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## 10. Resistors II

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### Abstract

Lecture slides 10 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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# M. C. Escher: Waterfall



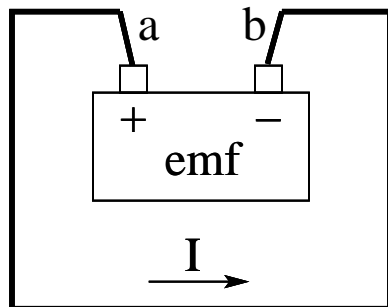
# Direct Current Circuit



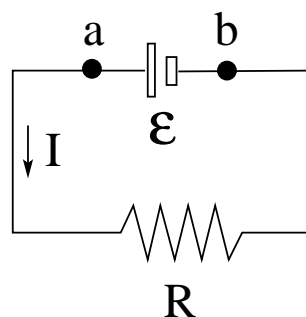
Consider a wire with resistance  $R = \rho\ell/A$  connected to a battery.

- **Resistor rule:** In the direction of  $I$  across a resistor with resistance  $R$ , the electric potential drops:  $\Delta V = -IR$ .
- **EMF rule:** From the  $(-)$  terminal to the  $(+)$  terminal in an ideal source of emf, the potential rises:  $\Delta V = \mathcal{E}$ .
- **Loop rule:** The algebraic sum of the changes in potential encountered in a complete traversal of any loop in a circuit must be zero:  $\sum \Delta V_i = 0$ .

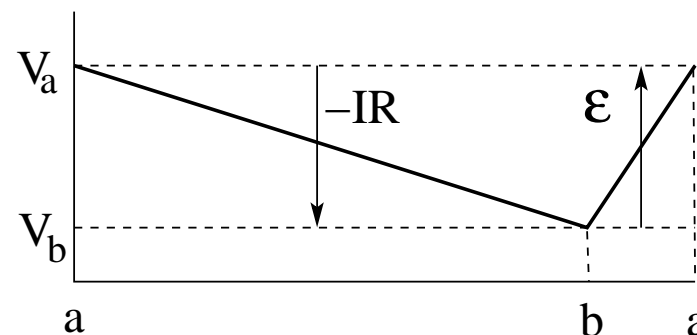
physical system



circuit diagram



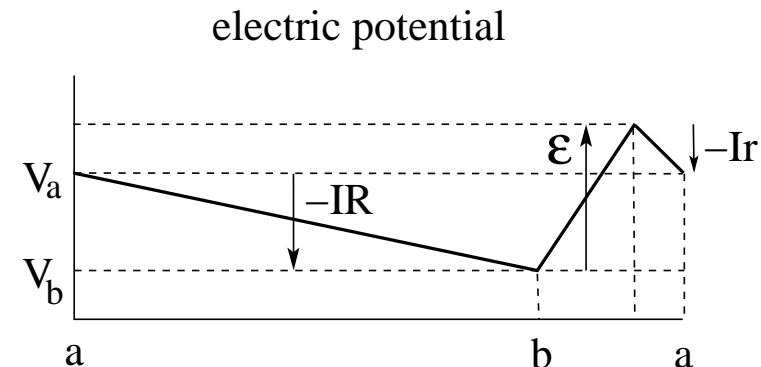
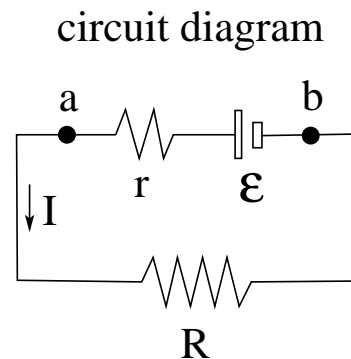
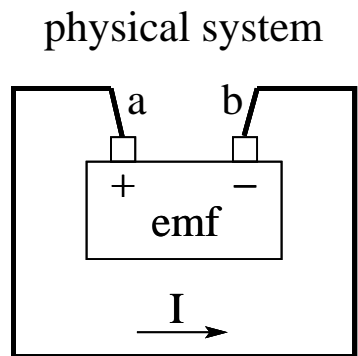
electric potential



