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10. Resistors II

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Abstract

Lecture slides 10 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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M. C. Escher: Waterfall



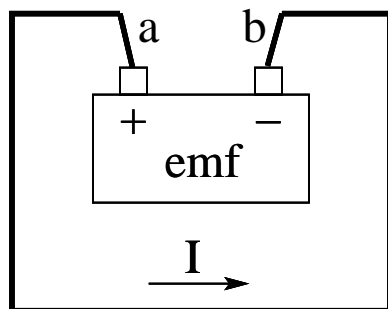
Direct Current Circuit



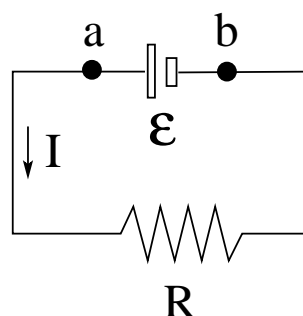
Consider a wire with resistance $R = \rho\ell/A$ connected to a battery.

- **Resistor rule:** In the direction of I across a resistor with resistance R , the electric potential drops: $\Delta V = -IR$.
- **EMF rule:** From the $(-)$ terminal to the $(+)$ terminal in an ideal source of emf, the potential rises: $\Delta V = \mathcal{E}$.
- **Loop rule:** The algebraic sum of the changes in potential encountered in a complete traversal of any loop in a circuit must be zero: $\sum \Delta V_i = 0$.

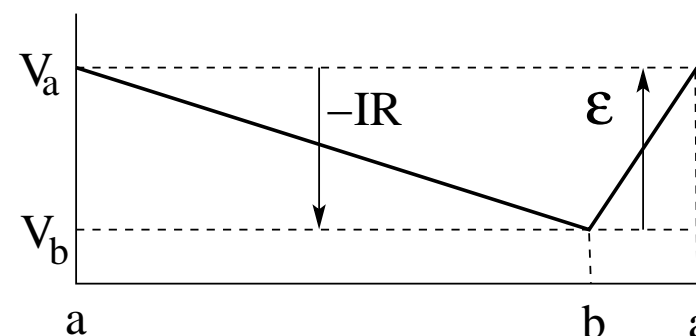
physical system



circuit diagram



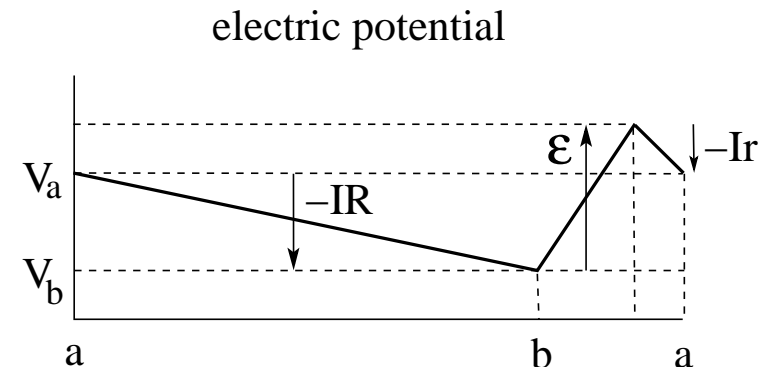
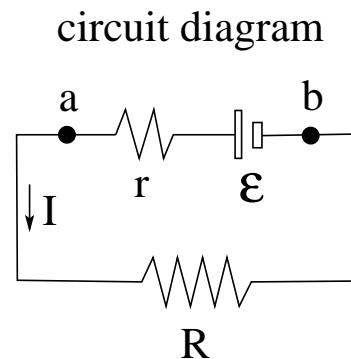
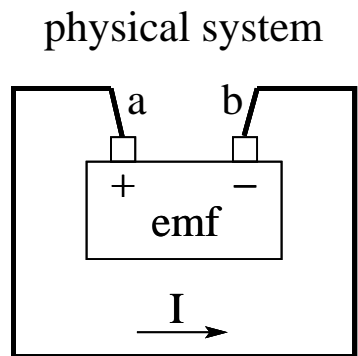
electric potential



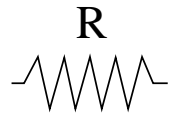
Battery with Internal Resistance



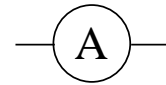
- Real batteries have an internal resistance r .
- The terminal voltage $V_{ba} \equiv V_a - V_b$ is smaller than the emf \mathcal{E} written on the label if a current flows through the battery.
- Usage of the battery increases its internal resistance.
- Current from loop rule: $\mathcal{E} - Ir - IR = 0 \Rightarrow I = \frac{\mathcal{E}}{R + r}$
- Current from terminal voltage: $V_{ba} = \mathcal{E} - Ir = IR \Rightarrow I = \frac{V_{ba}}{R}$



Symbols Used in Circuit Diagrams



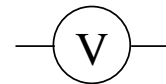
resistor



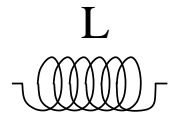
ammeter (connect in series)



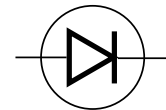
capacitor



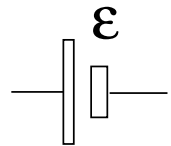
voltmeter (connect in parallel)



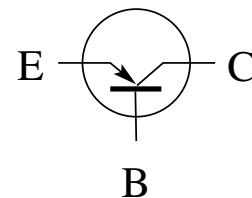
inductor



diode



emf source



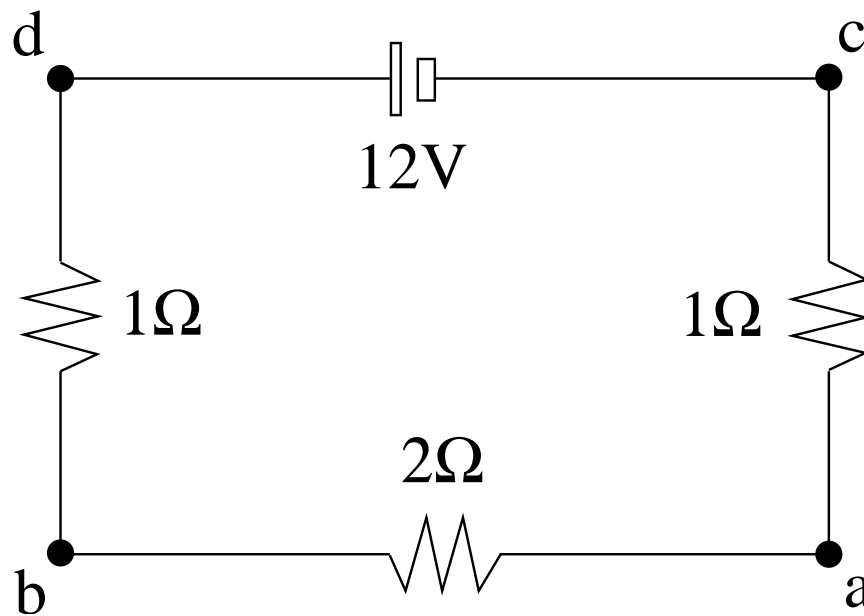
transistor

Resistor Circuit (4)



Consider the resistor circuit shown.

- (a) Find the direction of the current I (cw/ccw).
- (b) Find the magnitude of the current I .
- (c) Find the voltage $V_{ab} = V_b - V_a$.
- (d) Find the voltage $V_{cd} = V_d - V_c$.

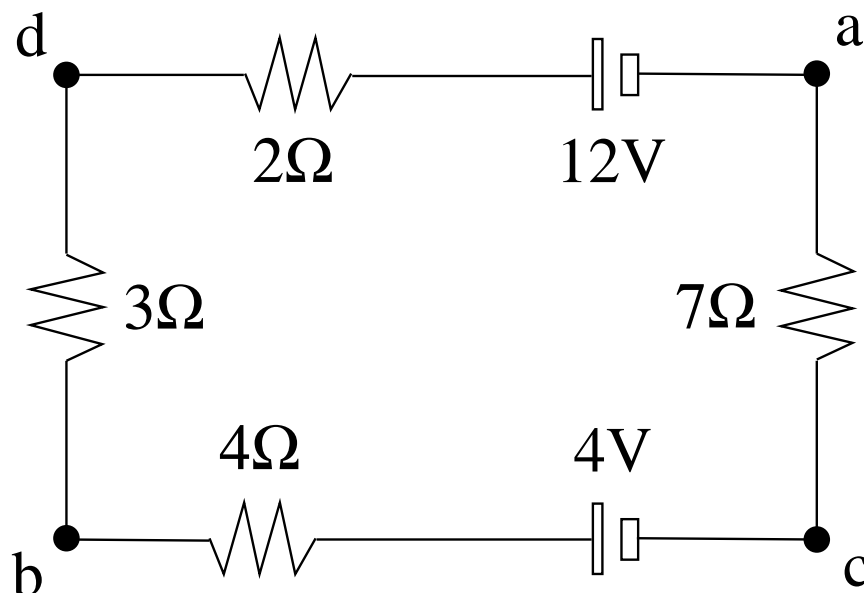


Resistor Circuit (5)



Consider the resistor circuit shown.

- (a) Find the direction (cw/ccw) of the current I in the loop.
- (b) Find the magnitude of the current I in the loop.
- (c) Find the potential difference $V_{ab} = V_b - V_a$.
- (d) Find the voltage $V_{cd} = V_d - V_c$.

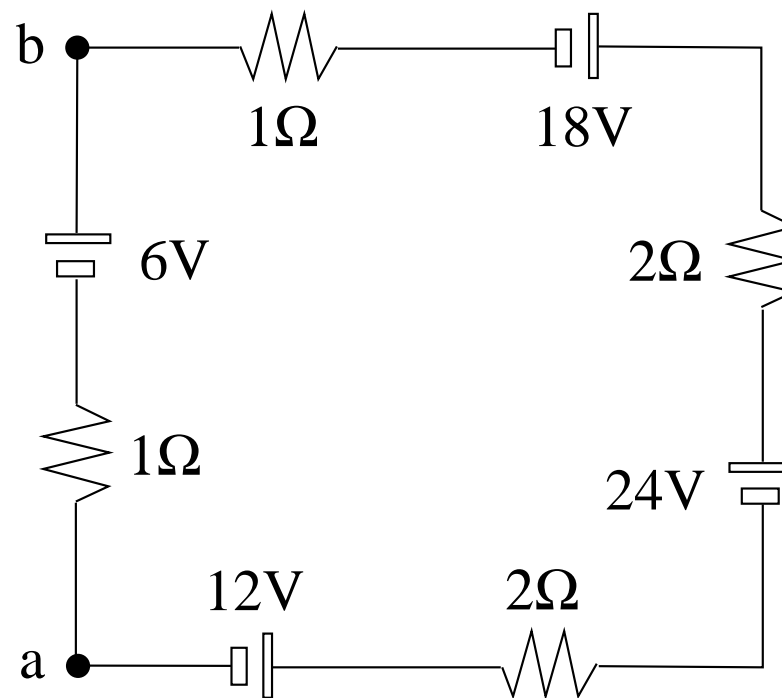


Resistor Circuit (6)



Consider the resistor circuit shown.

- (a) Guess the current direction (cw/ccw).
- (b) Use the loop rule to determine the current.
- (c) Find $V_{ab} \equiv V_b - V_a$ along two different paths.

