11-19-2015

12. Magnetic Field I

Gerhard Müller

University of Rhode Island, gmuller@uri.edu

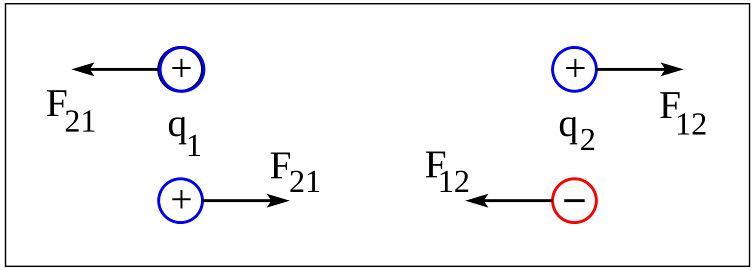
Abstract

Lecture slides 12 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

Some of the slides contain figures from the textbook, Paul A. Tipler and Gene Mosca. *Physics for Scientists and Engineers*, 5th/6th editions. The copyright to these figures is owned by W.H. Freeman. We acknowledge permission from W.H. Freeman to use them on this course web page. The textbook figures are not to be used or copied for any purpose outside this class without direct permission from W.H. Freeman.

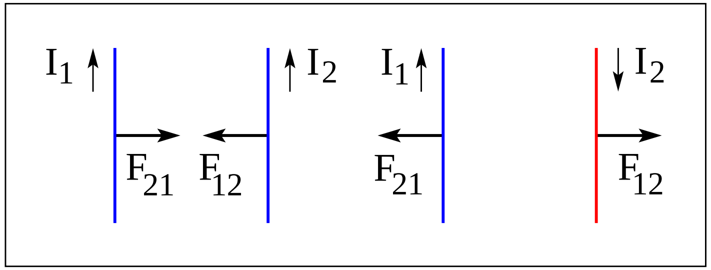
Electricity

- Electric charges generate an electric field.
- The electric field exerts a force on other electric charges.



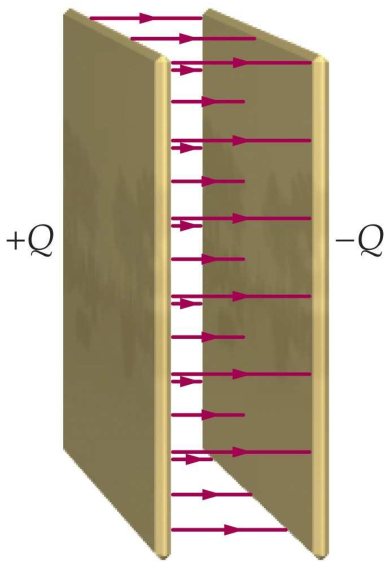
Magnetism

- Electric currents generate a magnetic field.
- The magnetic field exerts force on other electric currents.



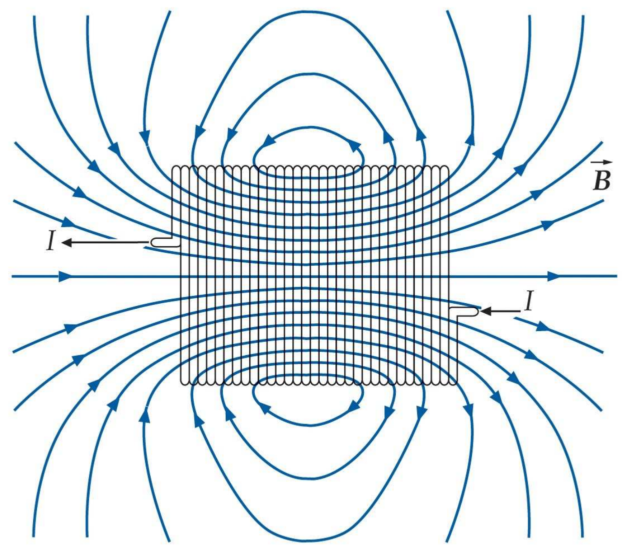
Capacitor

The parallel-plate capacitor generates a near uniform electric field provided the linear dimensions of the plates are large compared to the distance between them.



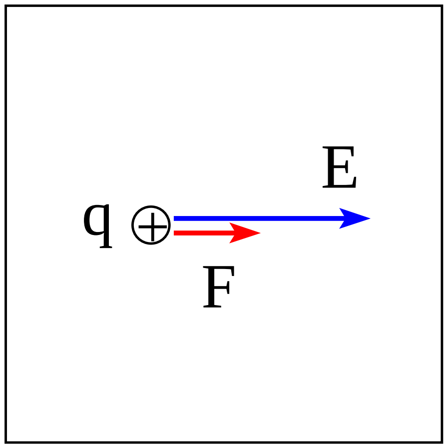
Solenoid

The solenoid (a tightly wound cylindrical coil) generates a near uniform magnetic field provided the length of the coil is large compared to its radius.



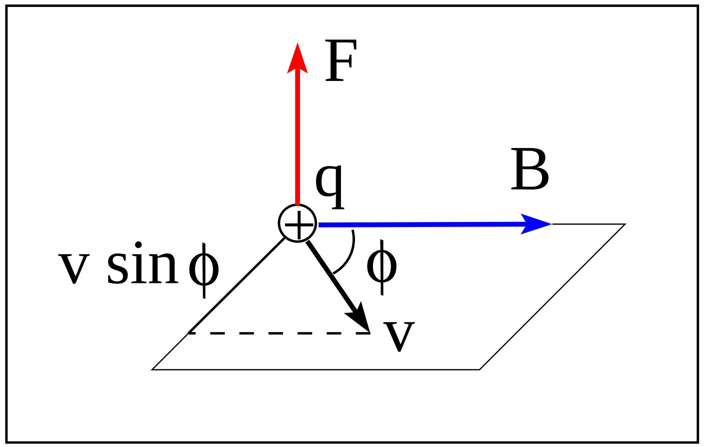
Electric Force

- $\vec{F} = q\vec{E}$
- electric force is parallel to electric field
- SI unit of E: 1N/C=1V/m



Magnetic Force

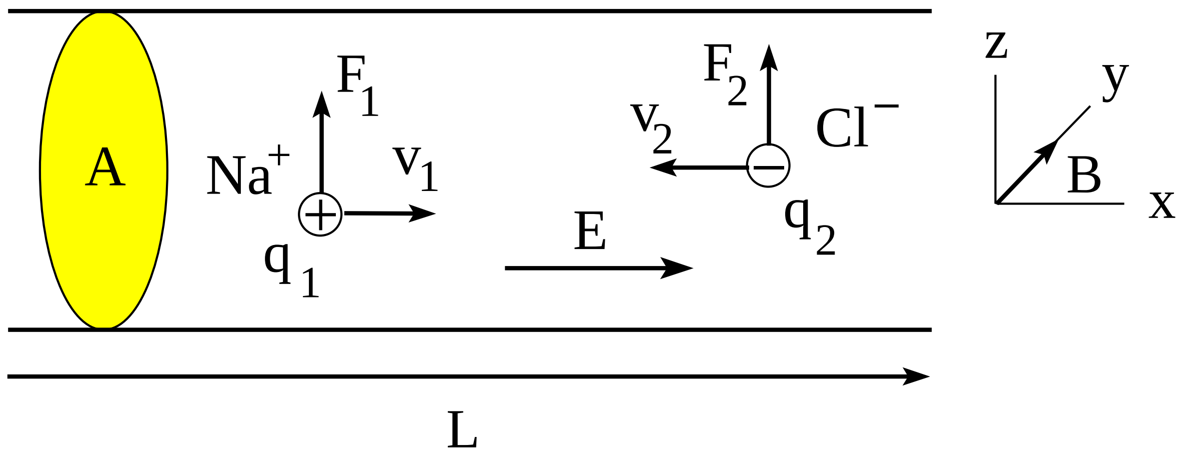
- $\vec{F} = q\vec{v} \times \vec{B}, \quad F = qvB\sin\phi$
- magnetic force is perpendicular to magnetic field
- SI unit of B: 1Ns/Cm=1T (Tesla)
- 1T=10^4G (Gauss)



Magnetic Force on Current-Carrying Conductor

Consider drift of Na$^+$ and Cl$^-$ ions in a plastic pipe filled with salt water.

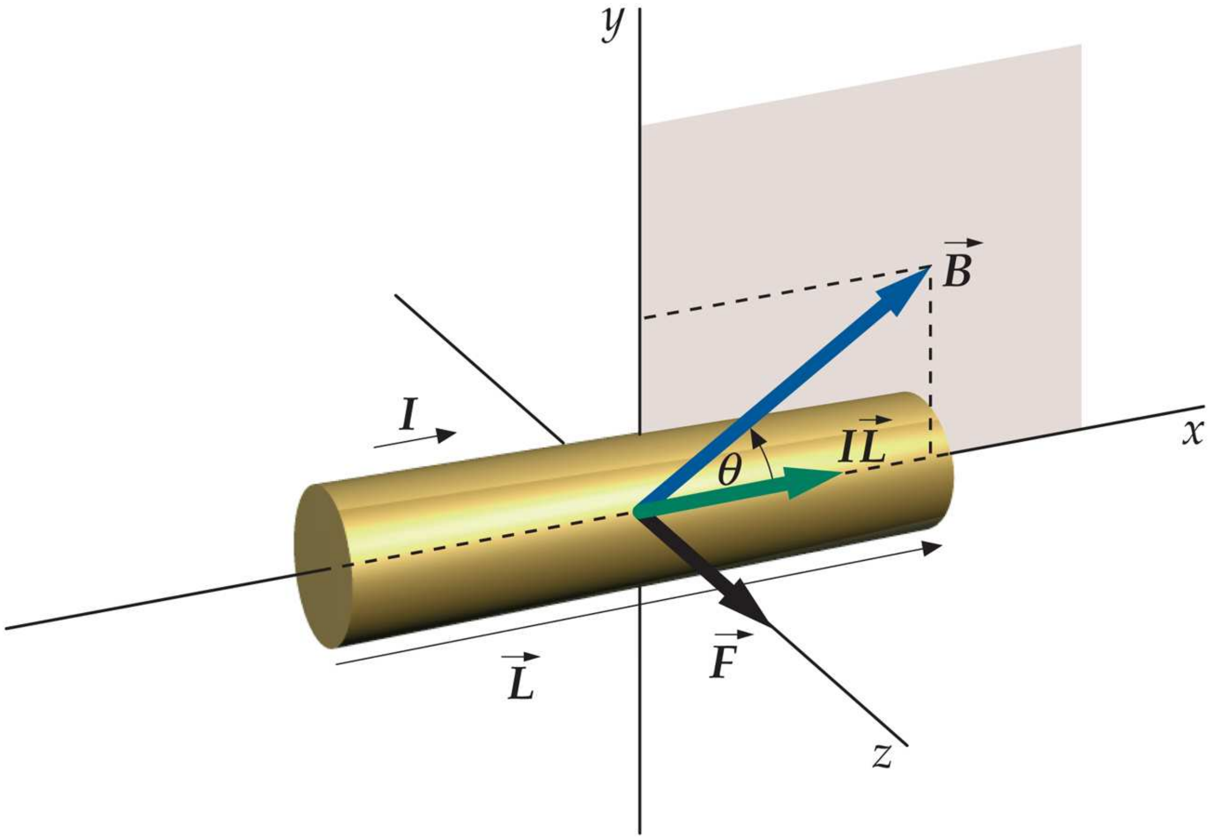
- $v_{1x} > 0,\ v_{2x} < 0$: drift velocities; $q_1 > 0,\ q_2 < 0$: charge on ions
- $n_1,\ n_2$: number of charge carriers per unit volume



