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11. RC Circuits

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Abstract

Lecture slides 11 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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RC Circuit: Fundamentals

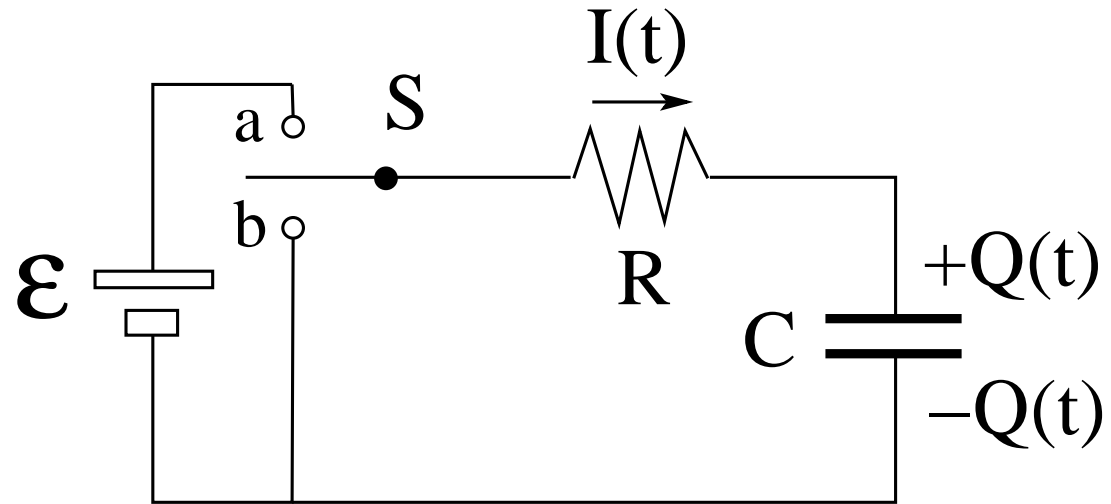


Specifications:

- \mathcal{E} (emf)
- R (resistance)
- C (capacitance)

Switch S :

- a: charging
- b: discharging



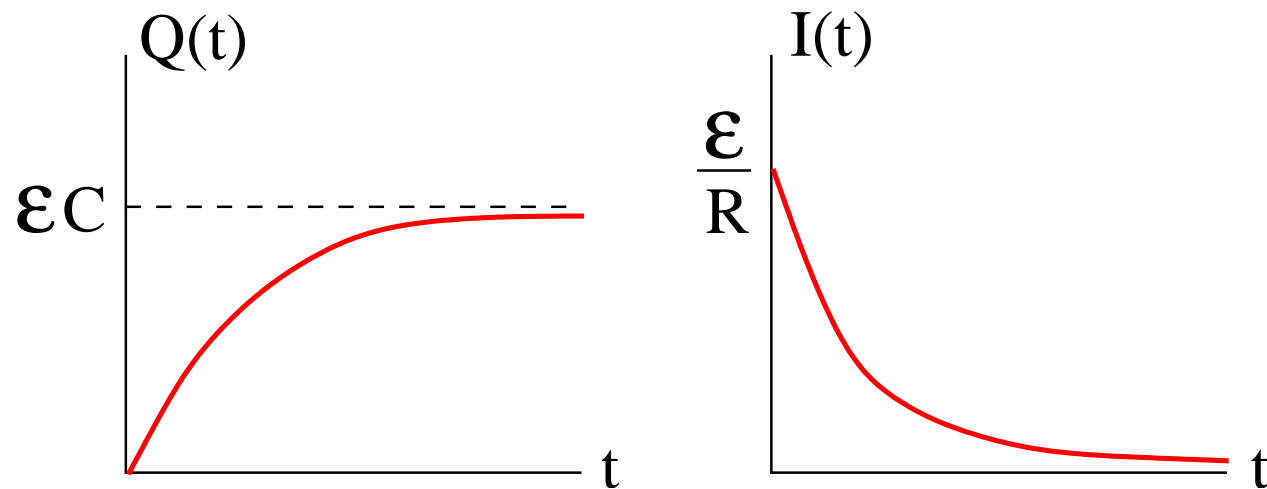
Time-dependent quantities:

- $Q(t)$: instantaneous charge on capacitor
- $I(t) = \frac{dQ}{dt}$: instantaneous current
- $V_R(t) = I(t)R$: instantaneous voltage across resistor
- $V_C(t) = \frac{Q(t)}{C}$: instantaneous voltage across capacitor