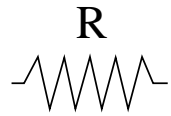
# Battery with Internal Resistance



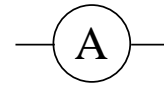
- Real batteries have an internal resistance  $r$ .
- The terminal voltage  $V_{ba} \equiv V_a - V_b$  is smaller than the emf  $\mathcal{E}$  written on the label if a current flows through the battery.
- Usage of the battery increases its internal resistance.
- Current from loop rule:  $\mathcal{E} - Ir - IR = 0 \Rightarrow I = \frac{\mathcal{E}}{R + r}$
- Current from terminal voltage:  $V_{ba} = \mathcal{E} - Ir = IR \Rightarrow I = \frac{V_{ba}}{R}$



# Symbols Used in Circuit Diagrams



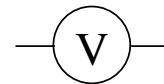
resistor



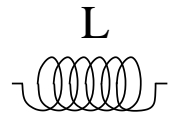
ammeter (connect in series)



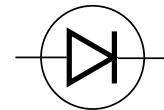
capacitor



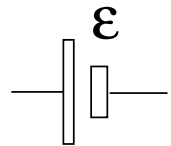
voltmeter (connect in parallel)



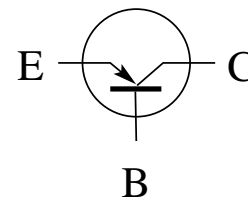
inductor



diode



emf source



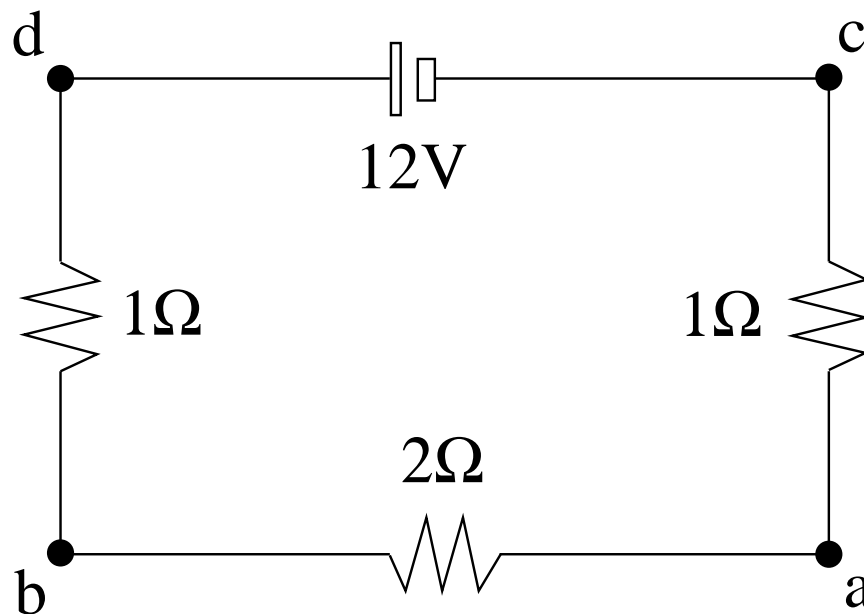
transistor

## Resistor Circuit (4)



Consider the resistor circuit shown.

- (a) Find the direction of the current  $I$  (cw/ccw).
- (b) Find the magnitude of the current  $I$ .
- (c) Find the voltage  $V_{ab} = V_b - V_a$ .
- (d) Find the voltage  $V_{cd} = V_d - V_c$ .

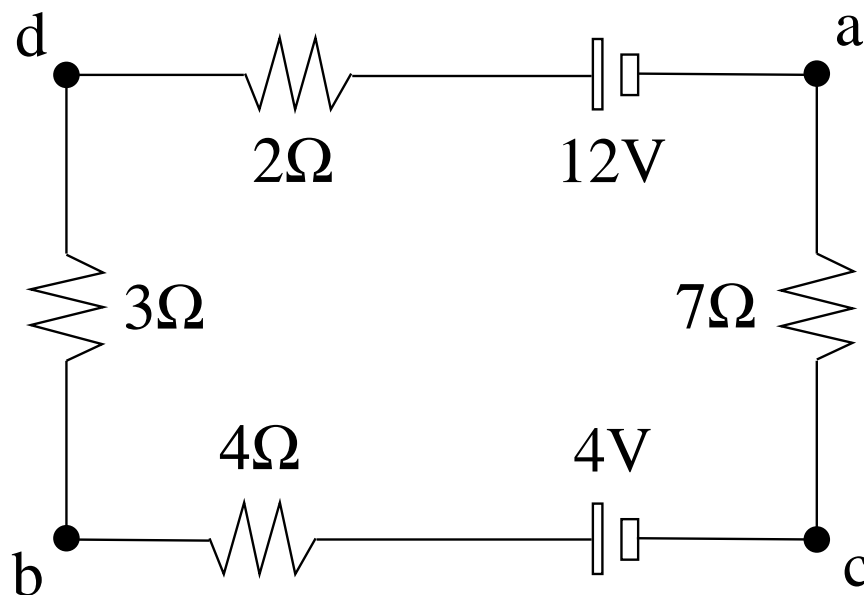


## Resistor Circuit (5)



Consider the resistor circuit shown.