- Electric current through A: $I = A(n_1 q_1 v_{1x} + n_2 q_2 v_{2x})$
- Force on Na$^+$: $\vec{F}_1 = q_1 \vec{v}_1 \times \vec{B} \ \Rightarrow \ F_{1z} = q_1 v_{1x} B_y$
- Force on Cl$^-$: $\vec{F}_2 = q_2 \vec{v}_2 \times \vec{B} \ \Rightarrow \ F_{2z} = q_2 v_{2x} B_y$
- Force on current-carrying pipe: $F_z = (n_1 q_1 v_{1x} + n_2 q_2 v_{2x}) A L B_y = I L B_y$
- Vector relation: $\vec{F} = I \vec{L} \times \vec{B}$

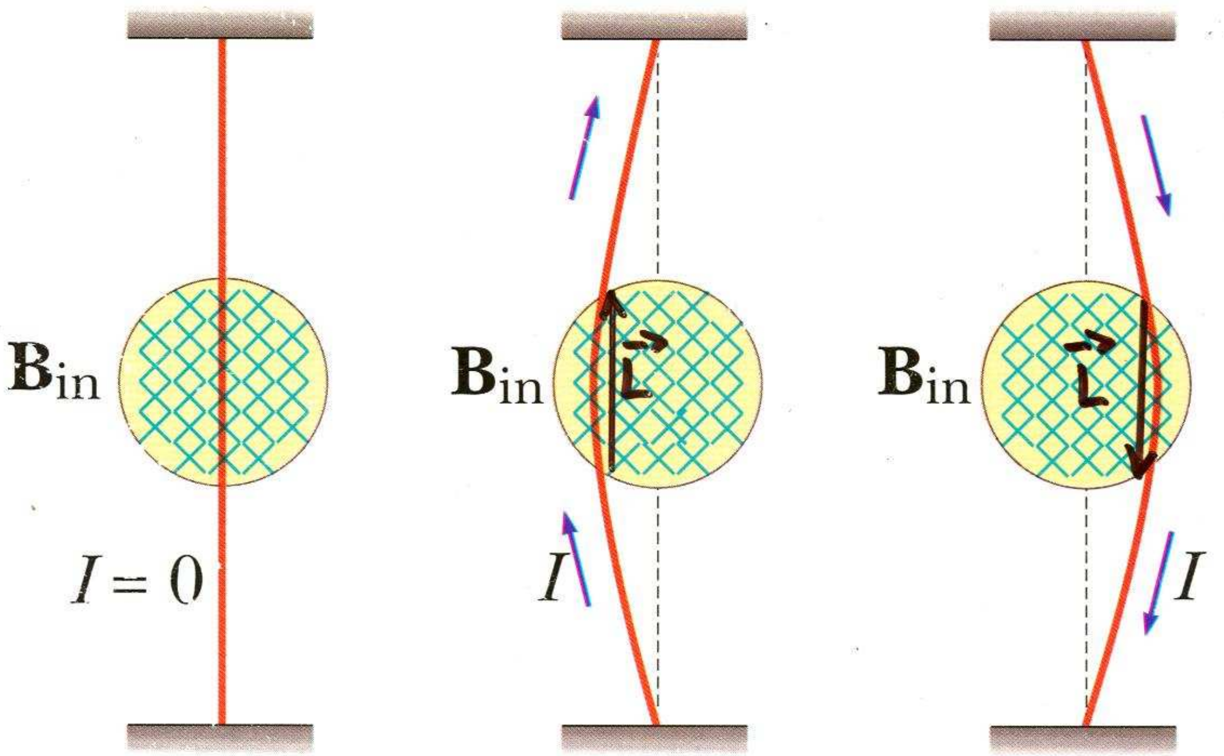
Direction of Magnetic Force

$$\vec{F} = I\vec{L} \times \vec{B}$$



Direction of Magnetic Force

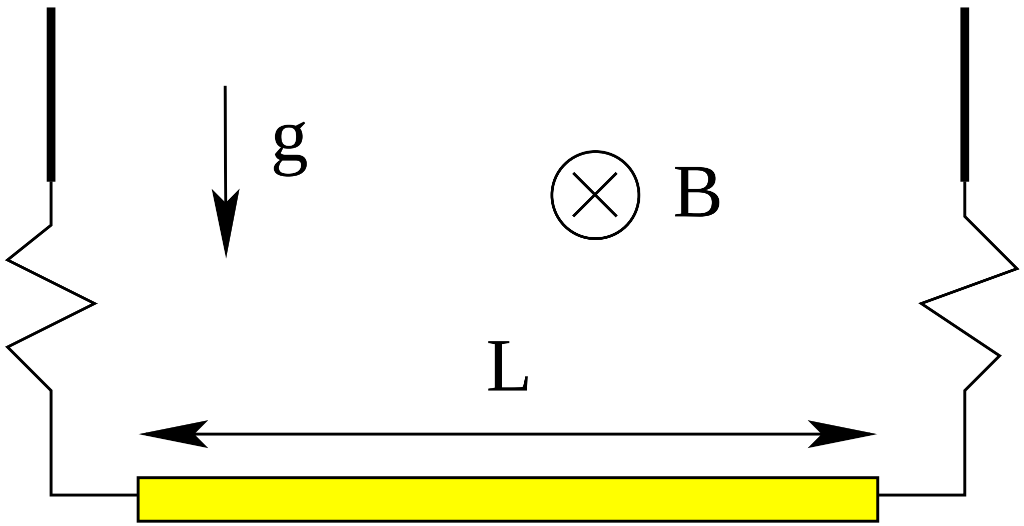
$$\vec{F} = I\vec{L} \times \vec{B}$$



Magnetic Force Application (1)

A wire of length $L = 62$cm and mass $m = 13$g is suspended by a pair of flexible leads in a uniform magnetic field $B = 0.440$T pointing in to the plane.

- What are the magnitude and direction of the current required to remove the tension in the supporting leads?



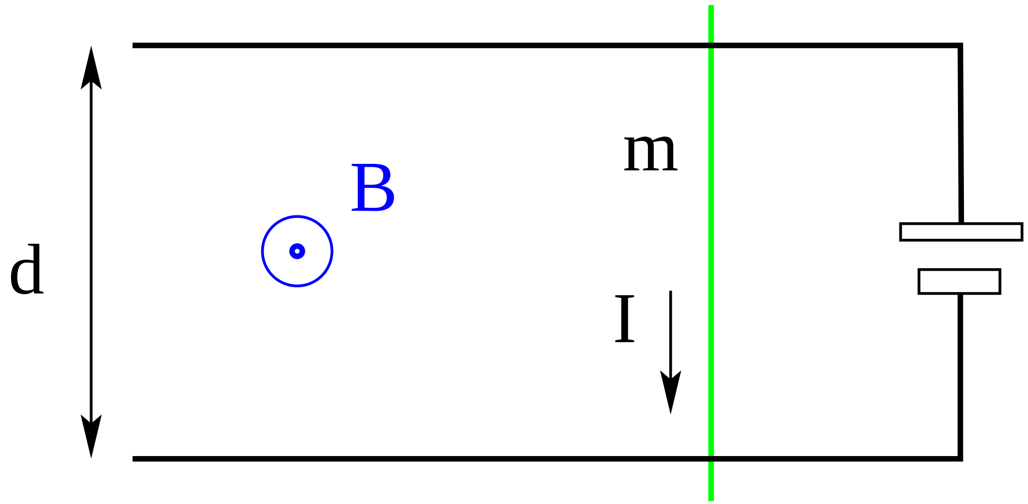
Magnetic Force Application (2)

A metal wire of mass $m = 1.5$kg slides without friction on two horizontal rails spaced a distance $d = 3$m apart.

The track lies in a vertical uniform magnetic field of magnitude $B = 24$mT pointing out of the plane.

A constant current $I = 12$A flows from a battery along one rail, across the wire, and back down the other rail. The wire starts moving from rest at $t = 0$.

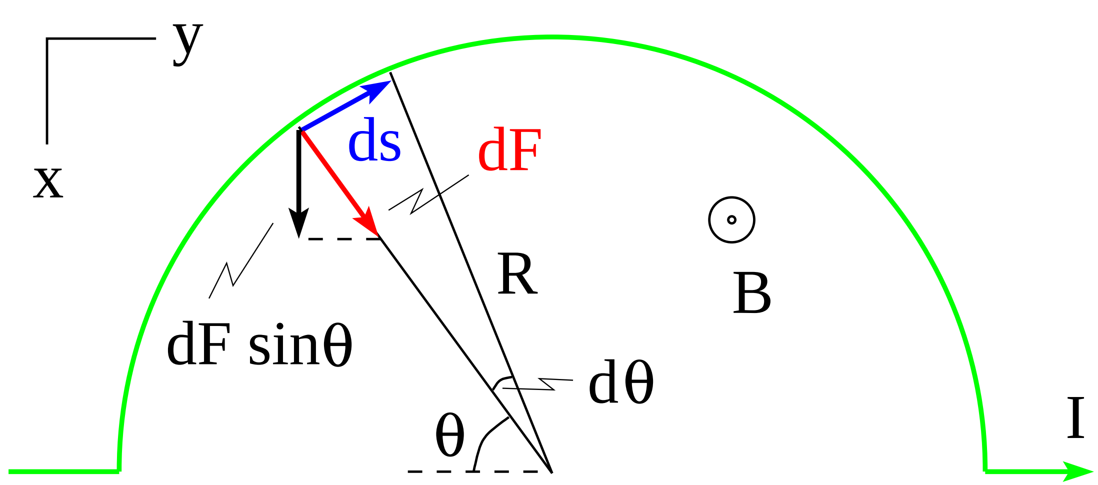
- Find the direction and magnitude of the velocity of the wire at time $t = 5$s.



