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11. RC Circuits

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Abstract
Lecture slides 11 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.
Some of the slides contain figures from the textbook, Paul A. Tipler and Gene Mosca. *Physics for Scientists and Engineers*, 5th/6th editions. The copyright to these figures is owned by W.H. Freeman. We acknowledge permission from W.H. Freeman to use them on this course web page. The textbook figures are not to be used or copied for any purpose outside this class without direct permission from W.H. Freeman.

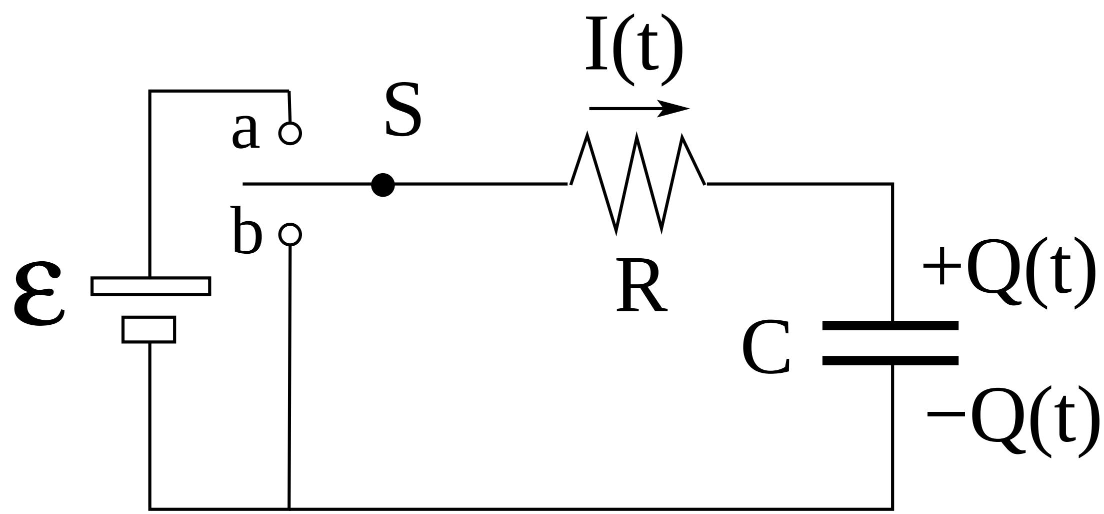
RC Circuit: Fundamentals

Specifications:

- $\mathcal{E}$ (emf)
- R (resistance)
- C (capacitance)

Switch S:

- a: charging
- b: discharging



Time-dependent quantities:

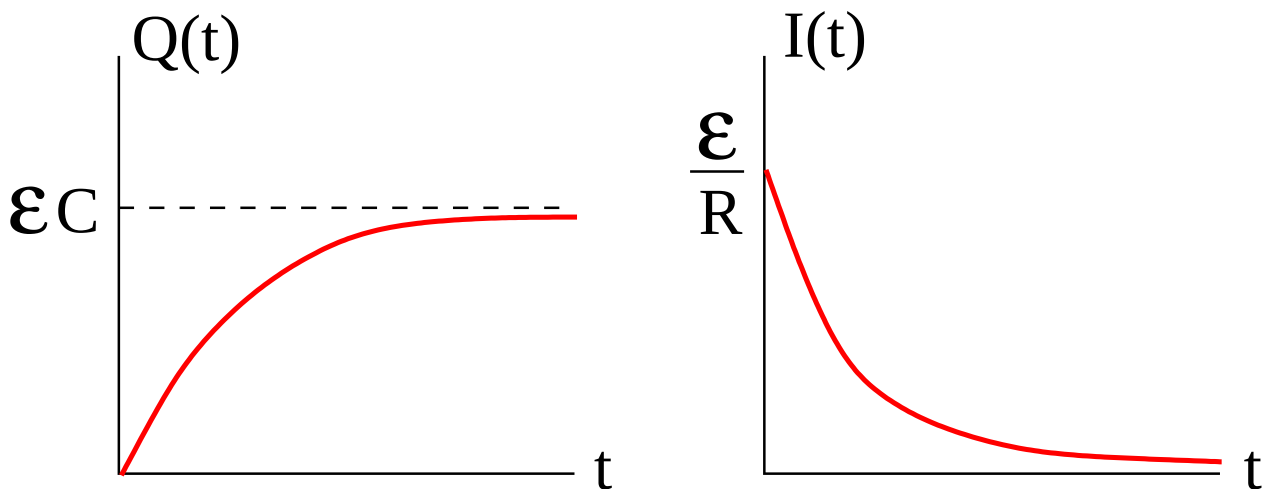
- $Q(t)$: instantaneous charge on capacitor
- $I(t) = \dfrac{dQ}{dt}$: instantaneous current
- $V_R(t) = I(t)R$: instantaneous voltage across resistor
- $V_C(t) = \dfrac{Q(t)}{C}$: instantaneous voltage across capacitor

RC Circuit: Charging the Capacitor

- Loop rule: $\mathcal{E} - IR - \dfrac{Q}{C} = 0$
- Differential equation: $R\dfrac{dQ}{dt} + \dfrac{Q}{C} = \mathcal{E} \quad \Rightarrow \quad \dfrac{dQ}{dt} = \dfrac{\mathcal{E}C - Q}{RC}$

$$\int_0^Q \frac{dQ}{\mathcal{E}C - Q} = \int_0^t \frac{dt}{RC} \quad \Rightarrow \quad -\ln\left(\frac{\mathcal{E}C - Q}{\mathcal{E}C}\right) = \frac{t}{RC} \quad \Rightarrow \quad \frac{\mathcal{E}C - Q}{\mathcal{E}C} = e^{-t/RC}$$

- Charge on capacitor: $Q(t) = \mathcal{E}C\left[1 - e^{-t/RC}\right]$
- Current through resistor: $I(t) \equiv \dfrac{dQ}{dt} = \dfrac{\mathcal{E}}{R}\, e^{-t/RC}$

