2008

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Citation/Publisher Attribution

Li, H., Zhang, M., & Hu, S. J. (2008). Refinement of reduced-models for dynamic systems. *Progress in Natural Science, 18*(8), 993-997. doi: 10.1016/j.pnsc.2008.03.011
Available at: https://doi.org/10.1016/j.pnsc.2008.03.011

Available online at www.sciencedirect.com

ScienceDirect

Progress in Natural Science 18 (2008) 993–997

Progress in Natural Science

www.elsevier.com/locate/pnsc

Refinement of reduced-models for dynamic systems

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Received 18 August 2007; received in revised form 25 March 2008; accepted 28 March 2008

Abstract

A refinement procedure for the reduced models of structural dynamic systems is presented in this article. The refinement procedure is to "tune" the parameters of a reduced model, which could be obtained from any traditional model reduction scheme, into an improved reduced model. Upon the completion of the refinement, the improved reduced model matches the dynamic characteristics – the chosen structural frequencies and their mode shapes – of the full order model. Mathematically, the procedure to implement the model refinement technique is an application of the recently developed *cross-model cross-mode* (CMCM) method for model updating. A numerical example of reducing a 5-DOF (degree-of-freedom) classical mass-spring (or shear-building) model into a 3-DOF generalized mass-spring model is demonstrated in this article.

Keywords: Model reduction; Model refinement; Finite element method; Modal analysis; Structural dynamics

1. Introduction

Finite element models of structures need to have many degrees of freedom to represent the geometrical detail of complex structures. For various reasons, engineers often want to simplify the complicated structural models prior to performing necessary tasks. Usually, carrying out an appropriate model reduction scheme allows one to create a lower order model that represents the dynamics of the full-order model in the considered loading/parameter conditions. In dealing with the limitations imposed by the available computing power, historical model reduction schemes, including static condensation [1], dynamic condensation [2], sub-structuring (component synthesis) [3], etc. were developed to gain computational efficiency in structural analysis. In particular, those model reduction schemes were performed prior to the eigen analysis, thus an expensive eigen analysis for the full-order model could be avoided. Nowadays, as the computing power increases enormously, performing model reduction to gain computational efficiency has become less necessary.

However, applying model reduction schemes remains very popular in the area of modal testing [4,5]. The main reason of this popularity is not due to any concern of computational efficiency, rather the necessity of compatible models. In modal testing, an obvious incompatibility lies in the difference in the order (the number of degrees of freedom) of the models derived respectively from tests (measured models) and theoretical analysis (analytical models). Usually the measured models – substantiated by measured degrees of freedom (DOFs) – are of relatively small order, while the analytical models – generally established via a finite-element procedure – are one or more order of magnitude larger. When comparisons are to be made between a measured model and its theoretical counterpart, this order incompatibility presents obstacles to meaningful interpretation. Therefore, there is a need to bring them both to the same order, which can be achieved by reducing the analytical model. Maintaining the essential eigen-properties of the full-order model unchanged for the reduced model is desired, how-

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1002-0071/$ - see front matter
doi:10.1016/j.pnsc.2008.03.011

ever, most traditional model reduction schemes can only achieve the goal approximately. In other words, traditional reduction schemes could only yield the eigen-properties of the reduced model approximating to those of the full-order model [4,5].

This article presents a refinement procedure for the reduced models. Implementing the refinement procedure is to tune the reduced model, obtained from one of the traditional model reduction schemes, into an improved reduced model (simply called *improved model* hereafter) that can maintain the equality of the selected modal properties of the full-order model. Mathematically, the proposed model refinement technique is similar to a model updating approach. By updating the chosen coefficients of the reduced model, one finds proper mass and stiffness matrices of the improved model, so that the mode shapes and frequencies of the improved model can agree with those of the full-order model. The mathematical kernel of the proposed model refinement technique is the *cross-model cross-mode* (CMCM) method [6], which is so named because it involves solving a set of linear simultaneous equations where each equation is formulated based on the product terms from two same/different modes associated with the reduced and improved models, respectively. In brief, the proposed refinement technique forms simultaneous linear equations in a matrix form, with the unknown vector being the correction factors which are used to correct the selected stiffness and/or mass sub-matrices.

