

11-19-2015

12. Magnetic Field I

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Abstract

Lecture slides 12 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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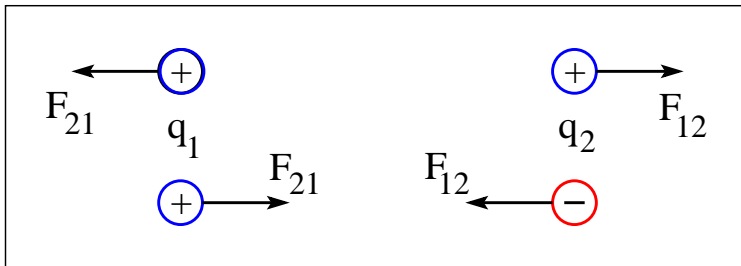
Müller, Gerhard, "12. Magnetic Field I" (2015). *PHY 204: Elementary Physics II (2015)*. Paper 9.
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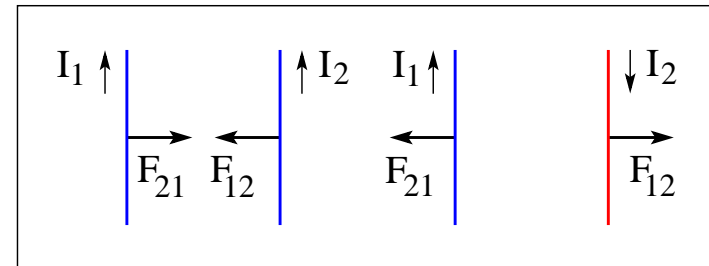
Electricity

- Electric charges generate an electric field.
- The electric field exerts a force on other electric charges.



Magnetism

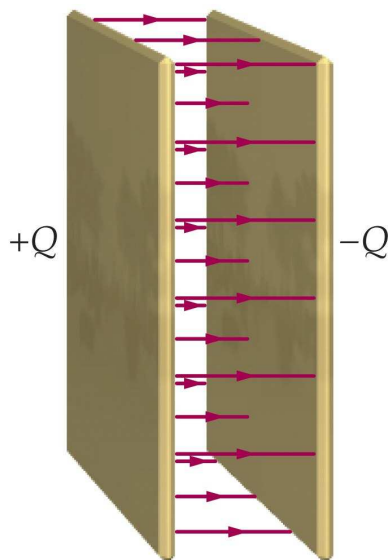
- Electric currents generate a magnetic field.
- The magnetic field exerts force on other electric currents.





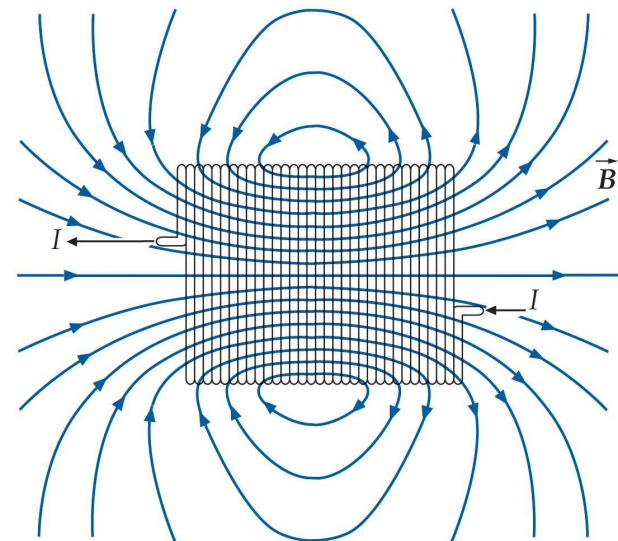
Capacitor

The parallel-plate capacitor generates a near uniform electric field provided the linear dimensions of the plates are large compared to the distance between them.



Solenoid

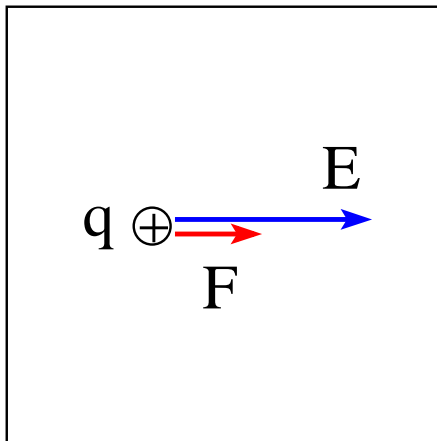
The solenoid (a tightly wound cylindrical coil) generates a near uniform magnetic field provided the length of the coil is large compared to its radius.





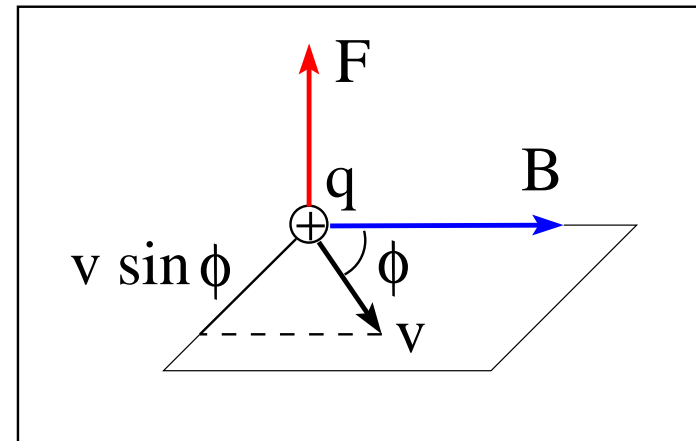
Electric Force

- $\vec{F} = q\vec{E}$
- electric force is parallel to electric field
- SI unit of E : $1\text{N/C}=1\text{V/m}$



Magnetic Force

- $\vec{F} = q\vec{v} \times \vec{B}$, $F = qvB \sin \phi$
- magnetic force is perpendicular to magnetic field
- SI unit of B : $1\text{Ns/Cm}=1\text{T}$ (Tesla)
- $1\text{T}=10^4\text{G}$ (Gauss)

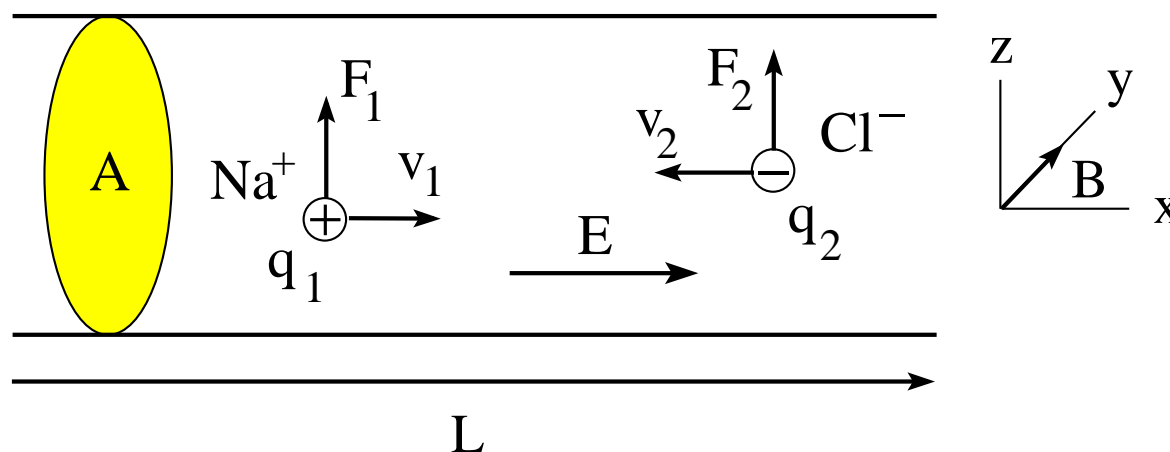


Magnetic Force on Current-Carrying Conductor



Consider drift of Na^+ and Cl^- ions in a plastic pipe filled with salt water.

- $v_{1x} > 0$, $v_{2x} < 0$: drift velocities; $q_1 > 0$, $q_2 < 0$: charge on ions
- n_1 , n_2 : number of charge carriers per unit volume