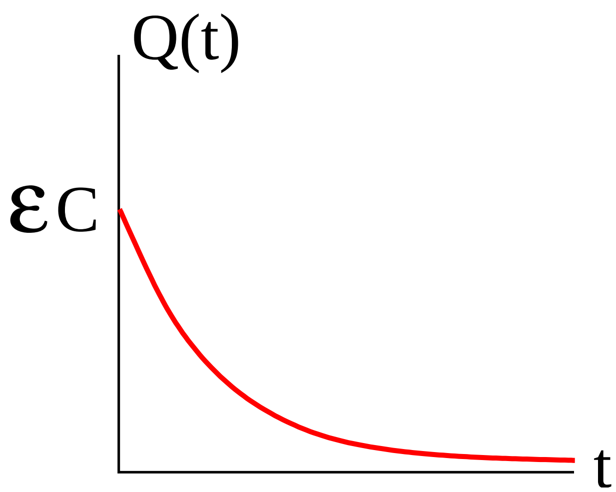


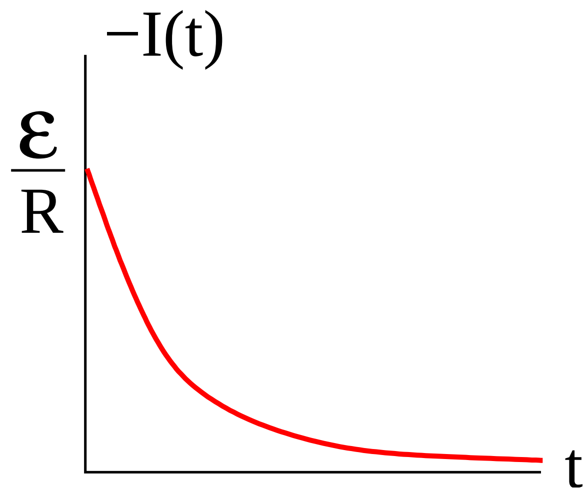
RC Circuit: Discharging the Capacitor

- Loop rule: $IR + \dfrac{Q}{C} = 0$
- Differential equation: $R\dfrac{dQ}{dt} + \dfrac{Q}{C} = 0 \quad \Rightarrow \dfrac{dQ}{dt} = -\dfrac{Q}{RC}$

$$\Rightarrow \int_{\mathcal{E}C}^{Q} \frac{dQ}{Q} = -\int_0^t \frac{dt}{RC} \quad \Rightarrow \ln\left(\frac{Q}{\mathcal{E}C}\right) = -\frac{t}{RC} \quad \Rightarrow \frac{Q}{\mathcal{E}C} = e^{-t/RC}$$

- Charge on capacitor: $Q(t) = \mathcal{E}Ce^{-t/RC}$
- Current through resistor: $I(t) \equiv \dfrac{dQ}{dt} = -\dfrac{\mathcal{E}}{R}\, e^{-t/RC}$



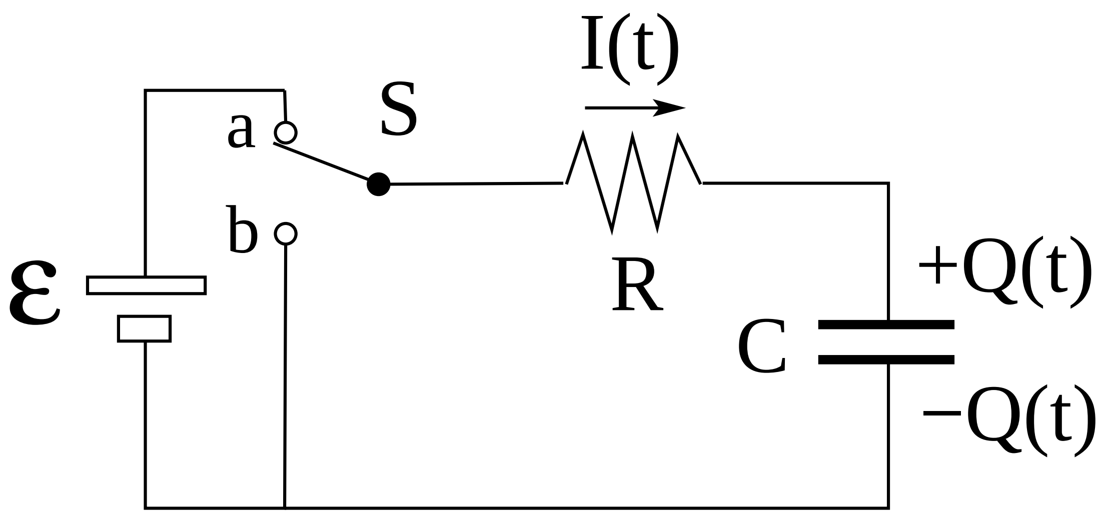


RC Circuit: Energy Transfer While Charging

Loop rule: $IR + \dfrac{Q}{C} = \mathcal{E}$ (I is positive)

- $I\mathcal{E}$: rate at which emf source delivers energy
- $IV_R = I^2R$: rate at which energy is dissipated in resistor
- $IV_C = \dfrac{IQ}{C}$: rate at which energy is stored in capacitor

