists of the dielectric and the plates with variable charge on it and the battery. The potential energy has two parts: the energy $U_{\rm cap}$ stored on the capacitor and the energy $U_{\rm bat}$ stored in the power source. The relevant information is found in the table on page 6.

Charge $\Delta Q = Q_0(\kappa - 1) > 0$ is added to the plates during insertion. The energy $U_{\rm cap}$ stored on the capacitor increases:

$$\Delta U_{\rm cap} = U_0(\kappa - 1) = \frac{1}{2}\Delta Q V_0 = \frac{1}{2}Q_0 V_0(\kappa - 1).$$

Moving the charge ΔQ onto the capacitor during insertion involves work $\Delta Q V_0$ by the battery. Its energy thus goes down by exactly that amount:

$$\Delta U_{\rm bat} = -\Delta Q V_0,$$

implying that

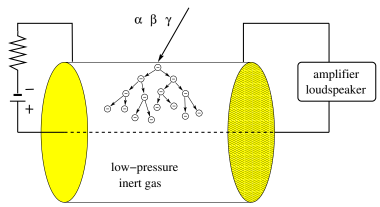
$$\Delta U = \Delta U_{\rm cap} + \Delta U_{\rm bat} = -\frac{1}{2}Q_0 V_0(\kappa - 1).$$

We again conclude that the force acting on the dielectric is toward the left. The dielectric is being pulled into the space between the plates.

Geiger Counter

Radioactive atomic nuclei produce high-energy particles of three different kinds:

- α-particles are ^{4}He nuclei.
- β-particles are electrons or positrons.
- γ-particles are high-energy photons.



- Free electrons produced by ionizing radiation are strongly accelerated toward the central wire.
- Collisions with gas atoms produce further free electrons, which are accelerated in the same direction.
- An avalanche of electrons reaching the wire produces a current pulse in the circuit.

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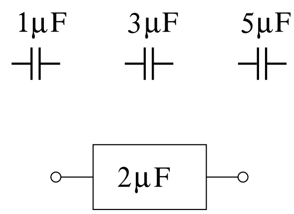
We finish this lecture with an application that is only loosely connected to capacitors and dielectric.

The Geiger counter is a sort of cylindrical capacitor with a low-pressure gaseous dielectric. Of importance in this case is the dielectric strength such as tabulated on page 4.

The electric field inside the cylinder must be chosen sufficiently high that ionizing events caused by radioactive decay products can initiate a chain reaction of further ionization events with a cumulative charge large enough to be convertible into an audible signal.

Capacitor Circuit (4)

Connect the three capacitors in such a way that the equivalent capacitance is $C_{eq} = 2\mu$F. Draw the circuit diagram.



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This is the quiz for lecture 14.

It is a variation of the exercise on page 2 of the previous lecture. There are eight different ways of connecting three capacitors that all have different capacitances.