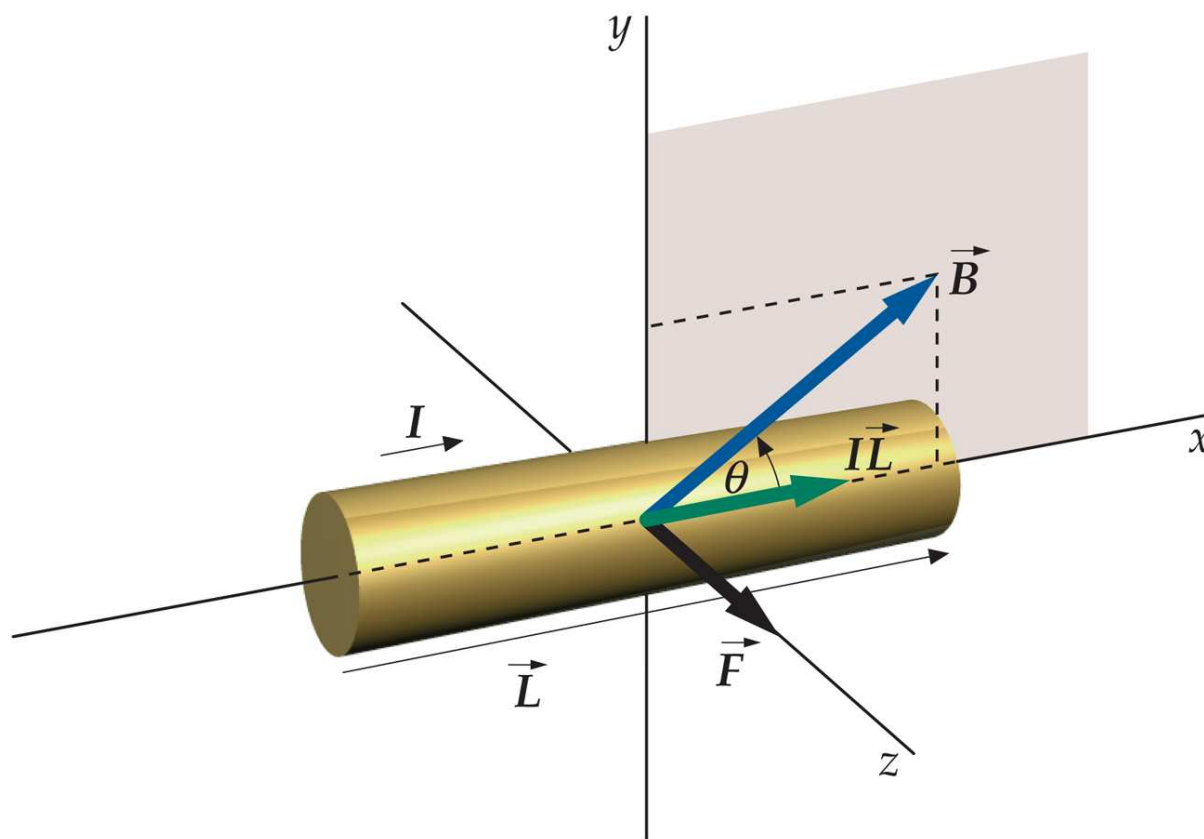


- Electric current through A : $I = A(n_1q_1v_{1x} + n_2q_2v_{2x})$
- Force on Na^+ : $\vec{F}_1 = q_1\vec{v}_1 \times \vec{B} \Rightarrow F_{1z} = q_1v_{1x}B_y$
- Force on Cl^- : $\vec{F}_2 = q_2\vec{v}_2 \times \vec{B} \Rightarrow F_{2z} = q_2v_{2x}B_y$
- Force on current-carrying pipe: $F_z = (n_1q_1v_{1x} + n_2q_2v_{2x})ALB_y = ILB_y$
- Vector relation: $\vec{F} = I\vec{L} \times \vec{B}$

Direction of Magnetic Force



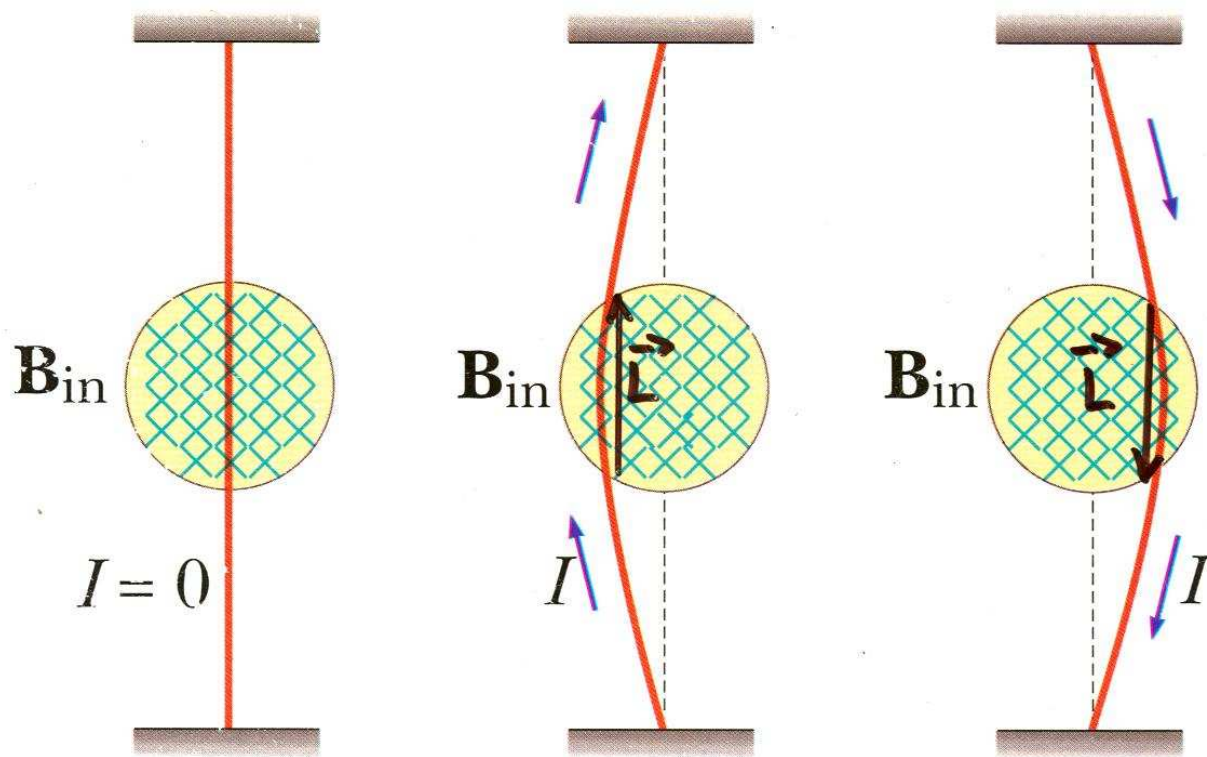
$$\vec{F} = I\vec{L} \times \vec{B}$$



Direction of Magnetic Force



$$\vec{F} = I\vec{L} \times \vec{B}$$

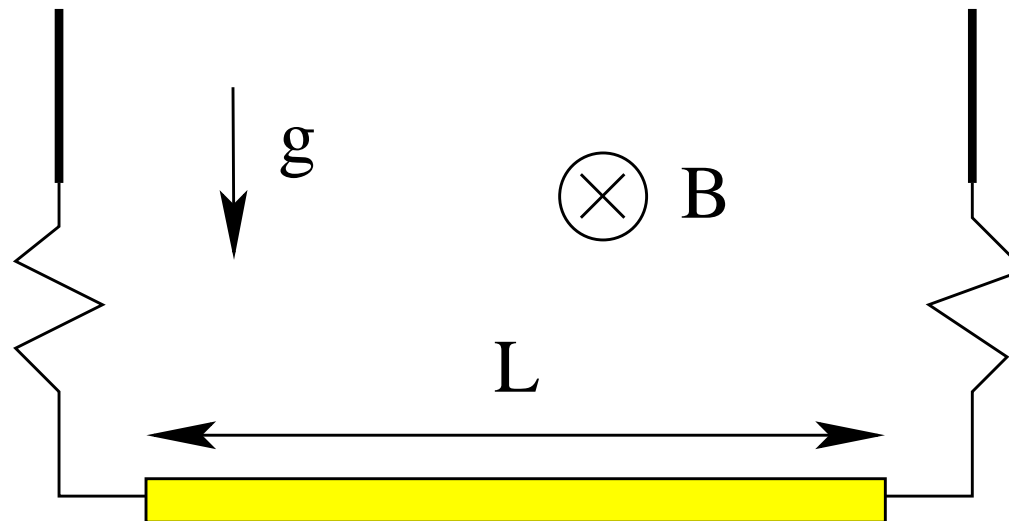


Magnetic Force Application (1)



A wire of length $L = 62\text{cm}$ and mass $m = 13\text{g}$ is suspended by a pair of flexible leads in a uniform magnetic field $B = 0.440\text{T}$ pointing in to the plane.

- What are the magnitude and direction of the current required to remove the tension in the supporting leads?



Magnetic Force Application (2)

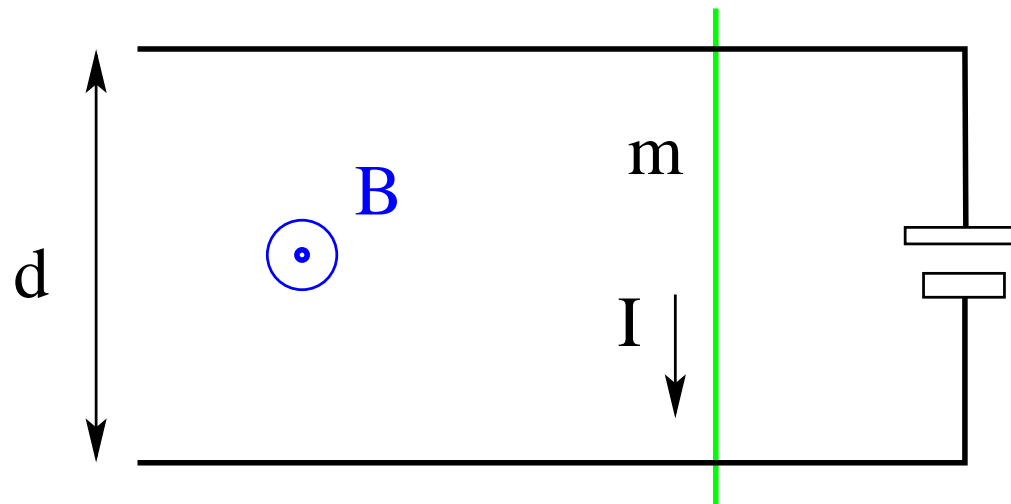


A metal wire of mass $m = 1.5\text{kg}$ slides without friction on two horizontal rails spaced a distance $d = 3\text{m}$ apart.

The track lies in a vertical uniform magnetic field of magnitude $B = 24\text{mT}$ pointing out of the plane.

A constant current $I = 12\text{A}$ flows from a battery along one rail, across the wire, and back down the other rail. The wire starts moving from rest at $t = 0$.

- Find the direction and magnitude of the velocity of the wire at time $t = 5\text{s}$.