RC Circuit: Charging the Capacitor



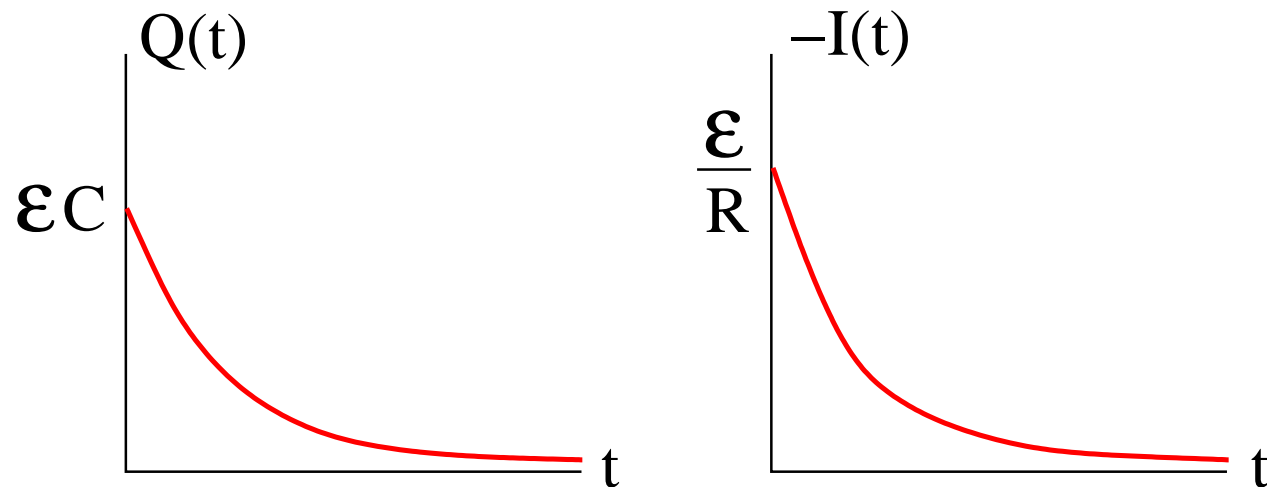
- Loop rule: $\mathcal{E} - IR - \frac{Q}{C} = 0$
- Differential equation: $R \frac{dQ}{dt} + \frac{Q}{C} = \mathcal{E} \Rightarrow \frac{dQ}{dt} = \frac{\mathcal{E}C - Q}{RC}$
 $\int_0^Q \frac{dQ}{\mathcal{E}C - Q} = \int_0^t \frac{dt}{RC} \Rightarrow -\ln\left(\frac{\mathcal{E}C - Q}{\mathcal{E}C}\right) = \frac{t}{RC} \Rightarrow \frac{\mathcal{E}C - Q}{\mathcal{E}C} = e^{-t/RC}$
- Charge on capacitor: $Q(t) = \mathcal{E}C \left[1 - e^{-t/RC}\right]$
- Current through resistor: $I(t) \equiv \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$



RC Circuit: Discharging the Capacitor



- Loop rule: $IR + \frac{Q}{C} = 0$
- Differential equation: $R \frac{dQ}{dt} + \frac{Q}{C} = 0 \Rightarrow \frac{dQ}{dt} = -\frac{Q}{RC}$
 $\Rightarrow \int_{\mathcal{E}C}^Q \frac{dQ}{Q} = -\int_0^t \frac{dt}{RC} \Rightarrow \ln\left(\frac{Q}{\mathcal{E}C}\right) = -\frac{t}{RC} \Rightarrow \frac{Q}{\mathcal{E}C} = e^{-t/RC}$
- Charge on capacitor: $Q(t) = \mathcal{E}C e^{-t/RC}$
- Current through resistor: $I(t) \equiv \frac{dQ}{dt} = -\frac{\mathcal{E}}{R} e^{-t/RC}$



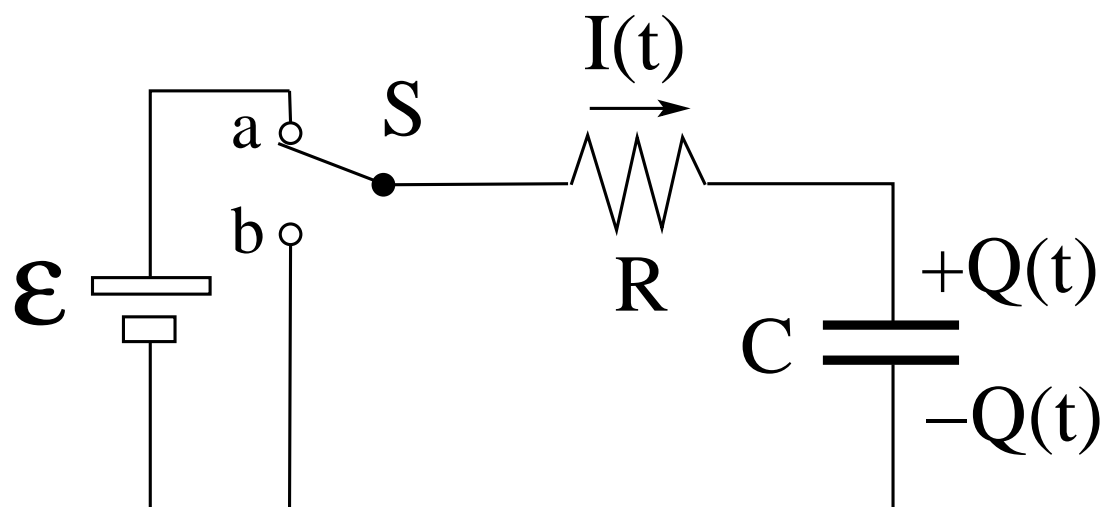
RC Circuit: Energy Transfer While Charging



Loop rule: $IR + \frac{Q}{C} = \mathcal{E}$ (I is positive)

- $I\mathcal{E}$: rate at which emf source delivers energy
- $IV_R = I^2R$: rate at which energy is dissipated in resistor
- $IV_C = \frac{IQ}{C}$: rate at which energy is stored in capacitor

