12. Magnetic Field I

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Abstract

Lecture slides 12 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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Electricity

- Electric charges generate an electric field.
- The electric field exerts a force on other electric charges.

Magnetism

- Electric currents generate a magnetic field.
- The magnetic field exerts force on other electric currents.
Sources of Electric and Magnetic Fields

Capacitor
The parallel-plate capacitor generates a near uniform electric field provided the linear dimensions of the plates are large compared to the distance between them.

Solenoid
The solenoid (a tightly wound cylindrical coil) generates a near uniform magnetic field provided the length of the coil is large compared to its radius.
Electric and Magnetic Forces on Point Charge

**Electric Force**
- $\vec{F} = q\vec{E}$
- electric force is parallel to electric field
- SI unit of $E$: 1N/C=1V/m

**Magnetic Force**
- $\vec{F} = q\vec{v} \times \vec{B}$, $F = qvB \sin \phi$
- magnetic force is perpendicular to magnetic field
- SI unit of $B$: 1Ns/Cm=1T (Tesla)
- 1T=10^4G (Gauss)
Consider drift of Na\(^+\) and Cl\(^-\) ions in a plastic pipe filled with salt water.

- \(v_{1x} > 0,\ v_{2x} < 0\): drift velocities; \(q_1 > 0,\ q_2 < 0\): charge on ions
- \(n_1,\ n_2\): number of charge carriers per unit volume

- Electric current through \(A\): \(I = A(n_1 q_1 v_{1x} + n_2 q_2 v_{2x})\)
- Force on Na\(^+\): \(\vec{F}_1 = q_1 \vec{v}_1 \times \vec{B} \Rightarrow F_{1z} = q_1 v_{1x} B_y\)
- Force on Cl\(^-\): \(\vec{F}_2 = q_2 \vec{v}_2 \times \vec{B} \Rightarrow F_{2z} = q_2 v_{2x} B_y\)
- Force on current-carrying pipe: \(F_z = (n_1 q_1 v_{1x} + n_2 q_2 v_{2x})ALB_y = ILB_y\)
- Vector relation: \(\vec{F} = I \vec{L} \times \vec{B}\)
\[ \vec{F} = I \vec{L} \times \vec{B} \]
Direction of Magnetic Force

\[ \vec{F} = I \vec{L} \times \vec{B} \]

- \( B_{in} \)
- \( I = 0 \)
- \( I \)
A wire of length $L = 62\text{cm}$ and mass $m = 13\text{g}$ is suspended by a pair of flexible leads in a uniform magnetic field $B = 0.440\text{T}$ pointing in to the plane.

- What are the magnitude and direction of the current required to remove the tension in the supporting leads?
Magnetic Force Application (2)

A metal wire of mass $m = 1.5\text{kg}$ slides without friction on two horizontal rails spaced a distance $d = 3\text{m}$ apart.

The track lies in a vertical uniform magnetic field of magnitude $B = 24\text{mT}$ pointing out of the plane. A constant current $I = 12\text{A}$ flows from a battery along one rail, across the wire, and back down the other rail. The wire starts moving from rest at $t = 0$.

- Find the direction and magnitude of the velocity of the wire at time $t = 5\text{s}$.
Fancy solution:

- Uniform magnetic field $\vec{B}$ points out of the plane.
- Magnetic force on segment $ds$: $dF = IBds = IBRd\theta$.
- Integrate $dF_x = dF \sin \theta$ and $dF_y = dF \cos \theta$ along semicircle.
- $F_x = IBR \int_0^{\pi} \sin \theta d\theta = 2IBR$, $F_y = IBR \int_0^{\pi} \cos \theta d\theta = 0$. 

\[ X = IBR \int_0^{\pi} \sin \theta d\theta = 2IBR, \quad Y = IBR \int_0^{\pi} \cos \theta d\theta = 0. \]
Clever solution:

- Replace the semicircle by symmetric staircase of tiny wire segments.
- Half the vertical segments experience a force to the left, the other half a force to the right. The resultant horizontal force is zero.
- All horizontal segments experience a downward force. The total length is $2R$. The total downward force is $2IBR$.
- Making the segments infinitesimally small does not change the result.
Inside the cube there is a magnetic field $\vec{B}$ directed vertically up.

