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13. Magnetic Field II

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Abstract

Lecture slides 13 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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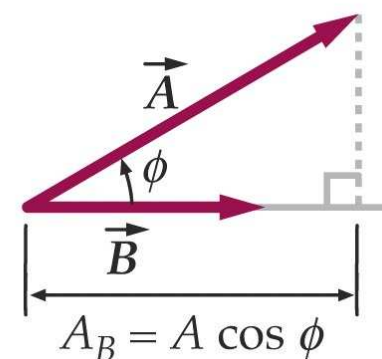
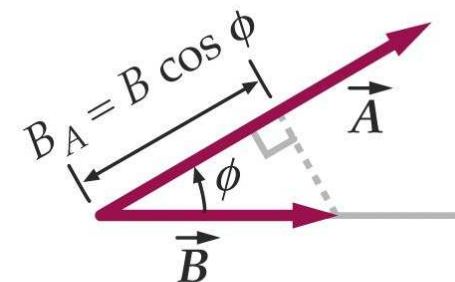
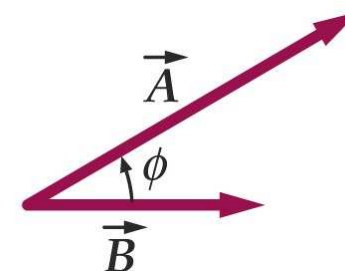
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Dot Product Between Vectors



Consider two vectors $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$.

- $\vec{A} \cdot \vec{B} = AB \cos \phi = AB_A = BA_B$.
- $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$.
- $\vec{A} \cdot \vec{B} = AB$ if $\vec{A} \parallel \vec{B}$.
- $\vec{A} \cdot \vec{B} = 0$ if $\vec{A} \perp \vec{B}$.
- $\vec{A} \cdot \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$
 $= A_xB_x(\hat{i} \cdot \hat{i}) + A_xB_y(\hat{i} \cdot \hat{j}) + A_xB_z(\hat{i} \cdot \hat{k})$
 $+ A_yB_x(\hat{j} \cdot \hat{i}) + A_yB_y(\hat{j} \cdot \hat{j}) + A_yB_z(\hat{j} \cdot \hat{k})$
 $+ A_zB_x(\hat{k} \cdot \hat{i}) + A_zB_y(\hat{k} \cdot \hat{j}) + A_zB_z(\hat{k} \cdot \hat{k})$.
- Use $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$,
 $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$.
- $\Rightarrow \vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$.

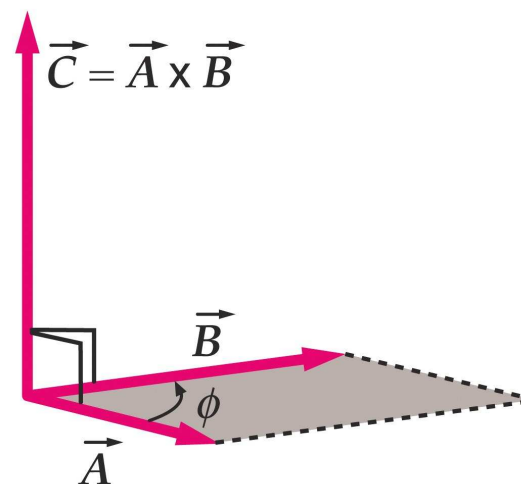


Cross Product Between Vectors



Consider two vectors $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$.

- $\vec{A} \times \vec{B} = AB \sin \phi \hat{n}$.
- $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$.
- $\vec{A} \times \vec{A} = 0$.
- $\vec{A} \times \vec{B} = AB \hat{n}$ if $\vec{A} \perp \vec{B}$.
- $\vec{A} \times \vec{B} = 0$ if $\vec{A} \parallel \vec{B}$.
- $$\begin{aligned}\vec{A} \times \vec{B} &= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\ &= A_xB_x(\hat{i} \times \hat{i}) + A_xB_y(\hat{i} \times \hat{j}) + A_xB_z(\hat{i} \times \hat{k}) \\ &\quad + A_yB_x(\hat{j} \times \hat{i}) + A_yB_y(\hat{j} \times \hat{j}) + A_yB_z(\hat{j} \times \hat{k}) \\ &\quad + A_zB_x(\hat{k} \times \hat{i}) + A_zB_y(\hat{k} \times \hat{j}) + A_zB_z(\hat{k} \times \hat{k}).\end{aligned}$$
- Use $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$,
 $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$.
- $\Rightarrow \vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$.



Magnetic Dipole Moment of Current Loop



N : number of turns

I : current through wire

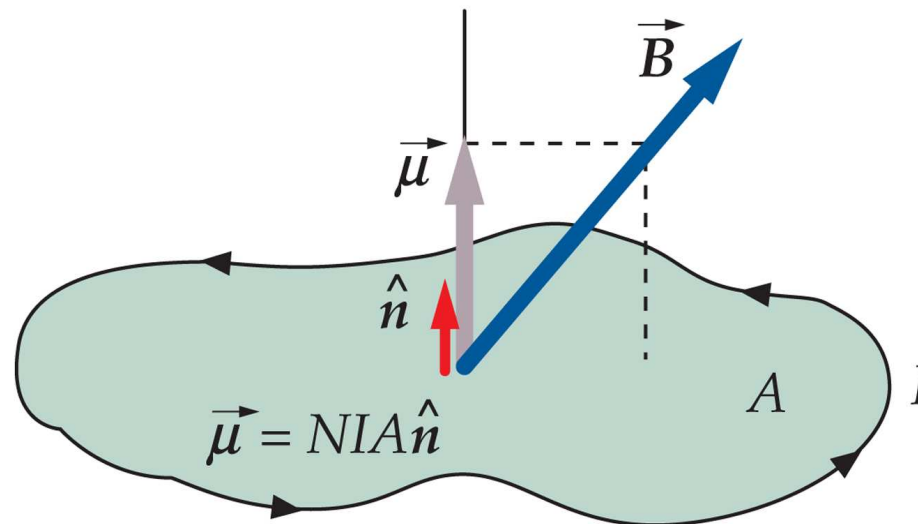
A : area of loop

\hat{n} : unit vector perpendicular to plane of loop

$\vec{\mu} = NIA\hat{n}$: magnetic dipole moment

\vec{B} : magnetic field

$\vec{\tau} = \vec{\mu} \times \vec{B}$: torque acting on current loop



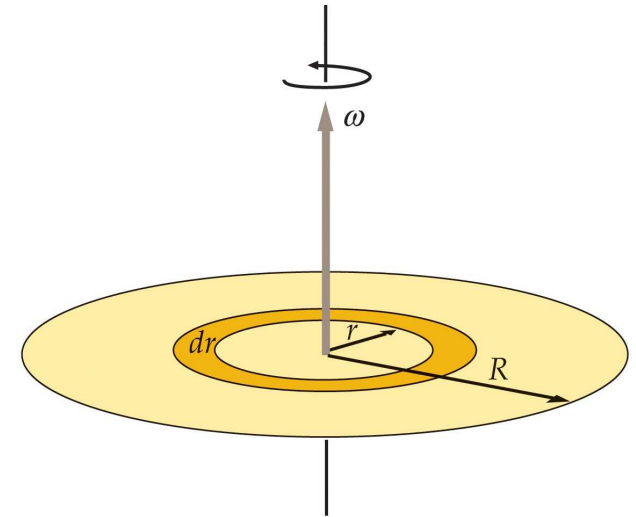
Magnetic Moment of a Rotating Disk



Consider a nonconducting disk of radius R with a uniform surface charge density σ . The disk rotates with angular velocity $\vec{\omega}$.