Magnetic Force on Semicircular Current (1)

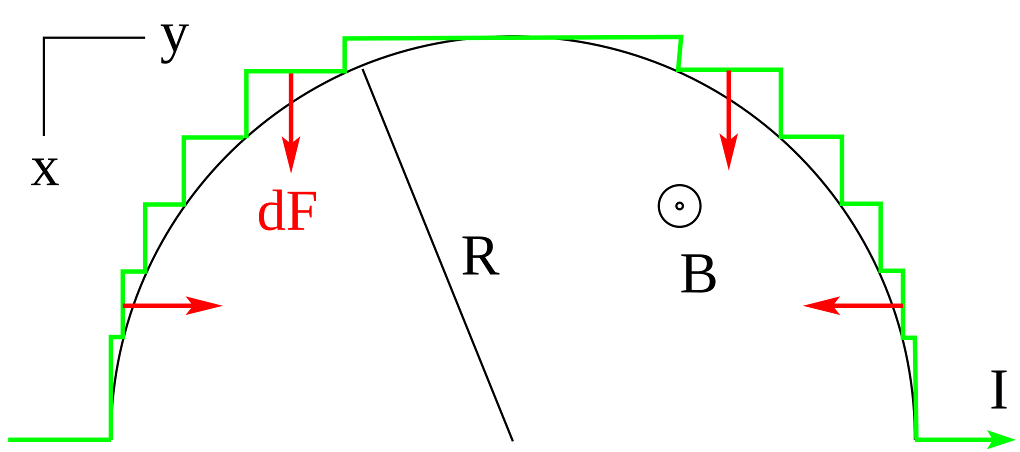
Fancy solution:

- Uniform magnetic field $\vec{B}$ points out of the plane.
- Magnetic force on segment ds: $dF = IBds = IBRd\theta$.
- Integrate $dF_x = dF \sin\theta$ and $dF_y = dF \cos\theta$ along semicircle.
- $F_x = IBR \int_0^\pi \sin\theta d\theta = 2IBR, \quad F_y = IBR \int_0^\pi \cos\theta d\theta = 0.$



Clever solution:

- Replace the semicircle by symmetric staircase of tiny wire segments.
- Half the vertical segments experience a force to the left, the other half a force to the right. The resultant horizontal force is zero.
- All horizontal segments experience a downward force. The total length is $2R$. The total downward force is $2IBR$.
- Making the segments infinitesimally small does not change the result.



Magnetic Force Application (5)

Inside the cube there is a magnetic field $\vec{B}$ directed vertically up.

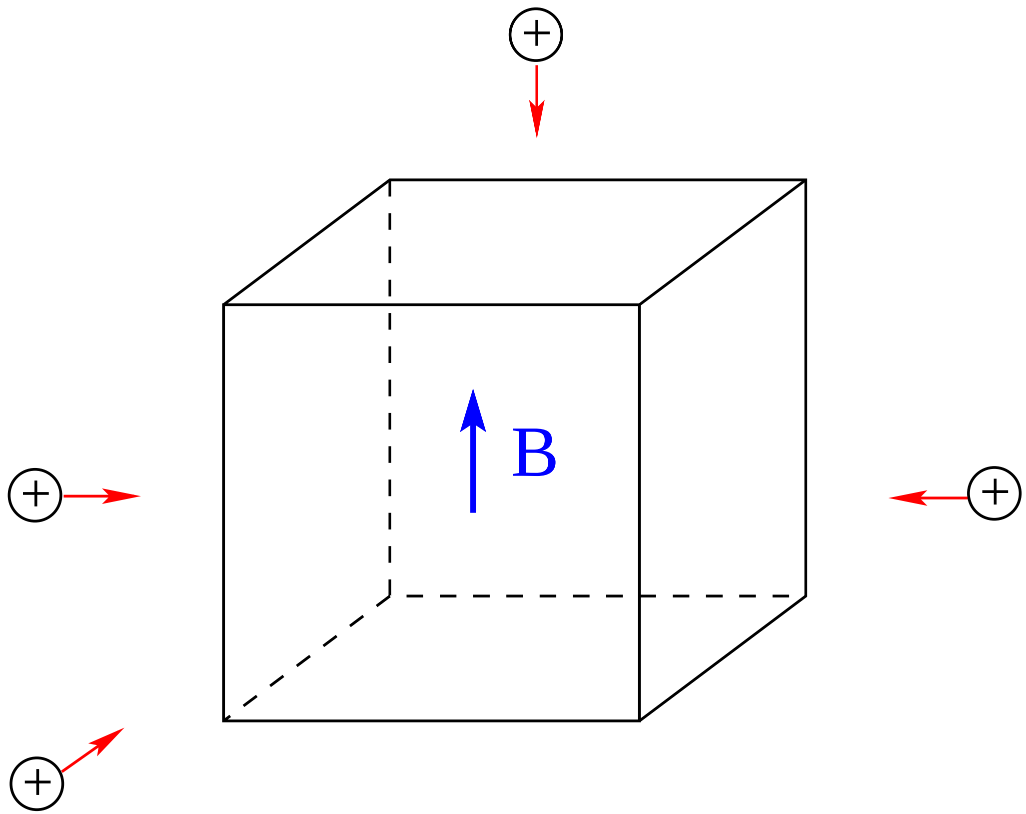
Find the direction of the magnetic force experienced by a proton entering the cube

(a) from the left,

(b) from the front,

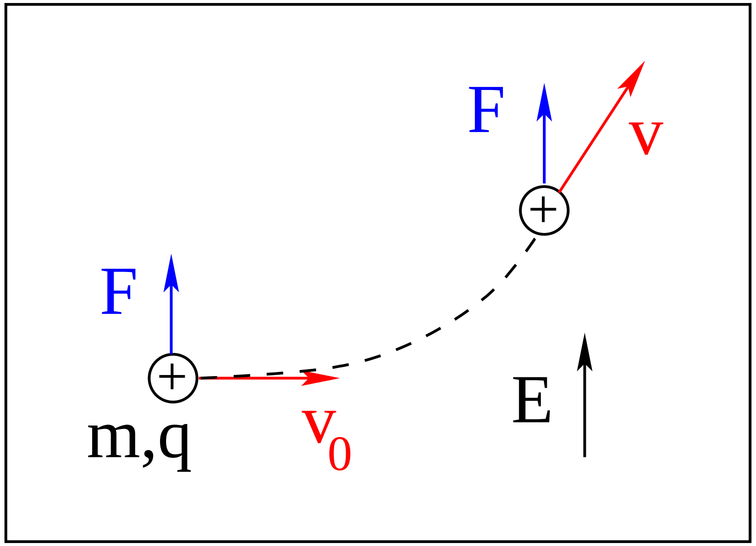
(c) from the right,

(d) from the top.



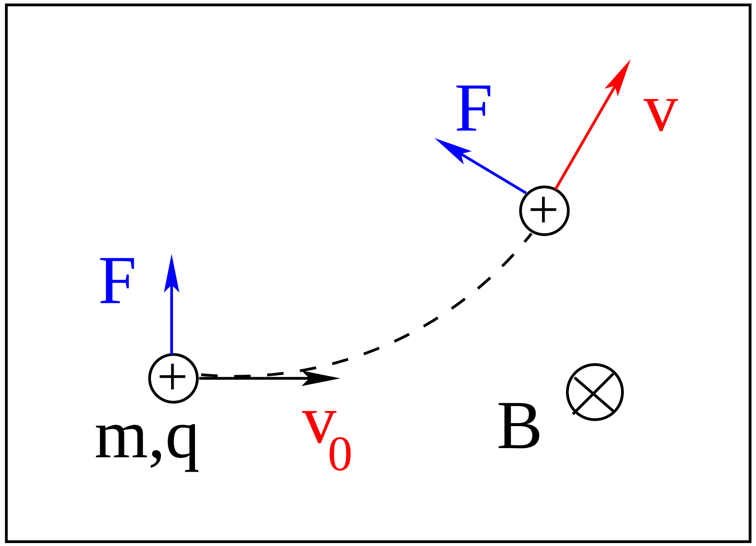
Charged Particle Moving in Uniform Electric Field

- Electric field $\vec{E}$ is directed up.
- Electric force: $\vec{F} = q\vec{E}$ (constant)
- Acceleration: $\vec{a} = \dfrac{\vec{F}}{m} = \dfrac{q}{m}\vec{E} =$ const.
- Horizontal motion: $a_x = 0 \;\Rightarrow\; v_x(t) = v_0 \;\Rightarrow\; x(t) = v_0 t$
- Vertical motion: $a_y = \dfrac{q}{m}E \;\Rightarrow\; v_y(t) = a_y t \;\Rightarrow\; y(t) = \dfrac{1}{2}a_y t^2$
- The path is parabolic: $y = \left(\dfrac{qE}{2mv_0^2}\right)x^2$
- $\vec{F}$ changes direction and magnitude of $\vec{v}$.



Charged Particle Moving in Uniform Magnetic Field

- Magnetic field $\vec{B}$ is directed into plane.
- Magnetic force: $\vec{F} = q\vec{v} \times \vec{B}$ (not constant)
- $\vec{F} \perp \vec{v} \;\Rightarrow\; \vec{F}$ changes direction of $\vec{v}$ only $\;\Rightarrow\; v = v_0$.
- $\vec{F}$ is the centripetal force of motion along circular path.
- Radius: $\dfrac{mv^2}{r} = qvB \;\Rightarrow\; r = \dfrac{mv}{qB}$
- Angular velocity: $\omega = \dfrac{v}{r} = \dfrac{qB}{m}$
- Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi m}{qB}$



Charged Particle in Circular Motion

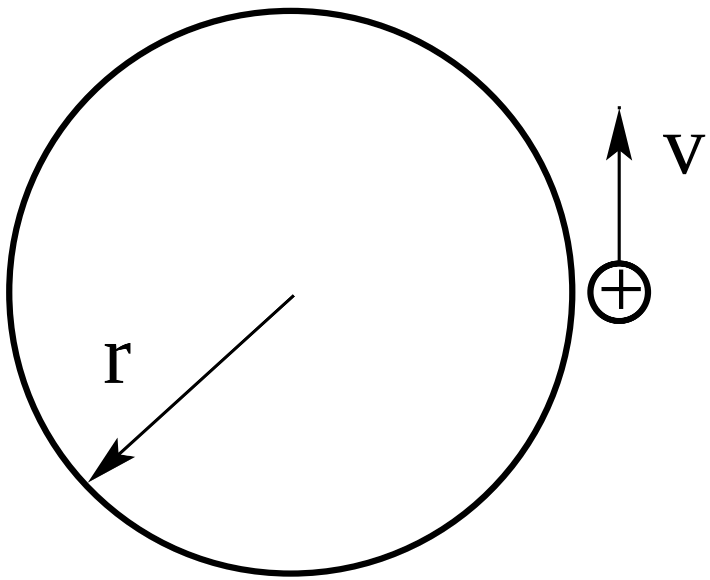
A proton with speed $v = 3.00 \times 10^5$m/s orbits just outside a charged conducting sphere of radius $r = 1.00$cm.