Balance of energy transfer: $I^2R + \frac{IQ}{C} = I\mathcal{E}$



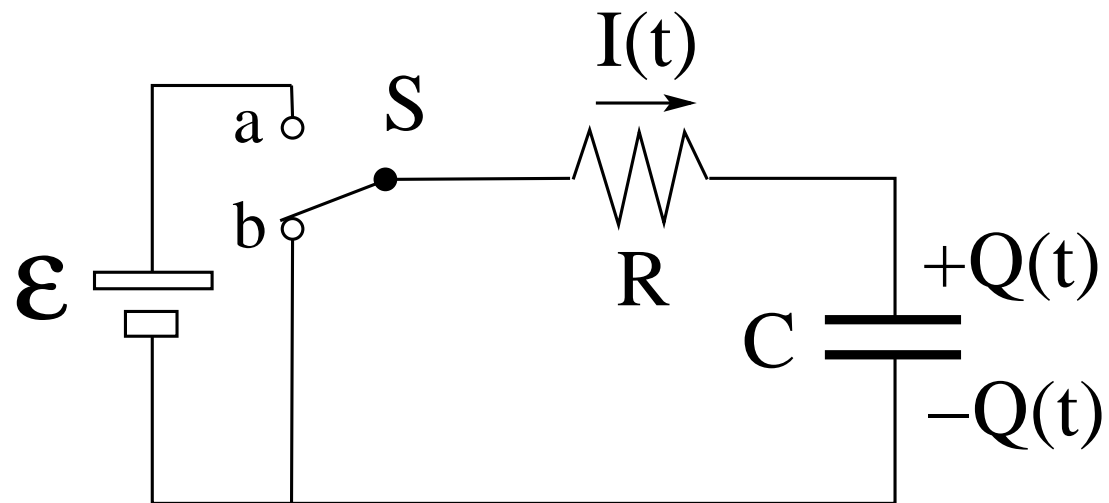
RC Circuit: Energy Transfer While Discharging



Loop rule: $IR + \frac{Q}{C} = 0$ (I is negative)

- $IV_R = I^2 R$: rate at which energy is dissipated in resistor
- $IV_C = \frac{IQ}{C}$: rate at which capacitor releases energy

Balance of energy transfer: $I^2 R + \frac{IQ}{C} = 0$

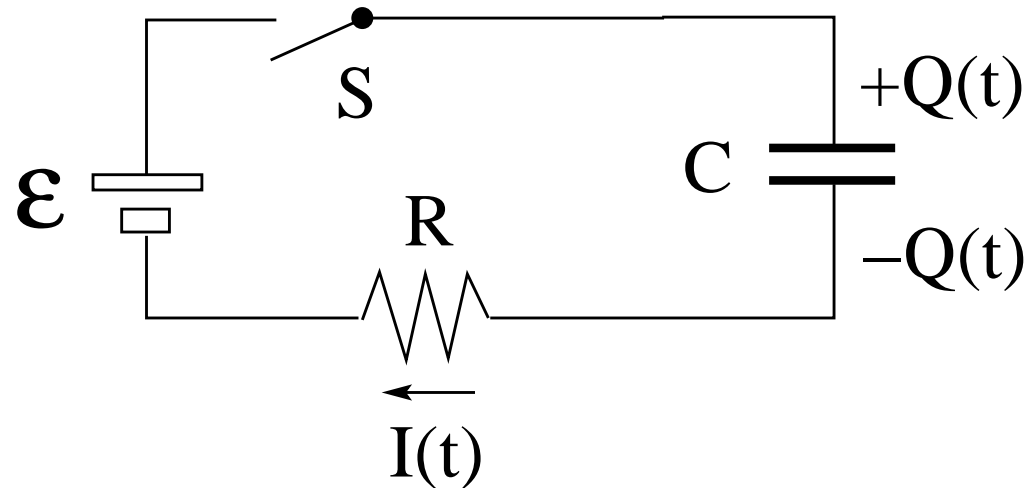


RC Circuit: Some Physical Properties



Specification of RC circuit by 3 device properties:

- \mathcal{E} [V] (emf)
- R [Ω] (resistance)
- C [F] (capacitance)



Physical properties of RC circuit during charging process determined by 3 combinations of the device properties:

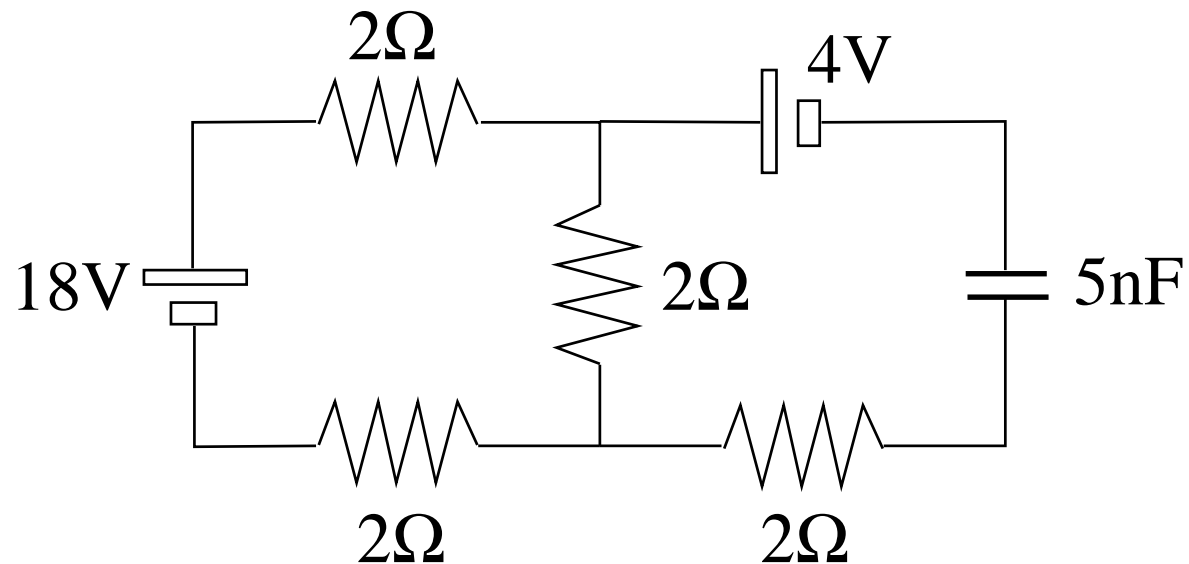
- $\mathcal{E}/R = I(t = 0)$: rate at which current flows onto capacitor initially
- $\mathcal{E}C = Q(t = \infty)$: total charge placed on capacitor ultimately
- $RC = \tau$: time it takes to place 63% of the charge onto the capacitor
[$1 - e^{-1} = 0.632 \dots$]

RC Circuit: Application (1)



This circuit has been running for a very long time.