Find the direction of the magnetic force experienced by a proton entering the cube

(a) from the left,
(b) from the front,
(c) from the right,
(d) from the top.
Charged Particle Moving in Uniform Electric Field

- Electric field $\vec{E}$ is directed up.
- Electric force: $\vec{F} = q\vec{E}$ (constant)
- Acceleration: $\vec{a} = \frac{\vec{F}}{m} = \frac{q}{m} \vec{E} = \text{const.}$
- Horizontal motion: $a_x = 0 \Rightarrow v_x(t) = v_0 \Rightarrow x(t) = v_0 t$
- Vertical motion: $a_y = \frac{q}{m} E \Rightarrow v_y(t) = a_y t \Rightarrow y(t) = \frac{1}{2} a_y t^2$
- The path is parabolic: $y = \left( \frac{qE}{2mv_0^2} \right) x^2$
- $\vec{F}$ changes direction and magnitude of $\vec{v}$. 
Charged Particle Moving in Uniform Magnetic Field

- Magnetic field $\vec{B}$ is directed into plane.
- Magnetic force: $\vec{F} = q\vec{v} \times \vec{B}$ (not constant)
- $\vec{F} \perp \vec{v} \Rightarrow \vec{F}$ changes direction of $\vec{v}$ only $\Rightarrow v = v_0$.
- $\vec{F}$ is the centripetal force of motion along circular path.
- Radius: $\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB}$
- Angular velocity: $\omega = \frac{v}{r} = \frac{qB}{m}$
- Period: $T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$
A proton with speed \( v = 3.00 \times 10^5 \text{m/s} \) orbits just outside a charged conducting sphere of radius \( r = 1.00 \text{cm} \).

(a) Find the force \( F \) acting on the proton.
(b) Find the charge per unit area \( \sigma \) on the surface of the sphere.
(c) Find the total charge \( Q \) on the sphere.

Note: Charged particles in circular motion lose energy through radiation. This effect is ignored here.
Magnetic Force Application (3)

The dashed rectangle marks a region of uniform magnetic field \( \vec{B} \) pointing out of the plane.

- Find the direction of the magnetic force acting on each loop with a ccw current \( I \).
A charged particle is moving horizontally into a region with “crossed” uniform fields:

- an electric field $\vec{E}$ pointing down,
- a magnetic field $\vec{B}$ pointing into the plane.

Forces experienced by particle:

- electric force $F = qE$ pointing down,
- magnetic force $B = qvB$ pointing up.

Forces in balance: $qE = qvB$.

Selected velocity: $v = \frac{E}{B}$.

Trajectories of particles with selected velocity are not bent.
Measurement of $e/m$ for Electron

First experiment by J. J. Thomson (1897)
Method used here: velocity selector

Equilibrium of forces: $eE = evB \Rightarrow v = \frac{E}{B}$

Work-energy relation: $eV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2eV}{m}}$

Eliminate $v$: $\frac{e}{m} = \frac{E^2}{2VB^2} \simeq 1.76 \times 10^{11} \text{C/kg}$
Measurement of $e$ and $m$ for Electron

First experiment by R. Millikan (1913)
Method used here: balancing weight and electric force on oil drop
Radius of oil drop: $r = 1.64\mu m$
Mass density of oil: $\rho = 0.851 g/cm^3$
Electric field: $E = 1.92 \times 10^5 N/C$

Mass of oil drop: $m = \frac{4\pi}{3} r^3 \rho = 1.57 \times 10^{-14} kg$

Equilibrium of forces: $neE = mg$
Number of excess elementary charges (integer): $n = 5$

Elementary charge: $e = \frac{mg}{nE} \simeq 1.6 \times 10^{-19} C$

Mass of electron: $m \simeq 9.1 \times 10^{-31} kg$
Mass Spectrometer

Purpose: measuring masses of ions.