(a) Find the force F acting on the proton.

(b) Find the charge per unit area σ on the surface of the sphere.

(c) Find the total charge Q on the sphere.

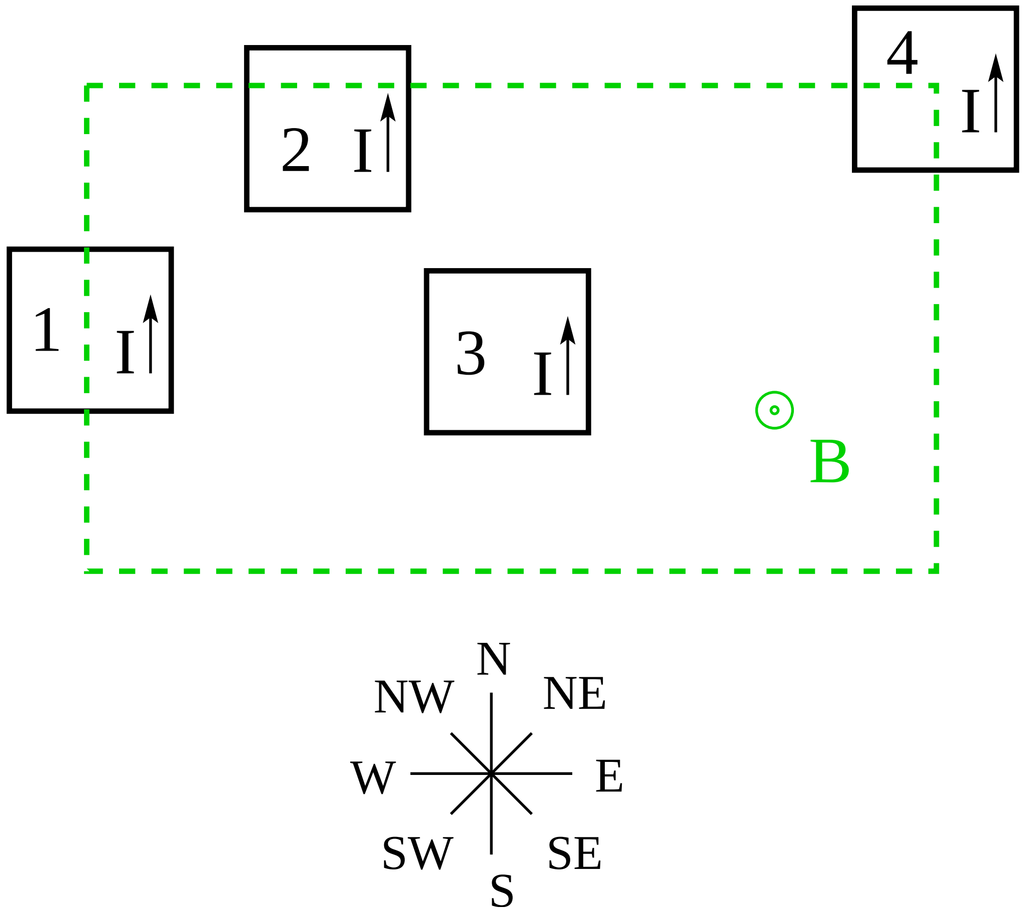
Note: Charged particles in circular motion lose energy through radiation. This effect is ignored here.



Magnetic Force Application (3)

The dashed rectangle marks a region of uniform magnetic field $\vec{B}$ pointing out of the plane.

- Find the direction of the magnetic force acting on each loop with a ccw current I.



Velocity Selector

A charged particle is moving horizontally into a region with “crossed” uniform fields:

- an electric field $\vec{E}$ pointing down,
- a magnetic field $\vec{B}$ pointing into the plane.

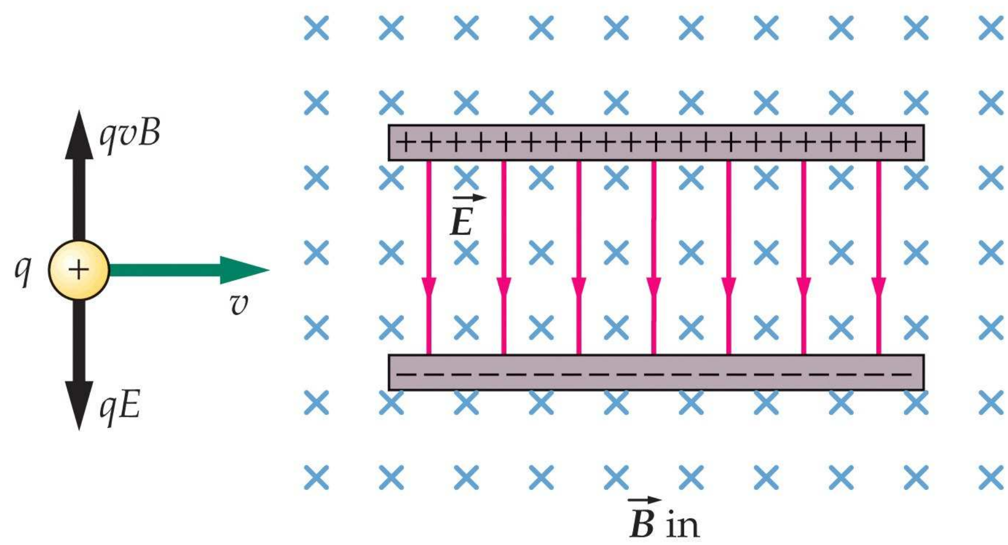
Forces experienced by particle:

- electric force $F = qE$ pointing down,
- magnetic force $B = qvB$ pointing up.

Forces in balance: $qE = qvB$.

Selected velocity: $v = \dfrac{E}{B}$.

Trajectories of particles
with selected velocity
are not bent.



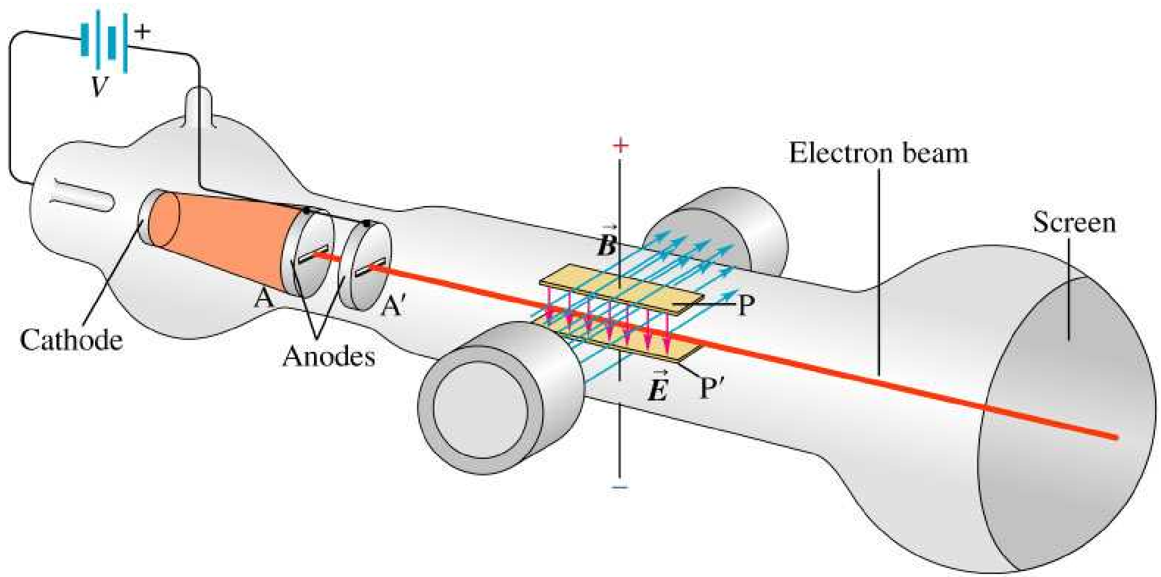
Measurement of e/m for Electron

First experiment by J. J. Thomson (1897)
Method used here: velocity selector

Equilibrium of forces: $eE = evB \quad \Rightarrow\ v = \dfrac{E}{B}$

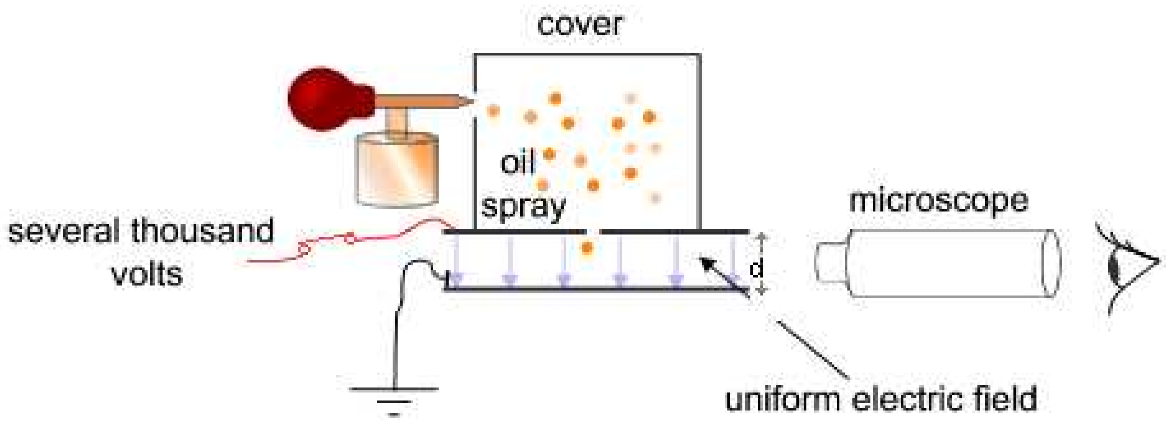
Work-energy relation: $eV = \dfrac{1}{2}mv^2 \quad \Rightarrow\ v = \sqrt{\dfrac{2eV}{m}}$