Calculation of the magnetic moment $\vec{\mu}$:

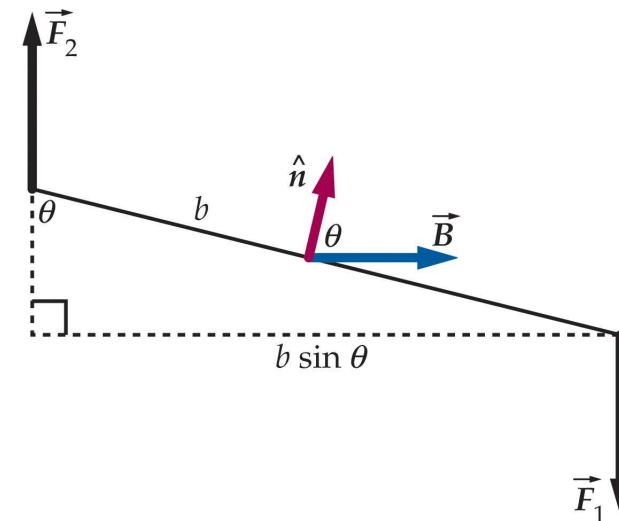
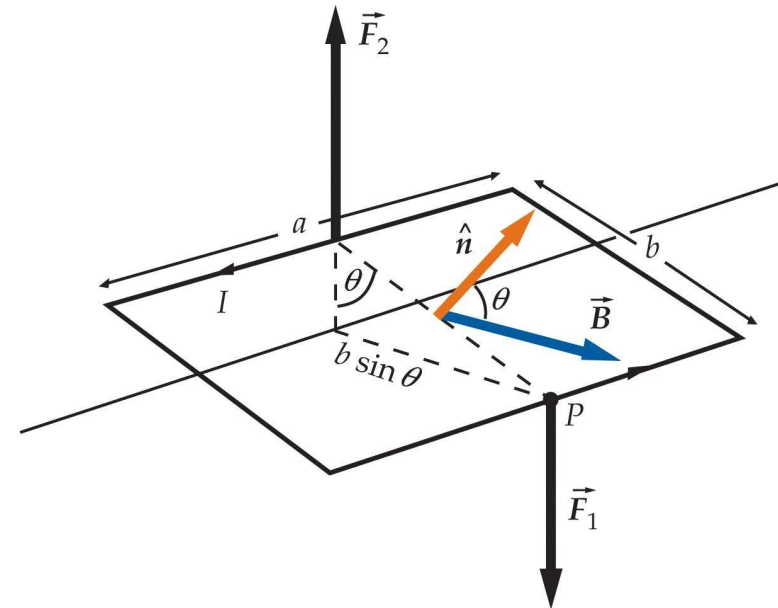
- Total charge on disk: $Q = \sigma(\pi R^2)$.
- Divide the disk into concentric rings of width dr .
- Period of rotation: $T = \frac{2\pi}{\omega}$.
- Current within ring: $dI = \frac{dQ}{T} = \sigma(2\pi r dr) \frac{\omega}{2\pi} = \sigma\omega r dr$.
- Magnetic moment of ring: $d\mu = dI(\pi r^2) = \pi\sigma\omega r^3 dr$.
- Magnetic moment of disk: $\mu = \int_0^R \pi\sigma\omega r^3 dr = \frac{\pi}{4}\sigma R^4\omega$.
- Vector relation: $\vec{\mu} = \frac{\pi}{4}\sigma R^4\vec{\omega} = \frac{1}{4}QR^2\vec{\omega}$.



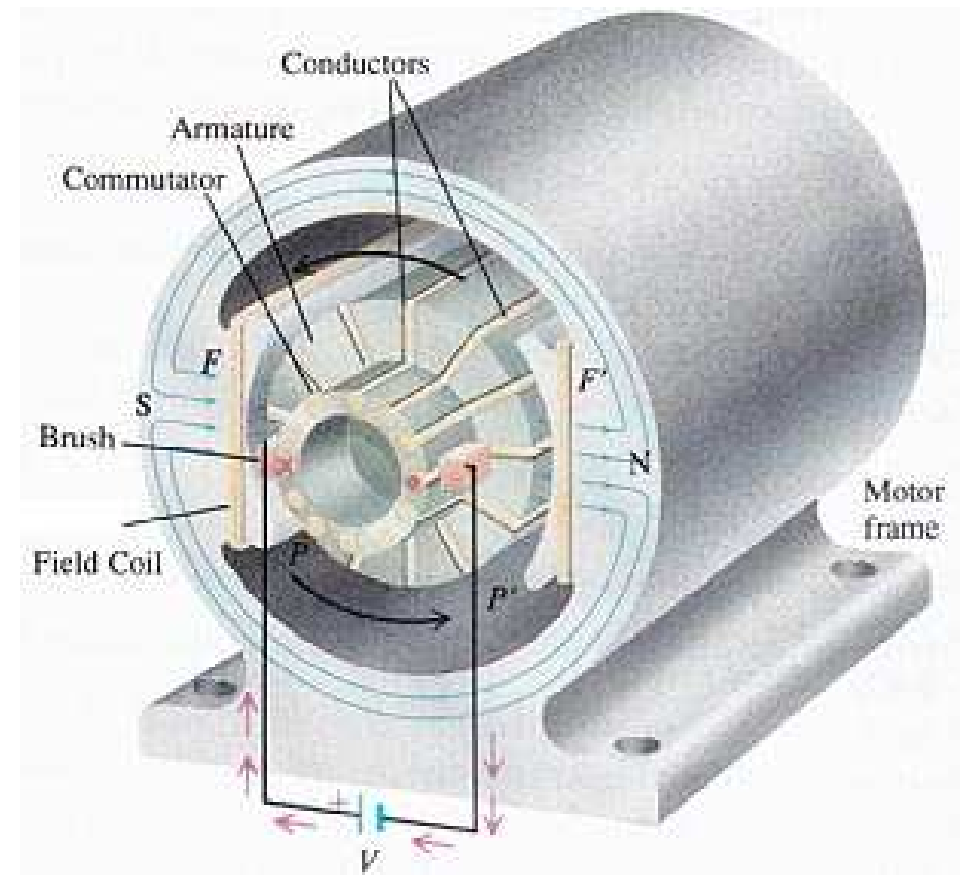
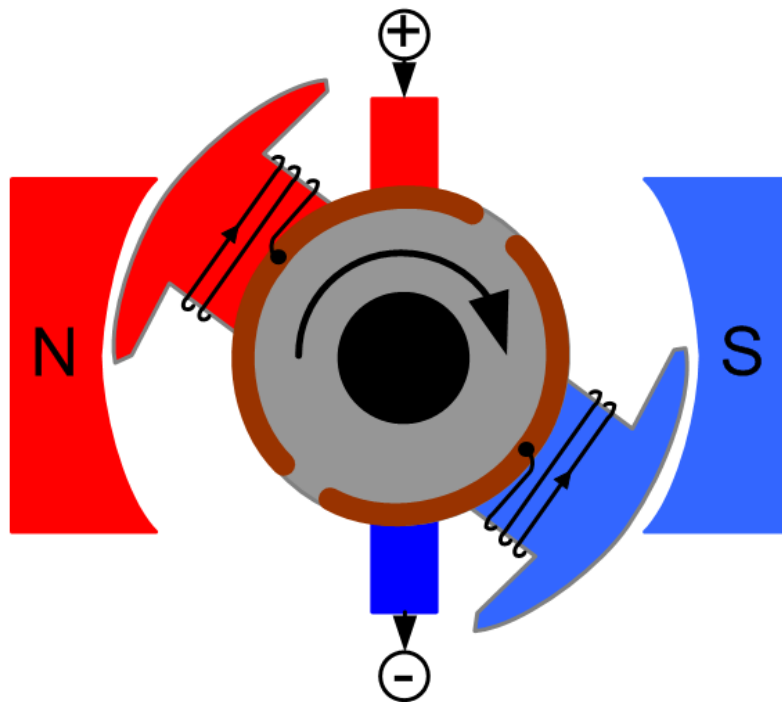
Torque on Current Loop