Balance of energy transfer: $I^2R + \dfrac{IQ}{C} = I\mathcal{E}$

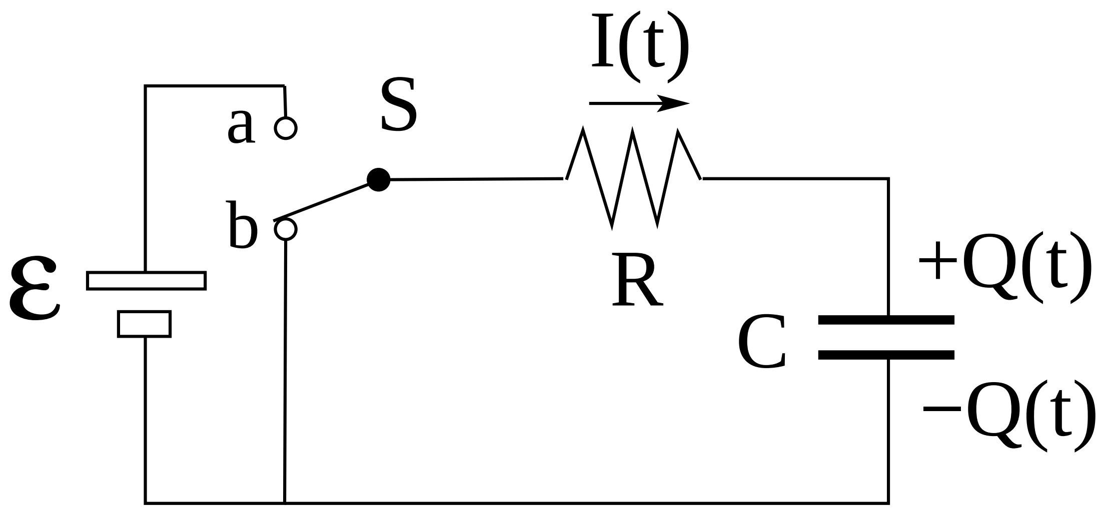


RC Circuit: Energy Transfer While Discharging

Loop rule: $IR + \dfrac{Q}{C} = 0$ (I is negative)

- $IV_R = I^2R$: rate at which energy is dissipated in resistor
- $IV_C = \dfrac{IQ}{C}$: rate at which capacitor releases energy

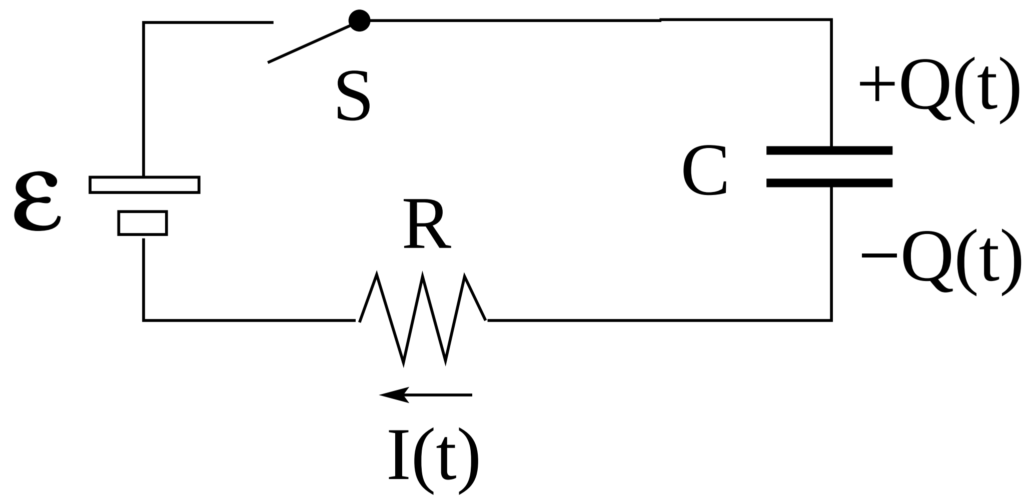
Balance of energy transfer: $I^2R + \dfrac{IQ}{C} = 0$



RC Circuit: Some Physical Properties

Specification of RC circuit by 3 device properties:

- $\mathcal{E}$ [V] (emf)
- R $[\Omega]$ (resistance)
- C [F] (capacitance)



Physical properties of RC circuit during charging process determined by 3 combinations of the device properties:

- $\mathcal{E}/R = I(t=0)$: rate at which current flows onto capacitor initially
- $\mathcal{E}C = Q(t=\infty)$: total charge placed on capacitor ultimately
- $RC = \tau$: time it takes to place 63% of the charge onto the capacitor
  $[1 - e^{-1} = 0.632\ldots]$

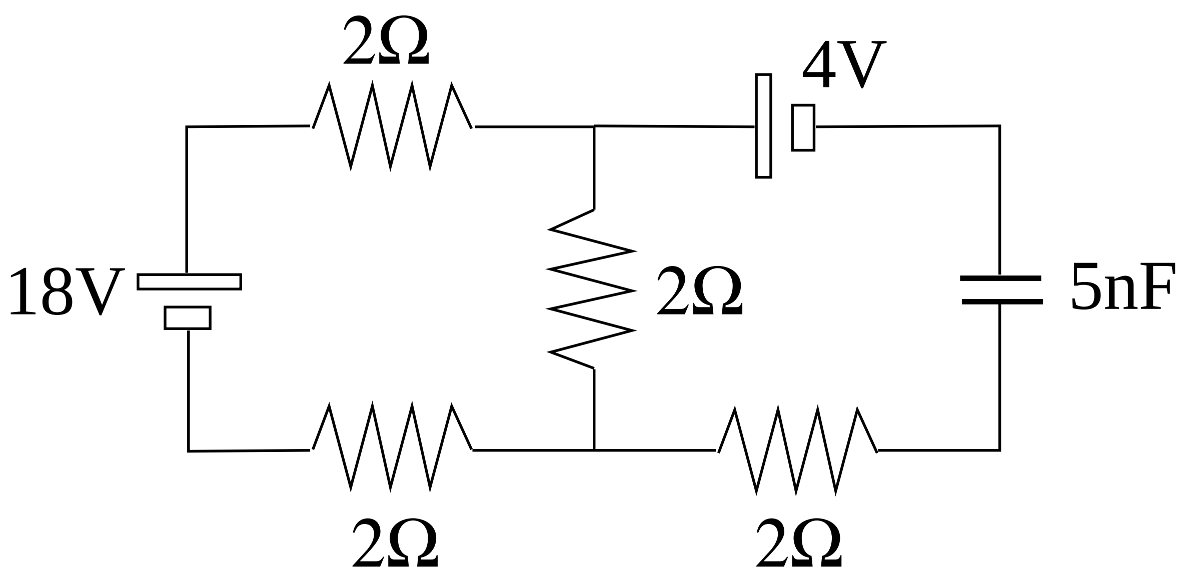
RC Circuit: Application (1)

This circuit has been running for a very long time.