The numerical example demonstrated in this paper is to reduce, then refine, a 5-DOF classical mass-spring model into a 3-DOF generalized mass-spring model, in which the initial model reduction is carried out by using the static condensation (Guyan reduction) technique.

2. Preliminaries

Most structural dynamic system modeling is performed with the finite-element (FE) method. The model consists of mass and stiffness matrices, which are required in an eigen analysis. The undamped free vibration of a structural dynamic system can be described by the second order differential equation as [7,8]:

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{0} \tag{1}$$

in which $\boldsymbol{M}$ and $\boldsymbol{K}$ are the mass and stiffness matrices, respectively, and $\boldsymbol{x}$ is the displacement vector. The eigen solution of this system consists of the eigenvalue matrix $\boldsymbol{\Lambda}$, which is a diagonal matrix of the squared natural frequencies, and the eigenvector matrix $\boldsymbol{\Phi}$.

2.1. Guyan reduction

The most widely adopted model reduction scheme is the static reduction introduced by Guyan [1]. This technique partitions the mass and stiffness matrices, and the displacement vector, in Eq. (1) into a set of *master* and *slave* degrees of freedom:

$$\begin{bmatrix} \boldsymbol{M}_m & \boldsymbol{M}_{ms} \\ \boldsymbol{M}_{sm} & \boldsymbol{M}_s \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{x}}_m \\ \ddot{\boldsymbol{x}}_s \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_m & \boldsymbol{K}_{ms} \\ \boldsymbol{K}_{sm} & \boldsymbol{K}_s \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_m \\ \boldsymbol{x}_s \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{Bmatrix} \tag{2}$$

The subscripts m and s relate to the master and slave coordinates, respectively. Neglecting the inertia terms for the second set of equations may be used to eliminate the slave degrees of freedom. It leads to

$$\begin{Bmatrix} \boldsymbol{x}_m \\ \boldsymbol{x}_s \end{Bmatrix} = \boldsymbol{T}_G \boldsymbol{x}_m \tag{3}$$

where

$$\boldsymbol{T}_G = \begin{bmatrix} \boldsymbol{I} \\ -\boldsymbol{K}_s^{-1}\boldsymbol{K}_{sm} \end{bmatrix} \tag{4}$$

is the Guyan transformation matrix. The reduced mass and stiffness matrices are then given by

$$\boldsymbol{M}^G = \boldsymbol{T}_G^{\mathrm{T}} \boldsymbol{M} \boldsymbol{T}_G \tag{5}$$

and

$$\boldsymbol{K}^G = \boldsymbol{T}_G^{\mathrm{T}} \boldsymbol{K} \boldsymbol{T}_G \tag{6}$$

where $\boldsymbol{M}^G$ and $\boldsymbol{K}^G$ denote the reduced mass and stiffness matrices associated with the Guyan reduction scheme.

3. Model refinement

Throughout this paper, to distinguish symbols for various models, the superscript "$\prime$" is used for the reduced model, superscript "$*$" for the improved model, and without a superscript for the full-order model. For instance, $\boldsymbol{M}'$, $\boldsymbol{M}^*$ and $\boldsymbol{M}$ represent the mass matrix of the reduced model, improved model, and full-order model, respectively. As the full-order model is usually formed by a finite-element procedure, the reduced model is obtained from the full-order model via a traditional model reduction scheme, and the improved model is tuned from the reduced model through the refinement technique presented below.

The undamped free vibration of the improved model can be described by the second order differential equation as

$$\boldsymbol{M}^*\ddot{\boldsymbol{x}}^* + \boldsymbol{K}^*\boldsymbol{x}^* = \boldsymbol{0} \tag{7}$$

where $\boldsymbol{x}^*$ is the displacement vector of the improved model, and should be corresponding to the master coordinates of the full model, i.e. $\boldsymbol{x}^* = \boldsymbol{x}_m$. The model refinement is in an attempt to have the mode shapes of the improved model match with those of the master coordinates of the full-order model, *i.e.,* attempting to make $\boldsymbol{\Phi}^* = (\boldsymbol{\Phi})_m$ in which the subscript "m" indicates the *master* coordinates only. In addition, the corresponding modal frequencies of the full-order and improved models must be matched as well.