- magnetic field: \vec{B} (horizontal)
- area of loop: $A = ab$
- unit vector \perp to plane of loop: \hat{n}
- right-hand rule: \hat{n} points up.
- forces on sides a : $F = IaB$ (vertical)
- forces on sides b : $F = IbB$ (horizontal, not shown)
- torque: $\tau = Fb \sin \theta = IAB \sin \theta$
- magnetic moment: $\vec{\mu} = IA\hat{n}$
- torque (vector): $\vec{\tau} = \vec{\mu} \times \vec{B}$



Direct-Current Motor

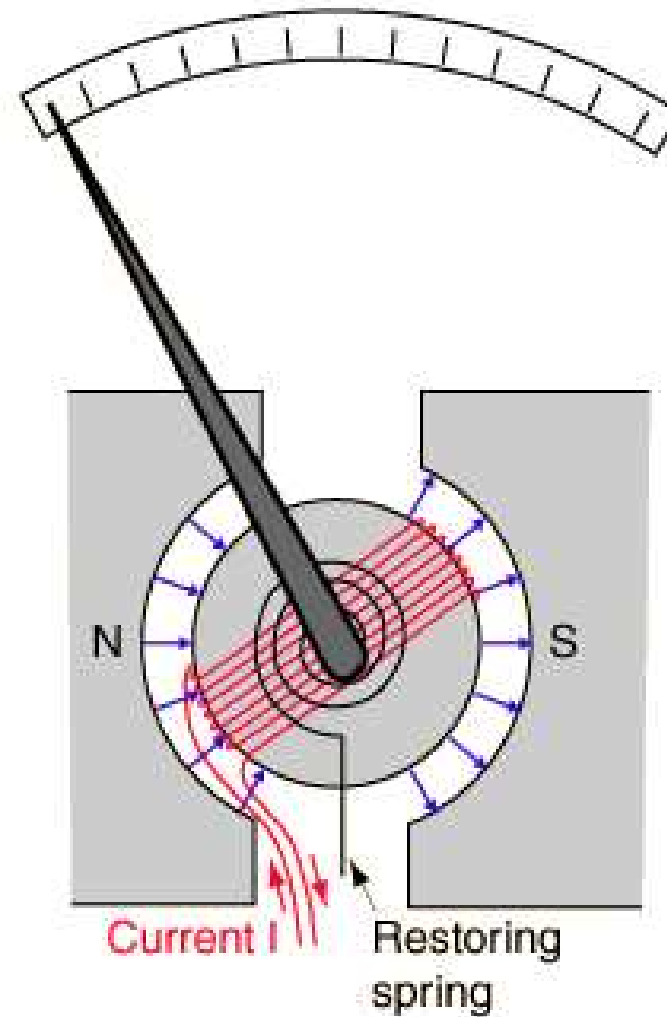


Galvanometer



Measuring direct currents.

- magnetic moment $\vec{\mu}$ (along needle)
- magnetic field \vec{B} (toward right)
- torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ (into plane)



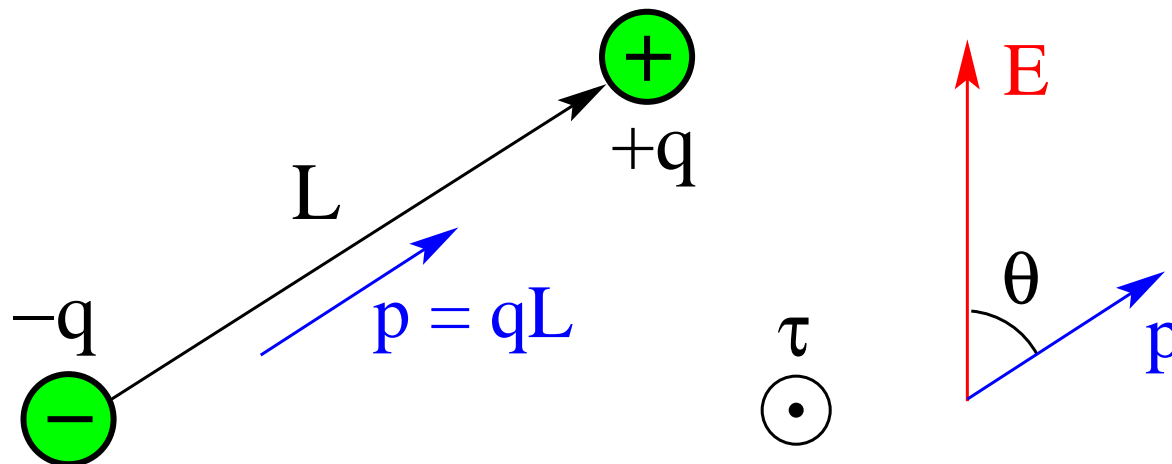
Electric Dipole in Uniform Electric Field



- Electric dipole moment: $\vec{p} = q\vec{L}$
- Torque exerted by electric field: $\vec{\tau} = \vec{p} \times \vec{E}$
- Potential energy: $U = -\vec{p} \cdot \vec{E}$

$$U(\theta) = - \int_{\pi/2}^{\theta} \tau(\theta) d\theta = pE \int_{\pi/2}^{\theta} \sin \theta d\theta = -pE \cos \theta$$

Note: $\tau(\theta)$ and $d\theta$ have opposite sign.



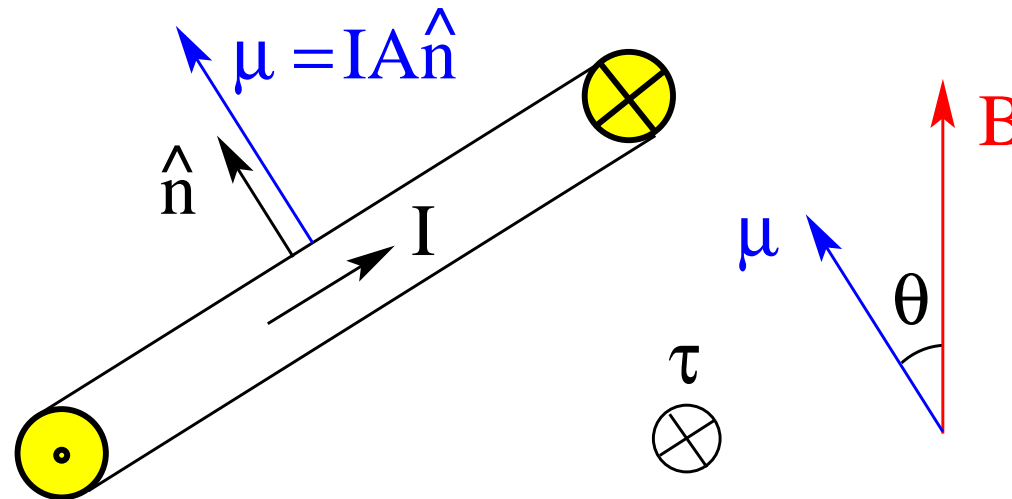
Magnetic Dipole in Uniform Magnetic Field



- Magnetic dipole moment: $\vec{\mu} = IA\hat{n}$
- Torque exerted by magnetic field: $\vec{\tau} = \vec{\mu} \times \vec{B}$
- Potential energy: $U = -\vec{\mu} \cdot \vec{B}$

$$U(\theta) = - \int_{\pi/2}^{\theta} \tau(\theta) d\theta = \mu B \int_{\pi/2}^{\theta} \sin \theta d\theta = -\mu B \cos \theta$$

Note: $\tau(\theta)$ and $d\theta$ have opposite sign.