- (a) Find the direction (cw/ccw) of the current  $I$  in the loop.
- (b) Find the magnitude of the current  $I$  in the loop.
- (c) Find the potential difference  $V_{ab} = V_b - V_a$ .
- (d) Find the voltage  $V_{cd} = V_d - V_c$ .

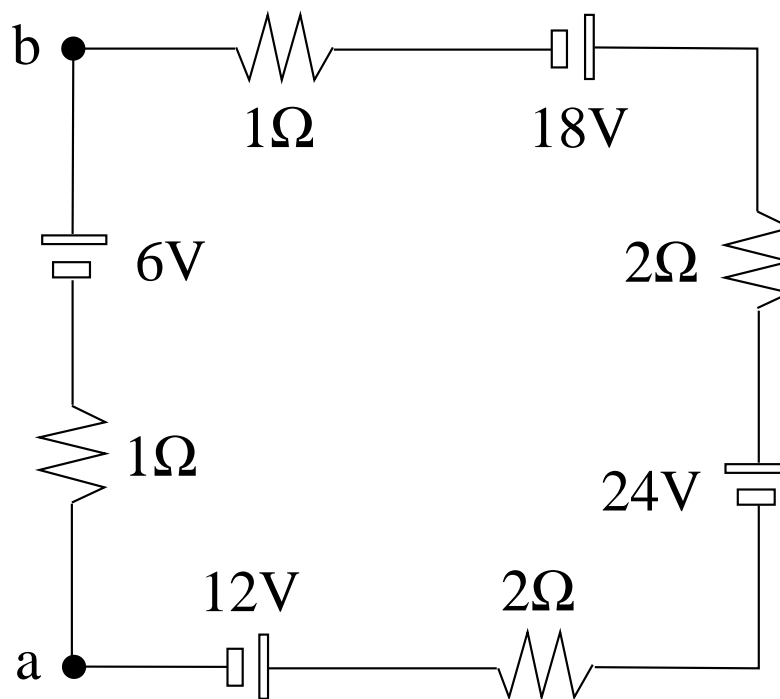


## Resistor Circuit (6)



Consider the resistor circuit shown.

- (a) Guess the current direction (cw/ccw).
- (b) Use the loop rule to determine the current.
- (c) Find  $V_{ab} \equiv V_b - V_a$  along two different paths.



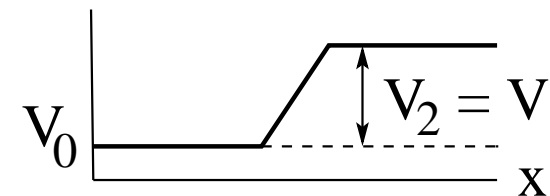
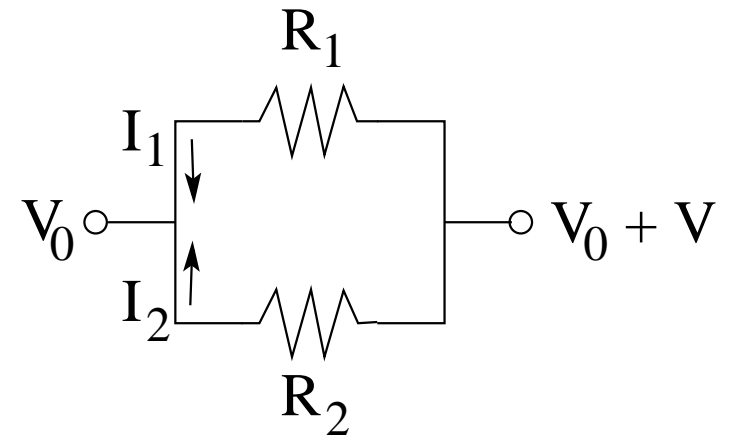
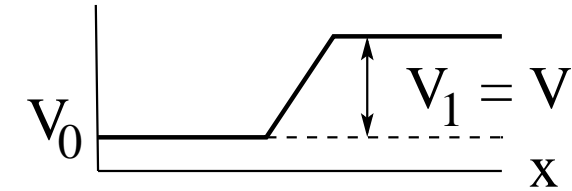


# Resistors Connected in Parallel



Find the equivalent resistance of two resistors connected in parallel.