(a) Find the current through the 18V battery.

(b) Find the total power dissipated in the resistors.

(c) Find the charge stored on the capacitor.



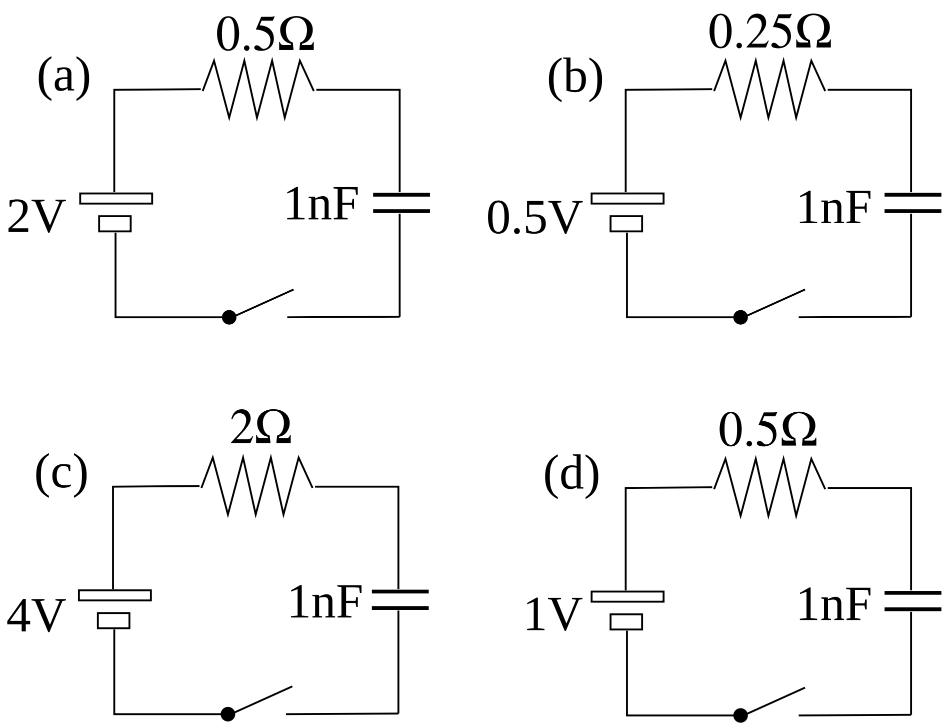
The switches are closed at $t = 0$. This begins the charging process in each RC circuit.

Name the circuit in which...

(i) the charge flows into the capacitor at the highest rate initially,

(ii) the capacitor has the most charge ultimately,

(iii) the capacitor is 63% full in the shortest time.



RC Circuit: Application (3)

At time $t = 0$ the capacitor in this circuit is discharged and the switch is being closed.

Find the current I_1

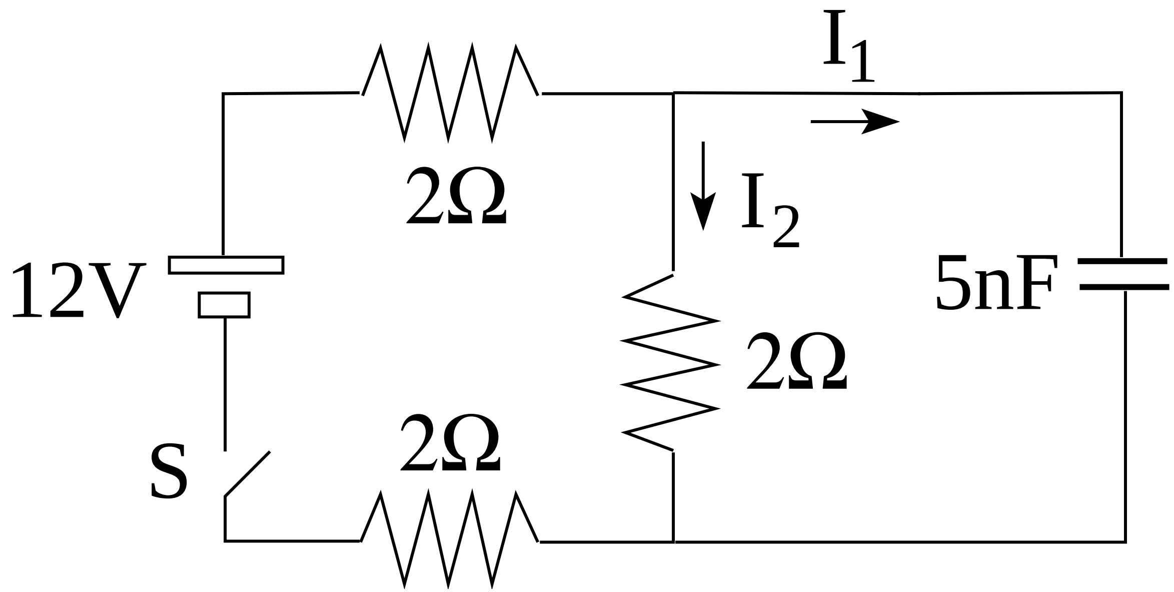
(a) at $t = 0$,

(b) at $t = \infty$.

Find the current I_2

(c) at $t = 0$,

(d) at $t = \infty$.