Eliminate v: $\dfrac{e}{m} = \dfrac{E^2}{2VB^2} \simeq 1.76 \times 10^{11}\mathrm{C/kg}$



Measurement of e and m for Electron

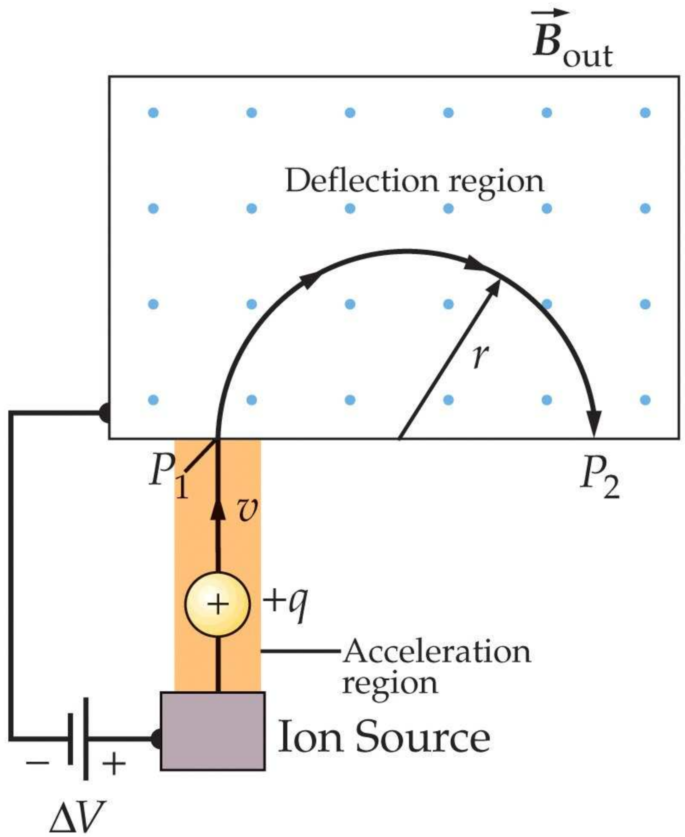
First experiment by R. Millikan (1913)
Method used here: balancing weight and electric force on oil drop
Radius of oil drop: $r = 1.64\mu\mathrm{m}$
Mass density of oil: $\rho = 0.851\mathrm{g/cm}^3$
Electric field: $E = 1.92 \times 10^5\mathrm{N/C}$
Mass of oil drop: $m = \dfrac{4\pi}{3} r^3 \rho = 1.57 \times 10^{-14}\mathrm{kg}$
Equilibrium of forces: $neE = mg$
Number of excess elementary charges (integer): $n = 5$
Elementary charge: $e = \dfrac{mg}{nE} \simeq 1.6 \times 10^{-19}\mathrm{C}$
Mass of electron: $m \simeq 9.1 \times 10^{-31}\mathrm{kg}$



Mass Spectrometer

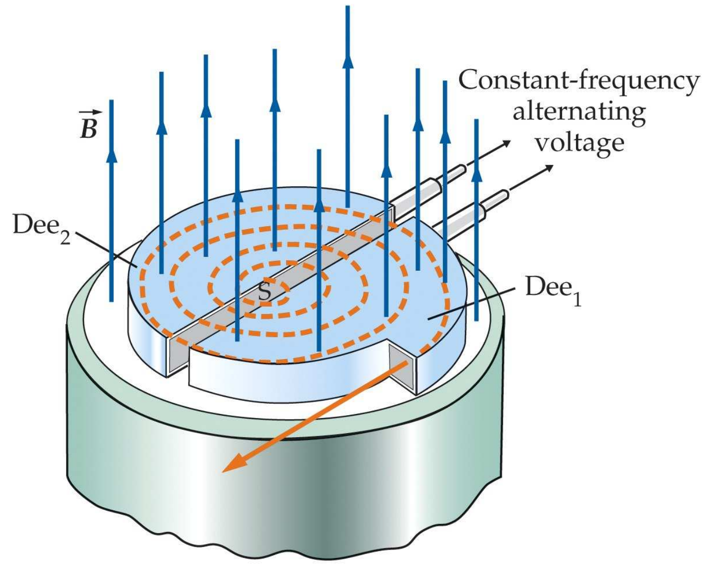
Purpose: measuring masses of ions.

- Charged particle is accelerated by moving through potential difference $|\Delta V|$.
- Trajectory is then bent into semicircle of radius r by magnetic field $\vec{B}$.
- Kinetic energy: $\frac{1}{2}mv^2 = q|\Delta V|$.
- Radius of trajectory: $r = \frac{mv}{qB}$.
- Charge: $q = e$
- Mass: $m = \frac{eB^2r^2}{2|\Delta V|}$.



Cyclotron

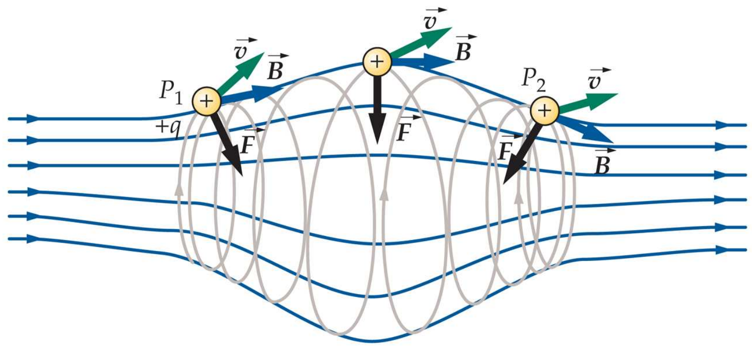
Purpose: accelerate charged particles to high energy.



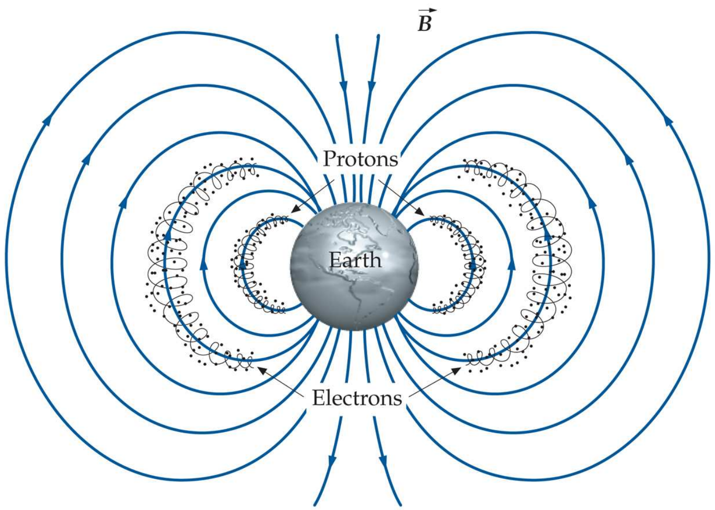
- Low-energy protons are injected at S.
- Path is bent by magnetic field $\vec{B}$.
- Proton is energized by alternating voltage ΔV between Dee_1 and Dee_2.
- Proton picks up energy $\Delta K = e\Delta V$ during each half cycle.
- Path spirals out as velocity of particle increases: Radial distance is proportional to velocity: $r = \dfrac{mv}{eB}$.
- Duration of cycle stays is independent of r or v: cyclotron period: $T = \dfrac{2\pi m}{eB}$.
- Cyclotron period is synchronized with alternation of accelerating voltage.
- High-energy protons exit at perimeter of $\vec{B}$-field region.

Magnetic Bottles

Moving charged particle
confined by
inhomogeneous magnetic field.

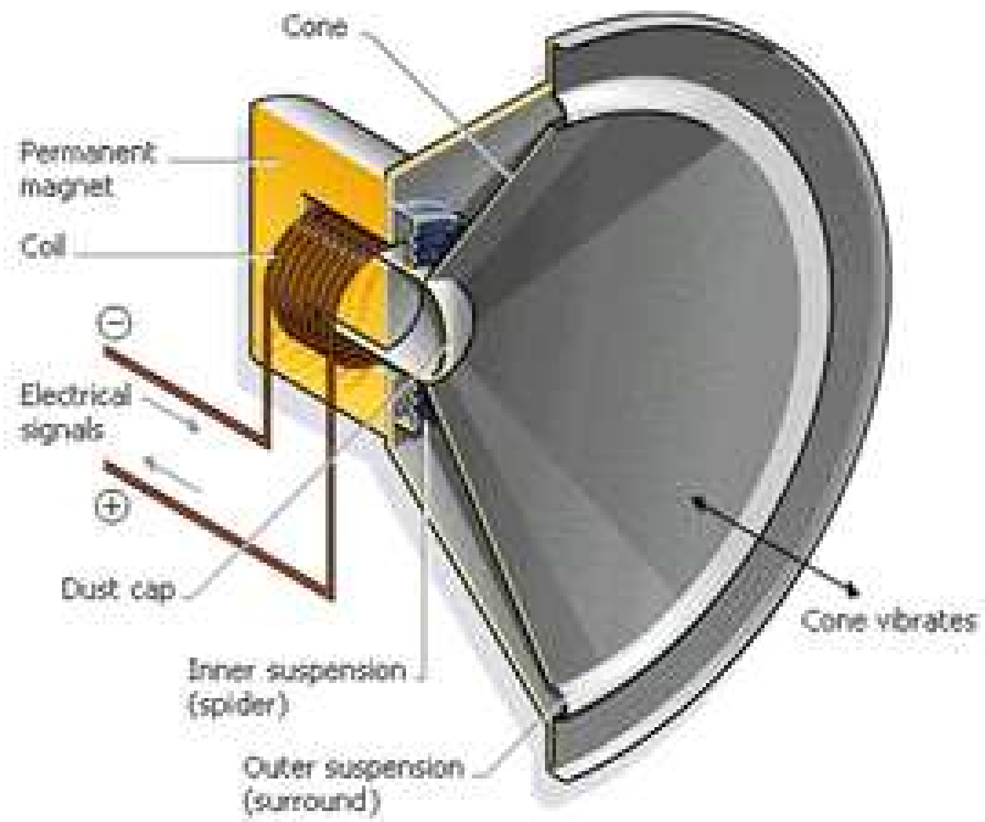


Van Allen belt:
trapped protons and electrons
in Earth's magnetic field.