- (a) Find the current through the 18V battery.
- (b) Find the total power dissipated in the resistors.
- (c) Find the charge stored on the capacitor.



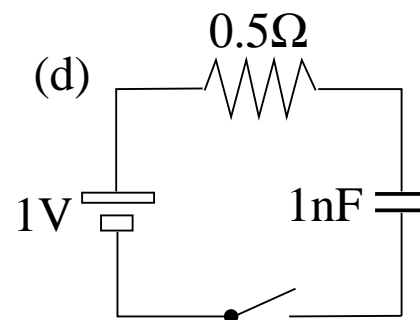
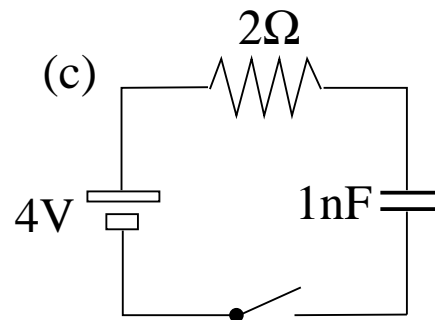
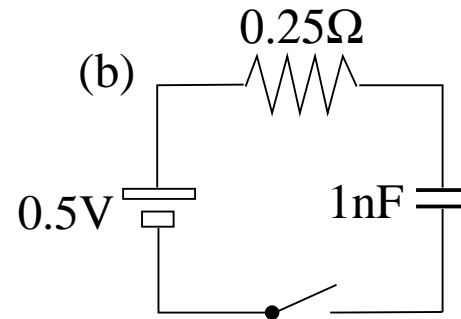
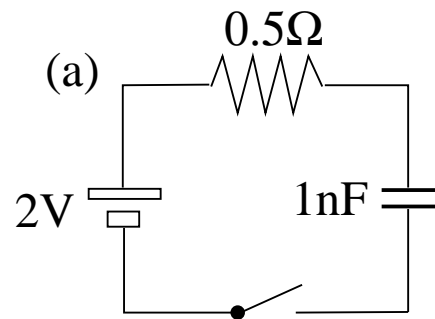
RC Circuit: Application (2)



The switches are closed at $t = 0$. This begins the charging process in each RC circuit.

Name the circuit in which...

- (i) the charge flows into the capacitor at the highest rate initially,
- (ii) the capacitor has the most charge ultimately,
- (iii) the capacitor is 63% full in the shortest time.



RC Circuit: Application (3)



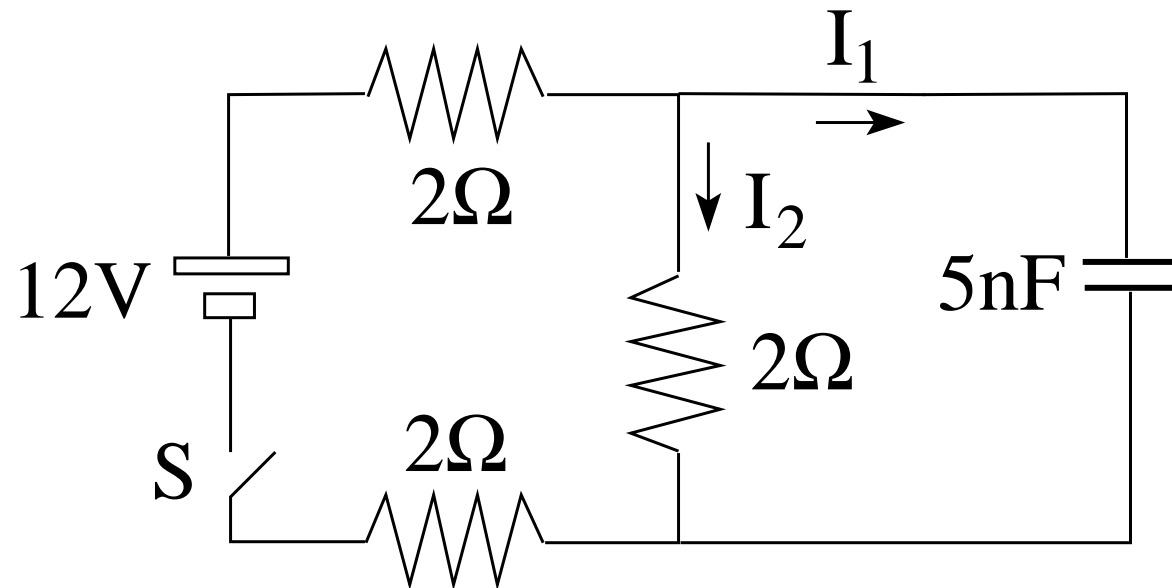
At time $t = 0$ the capacitor in this circuit is discharged and the switch is being closed.

Find the current I_1

- (a) at $t = 0$,
- (b) at $t = \infty$.

Find the current I_2

- (c) at $t = 0$,
- (d) at $t = \infty$.