- Current through resistors:  $I_1 + I_2 = I$
- Voltage across resistors:  $V_1 = V_2 = V$
- Equivalent resistance:  $\frac{1}{R} \equiv \frac{I}{V} = \frac{I_1}{V_1} + \frac{I_2}{V_2}$
- $\Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

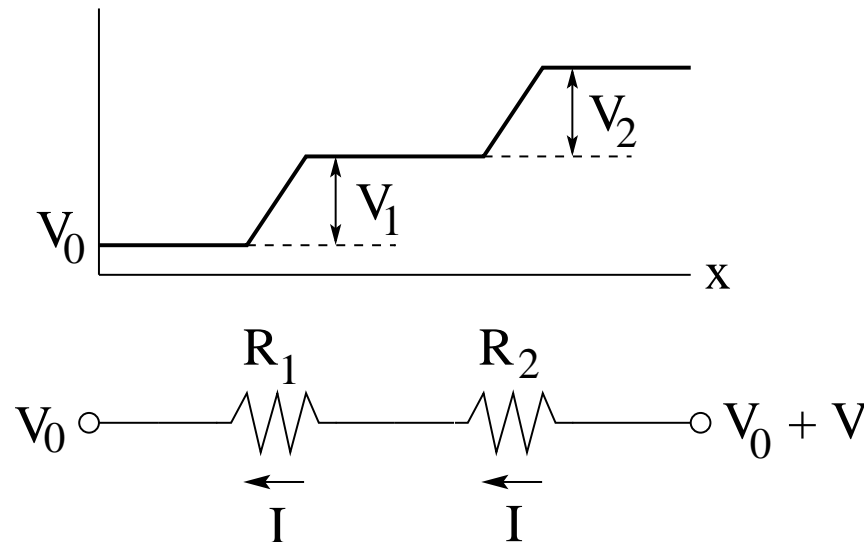


# Resistors Connected in Series



Find the equivalent resistance of two resistors connected in series.

- Current through resistors:  $I_1 = I_2 = I$
- Voltage across resistors:  $V_1 + V_2 = V$
- Equivalent resistance:  $R \equiv \frac{V}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2}$
- $\Rightarrow R = R_1 + R_2$

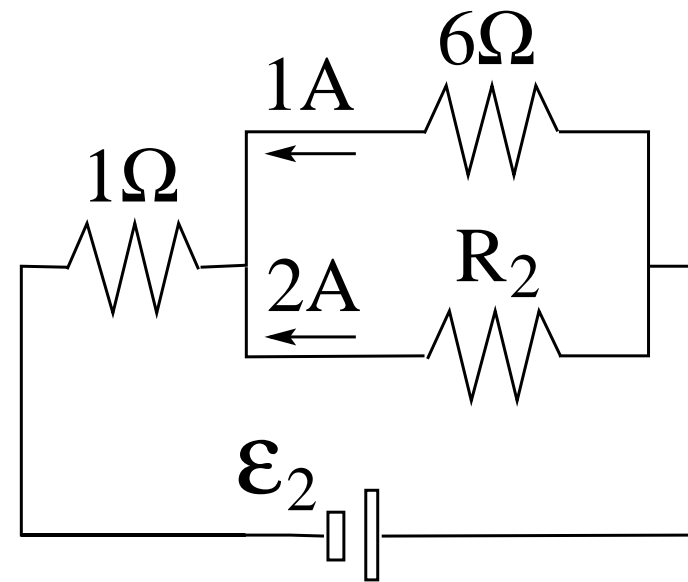
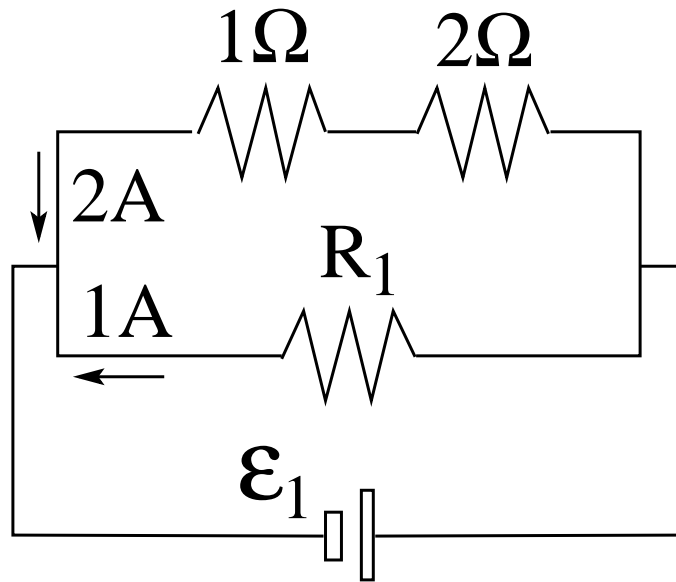