In the following derivation for the model refinement procedure, it is assumed that $\boldsymbol{M}$ and $\boldsymbol{K}$ have been formulated, thus one can perform the eigen analysis to get the corresponding mode shapes $\boldsymbol{\Phi}_j$ and modal frequencies ω_j, $j = 1, \cdots, N_s$, in which N_s is the number of modes for the full-order model. Furthermore, the stiffness $\boldsymbol{K}'$ and mass $\boldsymbol{M}'$ matrices of the reduced model have been obtained using one of the traditional model reduction methods, and the corresponding mode shapes $\mathbf{\Phi}'_i$ and modal frequencies ω'_i, $i = 1, \cdots, N_t$, where N_t is the number of modes for the reduced model, can be computed accordingly. The specific task of the proposed refinement procedure is to refine the stiffness and mass matrices from $\boldsymbol{K}'$ and $\mathbf{M}'$ to $\boldsymbol{K}^*$ and $\boldsymbol{M}^*$ as several $\mathbf{\Phi}^*_j$ and ω^*_j associated with $\boldsymbol{K}^*$ and $\boldsymbol{M}^*$ must match well with several $(\mathbf{\Phi}_j)_m$ and ω_j.

In the proposed refinement method, the stiffness matrix $\boldsymbol{K}^*$ of the improved model is a correction of $\boldsymbol{K}'$ via

$$\boldsymbol{K}^* = \boldsymbol{K}' + \sum_{n=1}^{N_K} \alpha_n \mathbf{K}'_n \tag{8}$$

where any individual $\boldsymbol{K}'_n$ is a pre-selected stiffness sub-matrix of the reduced model, N_K is the number of stiffness correction terms, and α_n are unknown stiffness correction factors to be determined. Likewise, one writes the corresponding expression for the mass matrix $\boldsymbol{M}^*$ as

$$\boldsymbol{M}^* = \boldsymbol{M}' + \sum_{n=1}^{N_M} \beta_n \mathbf{M}'_n \tag{9}$$

in which the individual $\boldsymbol{M}'_n$ is a pre-selected mass sub-matrix of the reduced model, N_M is the number of correction coefficients for the mass matrix, and β_n are mass correction coefficients to be determined.

For the ith eigenvalue λ'_i and eigenvector $\mathbf{\Phi}'_i$ associated with $\mathbf{K}'$ and $\mathbf{M}'$, one has

$$\boldsymbol{K}'\mathbf{\Phi}'_i = \lambda'_i \boldsymbol{M}'\mathbf{\Phi}'_i \tag{10}$$

Similarly, for the jth eigenvalue λ^*_j and eigenvector $\mathbf{\Phi}^*_j$ associated with $\boldsymbol{K}^*$ and $\boldsymbol{M}^*$, one writes

$$\boldsymbol{K}^*\mathbf{\Phi}^*_j = \lambda^*_j \boldsymbol{M}^*\mathbf{\Phi}^*_j \tag{11}$$

In the following development, the mode shapes of the improved model should equal to those of master coordinates of the full-order model. Also, the corresponding modal frequencies of the full-order and improved models must be equal as well. One must treat λ^*_j and $\mathbf{\Phi}^*_j$ to be known quantities available from the full-order model, that is $\lambda^*_j = \lambda_j$ and $\mathbf{\Phi}^*_j = (\mathbf{\Phi}_j)_m$.

Denoting superscript "T" as the transpose operator, and premultiplying Eq. (11) by $(\mathbf{\Phi}'_i)^{\mathrm{T}}$ yields

$$(\mathbf{\Phi}'_i)^{\mathrm{T}}\boldsymbol{K}^*\mathbf{\Phi}^*_j = \lambda^*_j(\mathbf{\Phi}'_i)^{\mathrm{T}}\boldsymbol{M}^*\mathbf{\Phi}^*_j \tag{12}$$