- Charged particle is accelerated by moving through potential difference $|\Delta V|$.
- Trajectory is then bent into semicircle of radius $r$ by magnetic field $\vec{B}$.
- Kinetic energy: $\frac{1}{2}mv^2 = q|\Delta V|$.
- Radius of trajectory: $r = \frac{mv}{qB}$.
- Charge: $q = e$
- Mass: $m = \frac{eB^2r^2}{2|\Delta V|}$. 
Cyclotron

Purpose: accelerate charged particles to high energy.

- Low-energy protons are injected at \( S \).
- Path is bent by magnetic field \( \vec{B} \).
- Proton is energized by alternating voltage \( \Delta V \) between \( Dee_1 \) and \( Dee_2 \).
- Proton picks up energy \( \Delta K = e\Delta V \) during each half cycle.
- Path spirals out as velocity of particle increases:
  Radial distance is proportional to velocity: \( r = \frac{mv}{eB} \).
- Duration of cycle stays is independent of \( r \) or \( v \):
  cyclotron period: \( T = \frac{2\pi m}{eB} \).
- Cyclotron period is synchronized with alternation of accelerating voltage.
- High-energy protons exit at perimeter of \( \vec{B} \)-field region.
Magnetic Bottles

Moving charged particle confined by inhomogeneous magnetic field.

Van Allen belt: trapped protons and electrons in Earth's magnetic field.
Conversion of electric signal into mechanical vibration.
Consider a charged particle moving in a uniform magnetic field as shown. The velocity is in $y$-direction and the magnetic field in the $yz$-plane at $30^\circ$ from the $y$-direction.

(a) Find the direction of the magnetic force acting on the particle.

(b) Find the magnitude of the magnetic force acting on the particle.
Consider a charged particle moving in a uniform magnetic field as shown. The velocity is in $y$-direction and the magnetic field in the $yz$-plane at $30^\circ$ from the $y$-direction.

(a) Find the direction of the magnetic force acting on the particle.
(b) Find the magnitude of the magnetic force acting on the particle.

Solution:

(a) Use the right-hand rule: positive $x$-direction (front, out of page).
Consider a charged particle moving in a uniform magnetic field as shown. The velocity is in $y$-direction and the magnetic field in the $yz$-plane at $30^\circ$ from the $y$-direction.

(a) Find the direction of the magnetic force acting on the particle.
(b) Find the magnitude of the magnetic force acting on the particle.

Solution:

(a) Use the right-hand rule: positive $x$-direction (front, out of page).
(b) $F = qvB \sin 30^\circ = (5 \times 10^{-9}\text{C})(3\text{m/s})(4 \times 10^{-3}\text{T})(0.5) = 3 \times 10^{-11}\text{N}$. 
A current loop in the form of a right triangle is placed in a uniform magnetic field of magnitude $B = 30 \text{ mT}$ as shown. The current in the loop is $I = 0.4 \text{ A}$ in the direction indicated.

(a) Find magnitude and direction of the force $\vec{F}_1$ on side 1 of the triangle.

(b) Find magnitude and direction of the force $\vec{F}_2$ on side 2 of the triangle.
A current loop in the form of a right triangle is placed in a uniform magnetic field of magnitude $\mathcal{B} = 30\text{mT}$ as shown. The current in the loop is $I = 0.4\text{A}$ in the direction indicated.

(a) Find magnitude and direction of the force $\vec{F}_1$ on side 1 of the triangle.

(b) Find magnitude and direction of the force $\vec{F}_2$ on side 2 of the triangle.

**Solution:**

(a) $\vec{F}_1 = I\vec{L} \times \mathcal{B} = 0$ (angle between $\vec{L}$ and $\mathcal{B}$ is $180^\circ$).
A current loop in the form of a right triangle is placed in a uniform magnetic field of magnitude $B = 30 \text{ mT}$ as shown. The current in the loop is $I = 0.4 \text{ A}$ in the direction indicated.