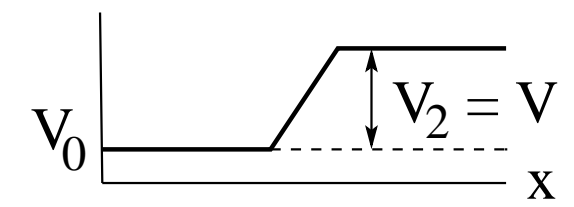
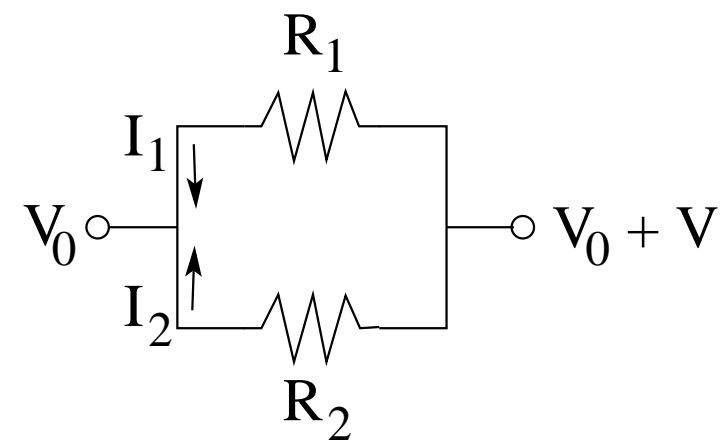
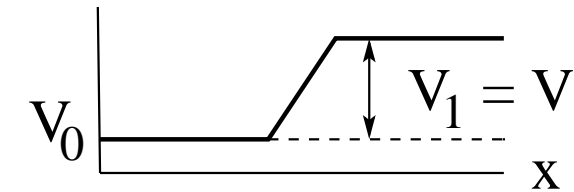


Resistors Connected in Parallel



Find the equivalent resistance of two resistors connected in parallel.

- Current through resistors: $I_1 + I_2 = I$
- Voltage across resistors: $V_1 = V_2 = V$
- Equivalent resistance: $\frac{1}{R} \equiv \frac{I}{V} = \frac{I_1}{V_1} + \frac{I_2}{V_2}$
- $\Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

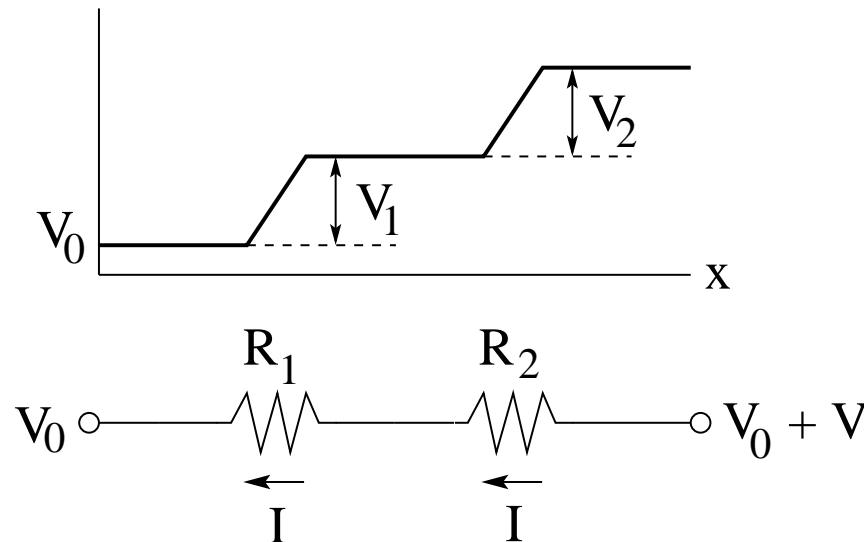


Resistors Connected in Series



Find the equivalent resistance of two resistors connected in series.

- Current through resistors: $I_1 = I_2 = I$
- Voltage across resistors: $V_1 + V_2 = V$
- Equivalent resistance: $R \equiv \frac{V}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2}$
- $\Rightarrow R = R_1 + R_2$

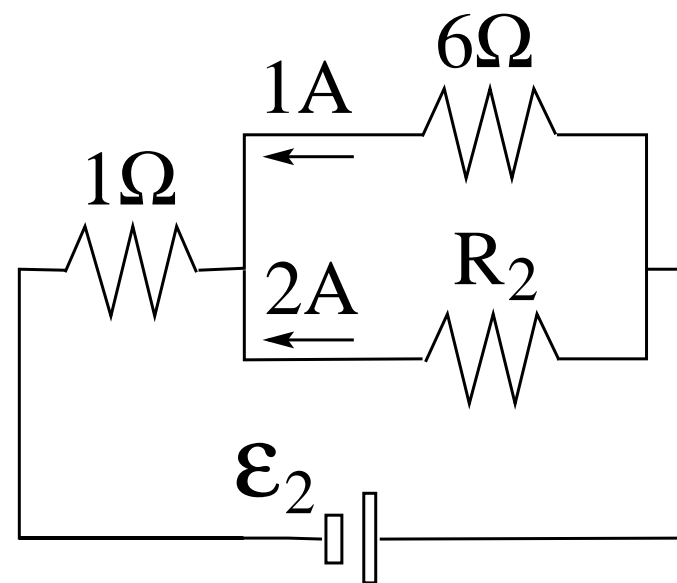
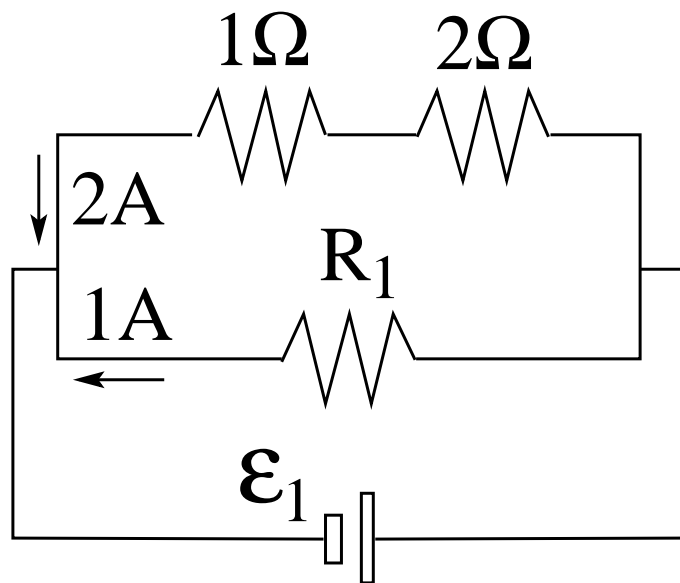


Resistor Circuit (1)



Consider the two resistor circuits shown.

- (a) Find the resistance R_1 .
- (b) Find the emf \mathcal{E}_1 .
- (c) Find the resistance R_2 .
- (d) Find the emf \mathcal{E}_2 .

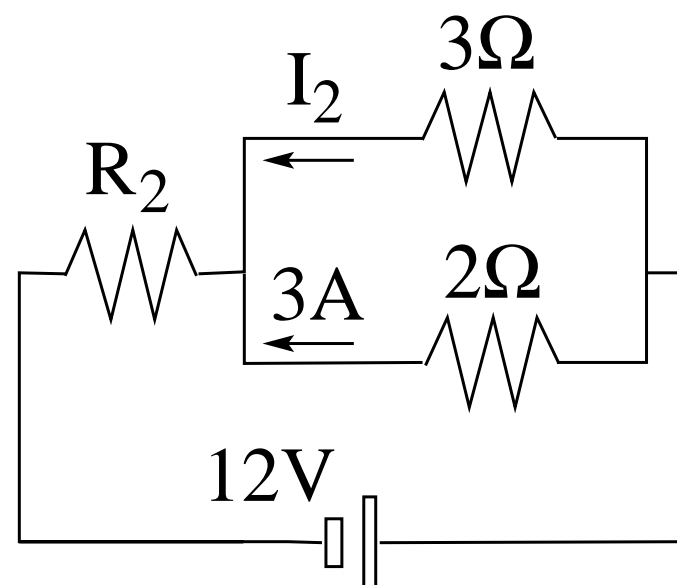
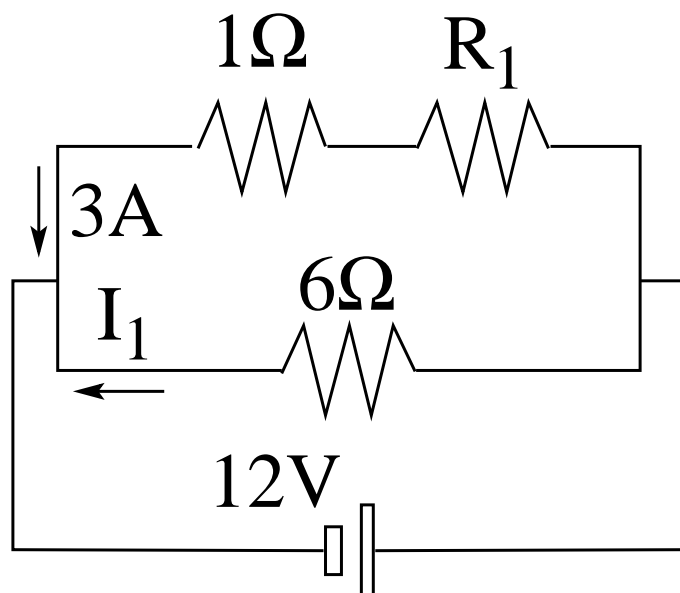


Resistor Circuit (2)



Consider the two resistor circuits shown.

- (a) Find the resistance R_1 .
- (b) Find the current I_1 .
- (c) Find the resistance R_2 .
- (d) Find the current I_2 .

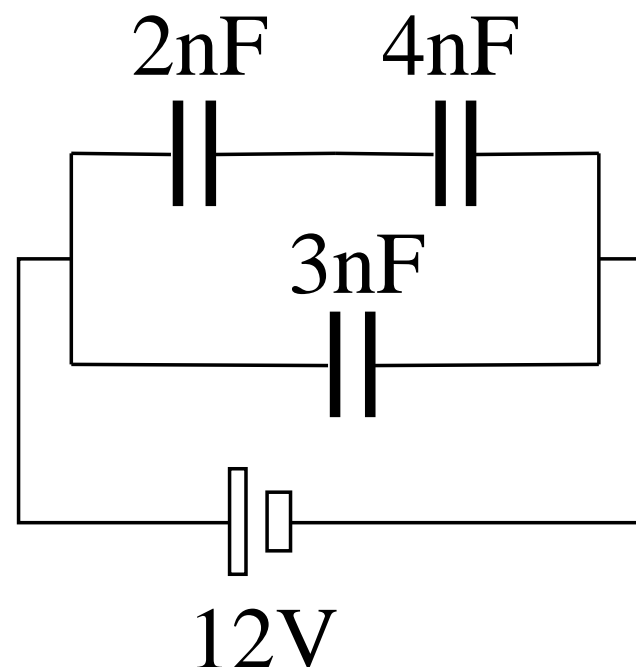
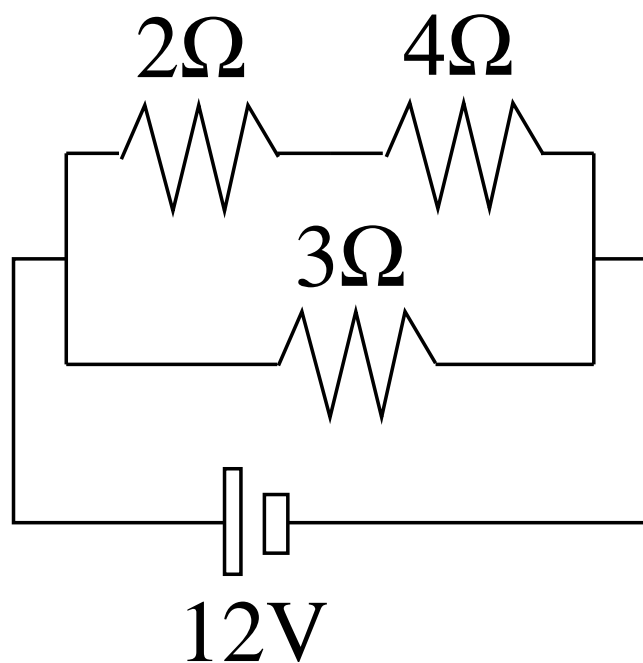


Resistor Circuit (3)



Consider the resistor and capacitor circuits shown.