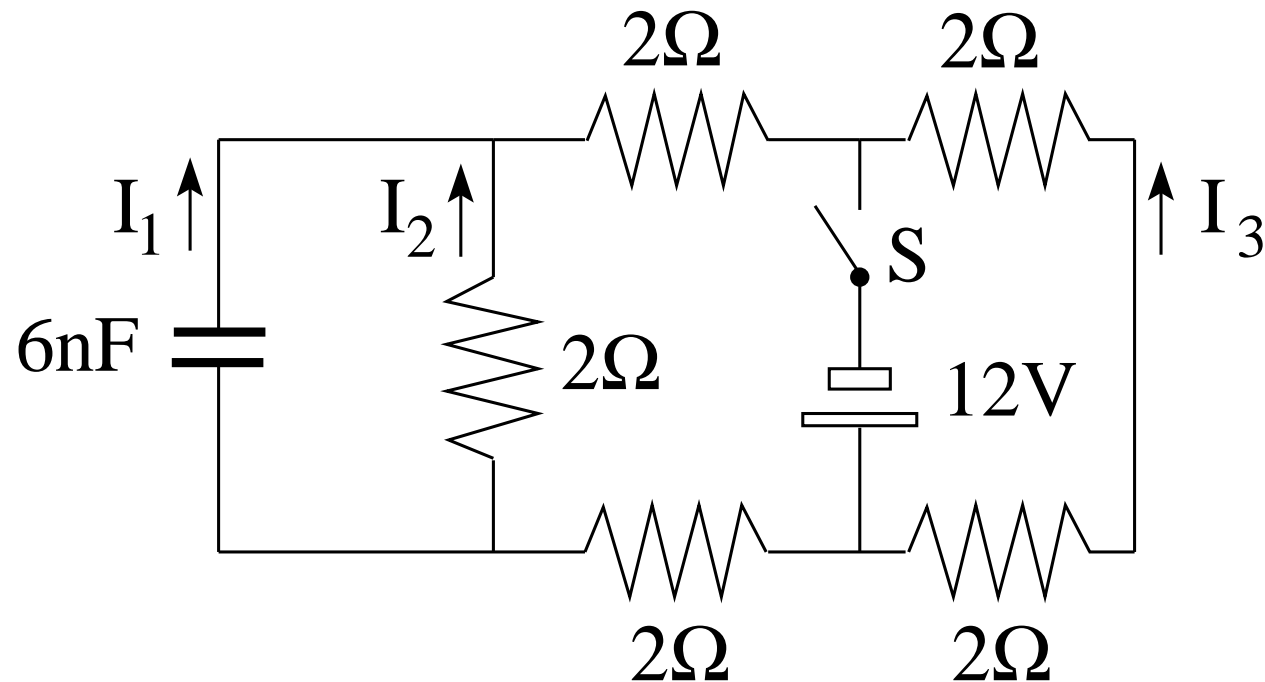


RC Circuit: Application (4)



In this 3-loop RC circuit, the switch S is closed at time $t = 0$.

- (a) Find the currents I_1, I_2, I_3 just after the switch has been closed.
- (b) Find the currents I_1, I_2, I_3 a very long time later.

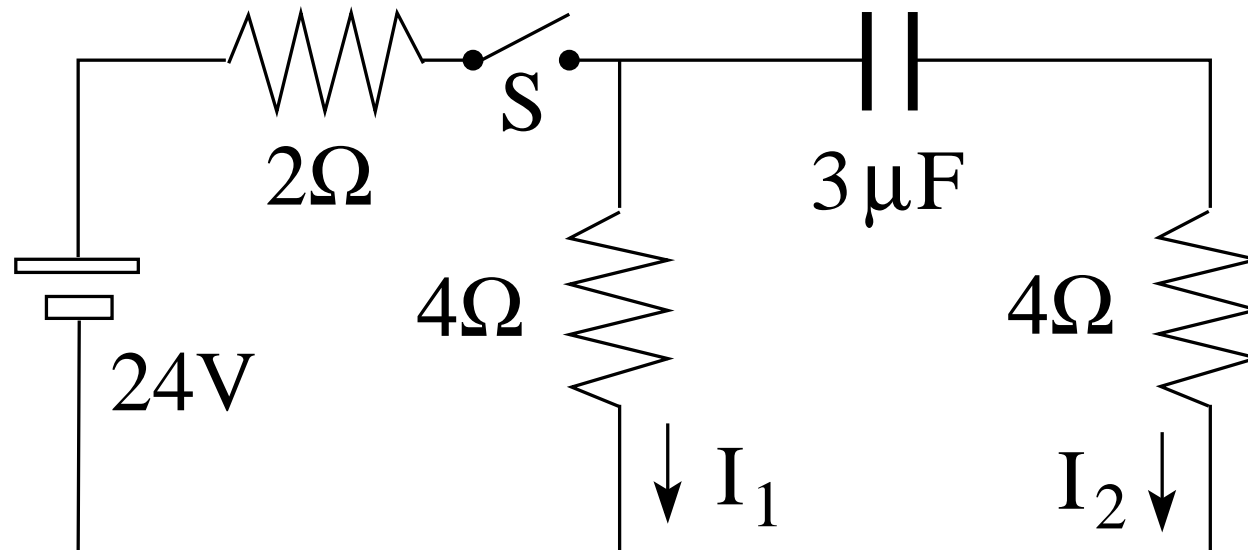


RC Circuit: Application (5)



In the RC circuit shown, the switch S has been open for a long time.

- Find the currents I_1 and I_2 immediately after the switch has been closed.
- Find the currents I_1 and I_2 a very long time later.