Substituting Eqs. (8) and (9) into the above equation leads to

$$K^{\dagger}_{ij} + \sum_{n=1}^{N_K} \alpha_n K^{\dagger}_{n,ij} = \lambda^*_j \left(M^{\dagger}_{ij} + \sum_{n=1}^{N_M} \beta_n M^{\dagger}_{n,ij} \right) \tag{13}$$

where $K^{\dagger}_{ij} = (\mathbf{\Phi}'_i)^{\mathrm{T}}\boldsymbol{K}\mathbf{\Phi}^*_j$, $K^{\dagger}_{n,ij} = (\mathbf{\Phi}'_i)^{\mathrm{T}}\mathbf{K}_n\mathbf{\Phi}^*_j$, $M^{\dagger}_{ij} = (\mathbf{\Phi}'_i)^{\mathrm{T}}\boldsymbol{M}\mathbf{\Phi}^*_j$, and $M^{\dagger}_{n,ij} = (\mathbf{\Phi}'_i)^{\mathrm{T}}\boldsymbol{M}_n\mathbf{\Phi}^*_j$. For clarity, symbols with superscript "†" throughout this paper are "cross" terms calculated from both reduced and improved models. Using a new index m to replace ij and rearranging Eq. (13) yields

$$\sum_{n=1}^{N_K} \alpha_n K^{\dagger}_{n,m} + \sum_{n=1}^{N_M} \beta_n(-\lambda^*_j M^{\dagger}_{n,m}) = f^{\dagger}_m \tag{14}$$

where $f^{\dagger}_m = \lambda^*_j M^{\dagger}_m - K^{\dagger}_m$. When N_i modes are taken from the reduced model, and N_j modes are taken from the full-order model, totally $N_m = N_i \times N_j$ equations can be formed from Eq. (14). Those equations are named the *cross-model cross-mode* (CMCM) equations in view of the fact that they are formed by crossing over two models, reduced and improved models, also crossing over various modes. Expressing Eq. (14) in a matrix form, one has

$$\boldsymbol{K}^{\dagger}\boldsymbol{\alpha} + \boldsymbol{M}^{\dagger}\boldsymbol{\beta} = \boldsymbol{f}^{\dagger} \tag{15}$$

in which $\boldsymbol{K}^{\dagger}$ and $\boldsymbol{M}^{\dagger}$ are N_m-by-N_K and N_m-by-N_M matrix, respectively; $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are column vectors of size N_K and N_M; and $\mathbf{f}^{\dagger}$ is a column vector of size N_m. Furthermore, one can rewrite Eq. (15) as

$$\boldsymbol{G}^{\dagger}\boldsymbol{\gamma} = \boldsymbol{f}^{\dagger} \tag{16}$$

where

$$\boldsymbol{G}^{\dagger} = [\,\boldsymbol{K}^{\dagger} \quad \boldsymbol{M}^{\dagger}\,], \text{and } \boldsymbol{\gamma} = \begin{Bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{Bmatrix}.$$

Analytically, one can solve $\boldsymbol{\gamma}$ in Eq. (16) by a standard inverse operation, $\boldsymbol{\gamma} = \boldsymbol{G}^{\dagger^{-1}}\mathbf{f}^{\dagger}$, if $\boldsymbol{G}^{\dagger}$ is a non-singular square matrix. For a non-square matrix $\boldsymbol{G}^{\dagger}$ where the number of equations does not equal the number of unknowns, the equivalent operator is the pseudo-inverse. If $\boldsymbol{G}^{\dagger}$ has more rows than columns, an over-determined case where there are more equations than unknowns the pseudo-inverse is defined as

$$\boldsymbol{G}^{\dagger\sharp} = (\boldsymbol{G}^{\dagger\mathrm{T}}\boldsymbol{G}^{\dagger})^{-1}\boldsymbol{G}^{\dagger\mathrm{T}} \tag{17}$$

for nonsingular $(\boldsymbol{G}^{\dagger\mathrm{T}}\boldsymbol{G}^{\dagger})$. The resulting solution, $\boldsymbol{\gamma} = \boldsymbol{G}^{\dagger\sharp}\mathbf{f}^{\dagger}$ is optimal in a least-squares sense.

4. Numerical example

A numerical example is given below to illustrate the procedure of applying the proposed model refinement technique. The full-order model is a 5-DOF mass-spring system shown in Fig. 1(a). Mathematically, a mass-spring model is equivalent to a shear building model [8], to a lumped-mass finite-element model of a rod in longitudinal vibration, to a set of point masses vibrating transversely on a taut string, and to a finite-difference or finite-element approximation to a Sturm–Liouville problem [9].