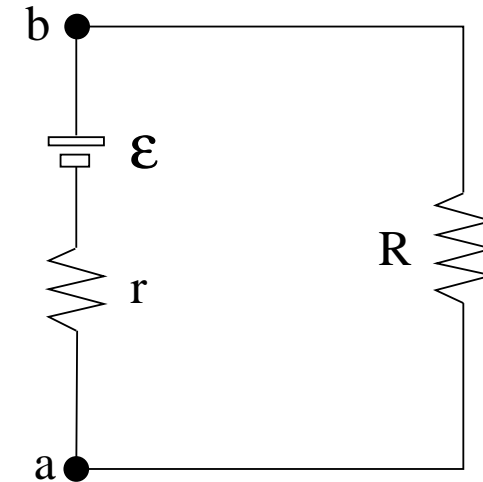
- (a) Find the equivalent resistance R_{eq} .
- (b) Find the power P_2, P_3, P_4 dissipated in each resistor.
- (c) Find the equivalent capacitance C_{eq} .
- (d) Find the energy U_2, U_3, U_4 stored in each capacitor.





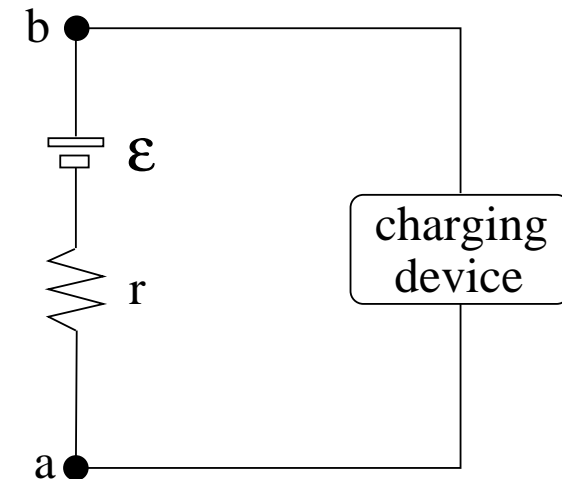
Battery in use

- Terminal voltage: $V_{ab} = \mathcal{E} - Ir = IR$
- Power output of battery: $P = V_{ab}I = \mathcal{E}I - I^2r$
 - Power generated in battery: $\mathcal{E}I$
 - Power dissipated in battery: I^2r
- Power dissipated in resistor: $P = I^2R$



Battery being charged:

- Terminal voltage: $V_{ab} = \mathcal{E} + Ir$
- Power supplied by charging device: $P = V_{ab}I$
- Power input into battery: $P = \mathcal{E}I + I^2r$
 - Power stored in battery: $\mathcal{E}I$
 - Power dissipated in battery: I^2r

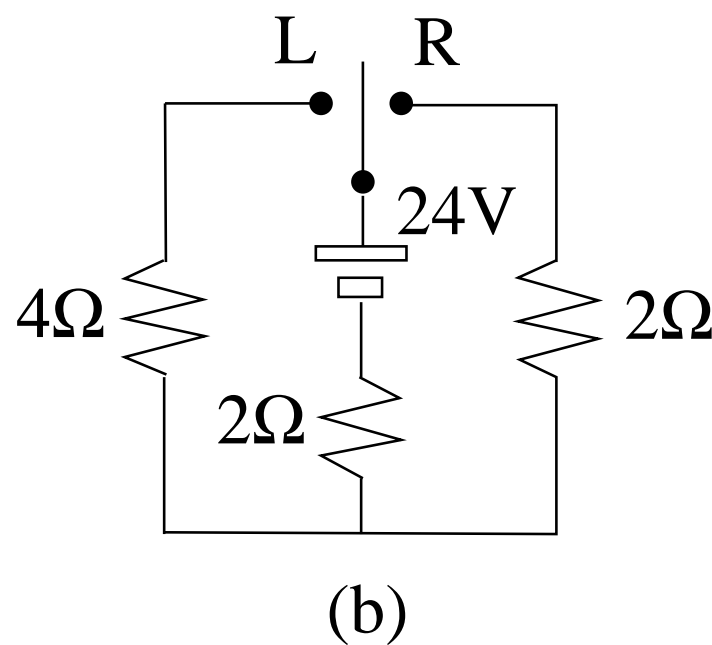
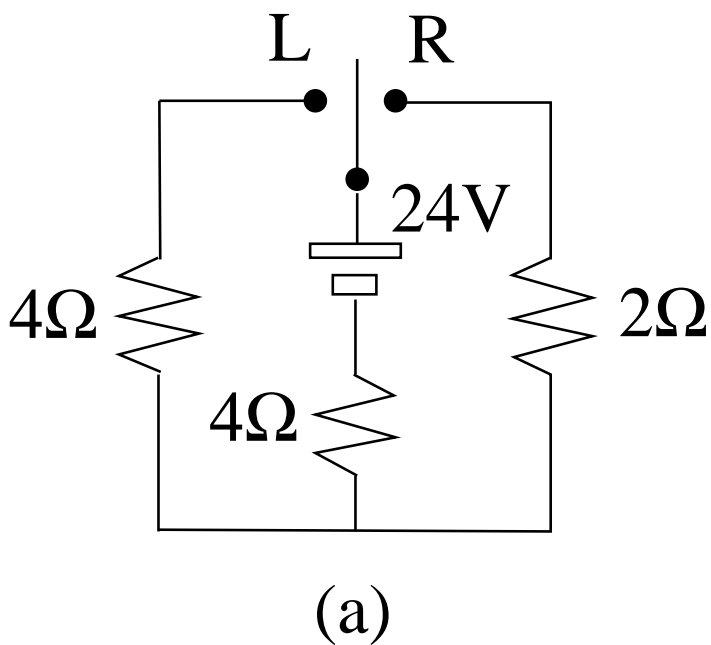


Resistor Circuit (7)



Consider two 24V batteries with internal resistances (a) $r = 4\Omega$, (b) $r = 2\Omega$.

- Which setting of the switch (L/R) produces the larger power dissipation in the resistor on the side?



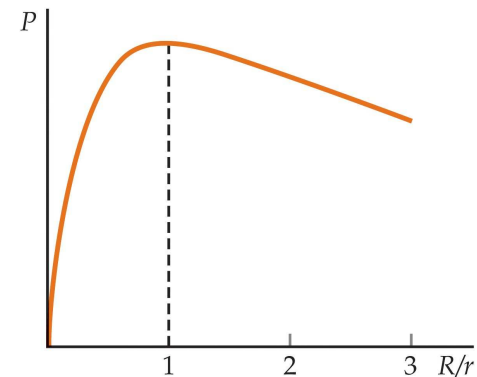
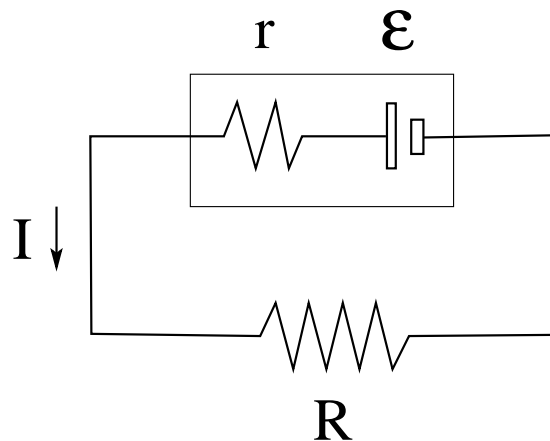
Impedance Matching



A battery providing an emf \mathcal{E} with internal resistance r is connected to an external resistor of resistance R as shown.

For what value of R does the battery deliver the maximum power to the external resistor?

- Electric current: $\mathcal{E} - Ir - IR = 0 \Rightarrow I = \frac{\mathcal{E}}{R + r}$
- Power delivered to external resistor: $P = I^2 R = \frac{\mathcal{E}^2 R}{(R + r)^2}$
- Condition for maximum power: $\frac{dP}{dR} = 0 \Rightarrow R = r$

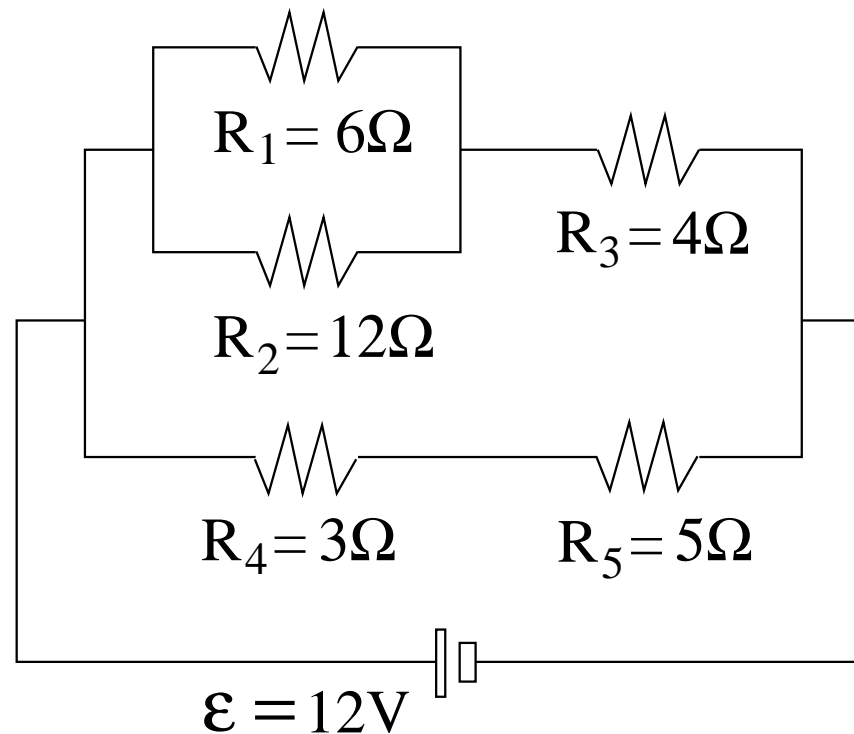


Resistor Circuit (8)



Consider the circuit of resistors shown.