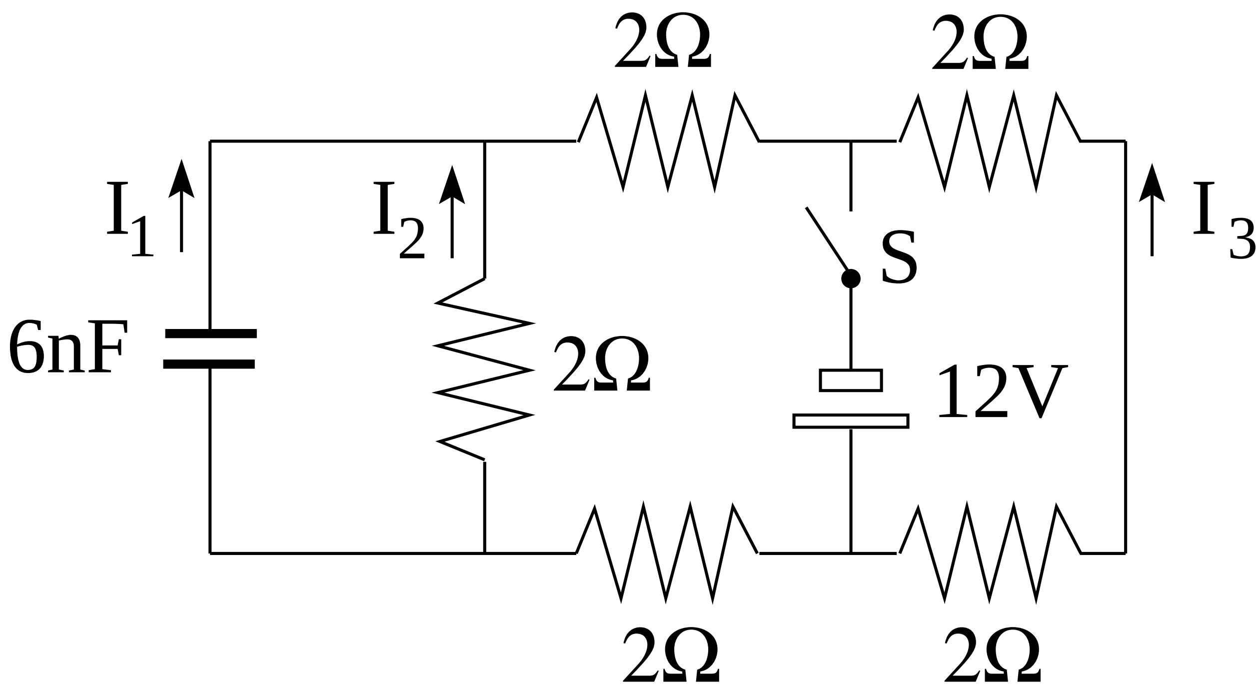


RC Circuit: Application (4)

In this 3-loop RC circuit, the switch S is closed at time $t = 0$.

(a) Find the currents I_1, I_2, I_3 just after the switch has been closed.

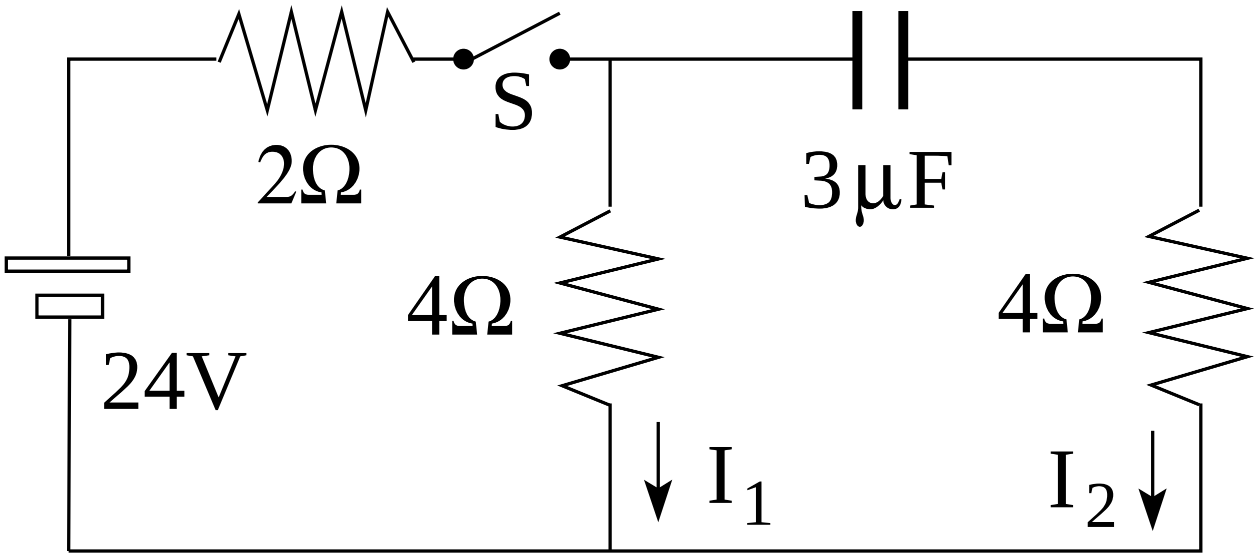
(b) Find the currents I_1, I_2, I_3 a very long time later.



In the RC circuit shown, the switch S has been open for a long time.

(a) Find the currents I_1 and I_2 immediately after the switch has been closed.

(b) Find the currents I_1 and I_2 a very long time later.