Loudspeaker

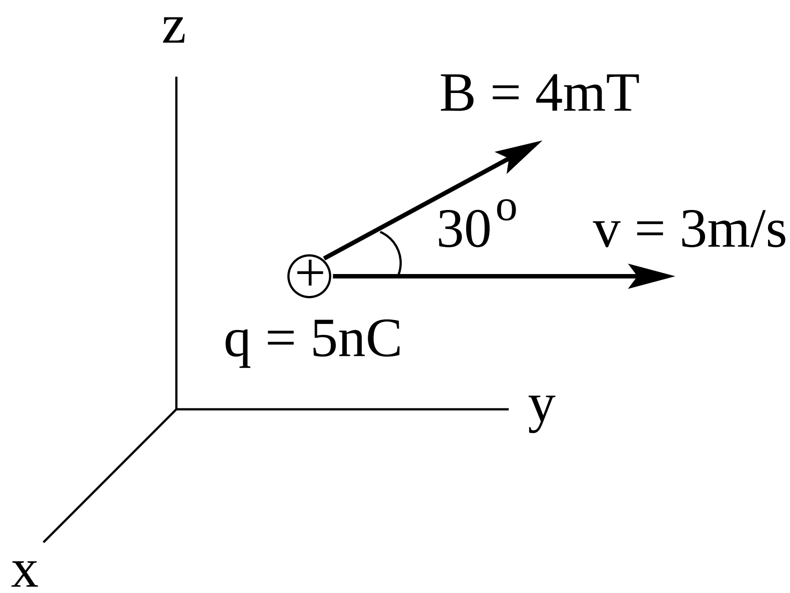
Conversion of electric signal into mechanical vibration.



Consider a charged particle moving in a uniform magnetic field as shown. The velocity is in y-direction and the magnetic field in the yz-plane at 30° from the y-direction.

(a) Find the direction of the magnetic force acting on the particle.

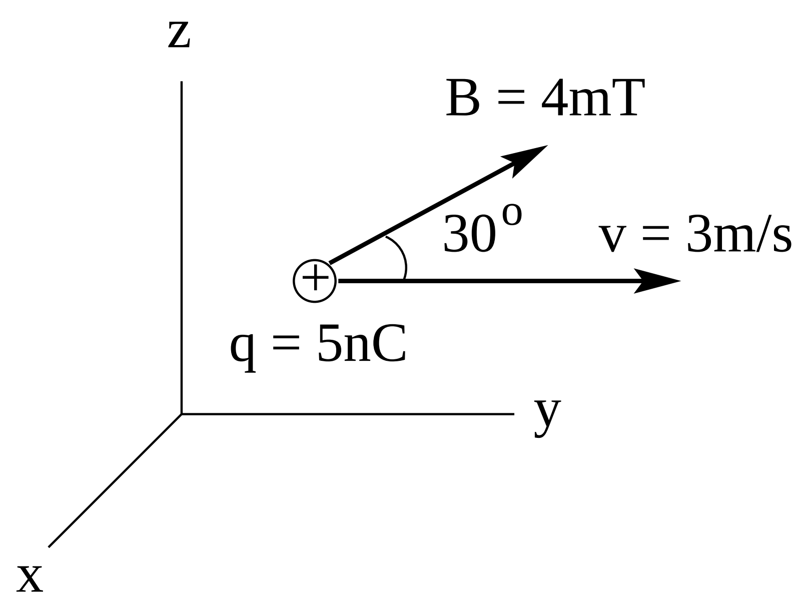
(b) Find the magnitude of the magnetic force acting on the particle.



Consider a charged particle moving in a uniform magnetic field as shown. The velocity is in y-direction and the magnetic field in the yz-plane at 30° from the y-direction.

(a) Find the direction of the magnetic force acting on the particle.

(b) Find the magnitude of the magnetic force acting on the particle.



Solution:

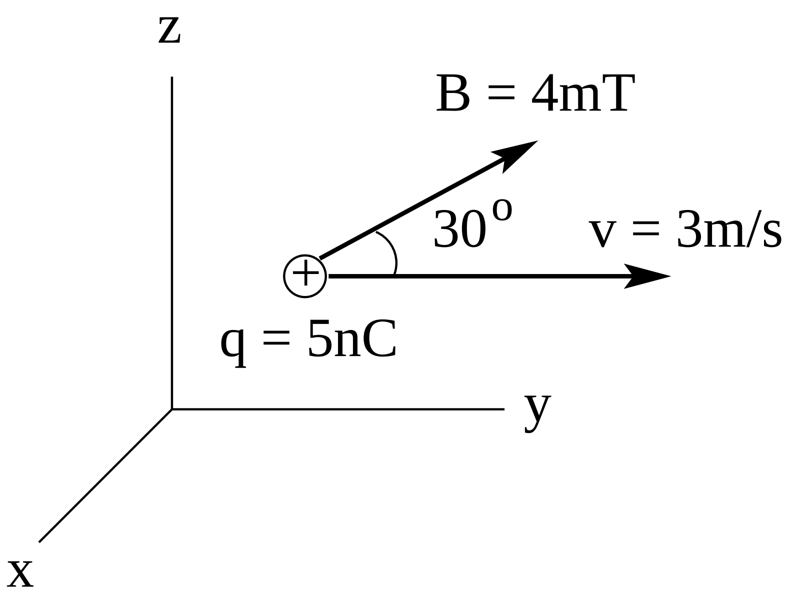
(a) Use the right-hand rule: positive x-direction (front, out of page).

Intermediate Exam II: Problem #4 (Spring '05)

Consider a charged particle moving in a uniform magnetic field as shown. The velocity is in y-direction and the magnetic field in the yz-plane at $30°$ from the y-direction.

(a) Find the direction of the magnetic force acting on the particle.

(b) Find the magnitude of the magnetic force acting on the particle.



Solution:

(a) Use the right-hand rule: positive x-direction (front, out of page).

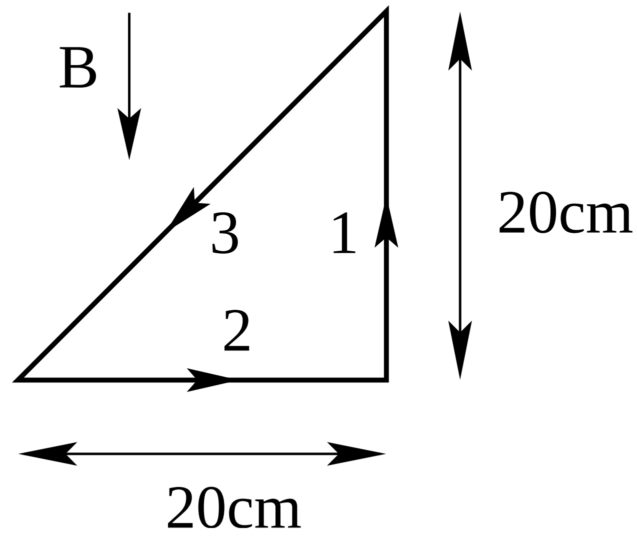
(b) $F = qvB\sin 30° = (5 \times 10^{-9}\mathrm{C})(3\mathrm{m/s})(4 \times 10^{-3}\mathrm{T})(0.5) = 3 \times 10^{-11}\mathrm{N}$.

Intermediate Exam II: Problem #4 (Spring ’06)

A current loop in the form of a right triangle is placed in a uniform magnetic field of magnitude $B = 30$mT as shown. The current in the loop is $I = 0.4$A in the direction indicated.

(a) Find magnitude and direction of the force $\vec{F}_1$ on side 1 of the triangle.

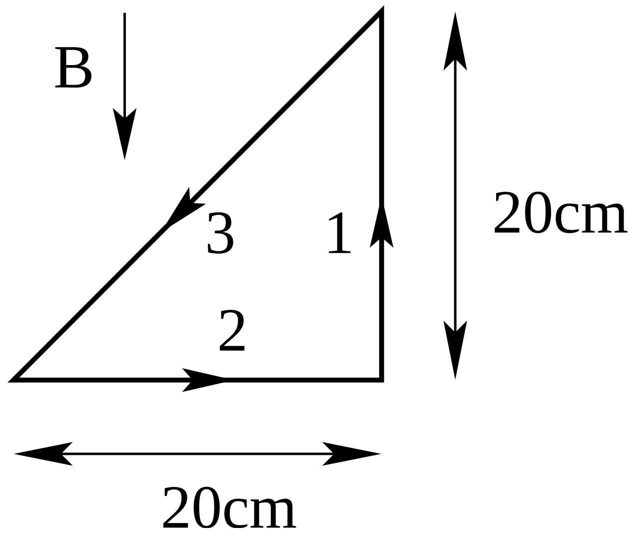
(b) Find magnitude and direction of the force $\vec{F}_2$ on side 2 of the triangle.



A current loop in the form of a right triangle is placed in a uniform magnetic field of magnitude $B = 30$mT as shown. The current in the loop is $I = 0.4$A in the direction indicated.

(a) Find magnitude and direction of the force $\vec{F}_1$ on side 1 of the triangle.

(b) Find magnitude and direction of the force $\vec{F}_2$ on side 2 of the triangle.



Solution:

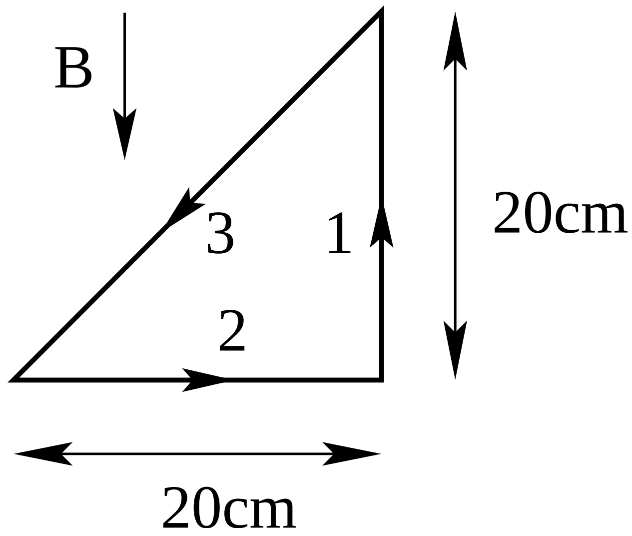
(a) $\vec{F}_1 = I\vec{L} \times \vec{B} = 0$ (angle between $\vec{L}$ and $\vec{B}$ is $180°$).