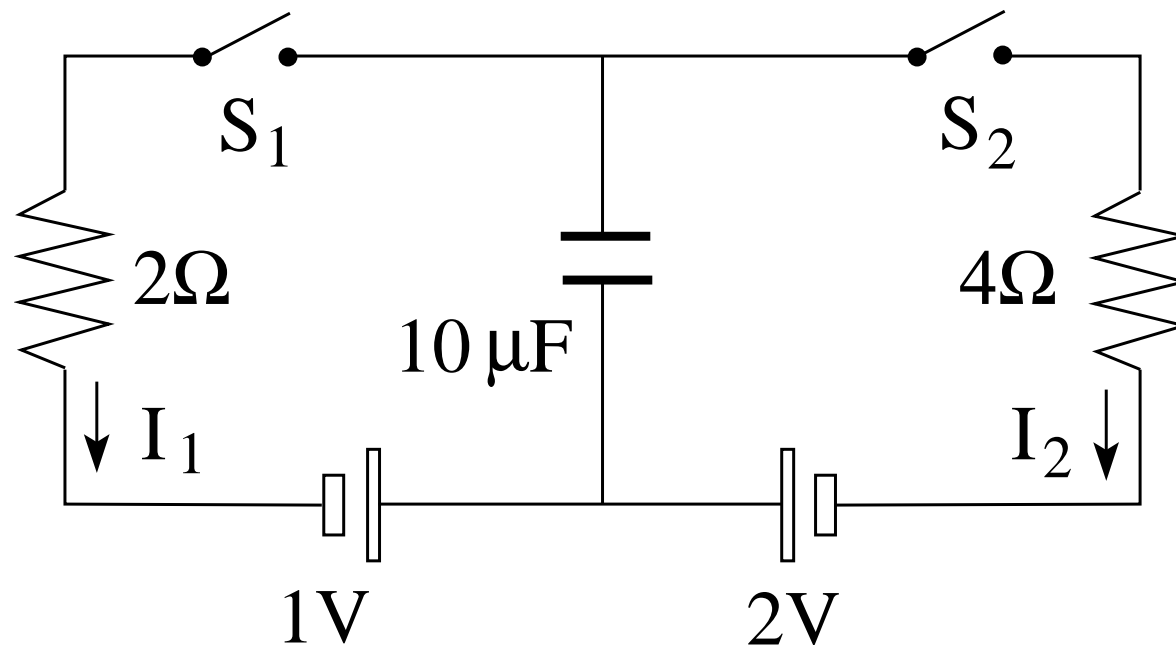


RC Circuit: Application (6)



In the RC circuit shown, both switches are initially open and the capacitor is discharged.

- Close switch S_1 and find the currents I_1 and I_2 immediately afterwards.
- Find the currents I_1 , I_2 and the charge Q on the capacitor a very long time later.
- Now close switch S_2 also and find the currents I_1 and I_2 immediately afterwards.
- Find the currents I_1 , I_2 and the charge Q on the capacitor a very long time later.



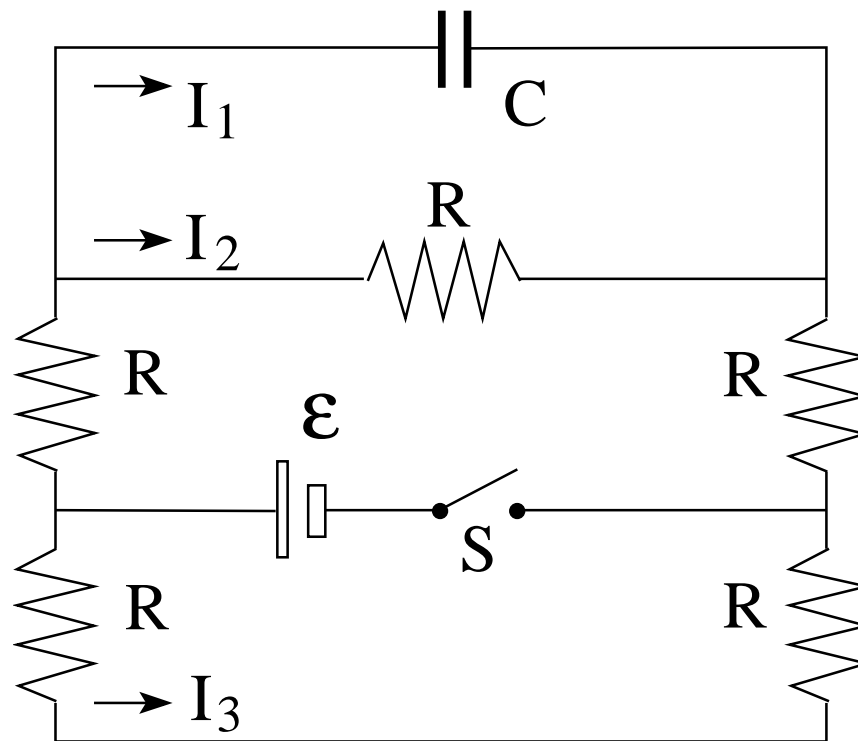
RC Circuit: Application (7)



In the RC circuit shown, the switch has been open for a long time.

Find the currents I_1 , I_2 , I_3 and the charge Q on the capacitor

- (a) right after the switch has been closed,
- (b) a very long time later.



$$R = 2\Omega$$

$$C = 6\mu\text{F}$$

$$\mathcal{E} = 12\text{V}$$

RC Circuit: Application (8)