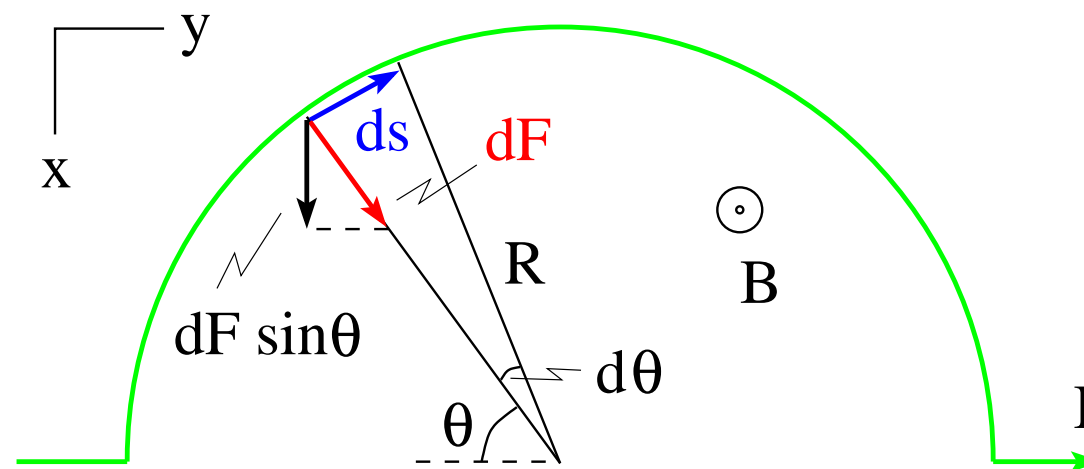


Magnetic Force on Semicircular Current (1)



Fancy solution:

- Uniform magnetic field \vec{B} points out of the plane.
- Magnetic force on segment ds : $dF = IBds = IBRd\theta$.
- Integrate $dF_x = dF \sin \theta$ and $dF_y = dF \cos \theta$ along semicircle.
- $F_x = IBR \int_0^\pi \sin \theta d\theta = 2IBR$, $F_y = IBR \int_0^\pi \cos \theta d\theta = 0$.

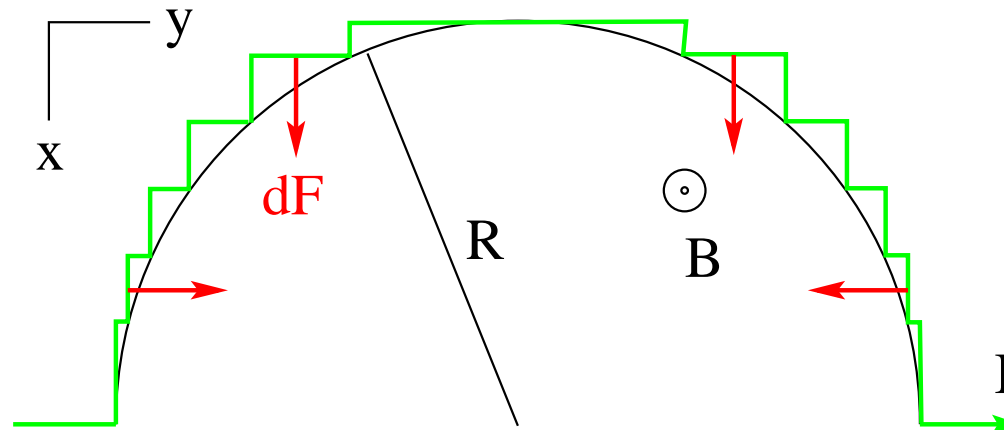


Magnetic Force on Semicircular Current (2)



Clever solution:

- Replace the semicircle by symmetric staircase of tiny wire segments.
- Half the vertical segments experience a force to the left, the other half a force to the right. The resultant horizontal force is zero.
- All horizontal segments experience a downward force. The total length is $2R$. The total downward force is $2IBR$.
- Making the segments infinitesimally small does not change the result.



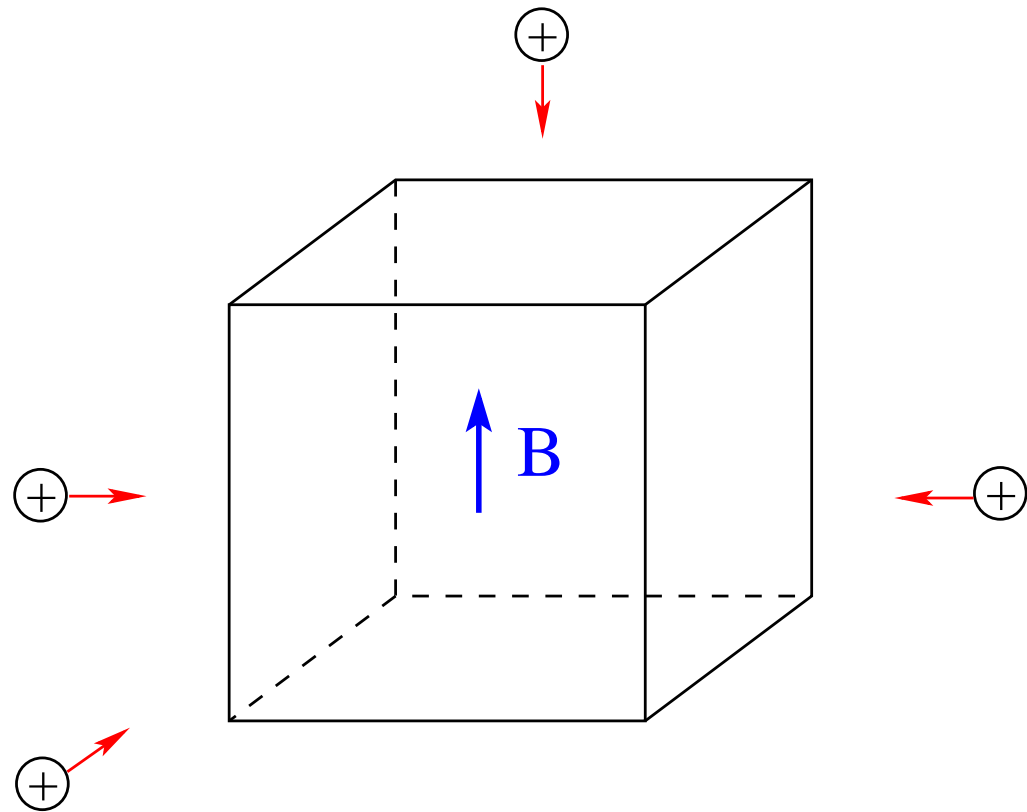
Magnetic Force Application (5)



Inside the cube there is a magnetic field \vec{B} directed vertically up.

Find the direction of the magnetic force experienced by a proton entering the cube

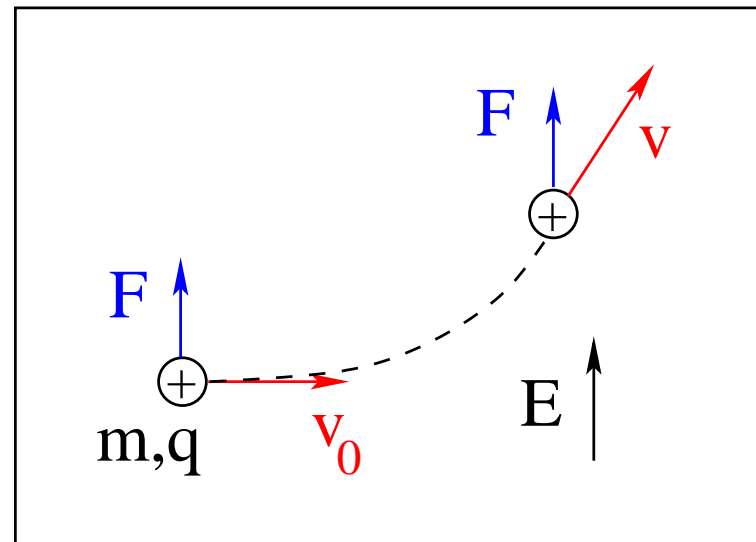
- (a) from the left,
- (b) from the front,
- (c) from the right,
- (d) from the top.



Charged Particle Moving in Uniform Electric Field



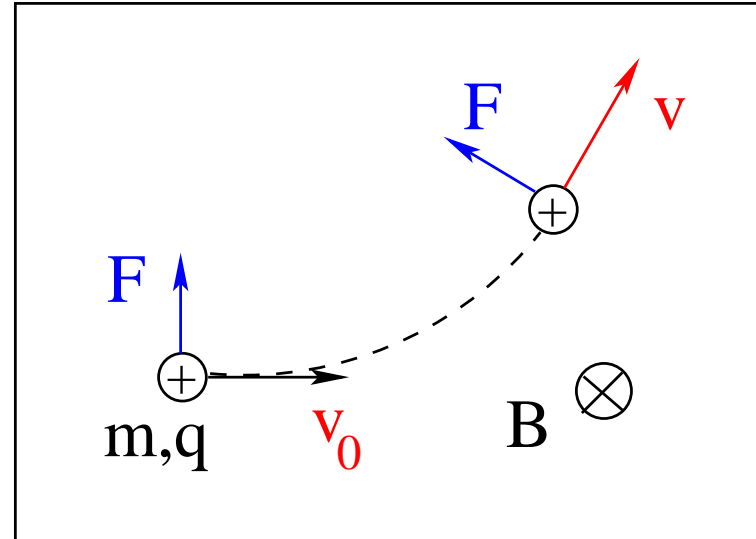
- Electric field \vec{E} is directed up.
- Electric force: $\vec{F} = q\vec{E}$ (constant)
- Acceleration: $\vec{a} = \frac{\vec{F}}{m} = \frac{q}{m}\vec{E} = \text{const.}$
- Horizontal motion: $a_x = 0 \Rightarrow v_x(t) = v_0 \Rightarrow x(t) = v_0 t$
- Vertical motion: $a_y = \frac{q}{m}E \Rightarrow v_y(t) = a_y t \Rightarrow y(t) = \frac{1}{2}a_y t^2$
- The path is parabolic: $y = \left(\frac{qE}{2mv_0^2}\right) x^2$
- \vec{F} changes direction and magnitude of \vec{v} .