The full-order model is taken to have $[m_1, \cdots, m_5]$ equal to $[3.5, 3.5, 2.5, 2.0, 1.5] \times 10^3$ Kg, and $[k_1, \cdots, k_5]$ all equal to 3×10^7 N/m. The displacements of the 5-DOF full-order model are denoted by x_i, $i = 1, \cdots, 5$. In this example, assuming the response data are measured at the first, third and fifth coordinates, thus the master coordinates are taken at x_1, x_3 and x_5. Following Eqs. (5) and (6), one can obtain

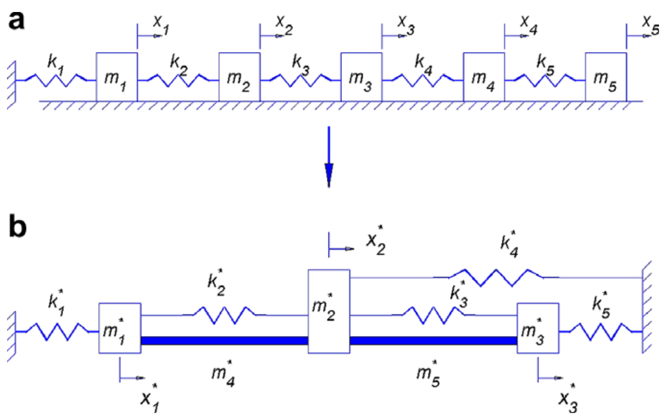

Fig. 1. (a) a 5-DOF mass-spring system, and (b) a 3-DOF generalized mass-spring system.

the reduced mass and stiffness matrices based on the Guyan reduction scheme as

$$M^G = \begin{bmatrix} 4375 & 875 & 0 \\ 875 & 3875 & 500 \\ 0 & 500 & 2000 \end{bmatrix} \text{kg}, \quad K^G = \begin{bmatrix} 45 & -15 & 0 \\ -15 & 30 & -15 \\ 0 & -15 & 15 \end{bmatrix} \times 10^6 \text{ N/m}$$

The above mass and stiffness matrices mathematically suggest a 3-DOF generalized mass-spring model shown in Fig. 1(b), in which no coupling terms are present between x_1^* and x_3^*. The resulting frequencies of this reduced model and the *modal assurance criterion* (MAC) values between the reduced and full-order models are shown in Table 1, where the MAC value between modes $\mathbf{\Phi}_i$ and $\mathbf{\Phi}_j$ has been defined as

$$\mathrm{MAC}(\mathbf{\Phi}_i, \mathbf{\Phi}_j) = \frac{|\mathbf{\Phi}_i \cdot \mathbf{\Phi}_j|}{|\mathbf{\Phi}_i||\mathbf{\Phi}_j|} \tag{18}$$

in which "·" represents the inner product operator, and $|\mathbf{\Phi}_i|$ denotes the length (norm) of $\mathbf{\Phi}_i$. The value of the MAC is always between 0 and 1, and a value of 1 indicates that the two modes have the same shape. From Table 1, one can observe that the Guyan reduction can only approximate the modal properties of the full system. The error for the first frequency is small (33.96 rad/s for full order model and 33.58 rad/s for Guyan reduction model), but the errors for the second and third frequencies/mode shapes (higher order modes) are relatively large (for example, 95.78 rad/s versus 88.98 rad/s for the second frequency, and the MAC value for the third mode degrading to 0.9844). Basically, results shown at Table 1 are in agreement with the conventional wisdom that the lower modes of a Guyan reduced model are more accurate whereas the higher modes can be more severe in error [5].

Table 1
Frequencies and MAC (modal assurance criterion) values for reduced and improved models

Mode number	Full order model	Reduced model (Guyan reduction)		Improved model	
n	ω_n(rad/s)	ω'_n (rad/s)	MAC	ω^*_n (rad/s)	MAC
1	33.58	33.96	1.0000	33.58	1.0000
2	88.98	95.78	0.9967	88.98	1.0000
3	140.92	142.80	0.9844	140.92	1.0000

Applying the proposed model refinement technique, one starts numerically with $K' = K^G$ and $M' = M^G$. For scaling purpose, a reference value associated with either stiffness or mass must be preset. Without losing generality, the (1,1) element of M^* is chosen to be the same as that of M'. The remaining issues include: (i) how to select the sub-matrices K'_n and M'_n? and (ii) how many K'_n and M'_n required?