# Resistor Circuit (1)



Consider the two resistor circuits shown.

- (a) Find the resistance  $R_1$ .
- (b) Find the emf  $\mathcal{E}_1$ .
- (c) Find the resistance  $R_2$ .
- (d) Find the emf  $\mathcal{E}_2$ .

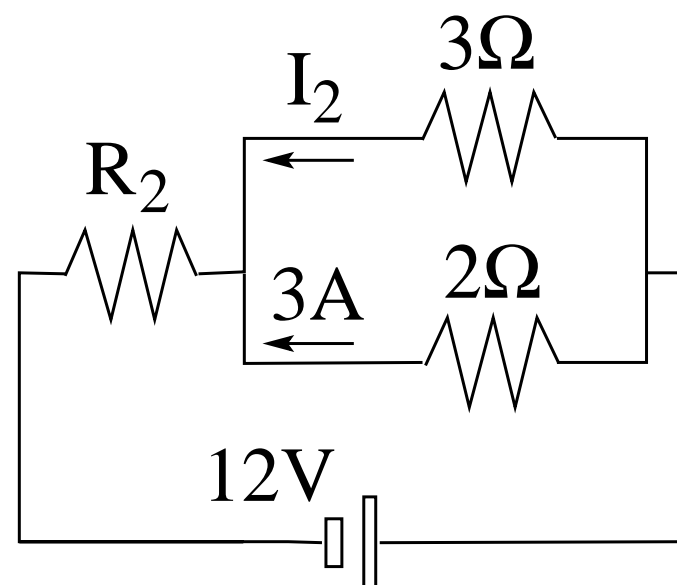
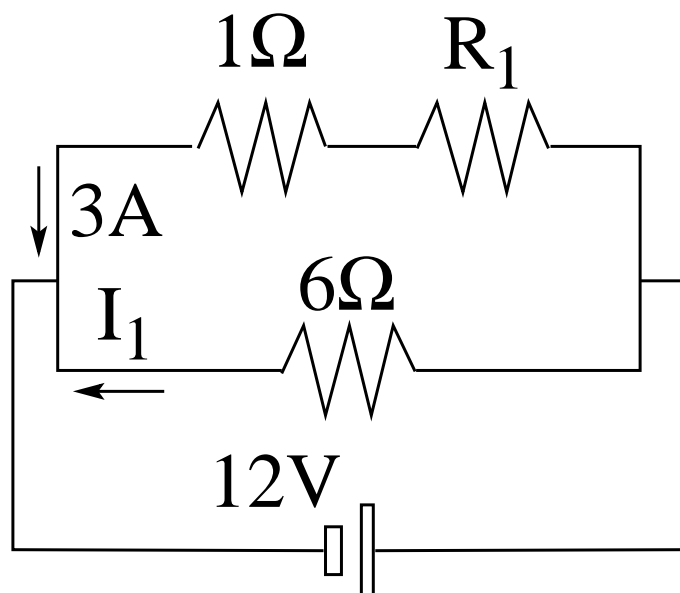


## Resistor Circuit (2)



Consider the two resistor circuits shown.

- (a) Find the resistance  $R_1$ .
- (b) Find the current  $I_1$ .
- (c) Find the resistance  $R_2$ .
- (d) Find the current  $I_2$ .