RC Circuit: Application (6)

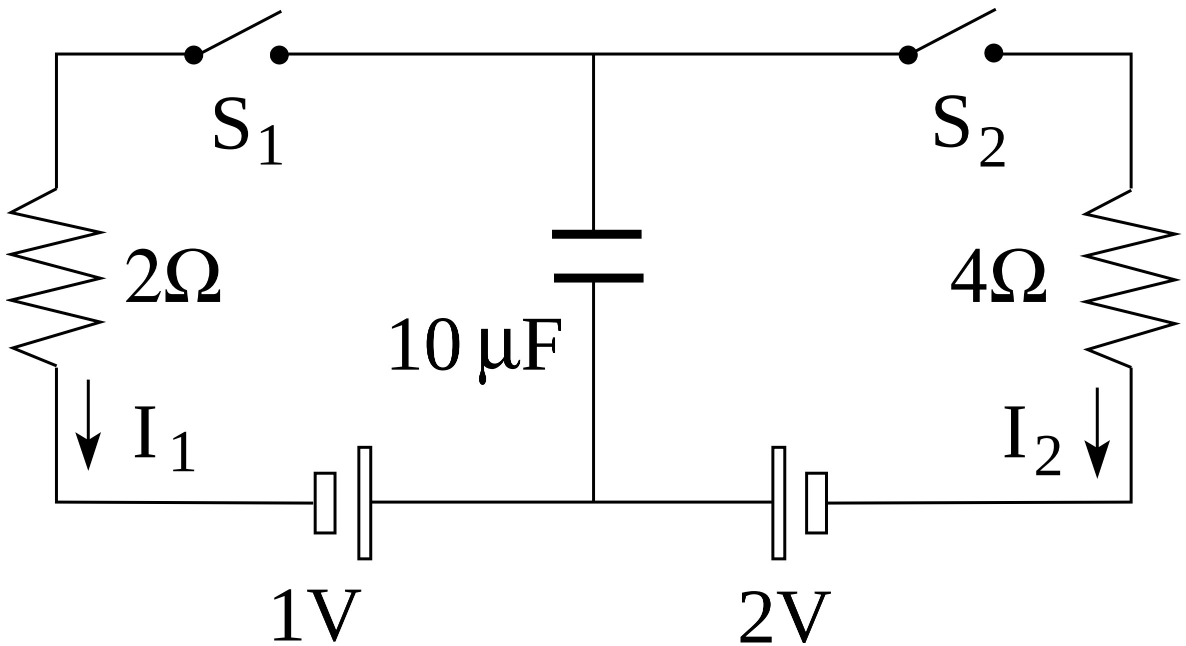
In the RC circuit shown, both switches are initially open and the capacitor is discharged.

(a) Close switch S_1 and find the currents I_1 and I_2 immediately afterwards.

(b) Find the currents $I_1 I_2$ and the charge Q on the capacitor a very long time later.

(c) Now close switch S_2 also and find the currents I_1 and I_2 immediately afterwards.

(d) Find the currents I_1, I_2 and the charge Q on the capacitor a very long time later.



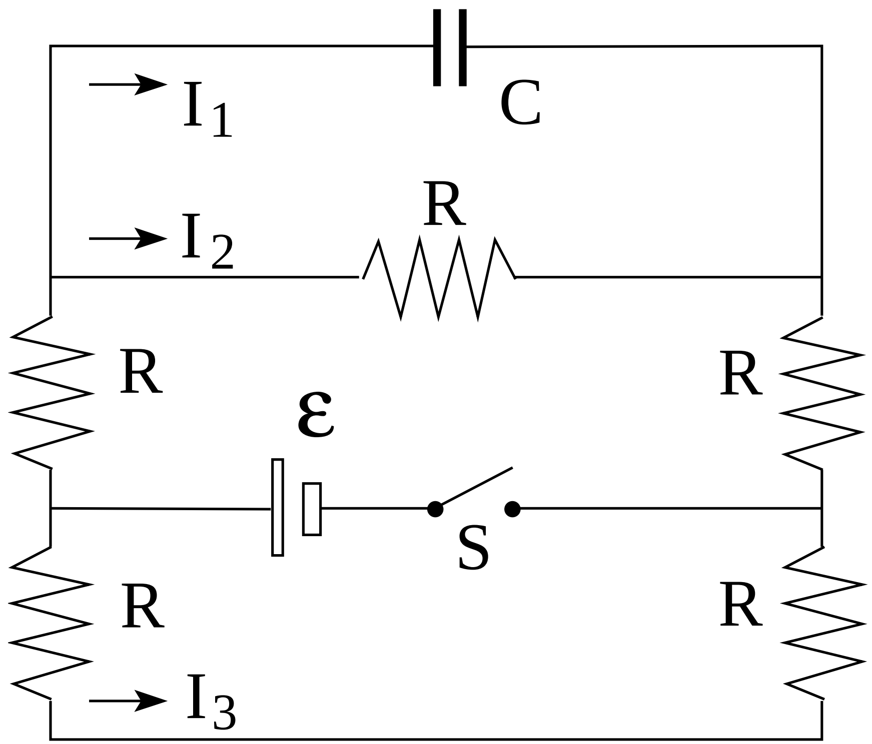
RC Circuit: Application (7)

In the RC circuit shown, the switch has been open for a long time.

Find the currents I_1, I_2, I_3 and the charge Q on the capacitor

(a) right after the switch has been closed,

(b) a very long time later.