Intermediate Exam II: Problem #4 (Spring '06)

A current loop in the form of a right triangle is placed in a uniform magnetic field of magnitude $B = 30$mT as shown. The current in the loop is $I = 0.4$A in the direction indicated.

(a) Find magnitude and direction of the force $\vec{F}_1$ on side 1 of the triangle.

(b) Find magnitude and direction of the force $\vec{F}_2$ on side 2 of the triangle.



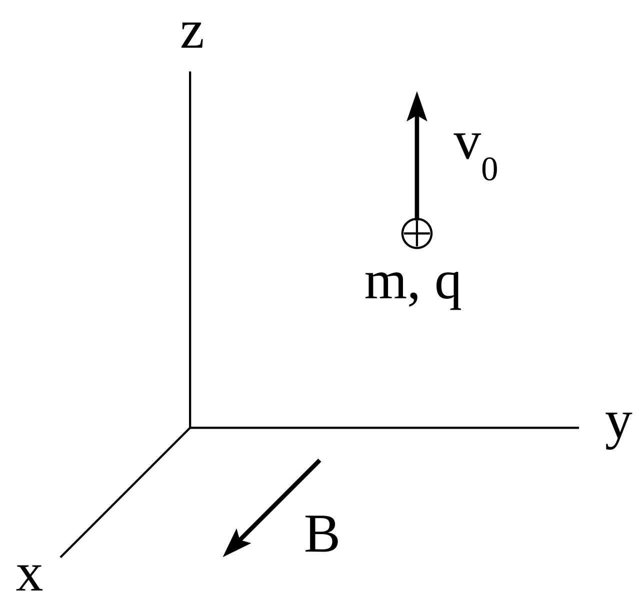
Solution:

(a) $\vec{F}_1 = I\vec{L} \times \vec{B} = 0$ (angle between $\vec{L}$ and $\vec{B}$ is $180°$).

(b) $F_2 = ILB = (0.4\text{A})(0.2\text{m})(30 \times 10^{-3}\text{T}) = 2.4 \times 10^{-3}\text{N}$.
Direction of $\vec{F}_2$: $\otimes$ (into plane).

In a region of uniform magnetic field $\mathbf{B} = 5\mathrm{mT}\hat{\mathbf{i}}$, a proton
$(m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C})$ is launched with velocity $\mathbf{v}_0 = 4000\mathrm{m/s}\hat{\mathbf{k}}$.

(a) Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.
(b) Calculate the radius r of the circular path.
(c) Calculate the time T it takes the proton to go around that circle once.
(d) Sketch the circular path of the proton in the graph.



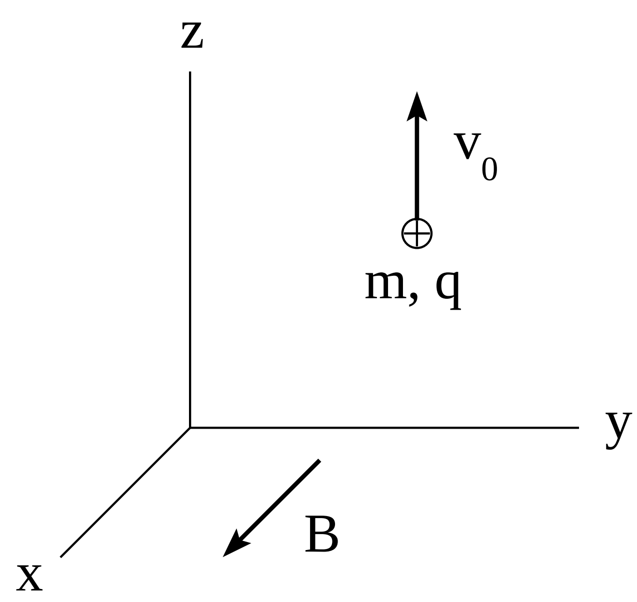
In a region of uniform magnetic field $\mathbf{B} = 5\mathrm{mT}\hat{\mathbf{i}}$, a proton $(m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C})$ is launched with velocity $\mathbf{v}_0 = 4000\mathrm{m/s}\hat{\mathbf{k}}$.

(a) Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.

(b) Calculate the radius r of the circular path.

(c) Calculate the time T it takes the proton to go around that circle once.

(d) Sketch the circular path of the proton in the graph.



Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\mathrm{N}.$

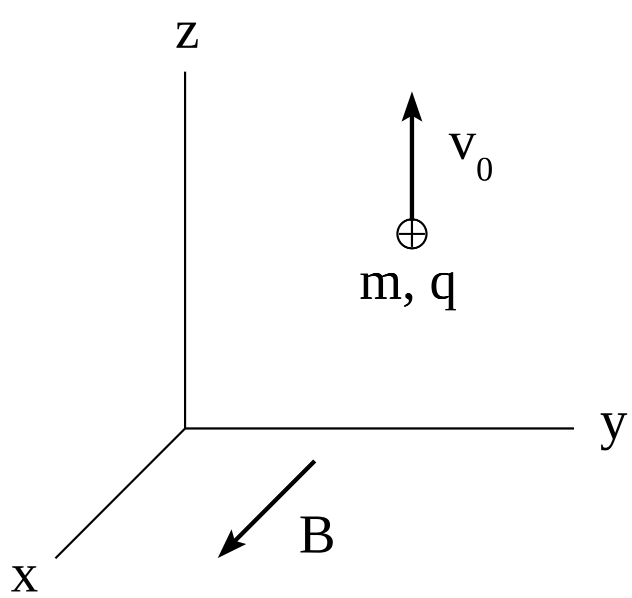
In a region of uniform magnetic field $\mathbf{B} = 5\text{mT}\hat{\mathbf{i}}$, a proton $(m = 1.67 \times 10^{-27}\text{kg},\ q = 1.60 \times 10^{-19}\text{C})$ is launched with velocity $\mathbf{v}_0 = 4000\text{m/s}\hat{\mathbf{k}}$.

(a) Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.

(b) Calculate the radius r of the circular path.

(c) Calculate the time T it takes the proton to go around that circle once.

(d) Sketch the circular path of the proton in the graph.

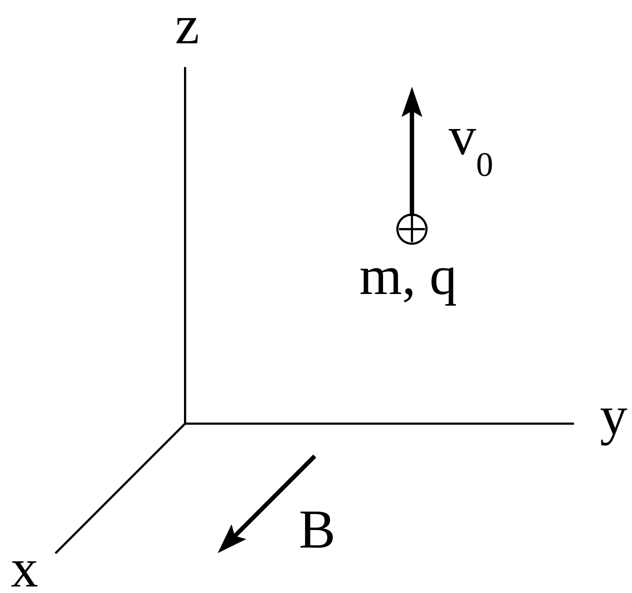


Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\text{N}.$

(b) $\dfrac{mv_0^2}{r} = qv_0B \quad \Rightarrow r = \dfrac{mv_0}{qB} = 8.35\text{mm}.$

In a region of uniform magnetic field $\mathbf{B} = 5\mathrm{mT}\hat{\mathbf{i}}$, a proton $(m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C})$ is launched with velocity $\mathbf{v}_0 = 4000\mathrm{m/s}\hat{\mathbf{k}}$.
(a) Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.
(b) Calculate the radius r of the circular path.
(c) Calculate the time T it takes the proton to go around that circle once.
(d) Sketch the circular path of the proton in the graph.



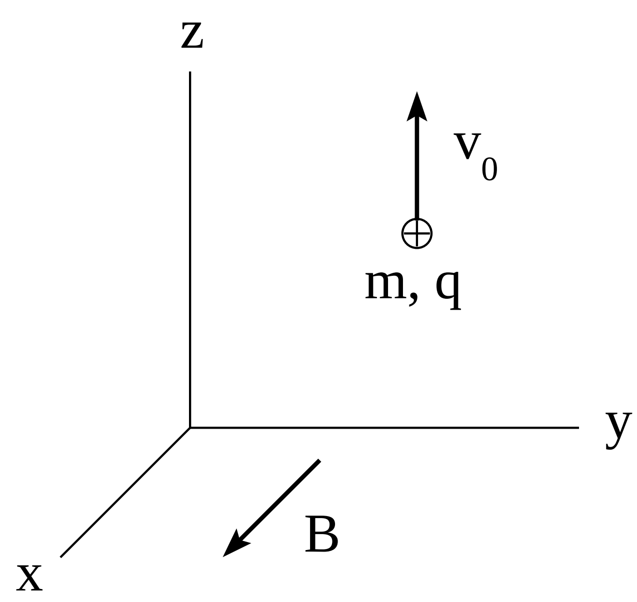
Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\mathrm{N}.$

(b) $\dfrac{mv_0^2}{r} = qv_0B \quad \Rightarrow r = \dfrac{mv_0}{qB} = 8.35\mathrm{mm}.$

(c) $T = \dfrac{2\pi r}{v_0} = \dfrac{2\pi m}{qB} = 13.1\mu\mathrm{s}.$

In a region of uniform magnetic field $\mathbf{B} = 5\mathrm{mT}\hat{\mathbf{i}}$, a proton ($m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C}$) is launched with velocity $\mathbf{v}_0 = 4000\mathrm{m/s}\hat{\mathbf{k}}$.
(a) Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.
(b) Calculate the radius r of the circular path.
(c) Calculate the time T it takes the proton to go around that circle once.
(d) Sketch the circular path of the proton in the graph.



Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\mathrm{N}.$

(b) $\dfrac{mv_0^2}{r} = qv_0B \quad \Rightarrow \ r = \dfrac{mv_0}{qB} = 8.35\mathrm{mm}.$

(c) $T = \dfrac{2\pi r}{v_0} = \dfrac{2\pi m}{qB} = 13.1\mu\mathrm{s}.$

(d) Center of circle to the right of proton’s initial position (cw motion).