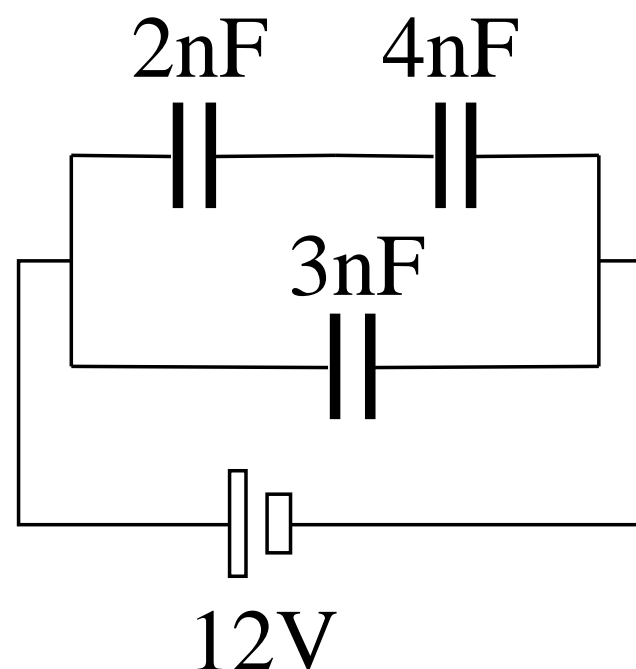
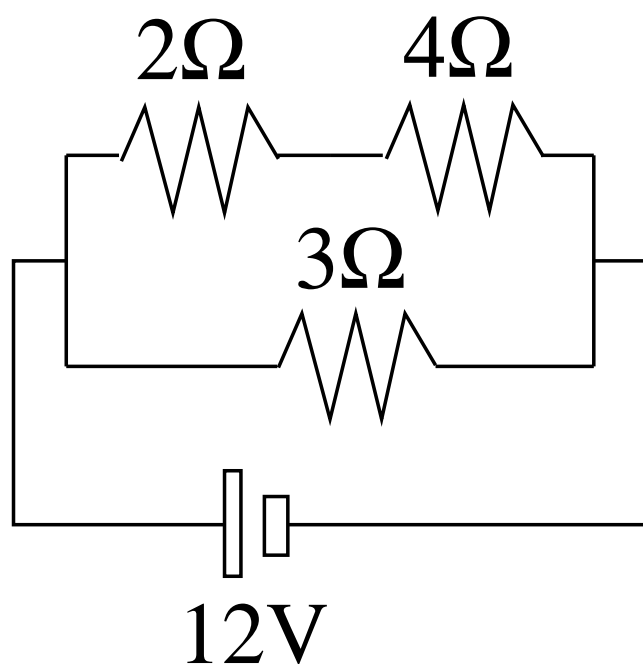


## Resistor Circuit (3)



Consider the resistor and capacitor circuits shown.

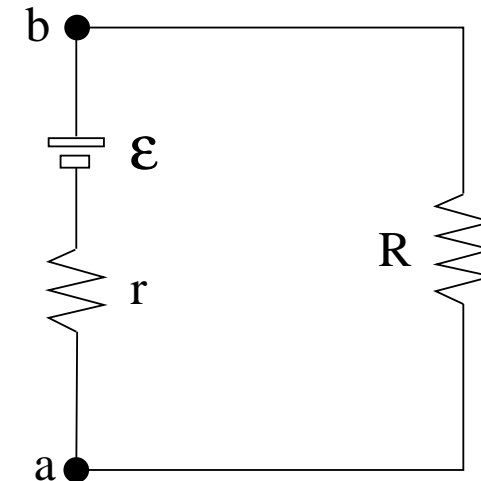
- Find the equivalent resistance  $R_{eq}$ .
- Find the power  $P_2, P_3, P_4$  dissipated in each resistor.
- Find the equivalent capacitance  $C_{eq}$ .
- Find the energy  $U_2, U_3, U_4$  stored in each capacitor.