(a) Find magnitude and direction of the force $\vec{F}_1$ on side 1 of the triangle.

(b) Find magnitude and direction of the force $\vec{F}_2$ on side 2 of the triangle.

![Diagram of a right triangle with magnetic field](image)

**Solution:**

(a) $\vec{F}_1 = I\vec{L} \times \vec{B} = 0$ (angle between $\vec{L}$ and $\vec{B}$ is $180^\circ$).

(b) $F_2 = ILB = (0.4 \text{ A})(0.2 \text{ m})(30 \times 10^{-3} \text{ T}) = 2.4 \times 10^{-3} \text{ N}$. Direction of $\vec{F}_2$: $\otimes$ (into plane).
In a region of uniform magnetic field $B = 5\text{mT} \hat{i}$, a proton ($m = 1.67 \times 10^{-27}\text{kg}$, $q = 1.60 \times 10^{-19}\text{C}$) is launched with velocity $v_0 = 4000\text{m/s} \hat{k}$.

(a) Calculate the magnitude $F$ of the magnetic force that keeps the proton on a circular path.
(b) Calculate the radius $r$ of the circular path.
(c) Calculate the time $T$ it takes the proton to go around that circle once.
(d) Sketch the circular path of the proton in the graph.
In a region of uniform magnetic field $\mathbf{B} = 5\text{mT}\hat{\mathbf{i}}$, a proton
($m = 1.67 \times 10^{-27}\text{kg}$, $q = 1.60 \times 10^{-19}\text{C}$) is launched with velocity $\mathbf{v}_0 = 4000\text{m/s}\hat{\mathbf{k}}$.
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Solution:

(a) $F = qv_0 B = 3.2 \times 10^{-18}\text{N}$.
In a region of uniform magnetic field $B = 5\text{mT} \hat{i}$, a proton $(m = 1.67 \times 10^{-27}\text{kg}, \; q = 1.60 \times 10^{-19}\text{C})$ is launched with velocity $v_0 = 4000\text{m/s} \hat{k}$.

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(d) Sketch the circular path of the proton in the graph.

Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\text{N}$.

(b) $\frac{mv_0^2}{r} = qv_0B \Rightarrow r = \frac{mv_0}{qB} = 8.35\text{mm}$. 
In a region of uniform magnetic field $\mathbf{B} = 5\text{mT} \hat{i}$, a proton $(m = 1.67 \times 10^{-27}\text{kg}, \, q = 1.60 \times 10^{-19}\text{C})$ is launched with velocity $\mathbf{v}_0 = 4000\text{m/s} \hat{k}$.

(a) Calculate the magnitude $F$ of the magnetic force that keeps the proton on a circular path.
(b) Calculate the radius $r$ of the circular path.
(c) Calculate the time $T$ it takes the proton to go around that circle once.
(d) Sketch the circular path of the proton in the graph.

Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\text{N}$.

(b) $\frac{mv_0^2}{r} = qv_0B \quad \Rightarrow \quad r = \frac{mv_0}{qB} = 8.35\text{mm}$.

(c) $T = \frac{2\pi r}{v_0} = \frac{2\pi m}{qB} = 13.1\mu\text{s}$.
In a region of uniform magnetic field $\mathbf{B} = 5 \text{mT} \hat{i}$, a proton
($m = 1.67 \times 10^{-27} \text{kg}$, $q = 1.60 \times 10^{-19} \text{C}$) is launched with velocity $\mathbf{v}_0 = 4000 \text{m/s} \hat{k}$.
(a) Calculate the magnitude $F$ of the magnetic force that keeps the proton on a circular path.
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(c) Calculate the time $T$ it takes the proton to go around that circle once.
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Solution:

(a) $F = q\mathbf{v}_0 \mathbf{B} = 3.2 \times 10^{-18} \text{N}$.

(b) $\frac{mv_0^2}{r} = q\mathbf{v}_0 \mathbf{B}$ $\Rightarrow$ $r = \frac{mv_0}{qB} = 8.35 \text{mm}$.