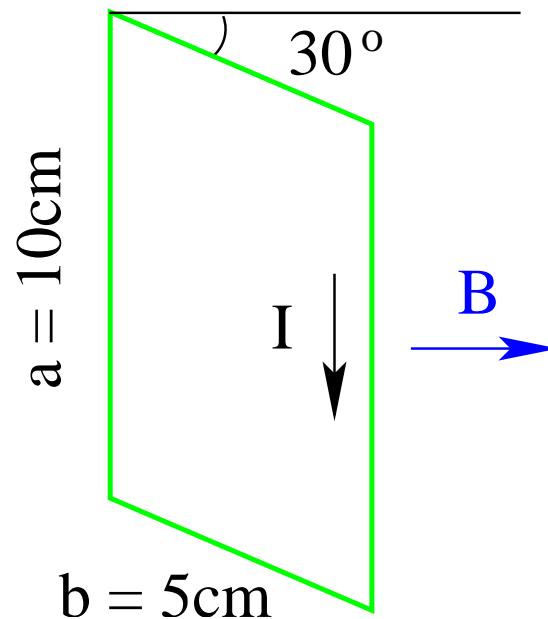


Magnetic Force Application (7)



The rectangular 20-turn loop of wire is 10cm high and 5cm wide. It carries a current $I = 0.1\text{A}$ and is hinged along one long side. It is mounted with its plane at an angle of 30° to the direction of a uniform magnetic field of magnitude $B = 0.50\text{T}$.

- Calculate the magnetic moment μ of the loop.
- Calculate the torque τ acting on the loop about the hinge line.



Magnetic Force Application (4)

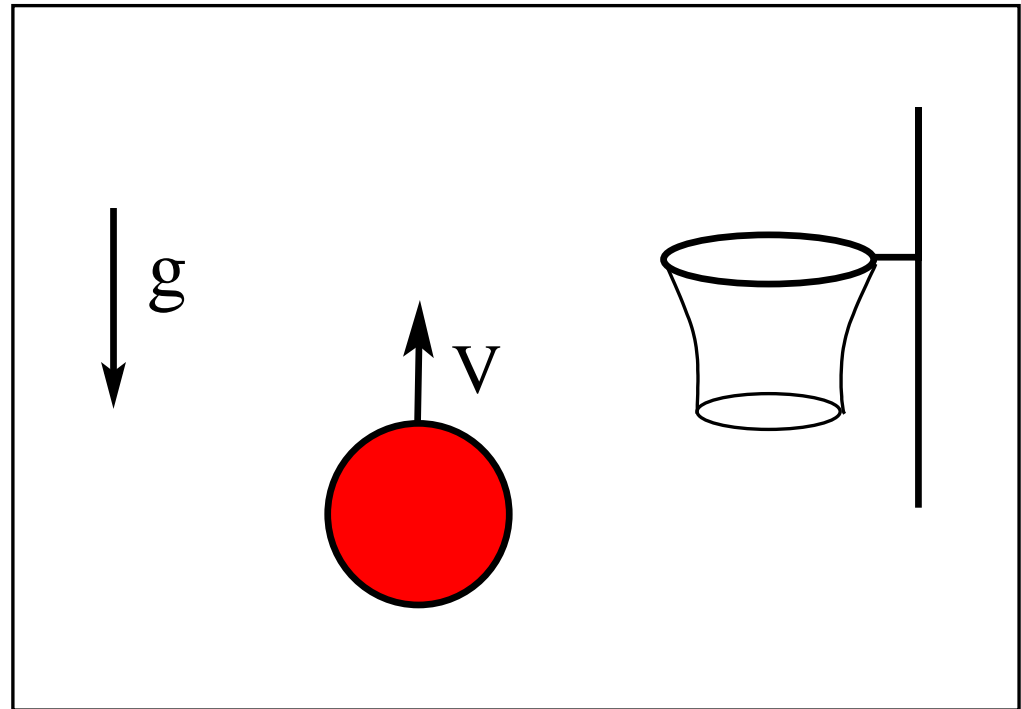


A negatively charged basketball is thrown vertically up against the gravitational field \vec{g} .

Which direction of

- (a) a uniform electric field \vec{E} ,
- (b) a uniform magnetic field \vec{B}

will give the ball a chance
to find its way into the basket?
(up/down/left/right/back/front)

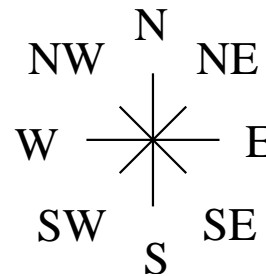
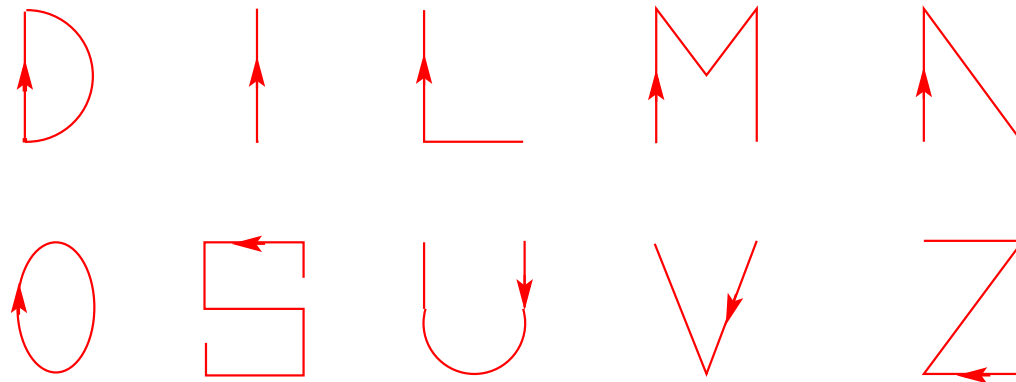


Magnetic Force Application (6)



An electric current flows through each of the letter-shaped wires in a region of uniform magnetic field pointing into the plane.

- Find the direction of the resultant magnetic force on each letter.



Magnetic Force Application (10)

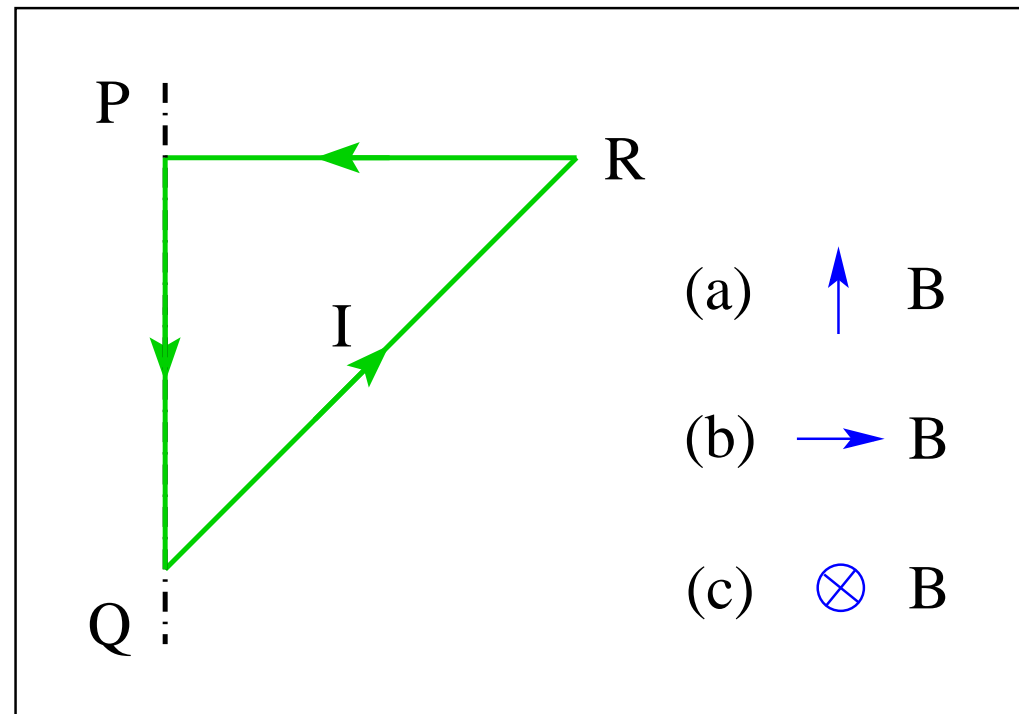


A triangular current loop is free to rotate around the vertical axis PQ .