$R = 2\Omega$

$C = 6\mu F$

$\mathcal{E} = 12V$

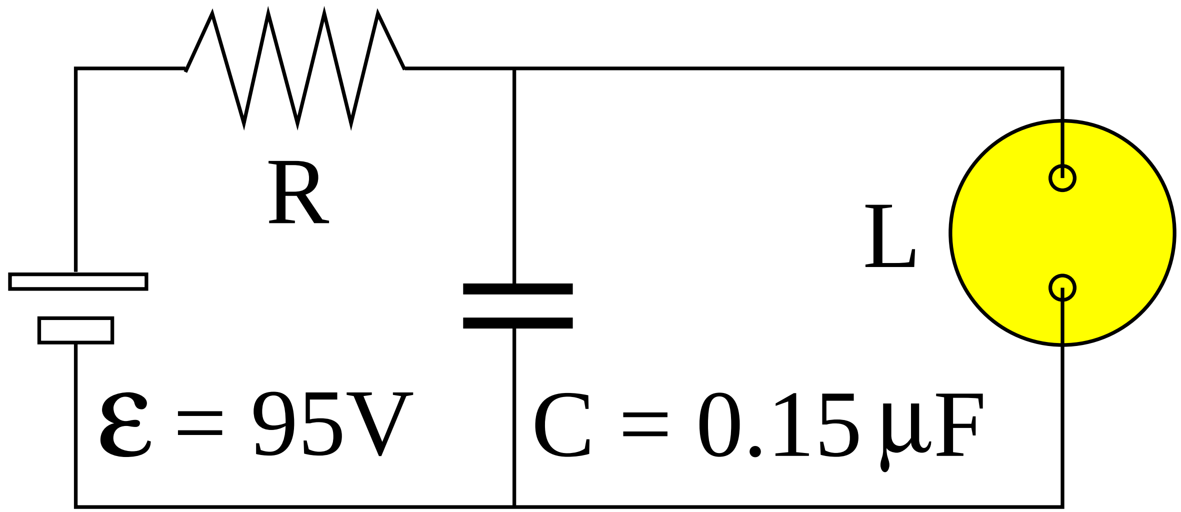
RC Circuit: Application (8)

The circuit shown is that of a flashing lamp, such as are attached to barrels at highway construction sites.

The power is supplied by a battery with $\mathcal{E} = 95V$. The fluorescent lamp L is connected in parallel to the capacitor with $C = 0.15\mu F$ of an RC circuit.

Current passes through the lamp only when the potential difference across it reaches the breakdown voltage $V_L = 72V$. In this event, the capacitor discharges through the lamp, and the lamp flashes briefly.

Suppose that two flashes per second are needed. What should the resistance R be?



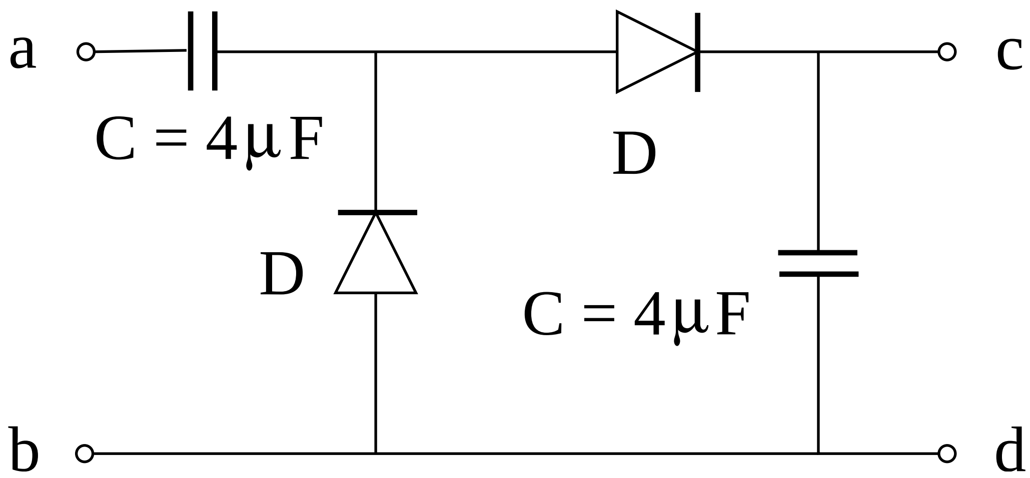
Circuit of Capacitors and Diodes

The circuit shown contains two identical capacitors and two ideal diodes. A 100V battery is connected to the two input terminals a and b. Find the voltage at the output terminals c and d

(1) if a is the positive input terminal,

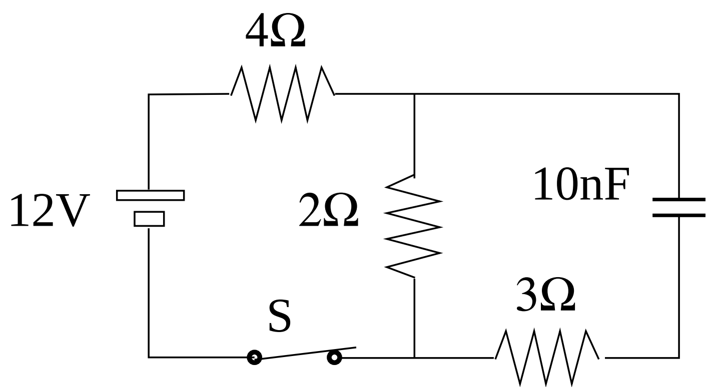
(2) if b is the positive input terminal.

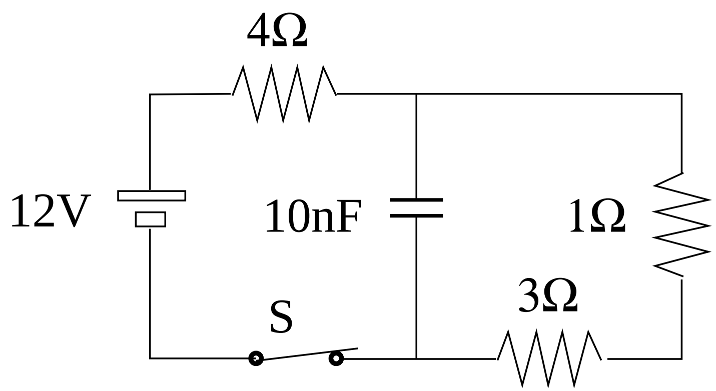
Note: An ideal diode is a perfect one-way street for electric currents. It lets a current through unimpeded in the direction of the arrow and totally blocks any current in the oppositie direction.