- Find the equivalent resistance R_{eq} .
- Find the currents I_1, \dots, I_5 through each resistor and the voltages V_1, \dots, V_5 across each resistor.
- Find the total power P dissipated in the circuit.





Loop Rule

- When any closed-circuit loop is traversed, the algebraic sum of the changes in electric potential must be zero.

Junction Rule

- At any junction in a circuit, the sum of the incoming currents must equal the sum of the outgoing currents.

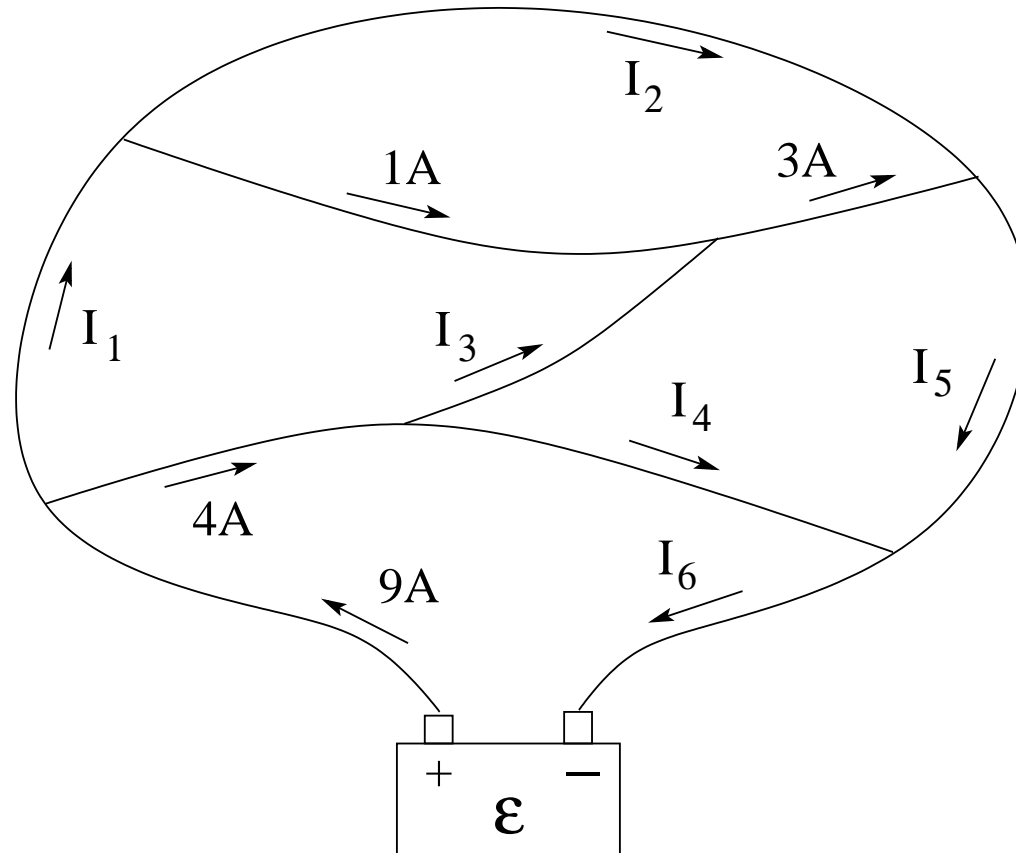
Strategy

- Use the junction rule to name all independent currents.
- Use the loop rule to determine the independent currents.

Applying the Junction Rule



In the circuit of steady currents, use the junction rule to find the unknown currents I_1, \dots, I_6 .

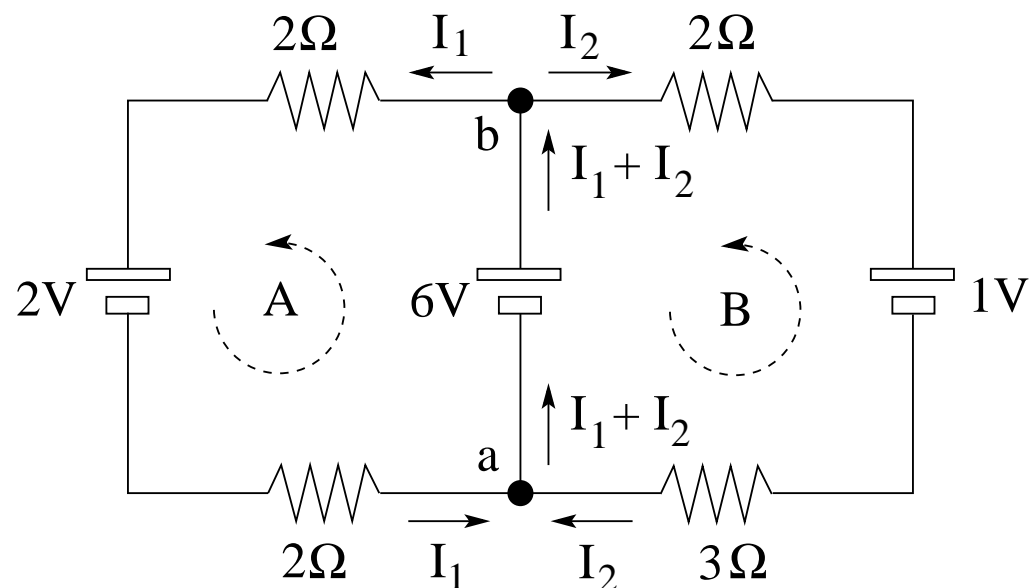


Applying Kirchhoff's Rules



Consider the circuit shown below.

- Junction a : I_1, I_2 (in); $I_1 + I_2$ (out)
- Junction b : $I_1 + I_2$ (in); I_1, I_2 (out)
- Two independent currents require the use of two loops.
- Loop A (ccw): $6V - (2\Omega)I_1 - 2V - (2\Omega)I_1 = 0$
- Loop B (ccw): $(3\Omega)I_2 + 1V + (2\Omega)I_2 - 6V = 0$
- Solution: $I_1 = 1A, I_2 = 1A$

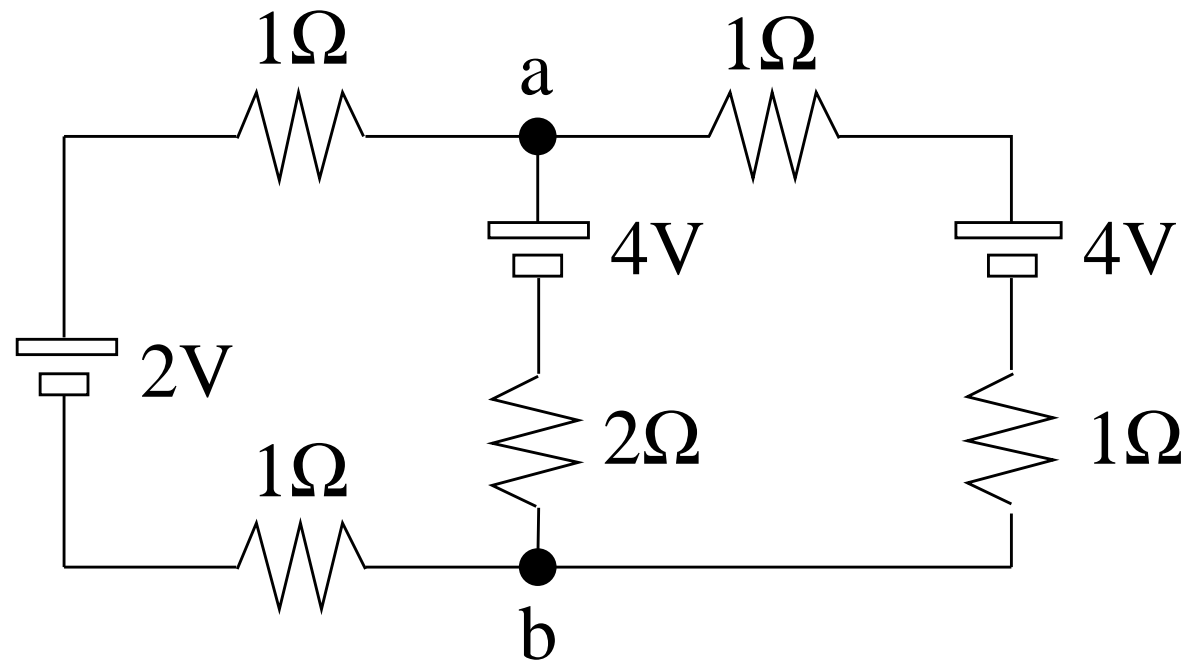


Resistor Circuit (11)



Consider the electric circuit shown.

- Identify all independent currents via junction rule.
- Determine the independent currents via loop rule.
- Find the Potential difference $V_{ab} = V_b - V_a$.

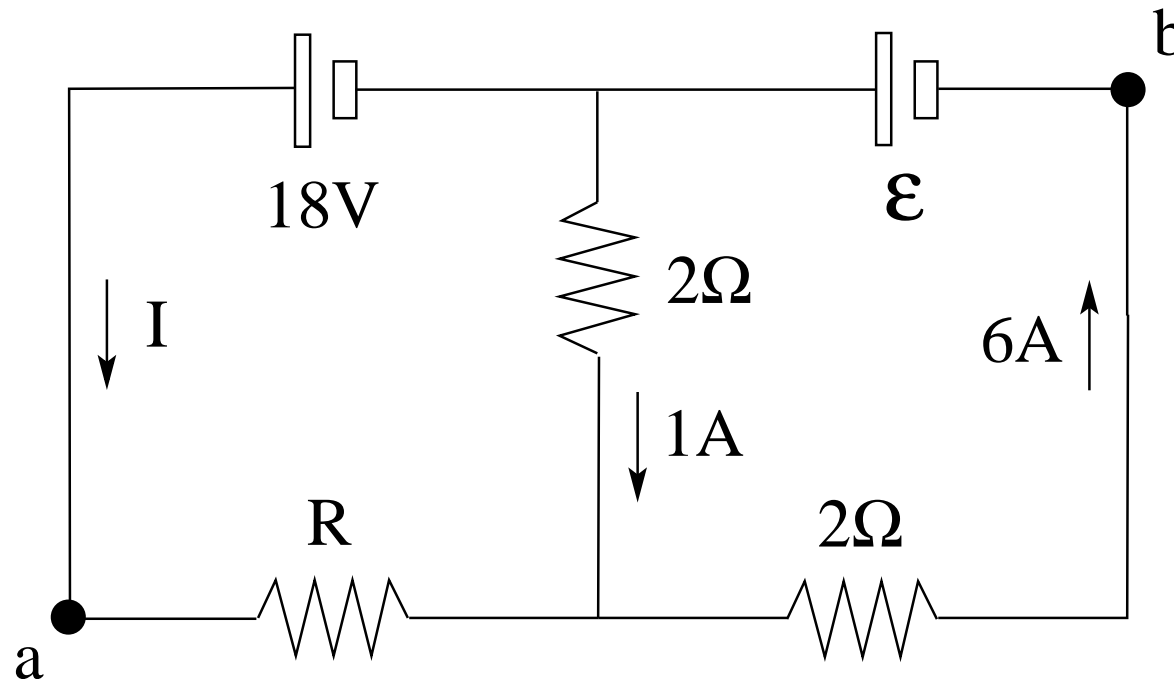


Resistor Circuit (9)



Use Kirchhoff's rules to find

- (a) the current I ,
- (b) the resistance R ,
- (c) the emf \mathcal{E} ,
- (d) the voltage $V_{ab} \equiv V_b - V_a$.