If a uniform magnetic field \vec{B} is switched on, will the corner R of the triangle start to move out of the plane, into the plane, or will it not move at all?

Find the answer for a field \vec{B} pointing

- (a) up,
- (b) to the right,
- (c) into the plane.

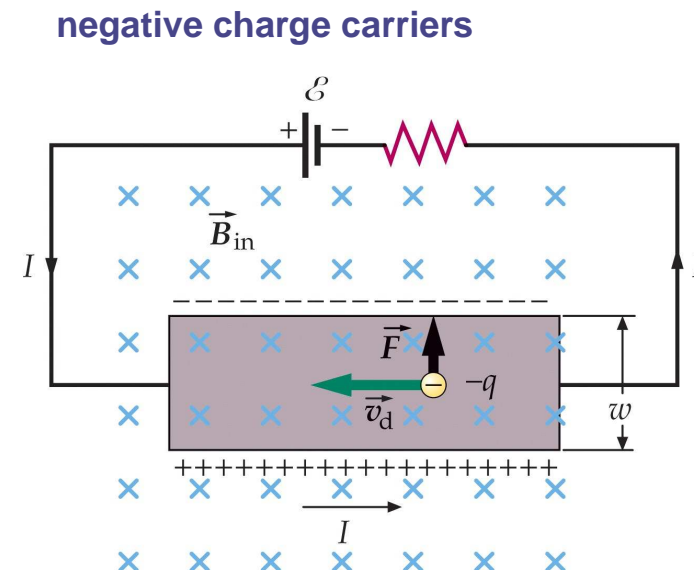
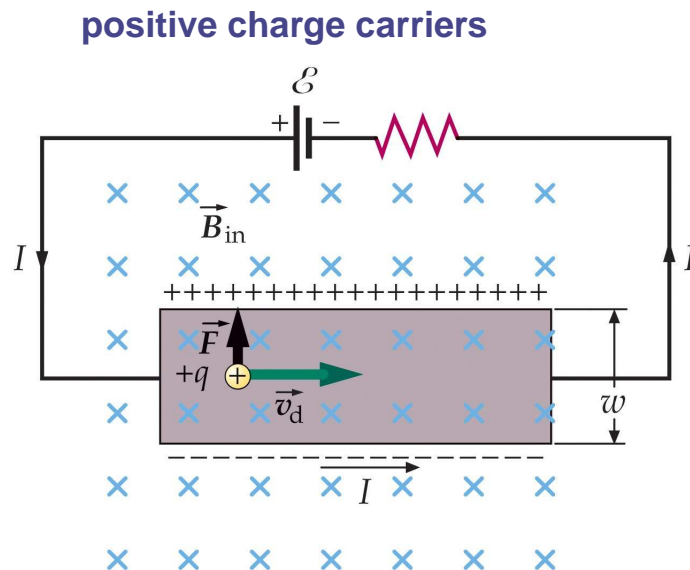


Hall Effect



Method for determining whether charge carriers are positively or negatively charged.

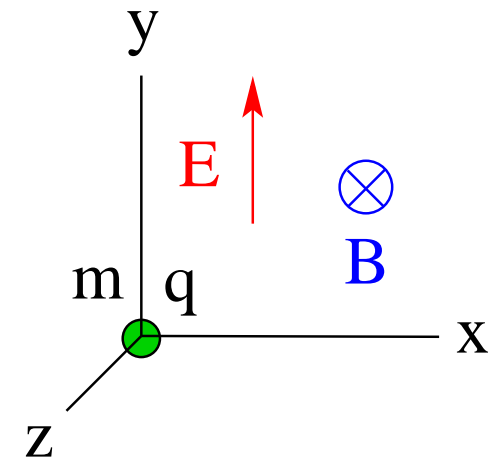
- Magnetic field \vec{B} pulls charge carriers to one side of conducting strip.
- Accumulation of charge carriers on that side and depletion on opposite side produce transverse electric field \vec{E} .
- Transverse forces on charge carrier: $F_E = qE$ and $F_B = qv_d B$.
- In steady state forces are balanced: $\vec{F}_E = -\vec{F}_B$.
- Hall voltage in steady state: $V_H = Ew = v_d Bw$.



Charged Particle in Crossed Electric and Magnetic Fields (1)



- Release particle from rest.
- Force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
- (1) $F_x = m \frac{dv_x}{dt} = -qv_y B \Rightarrow \frac{dv_x}{dt} = -\frac{qB}{m} v_y$
- (2) $F_y = m \frac{dv_y}{dt} = qv_x B + qE \Rightarrow \frac{dv_y}{dt} = \frac{qB}{m} v_x + \frac{qE}{m}$
- Ansatz: $v_x(t) = w_x \cos(\omega_0 t) + u_x$, $v_y(t) = w_y \sin(\omega_0 t) + u_y$
- Substitute ansatz into (1) and (2) to find $w_x, w_y, u_x, u_y, \omega_0$.
- (1) $-\omega_0 w_x \sin(\omega_0 t) = -\frac{qB}{m} w_y \sin(\omega_0 t) - \frac{qB}{m} u_y$
- (2) $\omega_0 w_y \cos(\omega_0 t) = \frac{qB}{m} w_x \cos(\omega_0 t) + \frac{qB}{m} u_x + \frac{qE}{m}$
- $\Rightarrow u_y = 0, \quad u_x = -\frac{E}{B}, \quad \omega_0 = \frac{qB}{m}, \quad w_x = w_y \equiv w$
- Initial condition: $v_x(0) = v_y(0) = 0 \Rightarrow w = \frac{E}{B}$



Charged Particle in Crossed Electric and Magnetic Fields (2)



- Solution for velocity of particle:

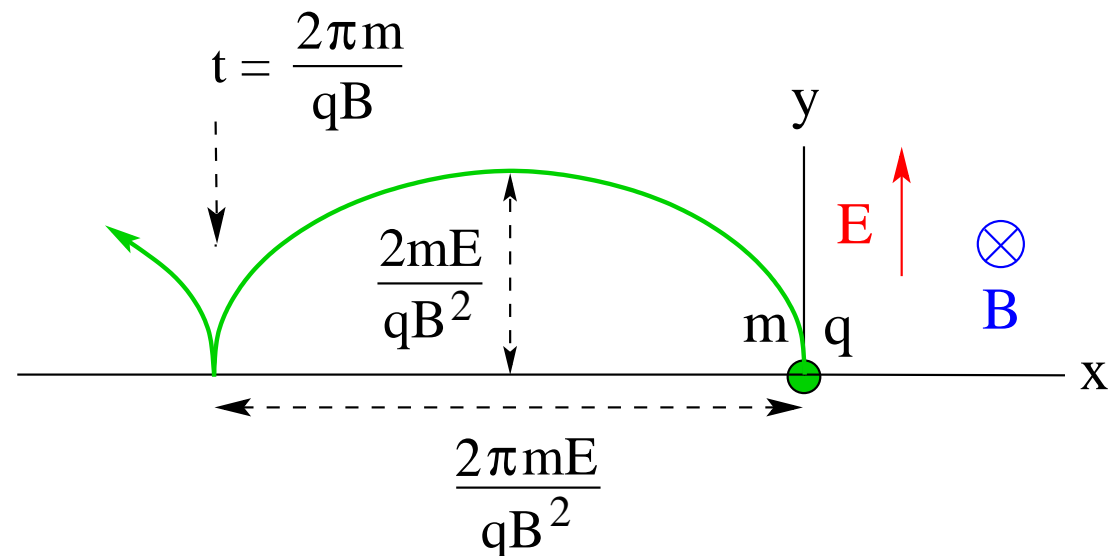
$$v_x(t) = \frac{E}{B} \left[\cos \left(\frac{qBt}{m} \right) - 1 \right], \quad v_y(t) = \frac{E}{B} \sin \left(\frac{qBt}{m} \right)$$

- Solution for position of particle:

$$x(t) = \frac{E}{B} \int_0^t \left[\cos \left(\frac{qBt}{m} \right) - 1 \right] dt = \frac{Em}{qB^2} \sin \left(\frac{qBt}{m} \right) - \frac{Et}{B}$$

$$y(t) = \frac{E}{B} \int_0^t \sin \left(\frac{qBt}{m} \right) dt = \frac{Em}{qB^2} \left[1 - \cos \left(\frac{qBt}{m} \right) \right]$$

- Path of particle in (x, y) -plane: cycloid

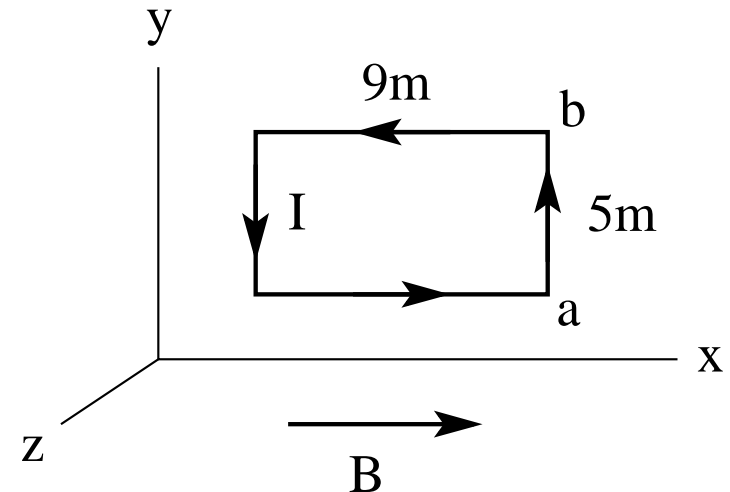


Intermediate Exam III: Problem #1 (Spring '07)



Consider a rectangular conducting loop in the xy -plane with a counterclockwise current $I = 7\text{A}$ in a uniform magnetic field $\vec{B} = 3\text{T}\hat{i}$.

- (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
- (b) Find the force \vec{F} (magnitude and direction) acting on the side ab of the rectangle.
- (c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.

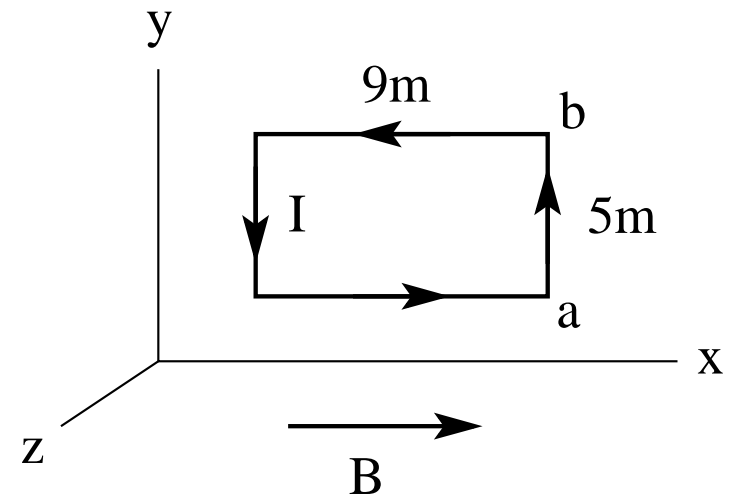


Intermediate Exam III: Problem #1 (Spring '07)



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Solution:

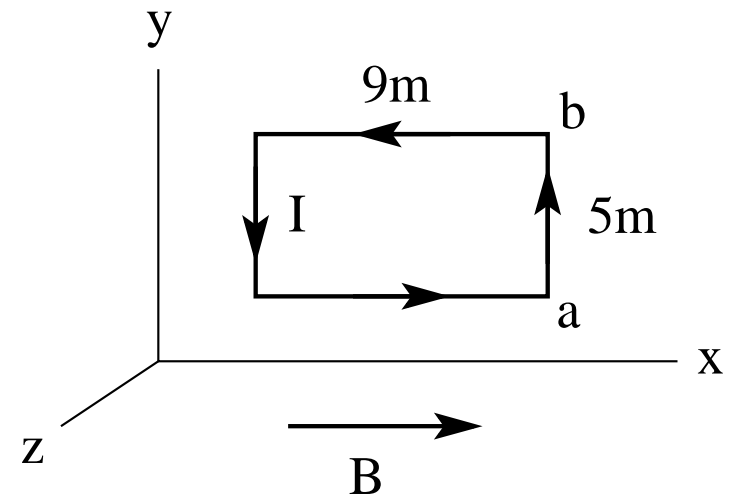
(a) $\vec{\mu} = (7\text{A})(45\text{m}^2)\hat{k} = 315\text{Am}^2\hat{k}.$

Intermediate Exam III: Problem #1 (Spring '07)



Consider a rectangular conducting loop in the xy -plane with a counterclockwise current $I = 7\text{A}$ in a uniform magnetic field $\vec{B} = 3\text{T}\hat{i}$.

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Solution:

(a) $\vec{\mu} = (7\text{A})(45\text{m}^2)\hat{k} = 315\text{Am}^2\hat{k}.$

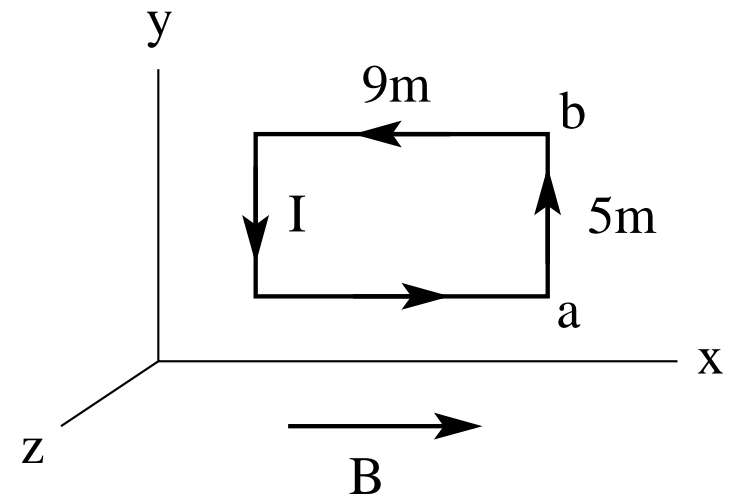
(b) $\vec{F} = I\vec{L} \times \vec{B} = (7\text{A})(5\text{m}\hat{j}) \times (3\text{T}\hat{i}) = -105\text{N}\hat{k}.$

Intermediate Exam III: Problem #1 (Spring '07)



Consider a rectangular conducting loop in the xy -plane with a counterclockwise current $I = 7\text{A}$ in a uniform magnetic field $\vec{B} = 3\text{T}\hat{i}$.