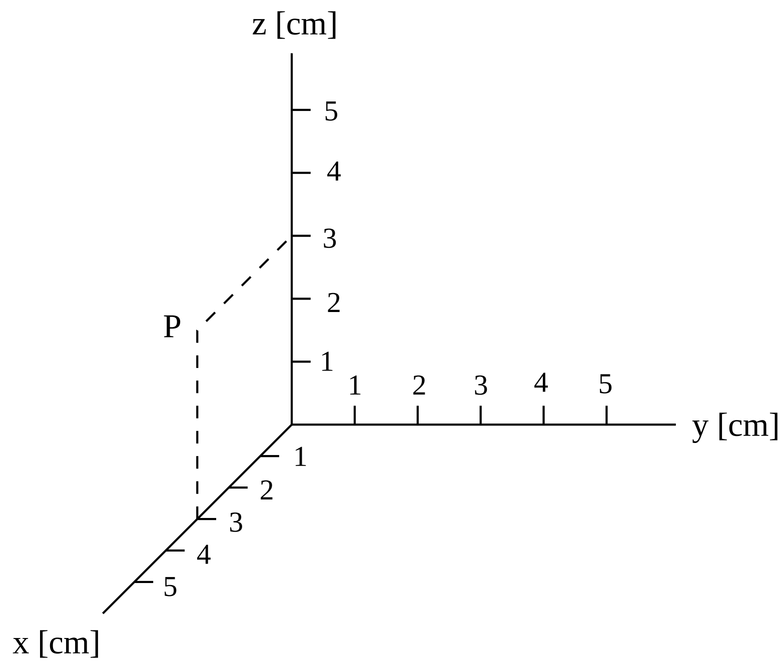
Unit Exam III: Problem #1 (Spring ’13)

In a region of uniform magnetic field $\mathbf{B}$ a proton $(m = 1.67 \times 10^{-27}\mathrm{kg},\ q = 1.60 \times 10^{-19}\mathrm{C})$ experiences a force $\mathbf{F} = 8.0 \times 10^{-19}\mathrm{N}\,\hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 2000\mathrm{m/s}\,\hat{\mathbf{k}}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).

(b) Calculate the radius r of the circular path.

(c) Locate the center C of the circular path in the coordinate system on the page.



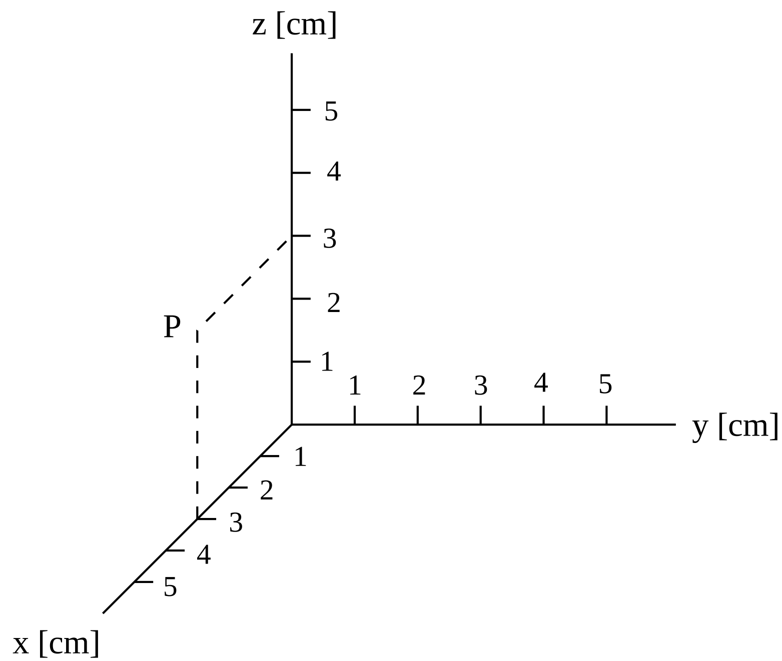
Unit Exam III: Problem #1 (Spring '13)

In a region of uniform magnetic field $\mathbf{B}$ a proton $(m = 1.67 \times 10^{-27}\text{kg},\ q = 1.60 \times 10^{-19}\text{C})$ experiences a force $\mathbf{F} = 8.0 \times 10^{-19}\text{N}\,\hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 2000\text{m/s}\,\hat{\mathbf{k}}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).

(b) Calculate the radius r of the circular path.

(c) Locate the center C of the circular path in the coordinate system on the page.



Solution:

(a) $B = \dfrac{F}{qv_0} = 2.50 \times 10^{-3}\text{T}, \quad \hat{\mathbf{i}} = \hat{\mathbf{k}} \times (-\hat{\mathbf{j}})$

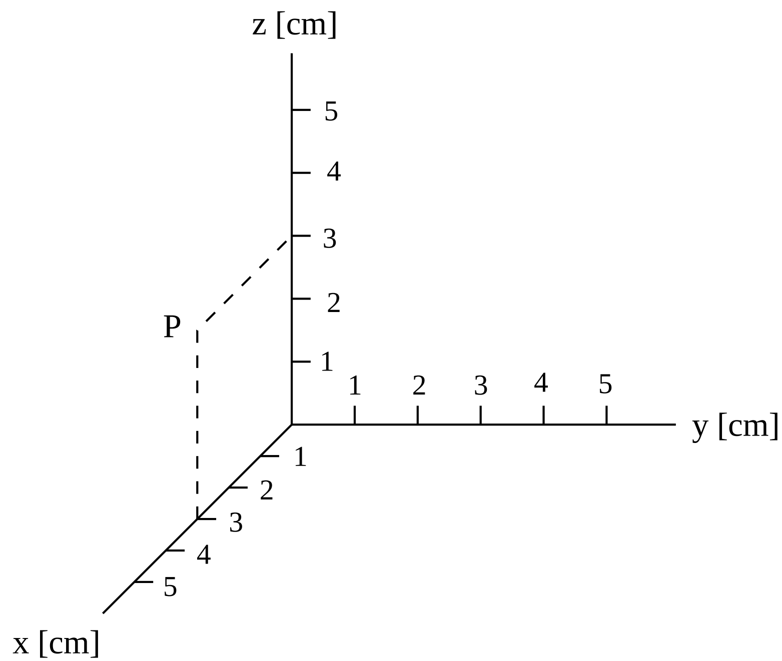
$\Rightarrow\ \mathbf{B} = -2.50 \times 10^{-3}\text{T}\,\hat{\mathbf{j}}.$

In a region of uniform magnetic field $\mathbf{B}$ a proton $(m = 1.67 \times 10^{-27}\text{kg},\ q = 1.60 \times 10^{-19}\text{C})$ experiences a force $\mathbf{F} = 8.0 \times 10^{-19}\text{N}\,\hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 2000\text{m/s}\,\hat{\mathbf{k}}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).

(b) Calculate the radius r of the circular path.

(c) Locate the center C of the circular path in the coordinate system on the page.



Solution:

(a) $B = \dfrac{F}{qv_0} = 2.50 \times 10^{-3}\text{T}, \quad \hat{\mathbf{i}} = \hat{\mathbf{k}} \times (-\hat{\mathbf{j}})$

$\Rightarrow\ \mathbf{B} = -2.50 \times 10^{-3}\text{T}\,\hat{\mathbf{j}}.$

(b) $F = \dfrac{mv_0^2}{r} = qv_0B$

$\Rightarrow\ r = \dfrac{mv_0^2}{F} = \dfrac{mv_0}{qB} = 0.835\text{cm}.$

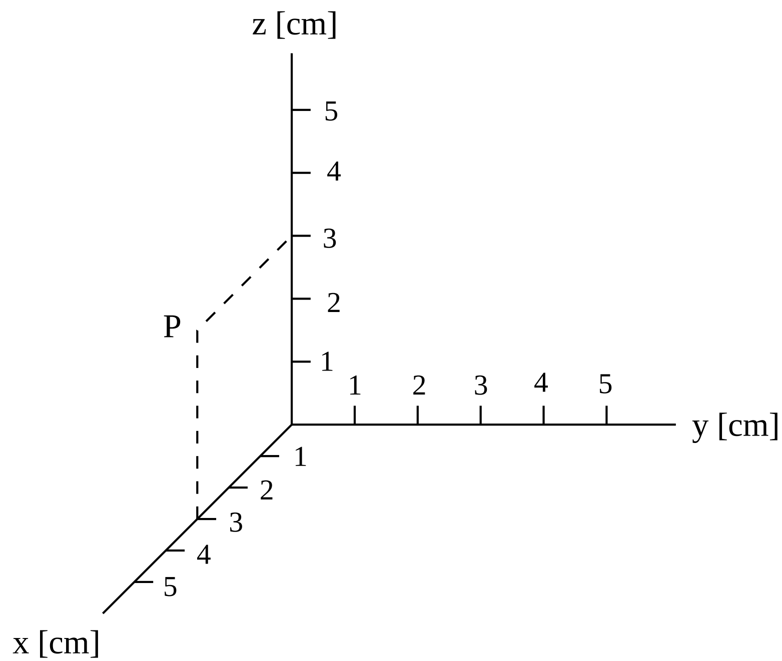
Unit Exam III: Problem #1 (Spring ’13)

In a region of uniform magnetic field $\mathbf{B}$ a proton $(m = 1.67 \times 10^{-27}\text{kg},\ q = 1.60 \times 10^{-19}\text{C})$ experiences a force $\mathbf{F} = 8.0 \times 10^{-19}\text{N}\,\hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 2000\text{m/s}\,\hat{\mathbf{k}}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).

(b) Calculate the radius r of the circular path.

(c) Locate the center C of the circular path in the coordinate system on the page.



Solution:

(a) $B = \dfrac{F}{qv_0} = 2.50 \times 10^{-3}\text{T}, \quad \hat{\mathbf{i}} = \hat{\mathbf{k}} \times (-\hat{\mathbf{j}})$

$\Rightarrow\ \mathbf{B} = -2.50 \times 10^{-3}\text{T}\,\hat{\mathbf{j}}.$

(b) $F = \dfrac{mv_0^2}{r} = qv_0B$

$\Rightarrow\ r = \dfrac{mv_0^2}{F} = \dfrac{mv_0}{qB} = 0.835\text{cm}.$

(c) $C = 3.84\text{cm}\,\hat{\mathbf{i}} + 3.00\text{cm}\,\hat{\mathbf{k}}.$