Charged Particle Moving in Uniform Magnetic Field



- Magnetic field \vec{B} is directed into plane.
- Magnetic force: $\vec{F} = q\vec{v} \times \vec{B}$ (not constant)
- $\vec{F} \perp \vec{v} \Rightarrow \vec{F}$ changes direction of \vec{v} only $\Rightarrow v = v_0$.
- \vec{F} is the centripetal force of motion along circular path.
- Radius: $\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB}$
- Angular velocity: $\omega = \frac{v}{r} = \frac{qB}{m}$
- Period: $T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$



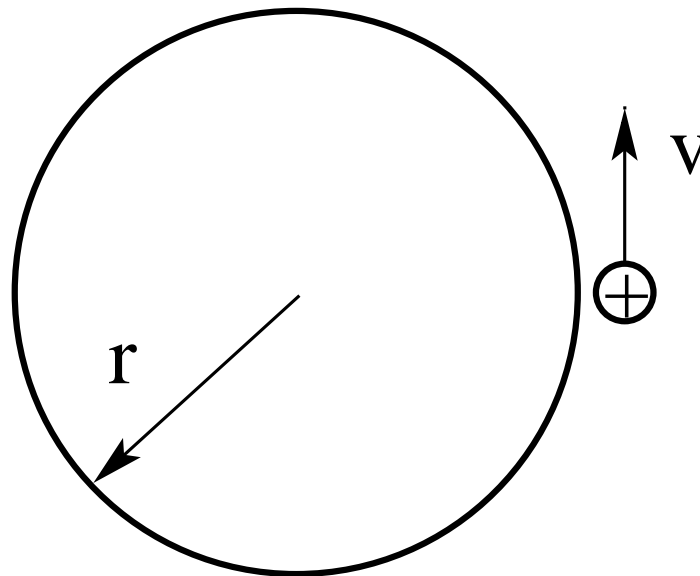
Charged Particle in Circular Motion



A proton with speed $v = 3.00 \times 10^5 \text{ m/s}$ orbits just outside a charged conducting sphere of radius $r = 1.00 \text{ cm}$.

- (a) Find the force F acting on the proton.
- (b) Find the charge per unit area σ on the surface of the sphere.
- (c) Find the total charge Q on the sphere.

Note: Charged particles in circular motion lose energy through radiation. This effect is ignored here.

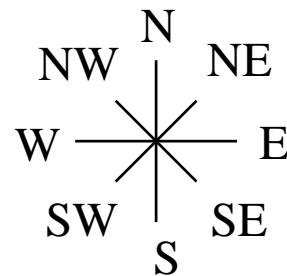
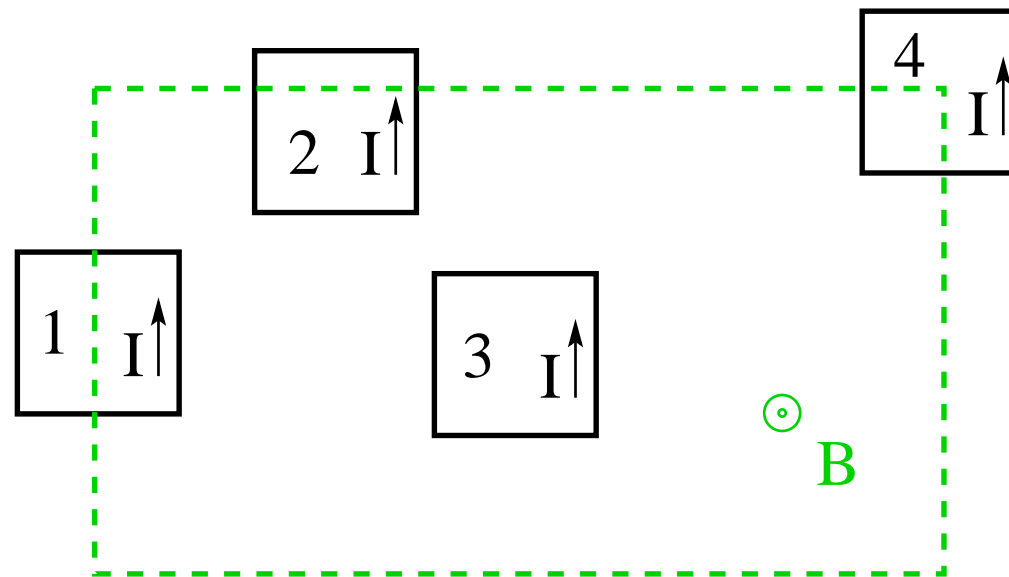


Magnetic Force Application (3)



The dashed rectangle marks a region of uniform magnetic field \vec{B} pointing out of the plane.

- Find the direction of the magnetic force acting on each loop with a ccw current I .



Velocity Selector



A charged particle is moving horizontally into a region with “crossed” uniform fields:

- an electric field \vec{E} pointing down,
- a magnetic field \vec{B} pointing into the plane.

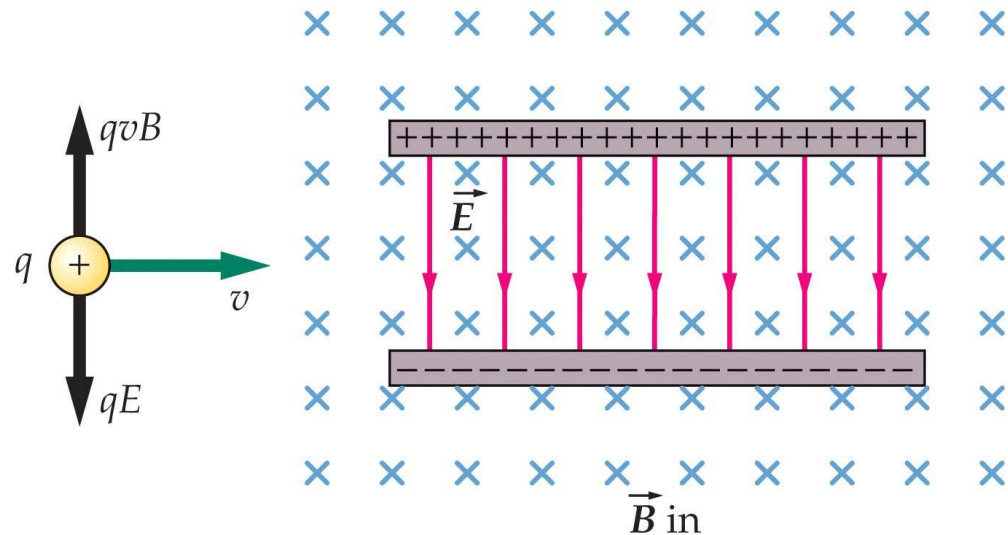
Forces experienced by particle:

- electric force $F = qE$ pointing down,
- magnetic force $F = qvB$ pointing up.

Forces in balance: $qE = qvB$.

Selected velocity: $v = \frac{E}{B}$.

Trajectories of particles with selected velocity are not bent.



Measurement of e/m for Electron



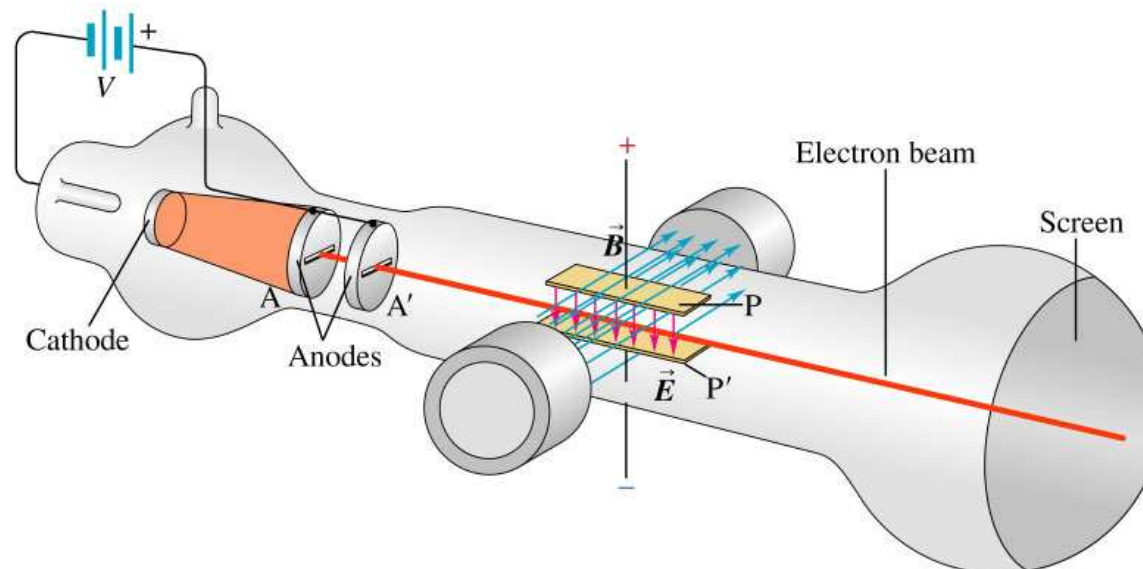
First experiment by J. J. Thomson (1897)