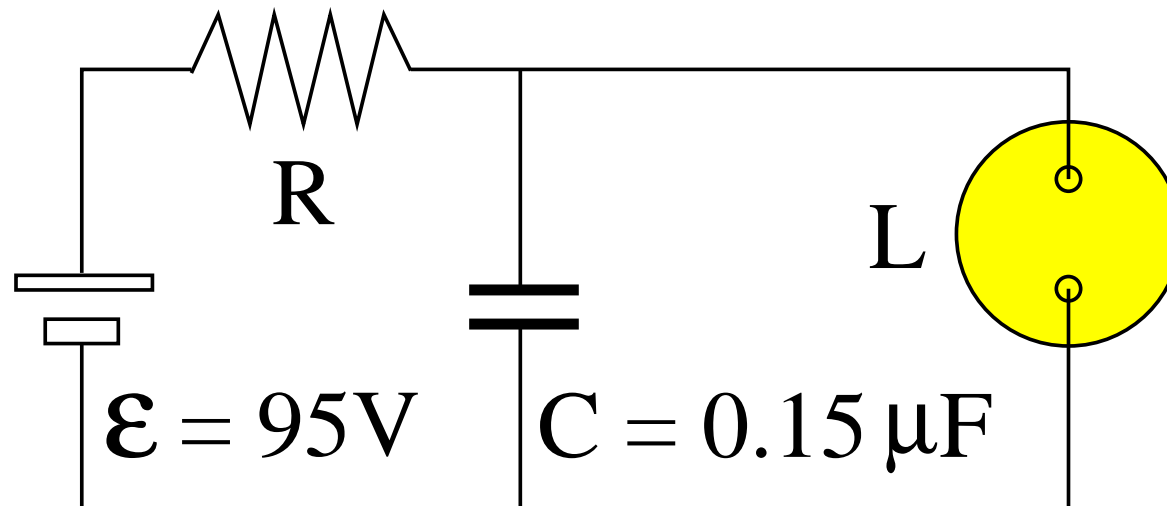


The circuit shown is that of a flashing lamp, such as are attached to barrels at highway construction sites.

The power is supplied by a battery with $\mathcal{E} = 95V$. The fluorescent lamp L is connected in parallel to the capacitor with $C = 0.15\mu F$ of an RC circuit.

Current passes through the lamp only when the potential difference across it reaches the breakdown voltage $V_L = 72V$. In this event, the capacitor discharges through the lamp, and the lamp flashes briefly.

Suppose that two flashes per second are needed. What should the resistance R be?



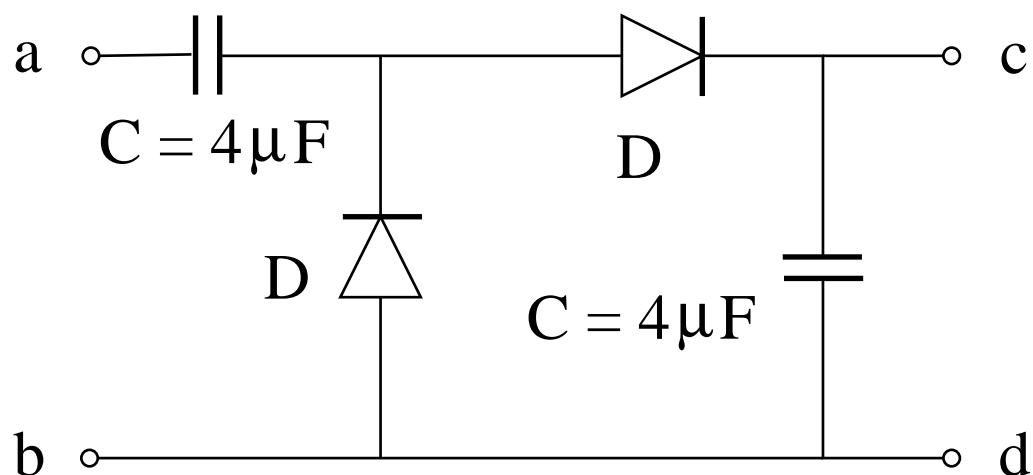
Circuit of Capacitors and Diodes



The circuit shown contains two identical capacitors and two ideal diodes. A 100V battery is connected to the two input terminals a and b . Find the voltage at the output terminals c and d

- (1) if a is the positive input terminal,
- (2) if b is the positive input terminal.

Note: An ideal diode is a perfect one-way street for electric currents. It lets a current through unimpeded in the direction of the arrow and totally blocks any current in the opposite direction.

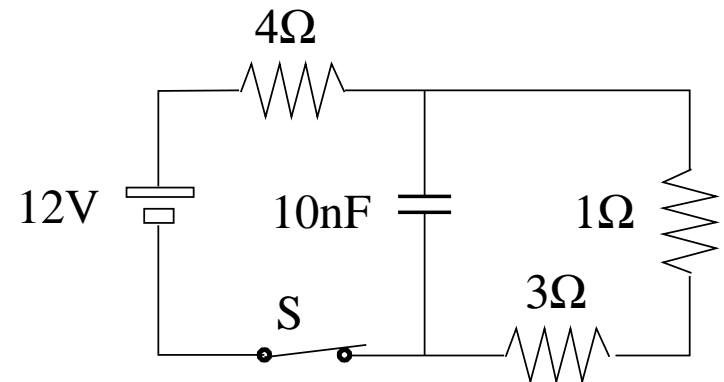
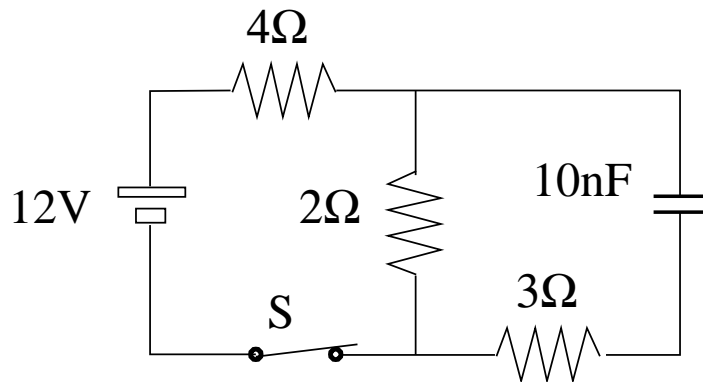


Unit Exam II: Problem #3 (Spring '13)



Consider the RC circuit shown. The switch has been closed for a long time.