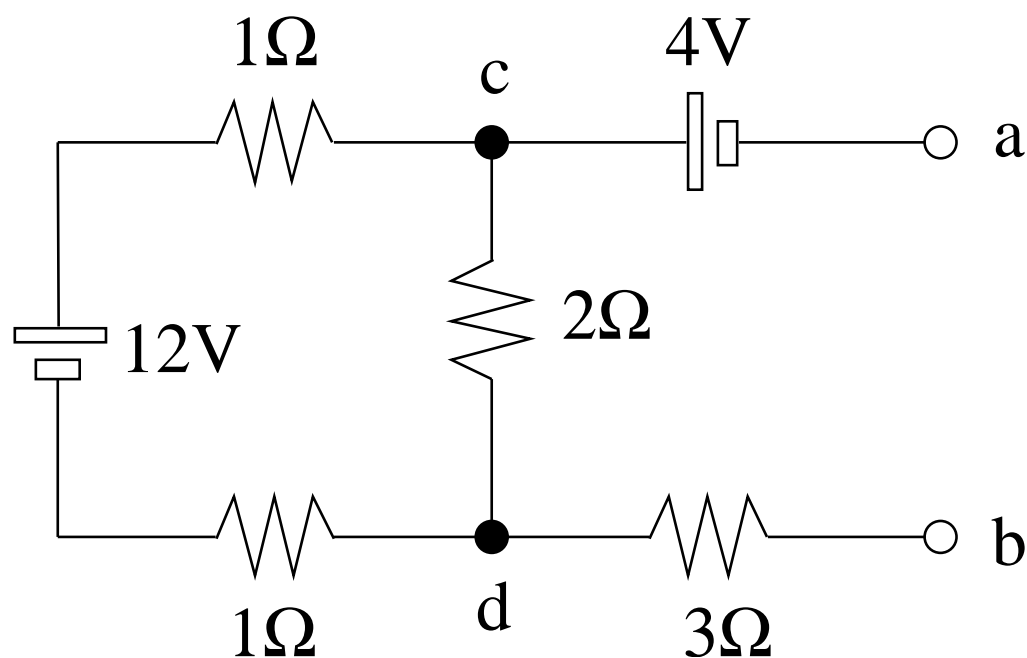


Resistor Circuit (10)



Consider the electric circuit shown.

- (a) Find the current through the 12V battery.
- (b) Find the current through the 2Ω resistor.
- (c) Find the total power dissipated.
- (d) Find the voltage $V_{cd} \equiv V_d - V_c$.
- (e) Find the voltage $V_{ab} \equiv V_b - V_a$.

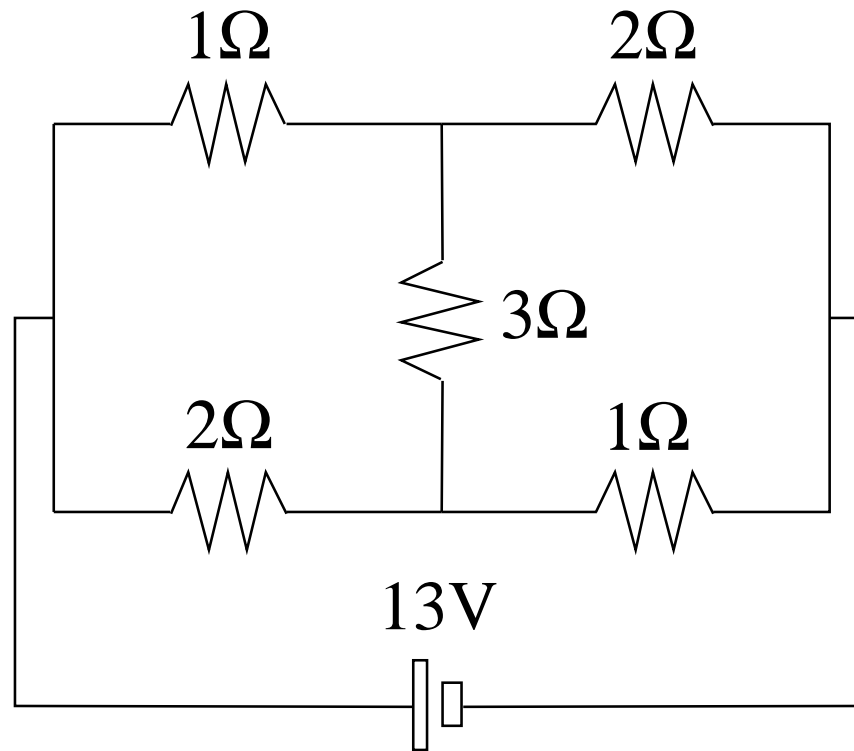


Resistor Circuit (12)



Consider the electric circuit shown.

- Find the equivalent resistance R_{eq} of the circuit.
- Find the total power P dissipated in the circuit.



More Complex Capacitor Circuit



No two capacitors are in parallel or in series.
Solution requires different strategy:

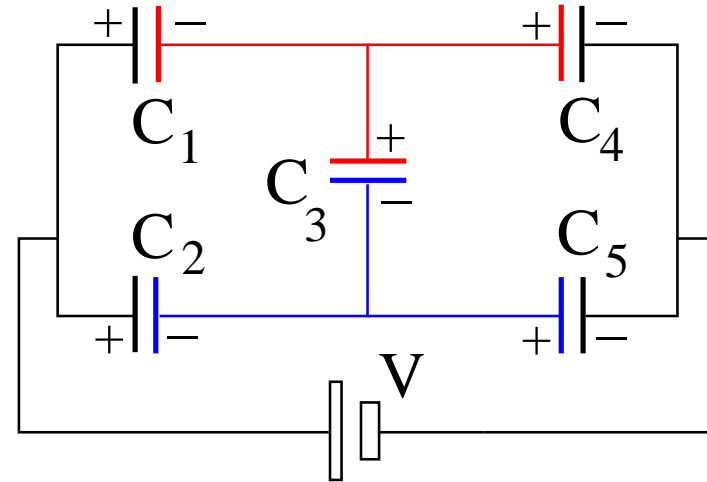
- zero charge on each conductor (here color coded),
- zero voltage around any closed loop.

Specifications: C_1, \dots, Q_5, V .

Five equations for unknowns Q_1, \dots, Q_5 :

- $Q_1 + Q_2 - Q_4 - Q_5 = 0$
- $Q_3 + Q_4 - Q_1 = 0$
- $\frac{Q_5}{C_5} + \frac{Q_3}{C_3} - \frac{Q_4}{C_4} = 0$
- $\frac{Q_2}{C_2} - \frac{Q_1}{C_1} - \frac{Q_3}{C_3} = 0$
- $V - \frac{Q_1}{C_1} - \frac{Q_4}{C_4} = 0$

Equivalent capacitance: $C_{eq} = \frac{Q_1 + Q_2}{V}$



(a) $C_m = 1\text{pF}, m = 1, \dots, 5$ and $V = 1\text{V}$:

$$C_{eq} = 1\text{pF}, Q_3 = 0,$$

$$Q_1 = Q_2 = Q_4 = Q_5 = \frac{1}{2}\text{pC}.$$

(b) $C_m = m\text{pF}, m = 1, \dots, 5$ and $V = 1\text{V}$:

$$C_{eq} = \frac{159}{71}\text{pF}, Q_1 = \frac{55}{71}\text{pC}, Q_2 = \frac{104}{71}\text{pC},$$

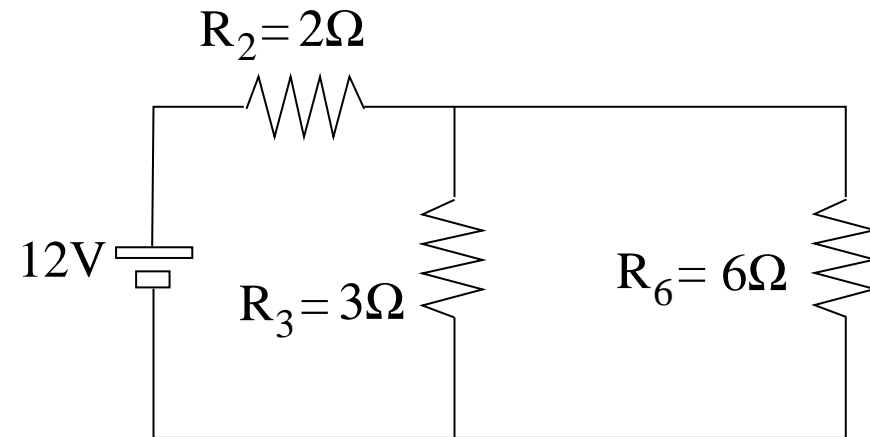
$$Q_3 = -\frac{9}{71}\text{pC}, Q_4 = \frac{64}{71}\text{pC}, Q_5 = \frac{95}{71}\text{pC}.$$

Intermediate Exam II: Problem #2 (Spring '05)



Consider the electrical circuit shown.

- (a) Find the equivalent resistance R_{eq} .
- (b) Find the current I_3 through resistor R_3 .

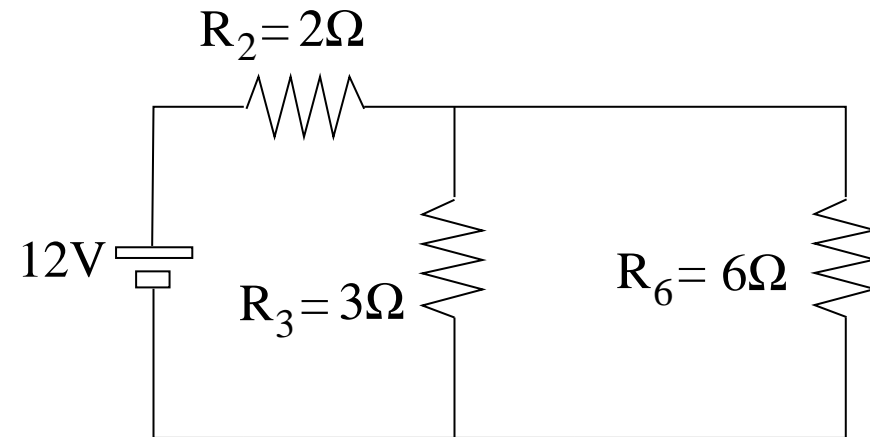


Intermediate Exam II: Problem #2 (Spring '05)



Consider the electrical circuit shown.

- (a) Find the equivalent resistance R_{eq} .
- (b) Find the current I_3 through resistor R_3 .