Method used here: velocity selector

$$\text{Equilibrium of forces: } eE = evB \Rightarrow v = \frac{E}{B}$$

$$\text{Work-energy relation: } eV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2eV}{m}}$$

$$\text{Eliminate } v: \frac{e}{m} = \frac{E^2}{2VB^2} \simeq 1.76 \times 10^{11} \text{ C/kg}$$



Measurement of e and m for Electron



First experiment by R. Millikan (1913)

Method used here: balancing weight and electric force on oil drop

Radius of oil drop: $r = 1.64\mu\text{m}$

Mass density of oil: $\rho = 0.851\text{g/cm}^3$

Electric field: $E = 1.92 \times 10^5\text{N/C}$

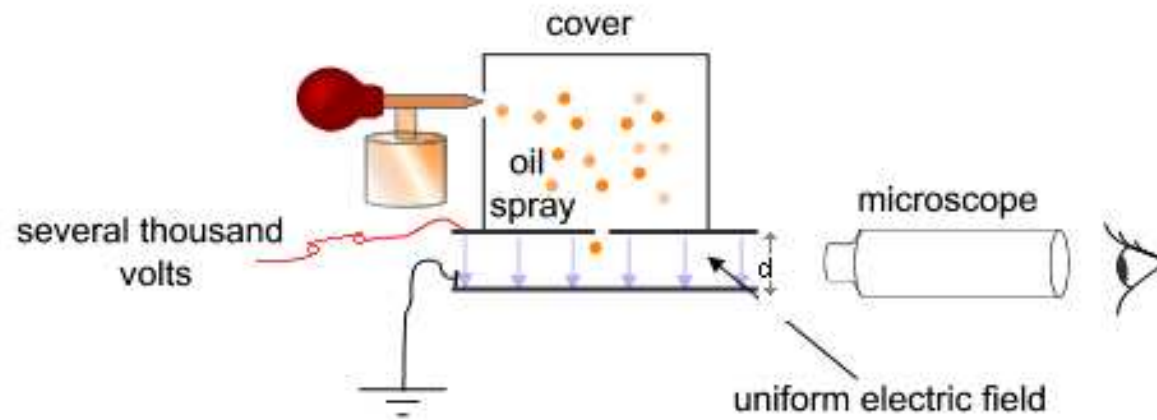
Mass of oil drop: $m = \frac{4\pi}{3}r^3\rho = 1.57 \times 10^{-14}\text{kg}$

Equilibrium of forces: $neE = mg$

Number of excess elementary charges (integer): $n = 5$

Elementary charge: $e = \frac{mg}{nE} \simeq 1.6 \times 10^{-19}\text{C}$

Mass of electron: $m \simeq 9.1 \times 10^{-31}\text{kg}$

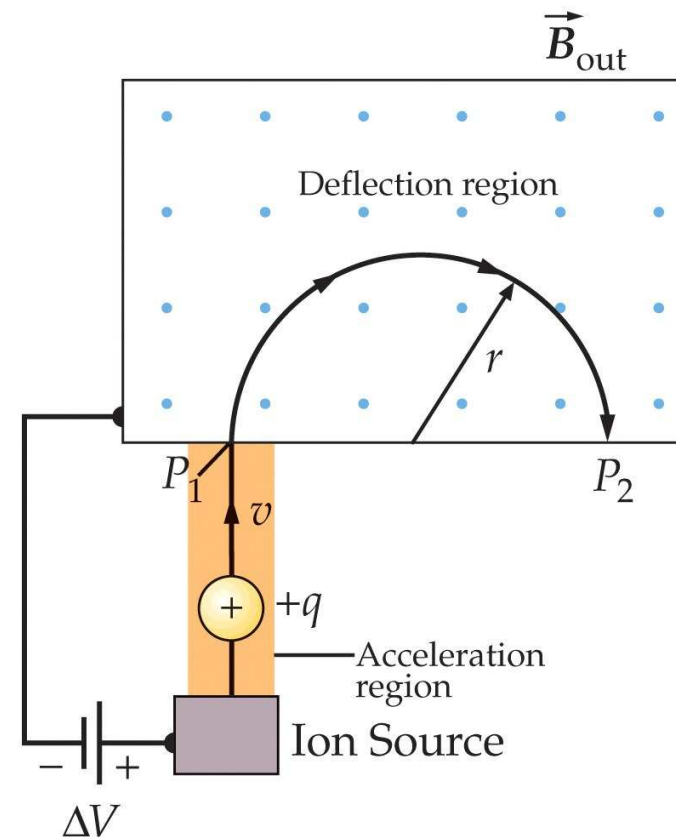


Mass Spectrometer



Purpose: measuring masses of ions.

- Charged particle is accelerated by moving through potential difference $|\Delta V|$.
- Trajectory is then bent into semicircle of radius r by magnetic field \vec{B} .
- Kinetic energy: $\frac{1}{2}mv^2 = q|\Delta V|$.
- Radius of trajectory: $r = \frac{mv}{qB}$.
- Charge: $q = e$
- Mass: $m = \frac{eB^2r^2}{2|\Delta V|}$.

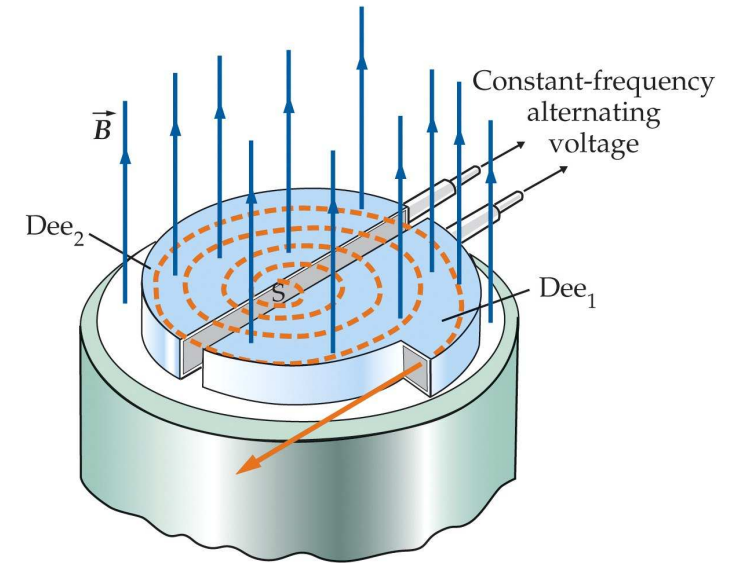


Cyclotron



Purpose: accelerate charged particles to high energy.

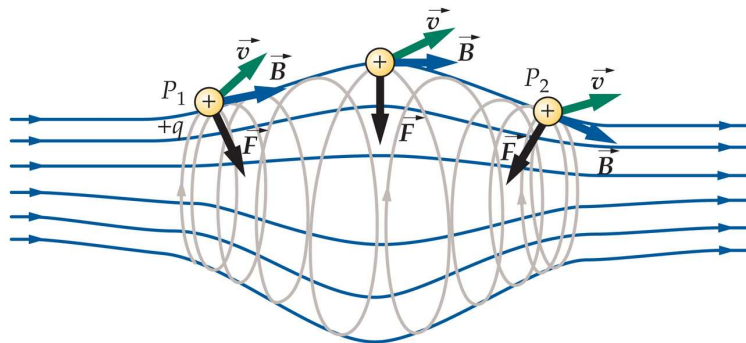
- Low-energy protons are injected at S .
- Path is bent by magnetic field \vec{B} .
- Proton is energized by alternating voltage ΔV between Dee_1 and Dee_2 .
- Proton picks up energy $\Delta K = e\Delta V$ during each half cycle.
- Path spirals out as velocity of particle increases:
Radial distance is proportional to velocity: $r = \frac{mv}{eB}$.
- Duration of cycle stays independent of r or v :
cyclotron period: $T = \frac{2\pi m}{eB}$.
- Cyclotron period is synchronized with alternation of accelerating voltage.
- High-energy protons exit at perimeter of \vec{B} -field region.



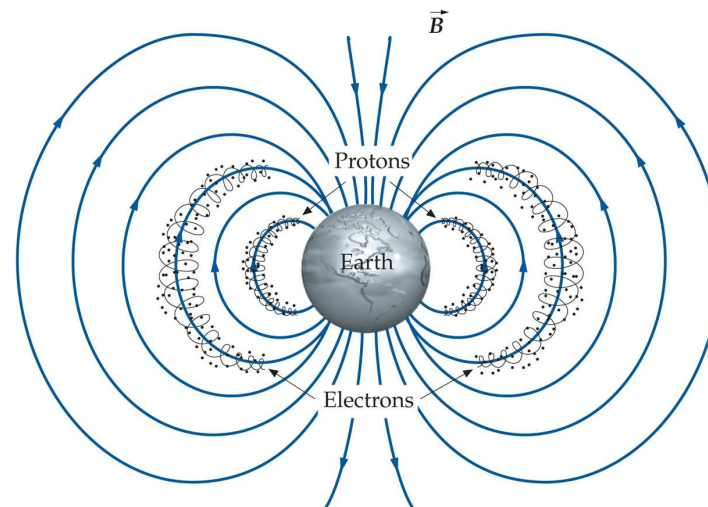
Magnetic Bottles



Moving charged particle confined by inhomogeneous magnetic field.