(c) $T = \frac{2\pi r}{v_0} = \frac{2\pi m}{qB} = 13.1 \mu\text{s}$.

(d) Center of circle to the right of proton’s initial position (cw motion).
In a region of uniform magnetic field \( B \) a proton \((m = 1.67 \times 10^{-27}\) kg, \( q = 1.60 \times 10^{-19}\) C\) experiences a force \( \mathbf{F} = 8.0 \times 10^{-19}\) N \( \hat{i} \) as it passes through point \( P \) with velocity \( \mathbf{v}_0 = 2000\) m/s \( \hat{k} \) on a circular path.
(a) Find the magnetic field \( B \) (magnitude and direction).
(b) Calculate the radius \( r \) of the circular path.
(c) Locate the center \( C \) of the circular path in the coordinate system on the page.
In a region of uniform magnetic field \( \mathbf{B} \) a proton \((m = 1.67 \times 10^{-27}\text{kg}, \ q = 1.60 \times 10^{-19}\text{C})\) experiences a force \( \mathbf{F} = 8.0 \times 10^{-19}\text{N} \hat{i} \) as it passes through point \( P \) with velocity \( v_0 = 2000\text{m/s} \hat{k} \) on a circular path.

(a) Find the magnetic field \( \mathbf{B} \) (magnitude and direction).

(b) Calculate the radius \( r \) of the circular path.

(c) Locate the center \( C \) of the circular path in the coordinate system on the page.

Solution:

\[
(a) \quad B = \frac{F}{qv_0} = 2.50 \times 10^{-3}\text{T}, \quad \hat{i} = \hat{k} \times (-\hat{j})
\]

\[
\Rightarrow \quad \mathbf{B} = -2.50 \times 10^{-3}\text{T} \hat{j}.
\]
In a region of uniform magnetic field $\mathbf{B}$ a proton ($m = 1.67 \times 10^{-27}\text{kg}$, $q = 1.60 \times 10^{-19}\text{C}$) experiences a force $\mathbf{F} = 8.0 \times 10^{-19}\text{N} \hat{i}$ as it passes through point $P$ with velocity $\mathbf{v}_0 = 2000\text{m/s} \hat{k}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).
(b) Calculate the radius $r$ of the circular path.
(c) Locate the center $C$ of the circular path in the coordinate system on the page.

**Solution:**

(a) $B = \frac{F}{qv_0} = 2.50 \times 10^{-3}\text{T}$, $\hat{i} = \hat{k} \times (-\hat{j})$

$\Rightarrow \mathbf{B} = -2.50 \times 10^{-3}\text{T} \hat{j}$.

(b) $F = \frac{mv_0^2}{r} = qv_0 B$

$\Rightarrow r = \frac{mv_0^2}{F} = \frac{mv_0}{qB} = 0.835\text{cm}.$
In a region of uniform magnetic field $\mathbf{B}$ a proton $(m = 1.67 \times 10^{-27} \text{kg}, \, q = 1.60 \times 10^{-19} \text{C})$ experiences a force $\mathbf{F} = 8.0 \times 10^{-19} \text{N} \, \hat{i}$ as it passes through point $P$ with velocity $v_0 = 2000 \text{m/s} \, \hat{k}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).
(b) Calculate the radius $r$ of the circular path.
(c) Locate the center $C$ of the circular path in the coordinate system on the page.

Solution:

(a) \[ B = \frac{F}{qv_0} = 2.50 \times 10^{-3} \text{T}, \quad \hat{i} = \hat{k} \times (-\hat{j}) \]
\[ \Rightarrow B = -2.50 \times 10^{-3} \text{T} \, \hat{j}. \]

(b) \[ F = \frac{mv_0^2}{r} = qv_0 B \]
\[ \Rightarrow r = \frac{mv_0^2}{F} = \frac{mv_0}{qB} = 0.835 \text{ cm}. \]

(c) \[ C = 3.84 \text{ cm} \, \hat{i} + 3.00 \text{ cm} \, \hat{k}. \]