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## Bifurcation in Deterministic Discrete Dynamical Systems: Advances in Theory and Applications

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## Editorial

# Bifurcation in Deterministic Discrete Dynamical Systems: Advances in Theory and Applications

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Bifurcation theory is a major focus of recent studies in continuous and discrete dynamical systems. Central to this area of research is the question whether the qualitative properties of a dynamical system change when one or more of the parameters change. Indeed, the parameters can be viewed as controllers which can be fine-tuned to force the system under study to behave in a desirable way.

In this special issue of the journal, we focused our attention on the different types of bifurcation in deterministic discrete dynamical systems. Specifically, the contributors investigated special types of bifurcation such as the global-period doubling bifurcation, discrete Hopf bifurcation, flip-bifurcation of some classes of bilinear rational difference equations of order two, and first-order systems of bilinear rational difference equations.

L. Gori et al. developed a cobweb model with discrete-time delays that characterize the length of production cycle. They assumed a market comprised of homogeneous producers that operate as adapters by taking the (expected) profit-maximizing quantity as a target to adjust production and consumers with a marginal willingness to pay captured by an isoelastic demand. The dynamics of the economy is characterized by a one-dimensional delay differential equation. They showed that if the elasticity of market demand is sufficiently

high, the steady-state equilibrium is locally asymptotically stable and if the elasticity of market demand is sufficiently low, quasiperiodic oscillations emerge when the time lag (that represents the length of production cycle) is large enough.

Although oligopoly theory is generally concerned with a single-product firm, what is true in the real world is that most of the firms offer multiproducts rather than single product in order to obtain cost-saving advantages, cater for the diversity of consumer tastes, and provide a barrier to entry. F. Wu and J. Ma developed a dynamical multiproduct Cournot duopoly model in discrete time, where each firm is an owner who delegates the output decision to a manager. The principle of decision-making is bounded rational and each firm has a nonlinear total cost function due to the multiproduct framework. The Cournot-Nash equilibrium and the local stability were investigated. The tangential bifurcation and intermittent chaos are reported by numerical simulations. The results show that high output adjustment speed can lead to output fluctuations which are characterized by phases of low volatility with small output changes and phases of high volatility with large output changes. The intermittent route to chaos of flip bifurcation and another intermittent route of flip bifurcation which contains Hopf bifurcation can exist in the system.

S. Kalabušić et al. investigated the local stability and the global asymptotic stability of the difference equation:

$$x_{n+1} = \frac{\alpha x_n^2 + \beta x_n x_{n-1} + \gamma x_{n-1}}{A x_n^2 + B x_n x_{n-1} + C x_{n-1}}, \quad (1)$$

where all the parameters and initial conditions are nonnegative and the denominator is always positive. They obtained the local stability of the equilibrium for all values of parameters and gave some global asymptotic stability results for some values of the parameters. They obtained global dynamics in the special case, where  $\beta = B = 0$ , in which case they showed that such equation exhibits a global period-doubling bifurcation.

R. Ma and C. Gao considered the spectrum of discrete linear second-order eigenvalue problems  $\Delta^2 u(t-1) + \lambda m(t)u(t) = 0$ ,  $t \in \mathbf{T} = \{1, 2, \dots, T\}$ , where  $T > 1$  is an integer and  $\lambda$  is a parameter;  $m : \mathbf{T} \rightarrow \mathbf{R}$  changes sign and  $m(t) \neq 0$  on  $\mathbf{T}$ . Furthermore, as an application of this spectrum result, they showed the existence of sign-changing solutions of discrete nonlinear second-order problems by using bifurcation technique.

In the paper entitled "Multiple Bifurcations and Chaos in a Discrete Prey-Predator System with Generalized Holling III Functional Response," X. Liu et al. considered a prey-predator system with the strong Allee effect, and generalized Holling type III functional response was presented and discretized. It was shown that the combined influences of Allee effect and step size have an important effect on the dynamics of the system. The existences of flip and Neimark-Sacker bifurcations and strange attractors and chaotic bands were investigated by using the center manifold theorem and bifurcation theory and some numerical methods.

In conclusion, we hope that these papers will enrich our readers and stimulate researchers to extend, generalize, and apply the established results.

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