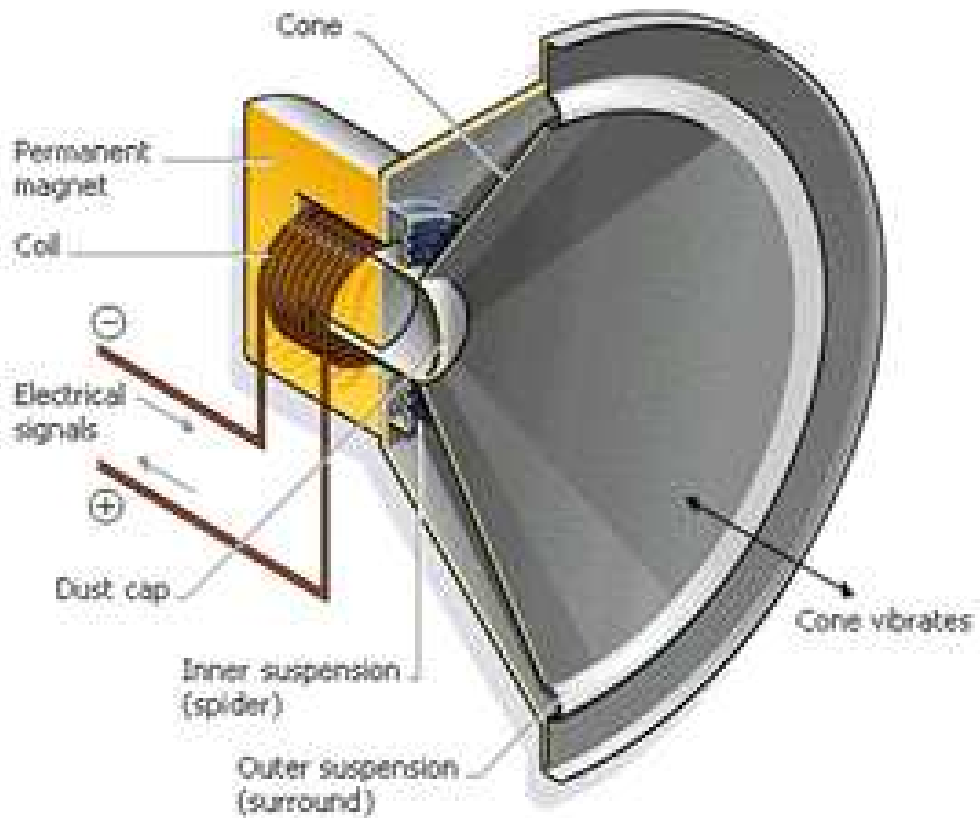
Van Allen belt: trapped protons and electrons in Earth's magnetic field.



Loudspeaker



Conversion of electric signal into mechanical vibration.

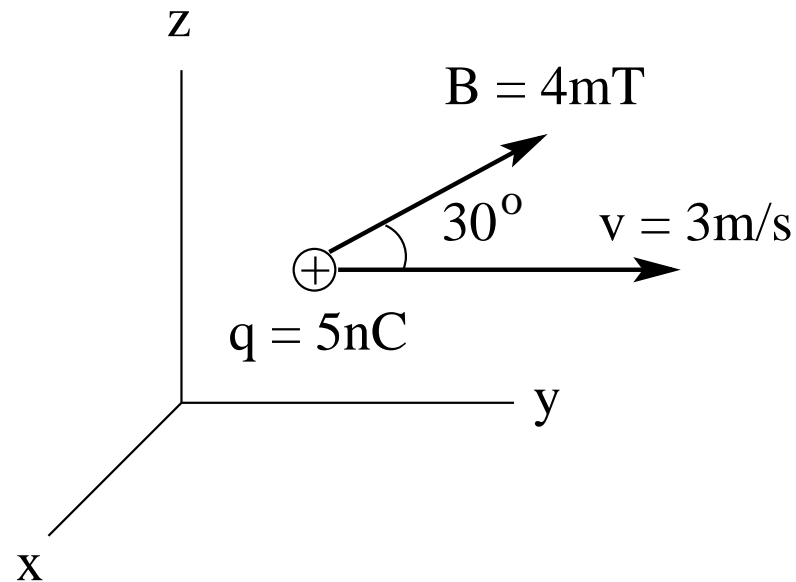


Intermediate Exam II: Problem #4 (Spring '05)



Consider a charged particle moving in a uniform magnetic field as shown. The velocity is in y -direction and the magnetic field in the yz -plane at 30° from the y -direction.

- Find the direction of the magnetic force acting on the particle.
- Find the magnitude of the magnetic force acting on the particle.

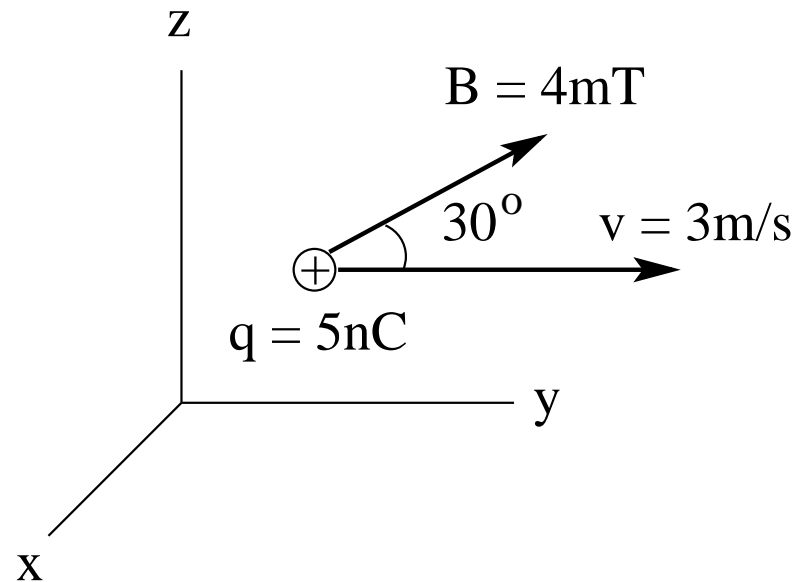


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Solution:

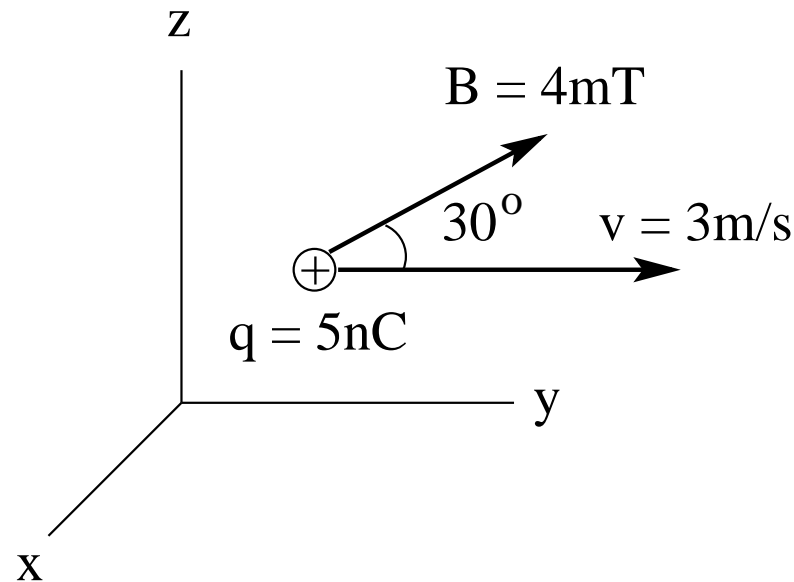
- Use the right-hand rule: positive x -direction (front, out of page).

Intermediate Exam II: Problem #4 (Spring '05)



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- Find the direction of the magnetic force acting on the particle.
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Solution:

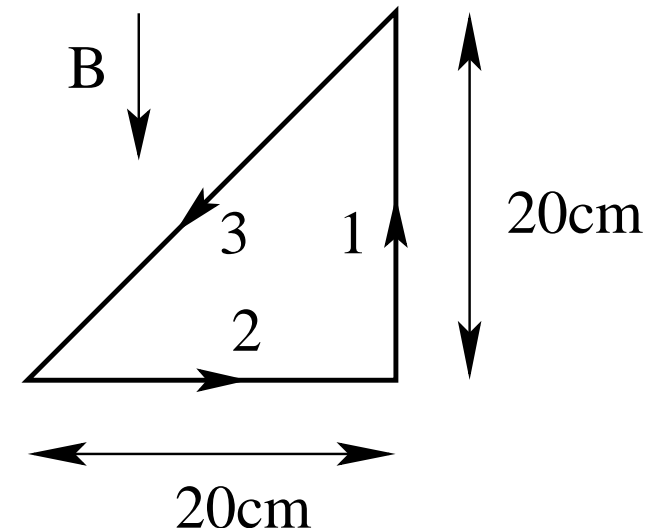
- Use the right-hand rule: positive x -direction (front, out of page).
- $F = qvB \sin 30^\circ = (5 \times 10^{-9}\text{C})(3\text{m/s})(4 \times 10^{-3}\text{T})(0.5) = 3 \times 10^{-11}\text{N}$.

Intermediate Exam II: Problem #4 (Spring '06)



A current loop in the form of a right triangle is placed in a uniform magnetic field of magnitude $B = 30\text{mT}$ as shown. The current in the loop is $I = 0.4\text{A}$ in the direction indicated.

- (a) Find magnitude and direction of the force \vec{F}_1 on side 1 of the triangle.
- (b) Find magnitude and direction of the force \vec{F}_2 on side 2 of the triangle.