Solution:

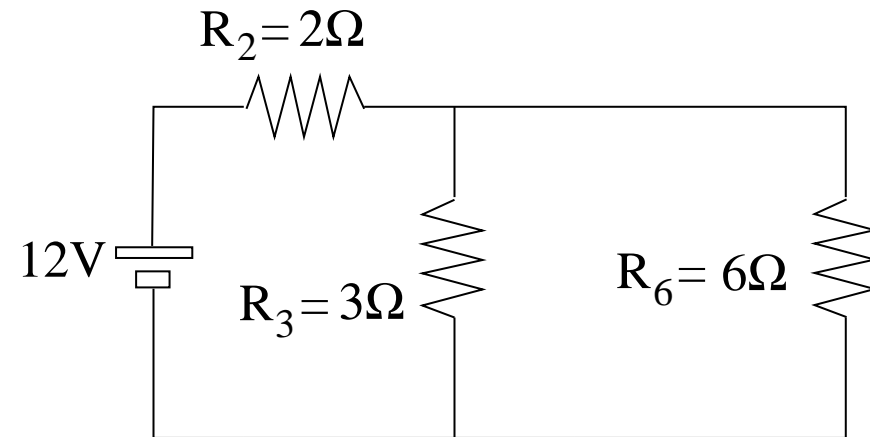
$$(a) \quad R_{36} = \left(\frac{1}{R_3} + \frac{1}{R_6} \right)^{-1} = 2\Omega, \quad R_{eq} = R_2 + R_{36} = 4\Omega.$$

Intermediate Exam II: Problem #2 (Spring '05)



Consider the electrical circuit shown.

- (a) Find the equivalent resistance R_{eq} .
- (b) Find the current I_3 through resistor R_3 .



Solution:

$$(a) \quad R_{36} = \left(\frac{1}{R_3} + \frac{1}{R_6} \right)^{-1} = 2\Omega, \quad R_{eq} = R_2 + R_{36} = 4\Omega.$$

$$(b) \quad I_2 = I_{36} = \frac{12V}{R_{eq}} = 3A$$

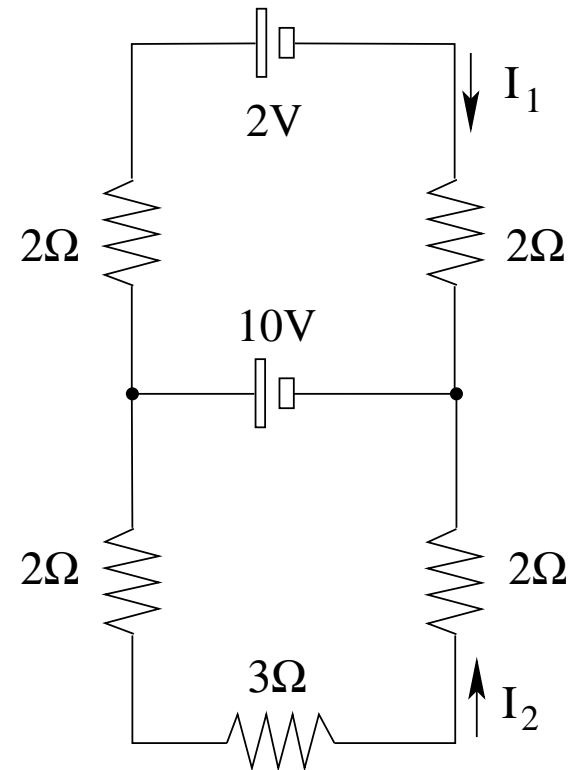
$$\Rightarrow V_3 = V_{36} = I_{36}R_{36} = 6V \quad \Rightarrow I_3 = \frac{V_3}{R_3} = 2A.$$

Intermediate Exam II: Problem #2 (Spring '06)



Consider the two-loop circuit shown.

- (a) Find the current I_1 .
- (b) Find the current I_2 .

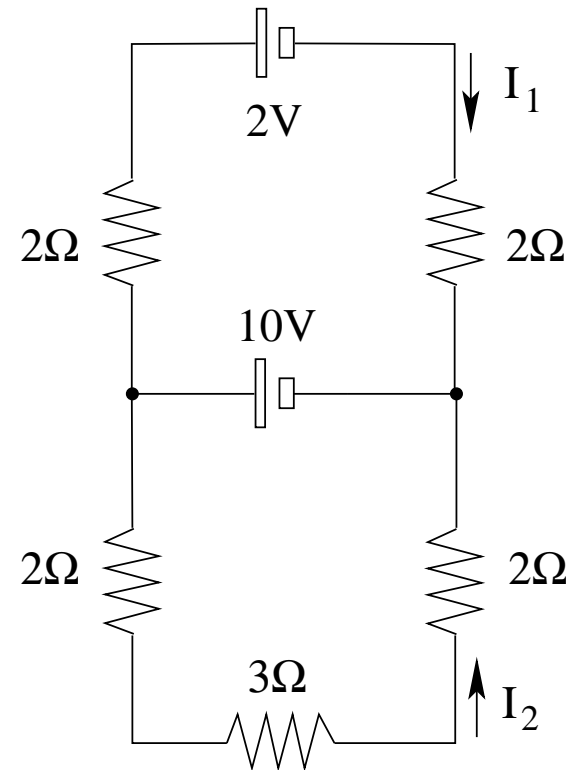


Intermediate Exam II: Problem #2 (Spring '06)



Consider the two-loop circuit shown.

- (a) Find the current I_1 .
- (b) Find the current I_2 .



Solution:

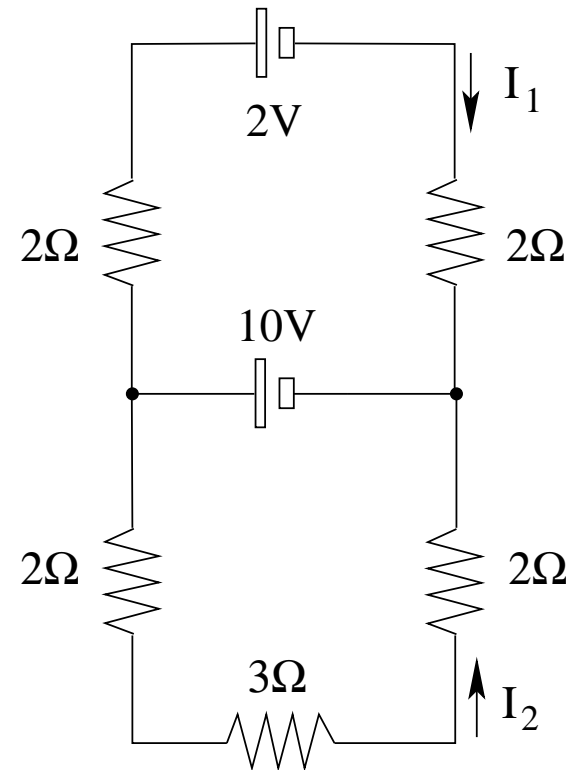
$$(a) \quad -(2\Omega)(I_1) + 10V - (2\Omega)(I_1) - 2V = 0 \quad \Rightarrow \quad I_1 = \frac{8V}{4\Omega} = 2A.$$

Intermediate Exam II: Problem #2 (Spring '06)



Consider the two-loop circuit shown.

- (a) Find the current I_1 .
- (b) Find the current I_2 .



Solution:

$$(a) \quad -(2\Omega)(I_1) + 10V - (2\Omega)(I_1) - 2V = 0 \quad \Rightarrow \quad I_1 = \frac{8V}{4\Omega} = 2A.$$

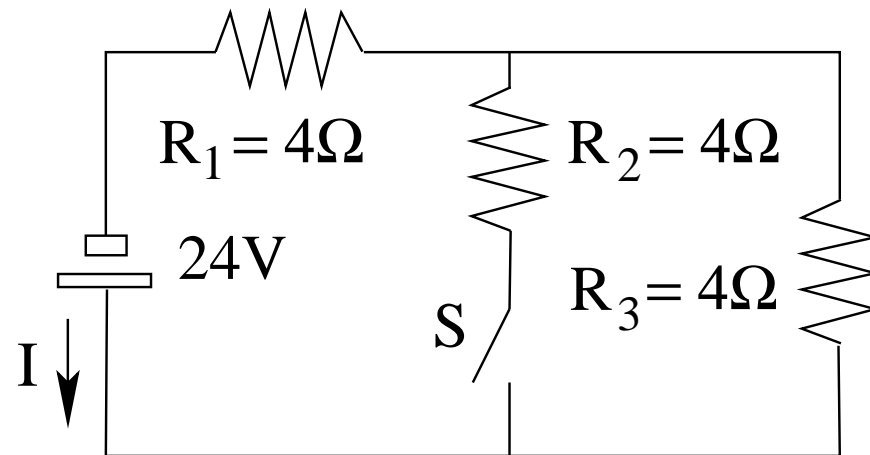
$$(b) \quad -(2\Omega)(I_2) + 10V - (2\Omega)(I_2) - (3\Omega)(I_2) = 0 \quad \Rightarrow \quad I_2 = \frac{10V}{7\Omega} = 1.43A.$$

Unit Exam II: Problem #2 (Spring '07)



Consider the electric circuit shown.

- (a) Find the current I when the switch S is open.
- (b) Find the power P_3 dissipated in resistor R_3 when the switch is open.
- (c) Find the current I when the switch S is closed.
- (d) Find the power P_3 dissipated in resistor R_3 when the switch is closed.



Unit Exam II: Problem #2 (Spring '07)

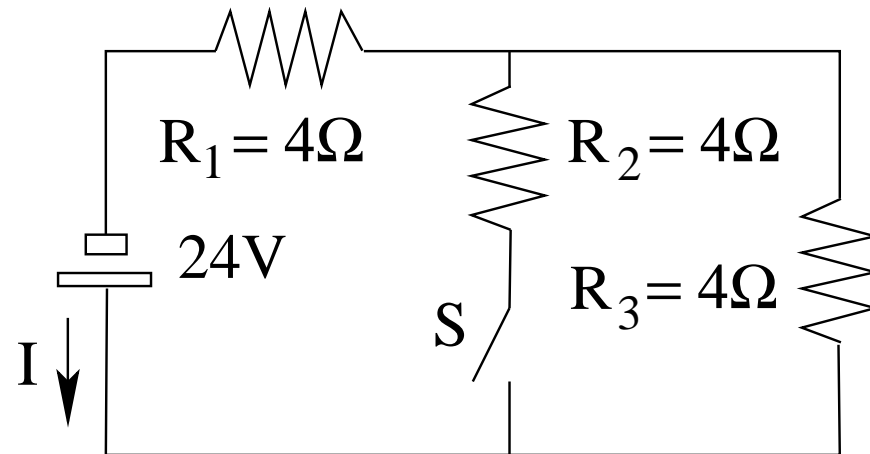


Consider the electric circuit shown.

- (a) Find the current I when the switch S is open.
- (b) Find the power P_3 dissipated in resistor R_3 when the switch is open.
- (c) Find the current I when the switch S is closed.
- (d) Find the power P_3 dissipated in resistor R_3 when the switch is closed.