- (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
- (b) Find the force \vec{F} (magnitude and direction) acting on the side ab of the rectangle.
- (c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



Solution:

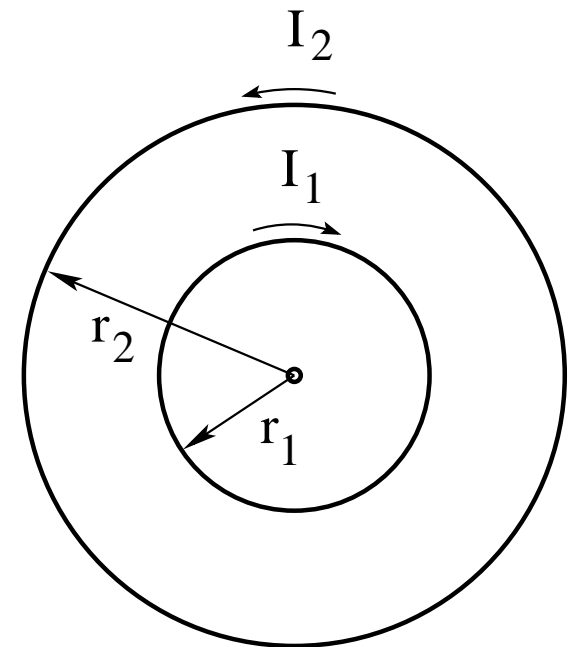
- (a) $\vec{\mu} = (7\text{A})(45\text{m}^2)\hat{k} = 315\text{Am}^2\hat{k}$.
- (b) $\vec{F} = I\vec{L} \times \vec{B} = (7\text{A})(5\text{m}\hat{j}) \times (3\text{T}\hat{i}) = -105\text{N}\hat{k}$.
- (c) $\vec{\tau} = \vec{\mu} \times \vec{B} = (315\text{Am}^2\hat{k}) \times (3\text{T}\hat{i}) = 945\text{Nm}\hat{j}$

Unit Exam III: Problem #1 (Spring '08)



Consider two circular currents $I_1 = 3\text{A}$ at radius $r_1 = 2\text{m}$ and $I_2 = 5\text{A}$ at radius $r_2 = 4\text{m}$ in the directions shown.

- (a) Find magnitude B and direction (\odot , \otimes) of the resultant magnetic field at the center.
- (b) Find magnitude μ and direction (\odot , \otimes) of the magnetic dipole moment generated by the two currents.



Unit Exam III: Problem #1 (Spring '08)

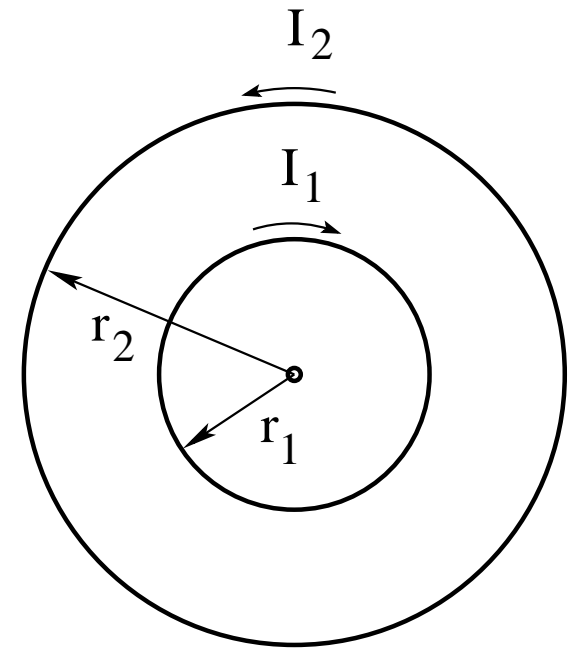


Consider two circular currents $I_1 = 3\text{A}$ at radius $r_1 = 2\text{m}$ and $I_2 = 5\text{A}$ at radius $r_2 = 4\text{m}$ in the directions shown.

- (a) Find magnitude B and direction (\odot , \otimes) of the resultant magnetic field at the center.
(b) Find magnitude μ and direction (\odot , \otimes) of the magnetic dipole moment generated by the two currents.

Solution:

$$\begin{aligned} \text{(a)} \quad B &= \frac{\mu_0(3\text{A})}{2(2\text{m})} - \frac{\mu_0(5\text{A})}{2(4\text{m})} = (9.42 - 7.85) \times 10^{-7}\text{T} \\ \Rightarrow B &= 1.57 \times 10^{-7}\text{T} \quad \otimes \end{aligned}$$



Unit Exam III: Problem #1 (Spring '08)



Consider two circular currents $I_1 = 3\text{A}$ at radius $r_1 = 2\text{m}$ and $I_2 = 5\text{A}$ at radius $r_2 = 4\text{m}$ in the directions shown.

- (a) Find magnitude B and direction (\odot , \otimes) of the resultant magnetic field at the center.
(b) Find magnitude μ and direction (\odot , \otimes) of the magnetic dipole moment generated by the two currents.

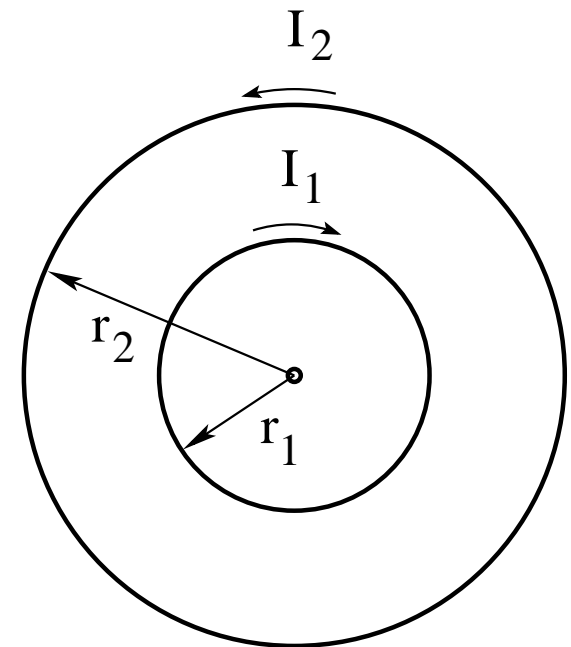
Solution:

$$(a) \quad B = \frac{\mu_0(3\text{A})}{2(2\text{m})} - \frac{\mu_0(5\text{A})}{2(4\text{m})} = (9.42 - 7.85) \times 10^{-7}\text{T}$$

$$\Rightarrow B = 1.57 \times 10^{-7}\text{T} \quad \otimes$$

$$(b) \quad \mu = \pi(4\text{m})^2(5\text{A}) - \pi(2\text{m})^2(3\text{A}) = (251 - 38)\text{Am}^2$$