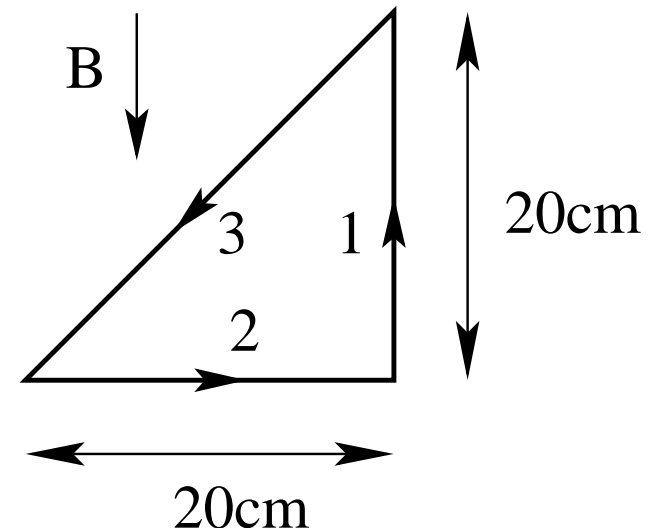


Intermediate Exam II: Problem #4 (Spring '06)



A current loop in the form of a right triangle is placed in a uniform magnetic field of magnitude $B = 30\text{mT}$ as shown. The current in the loop is $I = 0.4\text{A}$ in the direction indicated.

- (a) Find magnitude and direction of the force \vec{F}_1 on side 1 of the triangle.
- (b) Find magnitude and direction of the force \vec{F}_2 on side 2 of the triangle.



Solution:

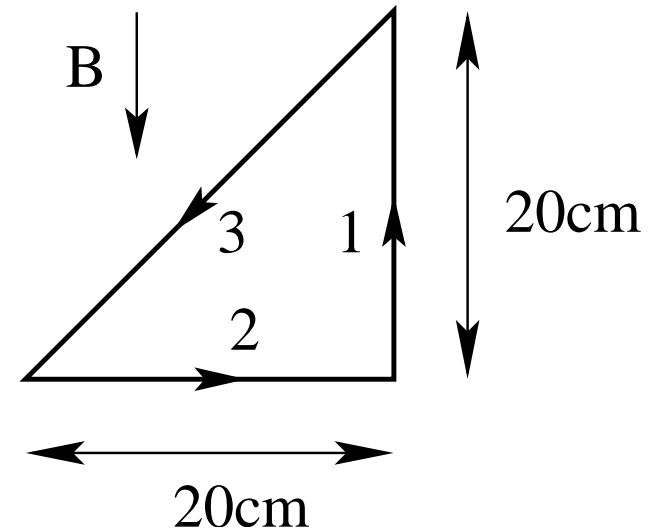
- (a) $\vec{F}_1 = I\vec{L} \times \vec{B} = 0$ (angle between \vec{L} and \vec{B} is 180°).

Intermediate Exam II: Problem #4 (Spring '06)



A current loop in the form of a right triangle is placed in a uniform magnetic field of magnitude $B = 30\text{mT}$ as shown. The current in the loop is $I = 0.4\text{A}$ in the direction indicated.

- (a) Find magnitude and direction of the force \vec{F}_1 on side 1 of the triangle.
- (b) Find magnitude and direction of the force \vec{F}_2 on side 2 of the triangle.



Solution:

- (a) $\vec{F}_1 = I\vec{L} \times \vec{B} = 0$ (angle between \vec{L} and \vec{B} is 180°).
- (b) $F_2 = ILB = (0.4\text{A})(0.2\text{m})(30 \times 10^{-3}\text{T}) = 2.4 \times 10^{-3}\text{N}$.
Direction of \vec{F}_2 : \otimes (into plane).

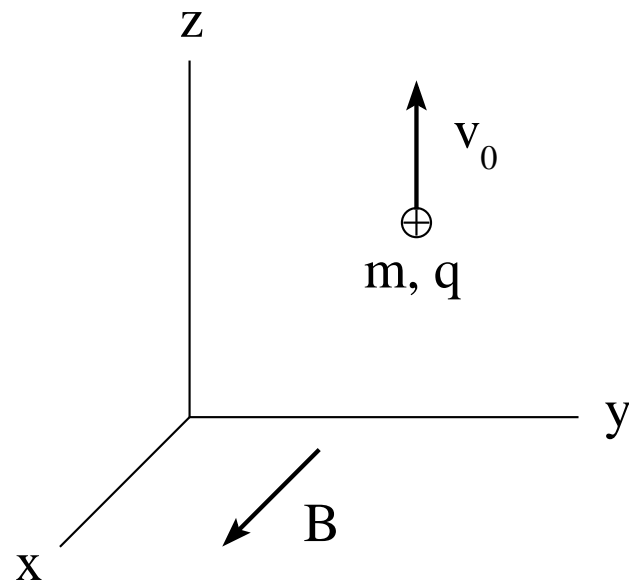
Unit Exam III: Problem #1 (Spring '12)



In a region of uniform magnetic field $\mathbf{B} = 5\text{mT}\hat{\mathbf{i}}$, a proton

($m = 1.67 \times 10^{-27}\text{kg}$, $q = 1.60 \times 10^{-19}\text{C}$) is launched with velocity $\mathbf{v}_0 = 4000\text{m/s}\hat{\mathbf{k}}$.

- (a) Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.
- (b) Calculate the radius r of the circular path.
- (c) Calculate the time T it takes the proton to go around that circle once.
- (d) Sketch the circular path of the proton in the graph.



Unit Exam III: Problem #1 (Spring '12)



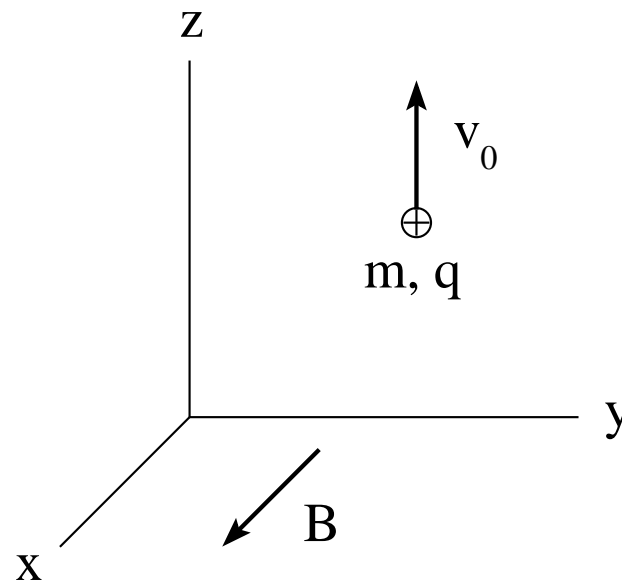
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- (a) Calculate the magnitude F of the magnetic force that keeps the proton on a circular path.
- (b) Calculate the radius r of the circular path.
- (c) Calculate the time T it takes the proton to go around that circle once.
- (d) Sketch the circular path of the proton in the graph.

Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\text{N}$.



Unit Exam III: Problem #1 (Spring '12)



In a region of uniform magnetic field $\mathbf{B} = 5\text{mT}\hat{\mathbf{i}}$, a proton

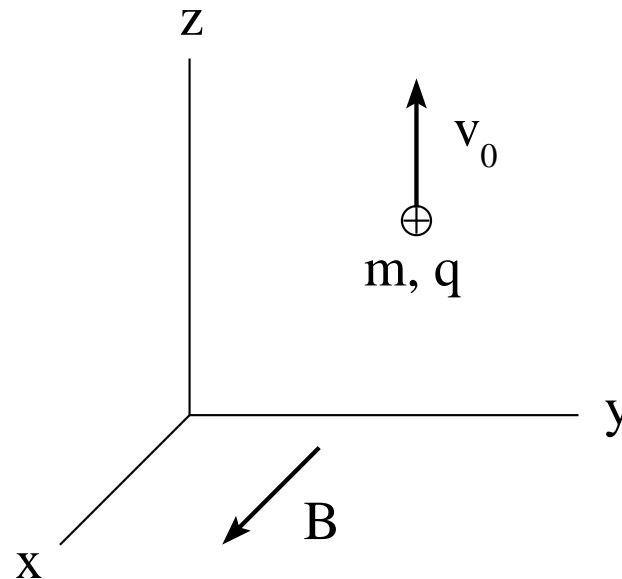
($m = 1.67 \times 10^{-27}\text{kg}$, $q = 1.60 \times 10^{-19}\text{C}$) is launched with velocity $\mathbf{v}_0 = 4000\text{m/s}\hat{\mathbf{k}}$.

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- (b) Calculate the radius r of the circular path.
- (c) Calculate the time T it takes the proton to go around that circle once.
- (d) Sketch the circular path of the proton in the graph.

Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\text{N}$.

(b) $\frac{mv_0^2}{r} = qv_0B \Rightarrow r = \frac{mv_0}{qB} = 8.35\text{mm}$.



Unit Exam III: Problem #1 (Spring '12)



In a region of uniform magnetic field $\mathbf{B} = 5\text{mT}\hat{\mathbf{i}}$, a proton

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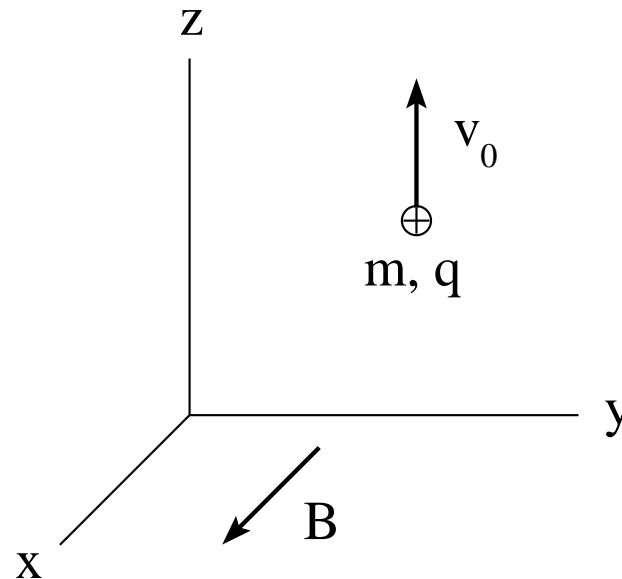
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- Calculate the radius r of the circular path.
- Calculate the time T it takes the proton to go around that circle once.
- Sketch the circular path of the proton in the graph.

Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\text{N}$.

(b) $\frac{mv_0^2}{r} = qv_0B \Rightarrow r = \frac{mv_0}{qB} = 8.35\text{mm}$.

(c) $T = \frac{2\pi r}{v_0} = \frac{2\pi m}{qB} = 13.1\mu\text{s}$.



Unit Exam III: Problem #1 (Spring '12)



In a region of uniform magnetic field $\mathbf{B} = 5\text{mT}\hat{\mathbf{i}}$, a proton

($m = 1.67 \times 10^{-27}\text{kg}$, $q = 1.60 \times 10^{-19}\text{C}$) is launched with velocity $\mathbf{v}_0 = 4000\text{m/s}\hat{\mathbf{k}}$.

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- Calculate the time T it takes the proton to go around that circle once.
- Sketch the circular path of the proton in the graph.

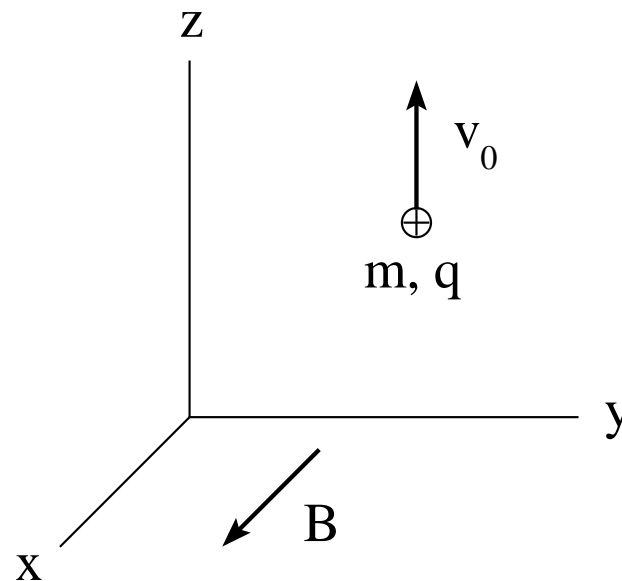
Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\text{N}$.

(b) $\frac{mv_0^2}{r} = qv_0B \Rightarrow r = \frac{mv_0}{qB} = 8.35\text{mm}$.

(c) $T = \frac{2\pi r}{v_0} = \frac{2\pi m}{qB} = 13.1\mu\text{s}$.

(d) Center of circle to the right of proton's initial position (cw motion).

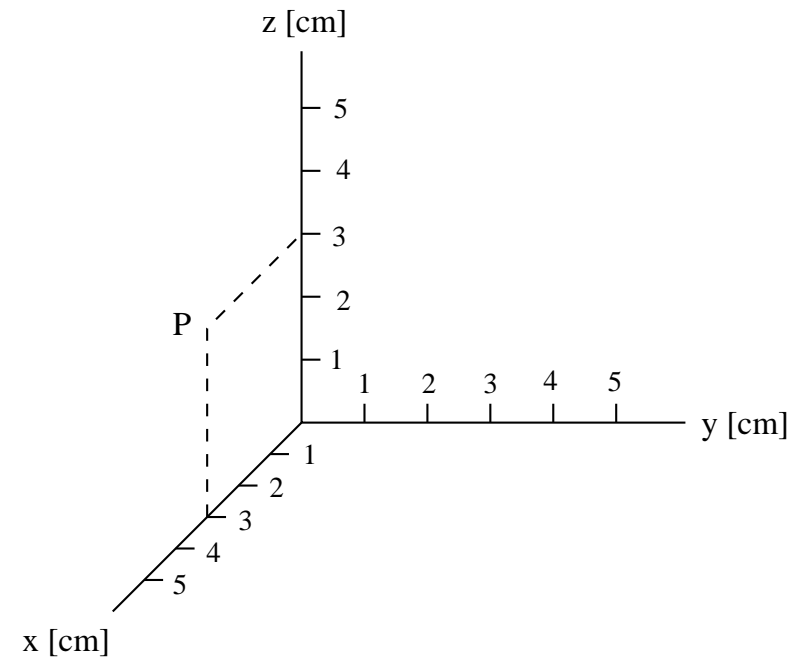


Unit Exam III: Problem #1 (Spring '13)



In a region of uniform magnetic field \mathbf{B} a proton ($m = 1.67 \times 10^{-27} \text{kg}$, $q = 1.60 \times 10^{-19} \text{C}$) experiences a force $\mathbf{F} = 8.0 \times 10^{-19} \text{N} \hat{\mathbf{i}}$ as it passes through point P with velocity $\mathbf{v}_0 = 2000 \text{m/s} \hat{\mathbf{k}}$ on a circular path.

- (a) Find the magnetic field \mathbf{B} (magnitude and direction).
- (b) Calculate the radius r of the circular path.
- (c) Locate the center C of the circular path in the coordinate system on the page.



Unit Exam III: Problem #1 (Spring '13)

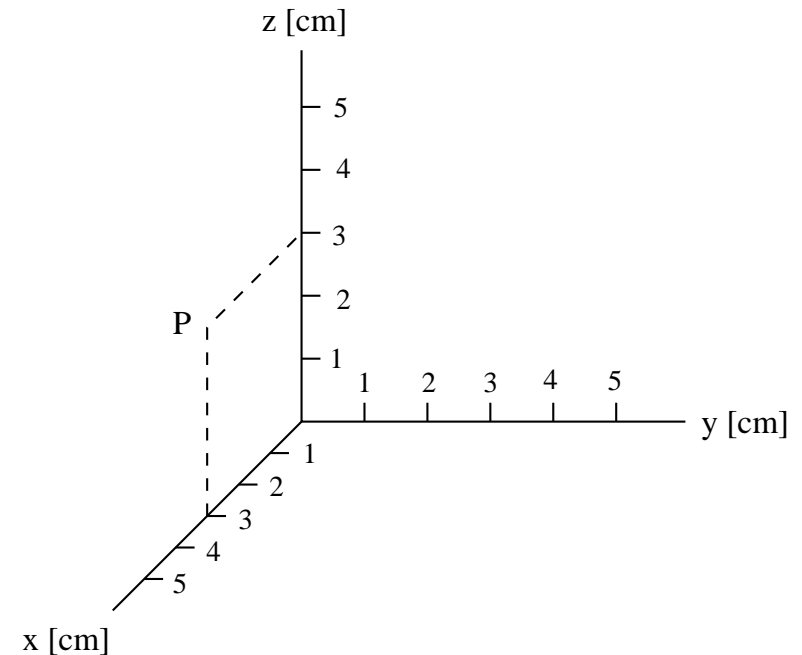


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- (b) Calculate the radius r of the circular path.
- (c) Locate the center C of the circular path in the coordinate system on the page.

Solution:

$$(a) \quad B = \frac{F}{qv_0} = 2.50 \times 10^{-3} \text{T}, \quad \hat{\mathbf{i}} = \hat{\mathbf{k}} \times (-\hat{\mathbf{j}})$$
$$\Rightarrow \mathbf{B} = -2.50 \times 10^{-3} \text{T} \hat{\mathbf{j}}.$$



Unit Exam III: Problem #1 (Spring '13)



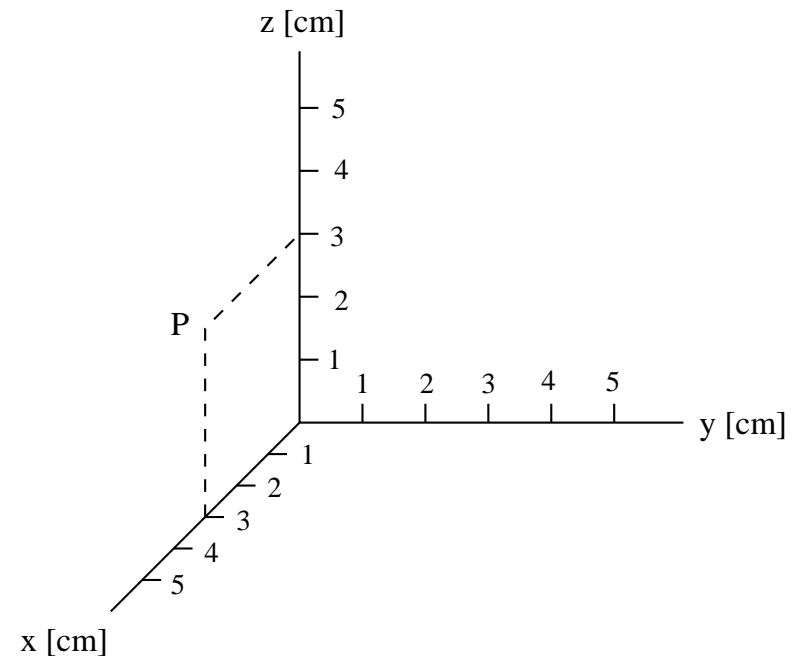
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$$(b) \quad F = \frac{mv_0^2}{r} = qv_0 B$$
$$\Rightarrow r = \frac{mv_0^2}{F} = \frac{mv_0}{qB} = 0.835 \text{cm}.$$



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$$(c) \quad C = 3.84 \text{cm} \hat{\mathbf{i}} + 3.00 \text{cm} \hat{\mathbf{k}}.$$