- (a) Find the current I_B flowing through the battery.
- (b) Find the voltage V_C across the capacitor.
- (c) Find the charge Q on the capacitor.
- (d) Find the current I_3 flowing through the 3Ω -resistor right after the switch has been opened.

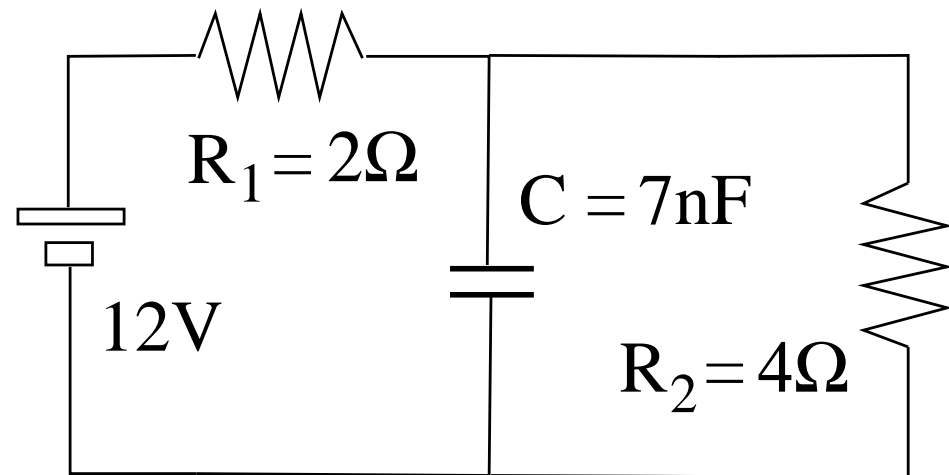


Intermediate Exam II: Problem #3 (Spring '05)



This RC circuit has been running for a long time.

- (a) Find the current I_2 through the resistor R_2 .
- (b) Find the voltage V_C across the capacitor.

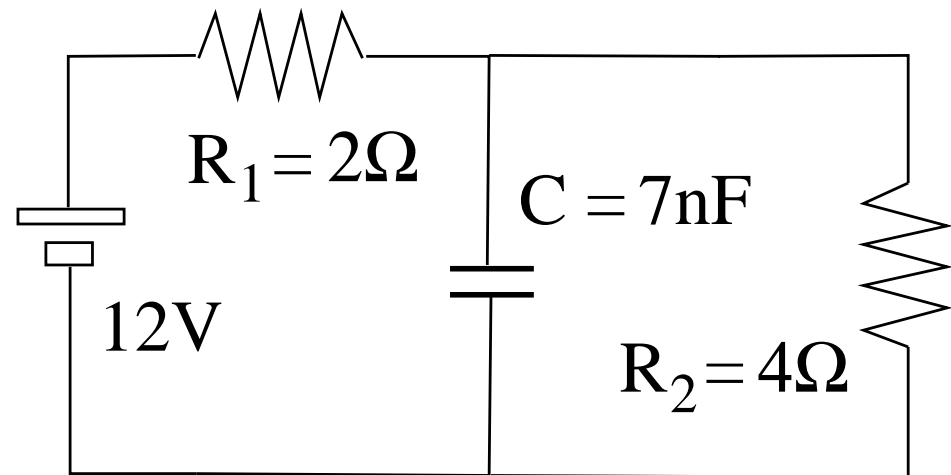


Intermediate Exam II: Problem #3 (Spring '05)



This RC circuit has been running for a long time.

- (a) Find the current I_2 through the resistor R_2 .
- (b) Find the voltage V_C across the capacitor.



Solution:

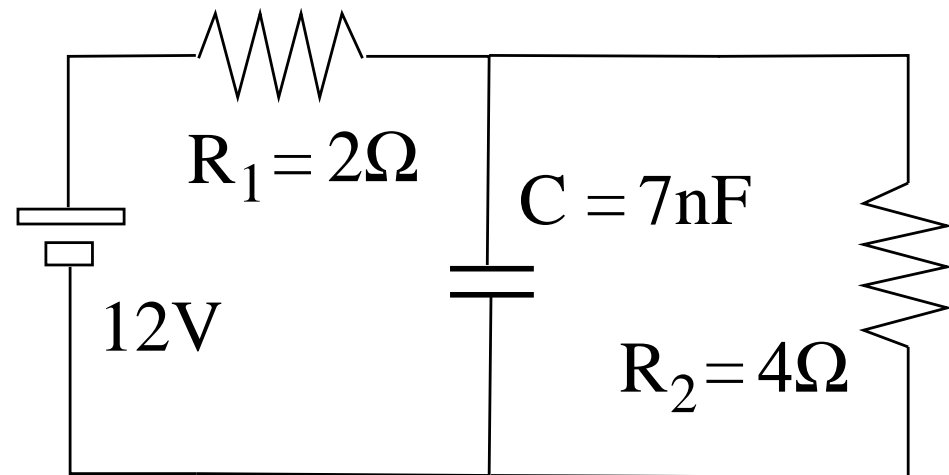
$$(a) \quad I_C = 0, \quad I_2 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{12\text{V}}{6\Omega} = 2\text{A}.$$

Intermediate Exam II: Problem #3 (Spring '05)



This RC circuit has been running for a long time.

- (a) Find the current I_2 through the resistor R_2 .
- (b) Find the voltage V_C across the capacitor.



Solution:

(a) $I_C = 0$, $I_2 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{12\text{V}}{6\Omega} = 2\text{A}$.

(b) $V_C = V_2 = I_2 R_2 = (2\text{A})(4\Omega) = 8\text{V}$.