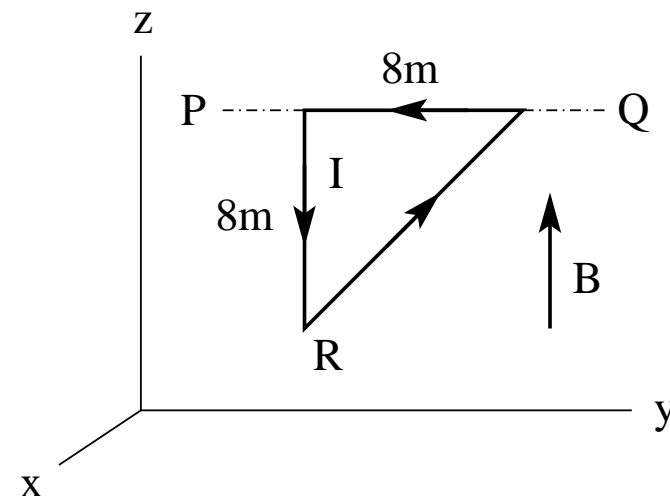
$$\Rightarrow \mu = 213\text{Am}^2 \quad \odot$$



Unit Exam III: Problem #1 (Spring '09)



- A triangular conducting loop in the yz -plane with a counterclockwise current $I = 3\text{A}$ is free to rotate about the axis PQ . A uniform magnetic field $\vec{B} = 0.5\text{T}\hat{k}$ is present. (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the triangle. (b) Find the magnetic torque $\vec{\tau}$ (magnitude and direction) acting on the triangle. (c) Find the magnetic force \vec{F}_H (magnitude and direction) acting on the long side (hypotenuse) of the triangle. (d) Find the force \vec{F}_R (magnitude and direction) that must be applied to the corner R to keep the triangle from rotating.



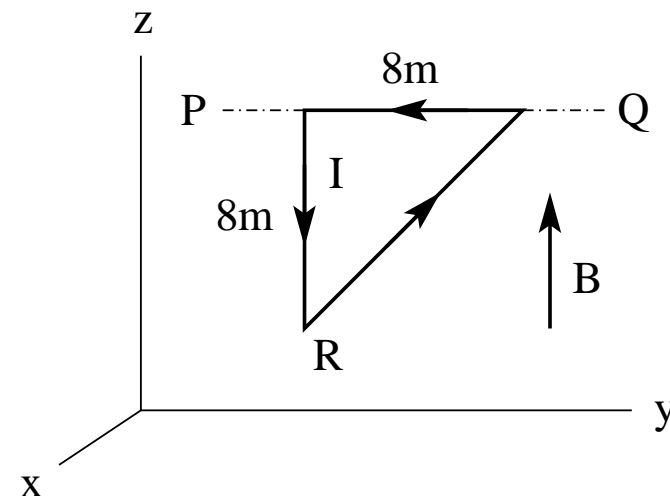
Unit Exam III: Problem #1 (Spring '09)



- A triangular conducting loop in the yz -plane with a counterclockwise current $I = 3\text{A}$ is free to rotate about the axis PQ . A uniform magnetic field $\vec{B} = 0.5\text{T}\hat{k}$ is present. (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the triangle. (b) Find the magnetic torque $\vec{\tau}$ (magnitude and direction) acting on the triangle. (c) Find the magnetic force \vec{F}_H (magnitude and direction) acting on the long side (hypotenuse) of the triangle. (d) Find the force \vec{F}_R (magnitude and direction) that must be applied to the corner R to keep the triangle from rotating.

Solution:

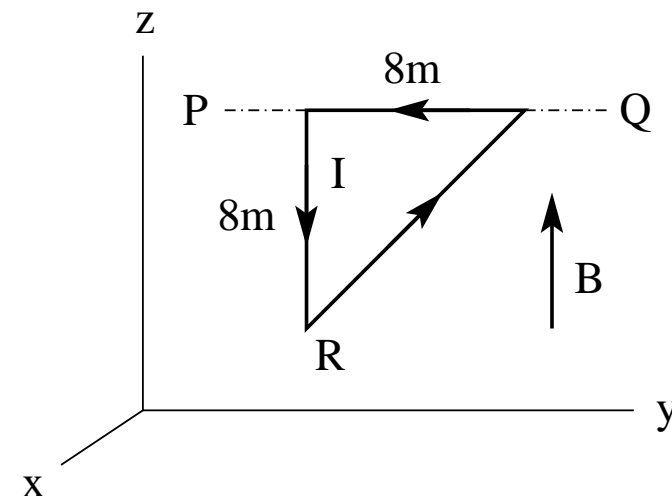
(a) $\vec{\mu} = (3\text{A})(32\text{m}^2)\hat{i} = 96\text{Am}^2\hat{i}.$



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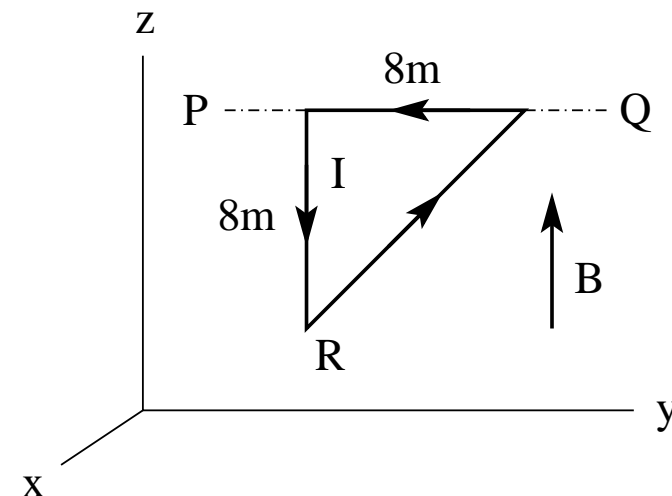
(a) $\vec{\mu} = (3\text{A})(32\text{m}^2)\hat{i} = 96\text{Am}^2\hat{i}.$

(b) $\vec{\tau} = \vec{\mu} \times \vec{B} = (96\text{Am}^2\hat{i}) \times (0.5\text{T}\hat{k}) = -48\text{Nm}\hat{j}.$

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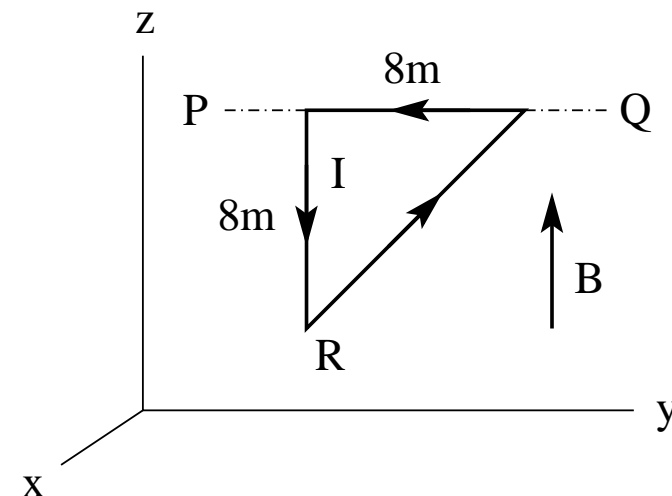
Solution:

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(c) $F_H = (3\text{A})(8\sqrt{2}\text{m})(0.5\text{T})(\sin 45^\circ) = 12\text{N} \quad \odot$.

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Solution:

- (a) $\vec{\mu} = (3\text{A})(32\text{m}^2)\hat{i} = 96\text{Am}^2\hat{i}$.
- (b) $\vec{\tau} = \vec{\mu} \times \vec{B} = (96\text{Am}^2\hat{i}) \times (0.5\text{T}\hat{k}) = -48\text{Nm}\hat{j}$.
- (c) $F_H = (3\text{A})(8\sqrt{2}\text{m})(0.5\text{T})(\sin 45^\circ) = 12\text{N} \quad \odot$.
- (d) $(-8\text{m}\hat{k}) \times \vec{F}_R = -\vec{\tau} = 48\text{Nm}\hat{j} \Rightarrow \vec{F}_R = -6\text{N}\hat{i}$.