The number of K'_n and M'_n terms to be included is mainly depending on the number of modes intended to be matched. While it is always desired to let the improved model match the modal properties of the full-order model as much as possible, there is a theoretical limitation because only a finite number of correction terms are involved. One analytical way to know how many modes could be matched is via counting the number of modal coefficients to be fitted and the number of correction terms adopted. From the one-to-one mapping principle, it is realized that matching each additional modal coefficient must have an extra correction term available. In order to match N modes of the improved model with those of the full-order model, one must include total $3N$ terms for K'_n and M'_n, in which "3" is for the reason that each mode of a 3-DOF system needs 3 independent quantities to characterize the mode—one quantity for the frequency and two independent quantities for the mode shape. In the present example, 9 correction terms are needed when all 3 modes are to be fitted.

Apparently, K'_n and M'_n both must be symmetric in order to maintain the symmetry of K^* and M^*. An obvious, but not the unique, way to choose K'_n is the sub-matrix associated with each independent parameter of the stiffness matrix of the reduced model

$$K'_1 = 10^6 \begin{bmatrix} 45 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{N/m}, \quad K'_2 = 10^6 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{N/m},$$

$$K'_3 = 10^6 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 15 \end{bmatrix} \text{N/m}, \quad K'_4 = 10^6 \begin{bmatrix} 0 & -15 & 0 \\ -15 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{N/m},$$

$$K'_5 = 10^6 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -15 \\ 0 & -15 & 0 \end{bmatrix} \text{N/m}$$

Likewise, M'_n could be associated with each parameter of the mass matrix of the reduced model, except the (1,1) entry which has been preset to be unchanged. There are 4 possible terms:

$$M'_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3875 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{kg}, \quad M'_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2000 \end{bmatrix} \text{kg},$$

$$M'_3 = \begin{bmatrix} 0 & 875 & 0 \\ 875 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{kg}, \quad M'_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 500 \\ 0 & 500 & 0 \end{bmatrix} \text{kg}$$

In applying the CMCM method, all 3 modes of the reduced model and the first three modes chosen from the the full-order model are employed to form 9 equations. Because 9 unknowns are to be solved in 9 CMCM equations, it can be solved by a standard inverse operation. The resulting $\boldsymbol{M}^*$ and $\boldsymbol{K}^*$ are:

$$\boldsymbol{M}^* = \begin{bmatrix} 4375 & 2105.5 & 0 \\ 2105.5 & 6135 & 736.2 \\ 0 & 736.2 & 2315 \end{bmatrix} \text{kg},$$

$$\boldsymbol{K}^* = \begin{bmatrix} 39.07 & -11.33 & 0 \\ -11.33 & 32.46 & -15.53 \\ 0 & -15.53 & 15.97 \end{bmatrix} \times 10^6 \text{ N/m}$$

Performing the numerical eigen analysis based on the above $\boldsymbol{M}^*$ and $\boldsymbol{K}^*$, one finds that the resulting modal frequencies and mode shapes match perfectly with those of both lower mode (1st mode) and higher mode (2nd and 3rd modes) of the 5-DOF model (see Table 1).

5. Concluding remarks

As traditional model reduction schemes intend to create lower order models that represent the dynamics of the full-order model in the considered loading/parameter conditions, normally the obtained reduced models can only approximate the dynamic properties of the full-order model. The model refinement procedure introduced in this article is to fine-tune the reduced model, so that the resulting improved model can equal its modal properties to those of the full-order model. The traditional Guyan reduction scheme was performed in the numerical example to reduce a 5-DOF (degree-of-freedom) classical mass-spring model into a 3-DOF generalized mass-spring model. After applying the proposed model refinement procedure to the 3-DOF generalized mass-spring model, the improved model could match its 3 modes perfectly with the first 3 modes of the 5-DOF classical mass-spring model.

Acknowledgements

This work was supported by National Natural Science Foundation of China (Grant No. 50739004), and by the National High Technology Research and Development Program of China (Grant No. 2006AA09Z331).

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