Solution:

(a) $I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$



Unit Exam II: Problem #2 (Spring '07)



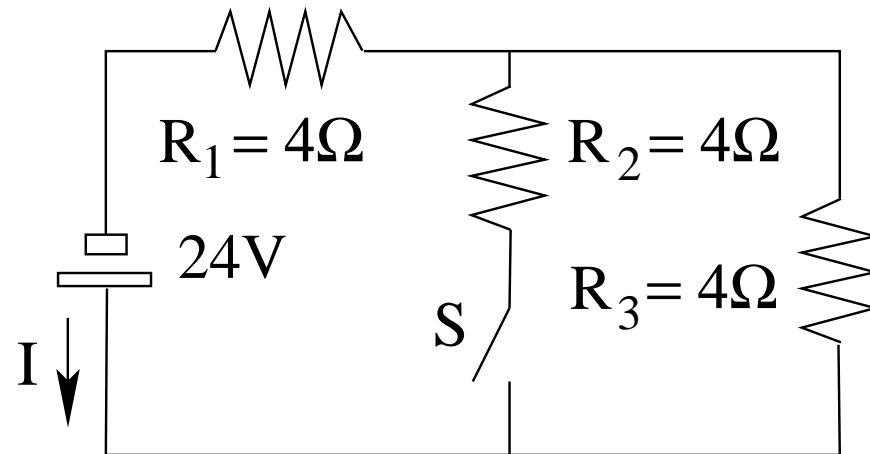
Consider the electric circuit shown.

- (a) Find the current I when the switch S is open.
- (b) Find the power P_3 dissipated in resistor R_3 when the switch is open.
- (c) Find the current I when the switch S is closed.
- (d) Find the power P_3 dissipated in resistor R_3 when the switch is closed.

Solution:

$$(a) \quad I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$$

$$(b) \quad P_3 = (3\text{A})^2(4\Omega) = 36\text{W}.$$



Unit Exam II: Problem #2 (Spring '07)



Consider the electric circuit shown.

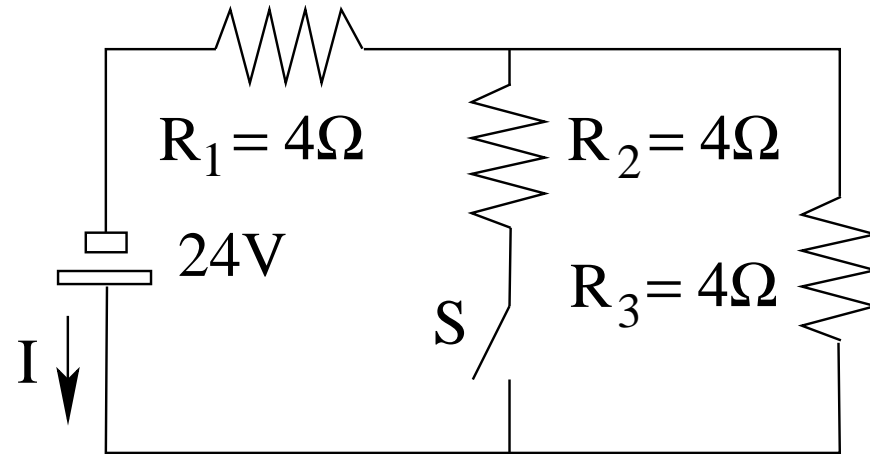
- Find the current I when the switch S is open.
- Find the power P_3 dissipated in resistor R_3 when the switch is open.
- Find the current I when the switch S is closed.
- Find the power P_3 dissipated in resistor R_3 when the switch is closed.

Solution:

$$(a) \quad I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$$

$$(b) \quad P_3 = (3\text{A})^2(4\Omega) = 36\text{W}.$$

$$(c) \quad I = \frac{24\text{V}}{6\Omega} = 4\text{A}.$$



Unit Exam II: Problem #2 (Spring '07)



Consider the electric circuit shown.

- (a) Find the current I when the switch S is open.
- (b) Find the power P_3 dissipated in resistor R_3 when the switch is open.
- (c) Find the current I when the switch S is closed.
- (d) Find the power P_3 dissipated in resistor R_3 when the switch is closed.

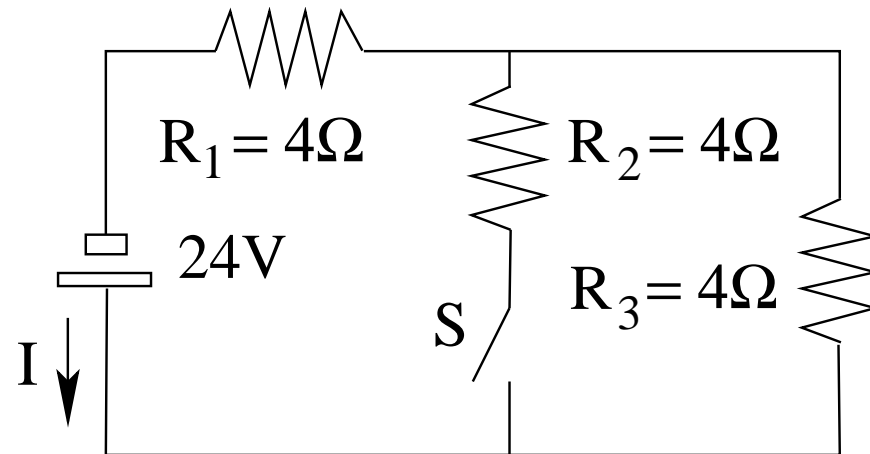
Solution:

$$(a) \quad I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$$

$$(b) \quad P_3 = (3\text{A})^2(4\Omega) = 36\text{W}.$$

$$(c) \quad I = \frac{24\text{V}}{6\Omega} = 4\text{A}.$$

$$(d) \quad P_3 = (2\text{A})^2(4\Omega) = 16\text{W}.$$

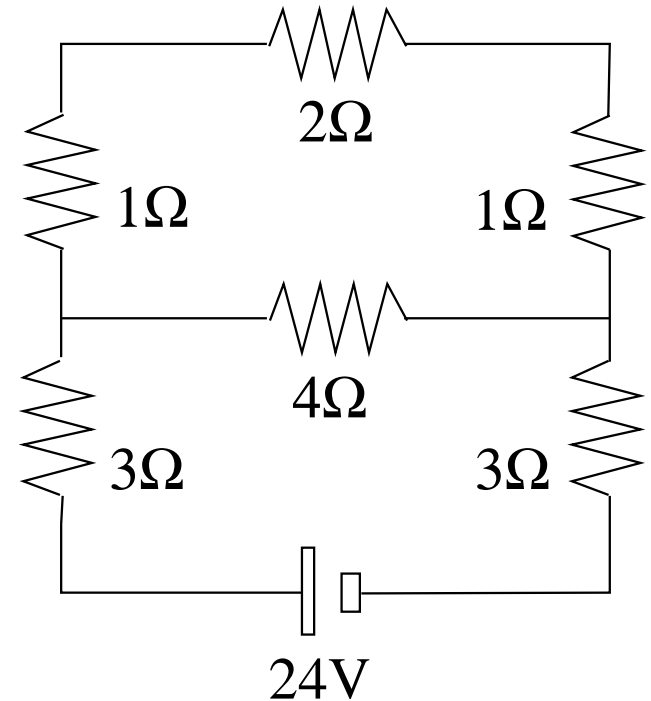


Unit Exam II: Problem #2 (Spring '09)



Consider the resistor circuit shown.

- (a) Find the equivalent resistance R_{eq} .
- (b) Find the power P supplied by the battery.
- (c) Find the current I_4 through the 4Ω -resistor.
- (d) Find the voltage V_2 across the 2Ω -resistor.



Unit Exam II: Problem #2 (Spring '09)

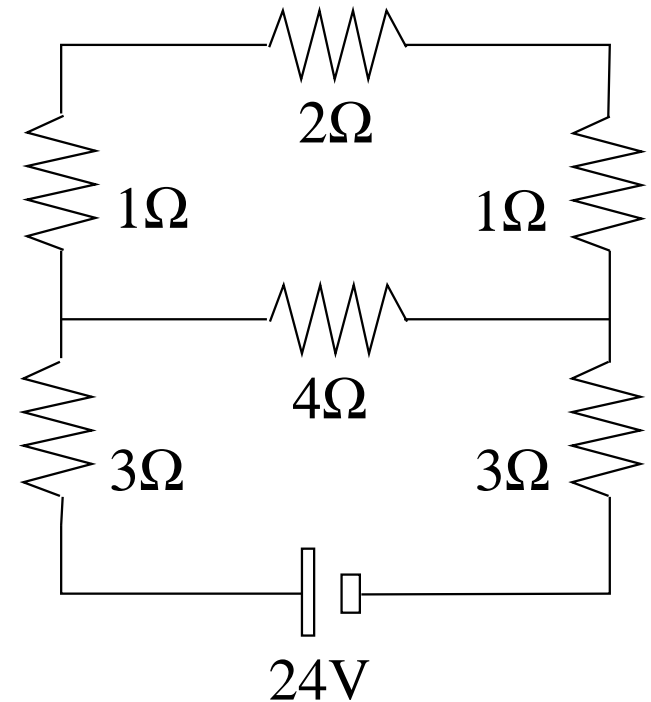


Consider the resistor circuit shown.

- (a) Find the equivalent resistance R_{eq} .
- (b) Find the power P supplied by the battery.
- (c) Find the current I_4 through the 4Ω -resistor.
- (d) Find the voltage V_2 across the 2Ω -resistor.

Solution:

(a) $R_{eq} = 8\Omega$.



Unit Exam II: Problem #2 (Spring '09)



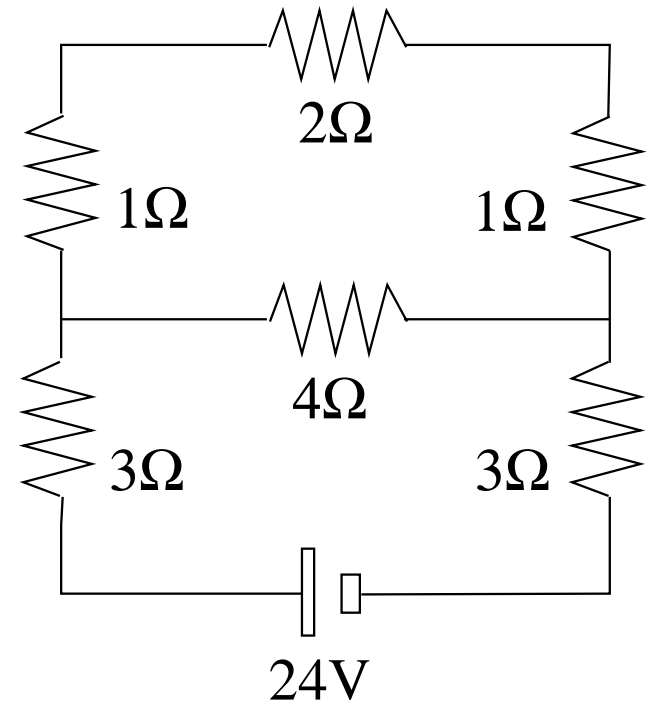
Consider the resistor circuit shown.

- (a) Find the equivalent resistance R_{eq} .
- (b) Find the power P supplied by the battery.
- (c) Find the current I_4 through the 4Ω -resistor.
- (d) Find the voltage V_2 across the 2Ω -resistor.

Solution:

(a) $R_{eq} = 8\Omega$.

(b) $P = \frac{(24V)^2}{8\Omega} = 72W$.



Unit Exam II: Problem #2 (Spring '09)



Consider the resistor circuit shown.