Unit Exam II: Problem #3 (Spring '13)

Consider the RC circuit shown. The switch has been closed for a long time.
(a) Find the current I_B flowing through the battery.
(b) Find the voltage V_C across the capacitor.
(c) Find the charge Q on the capacitor.
(d) Find the current I_3 flowing through the 3Ω-resistor right after the switch has been opened.

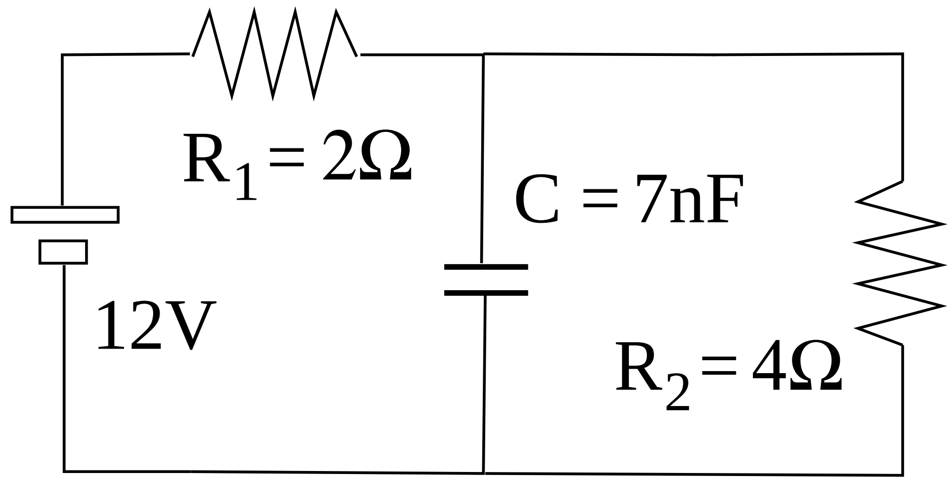




This RC circuit has been running for a long time.

(a) Find the current I_2 through the resistor R_2.

(b) Find the voltage V_C across the capacitor.

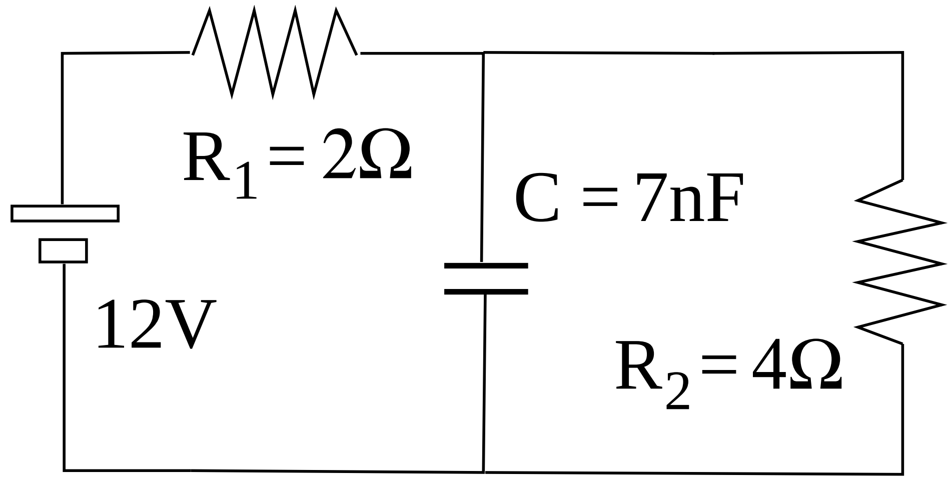


Intermediate Exam II: Problem #3 (Spring '05)

This RC circuit has been running for a long time.

(a) Find the current I_2 through the resistor R_2.

(b) Find the voltage V_C across the capacitor.



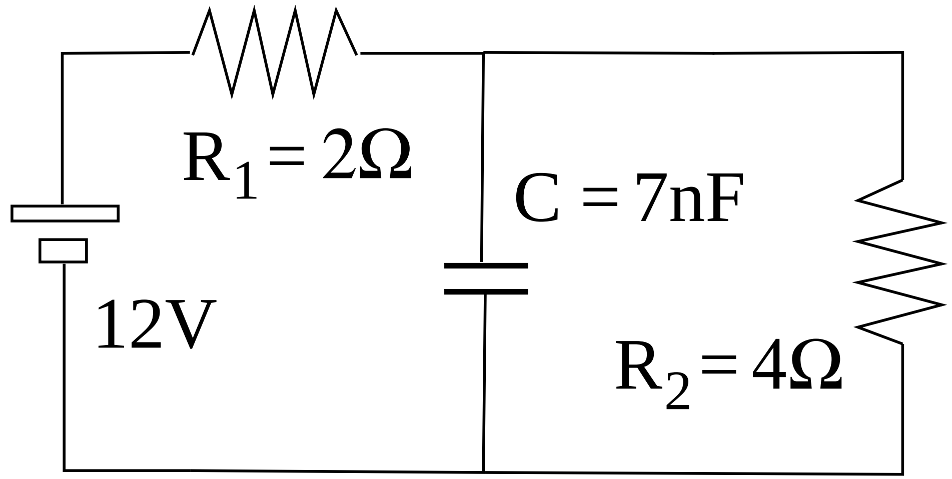
Solution:

(a) $I_C = 0, \quad I_2 = \dfrac{\mathcal{E}}{R_1 + R_2} = \dfrac{12\text{V}}{6\Omega} = 2\text{A}.$

This RC circuit has been running for a long time.

(a) Find the current I_2 through the resistor R_2.

(b) Find the voltage V_C across the capacitor.



Solution:

(a) $I_C = 0, \quad I_2 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{12\mathrm{V}}{6\Omega} = 2\mathrm{A}.$

(b) $V_C = V_2 = I_2 R_2 = (2\mathrm{A})(4\Omega) = 8\mathrm{V}.$