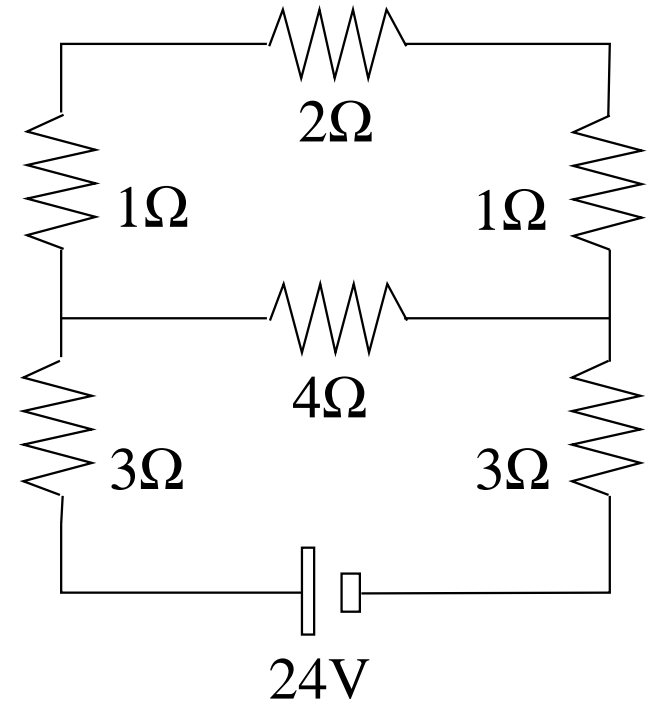
- (a) Find the equivalent resistance R_{eq} .
- (b) Find the power P supplied by the battery.
- (c) Find the current I_4 through the 4Ω -resistor.
- (d) Find the voltage V_2 across the 2Ω -resistor.

Solution:

(a) $R_{eq} = 8\Omega$.

(b) $P = \frac{(24V)^2}{8\Omega} = 72W$.

(c) $I_4 = \frac{1}{2} \frac{24V}{8\Omega} = 1.5A$.



Unit Exam II: Problem #2 (Spring '09)

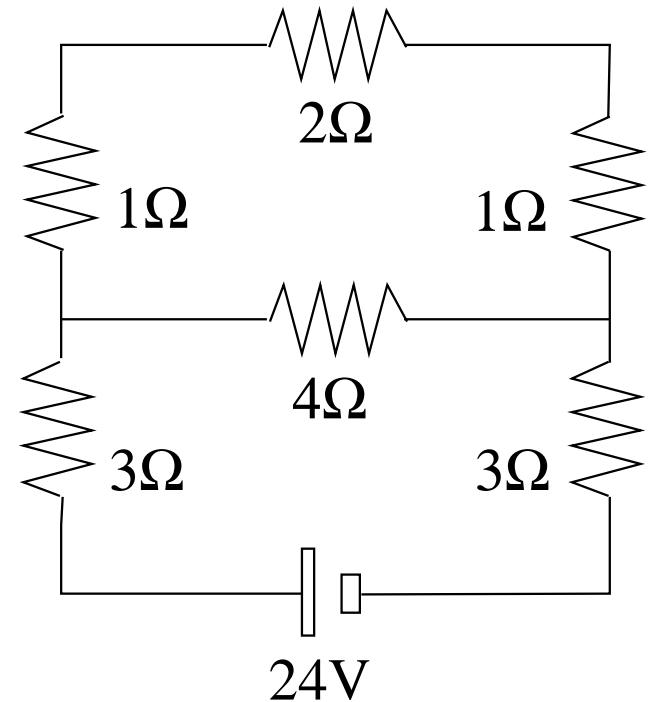


Consider the resistor circuit shown.

- (a) Find the equivalent resistance R_{eq} .
- (b) Find the power P supplied by the battery.
- (c) Find the current I_4 through the 4Ω -resistor.
- (d) Find the voltage V_2 across the 2Ω -resistor.

Solution:

- (a) $R_{eq} = 8\Omega$.
- (b) $P = \frac{(24V)^2}{8\Omega} = 72W$.
- (c) $I_4 = \frac{1}{2} \frac{24V}{8\Omega} = 1.5A$.
- (d) $V_2 = (1.5A)(2\Omega) = 3V$.

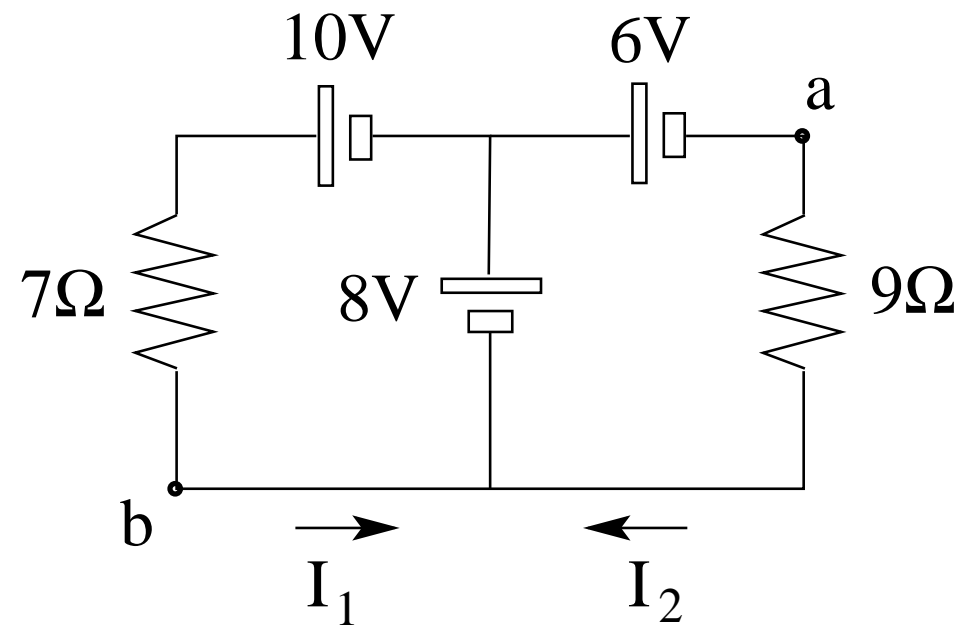


Unit Exam II: Problem #3 (Spring '07)



Consider the two-loop circuit shown.

- (a) Find the current I_1 .
- (b) Find the current I_2 .
- (c) Find the potential difference $V_a - V_b$.



Unit Exam II: Problem #3 (Spring '07)

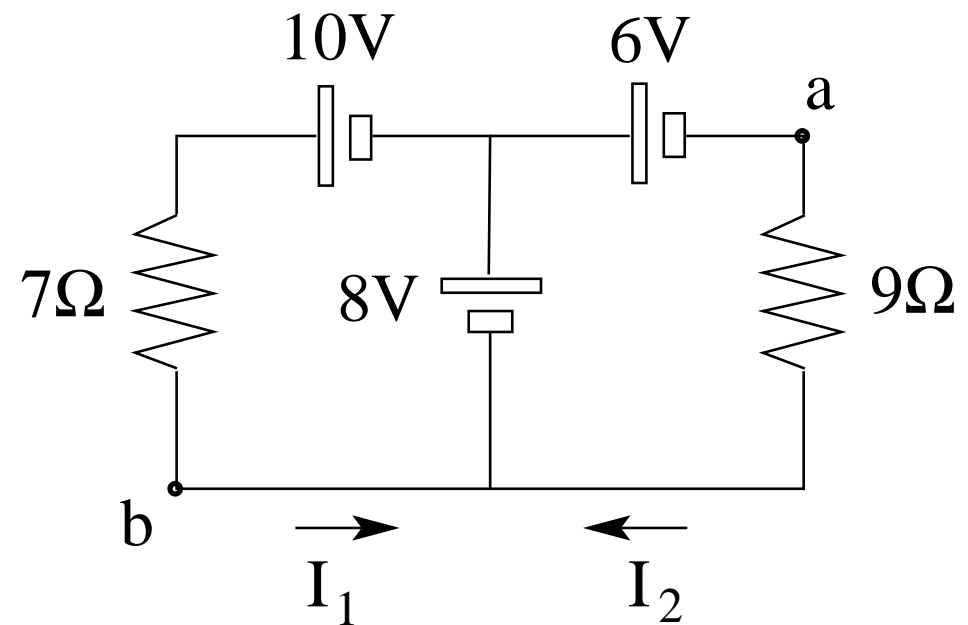


Consider the two-loop circuit shown.

- (a) Find the current I_1 .
- (b) Find the current I_2 .
- (c) Find the potential difference $V_a - V_b$.

Solution:

(a)
$$I_1 = \frac{8V + 10V}{7\Omega} = 2.57A.$$



Unit Exam II: Problem #3 (Spring '07)



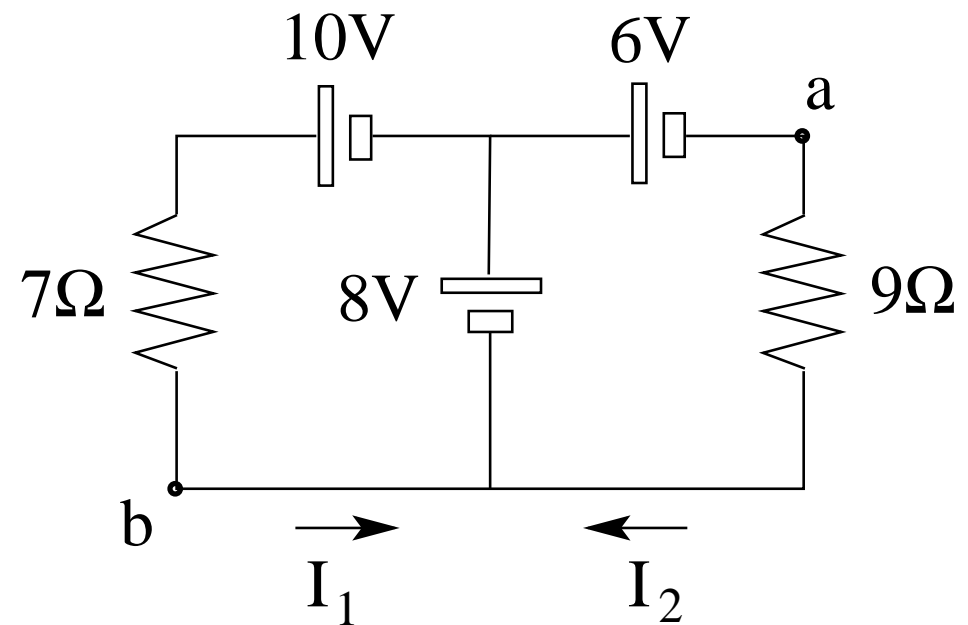
Consider the two-loop circuit shown.

- (a) Find the current I_1 .
- (b) Find the current I_2 .
- (c) Find the potential difference $V_a - V_b$.

Solution:

$$(a) \quad I_1 = \frac{8V + 10V}{7\Omega} = 2.57A.$$

$$(b) \quad I_2 = \frac{8V - 6V}{9\Omega} = 0.22A.$$



Unit Exam II: Problem #3 (Spring '07)



Consider the two-loop circuit shown.

- (a) Find the current I_1 .
- (b) Find the current I_2 .
- (c) Find the potential difference $V_a - V_b$.

Solution:

$$(a) I_1 = \frac{8V + 10V}{7\Omega} = 2.57A.$$

$$(b) I_2 = \frac{8V - 6V}{9\Omega} = 0.22A.$$

$$(c) V_a - V_b = 8V - 6V = 2V.$$