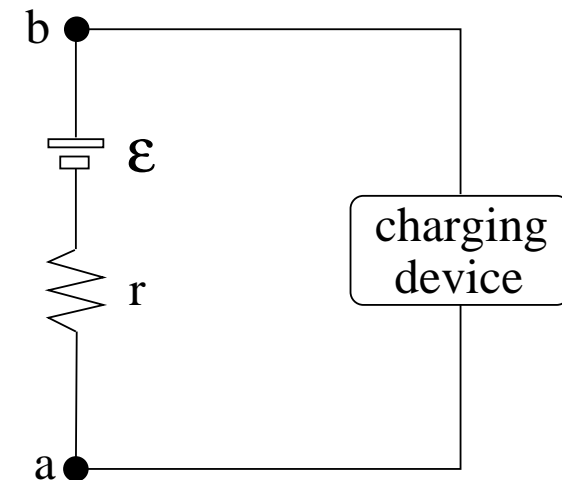
## Battery in use

- Terminal voltage:  $V_{ab} = \mathcal{E} - Ir = IR$
- Power output of battery:  $P = V_{ab}I = \mathcal{E}I - I^2r$ 
  - Power generated in battery:  $\mathcal{E}I$
  - Power dissipated in battery:  $I^2r$
- Power dissipated in resistor:  $P = I^2R$



## Battery being charged:

- Terminal voltage:  $V_{ab} = \mathcal{E} + Ir$
- Power supplied by charging device:  $P = V_{ab}I$
- Power input into battery:  $P = \mathcal{E}I + I^2r$ 
  - Power stored in battery:  $\mathcal{E}I$
  - Power dissipated in battery:  $I^2r$

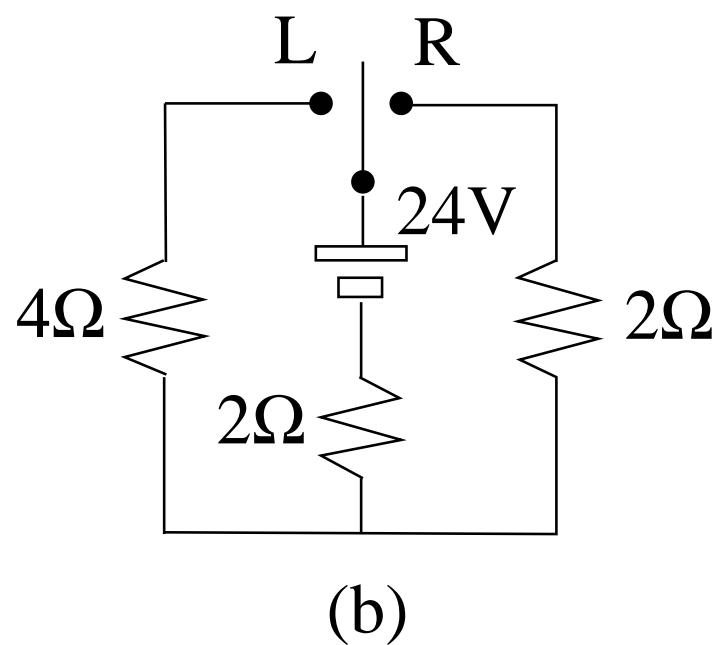
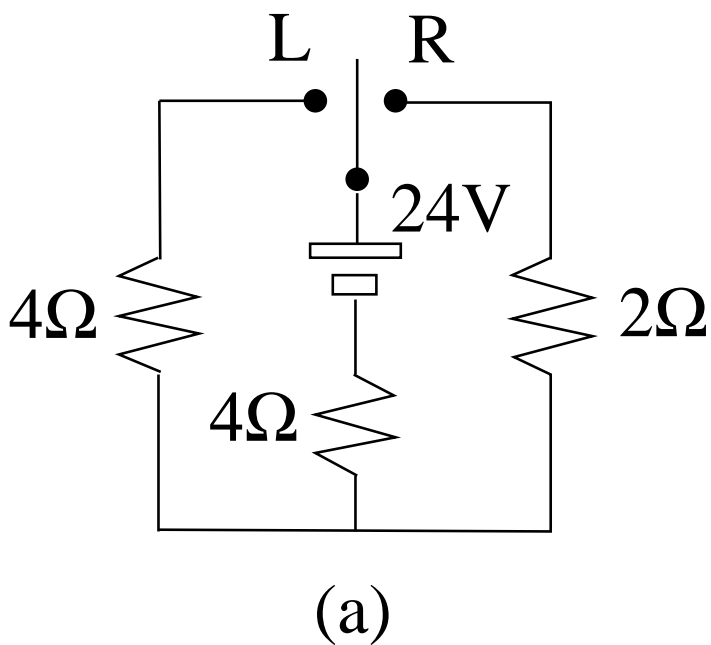


# Resistor Circuit (7)



Consider two 24V batteries with internal resistances (a)  $r = 4\Omega$ , (b)  $r = 2\Omega$ .

- Which setting of the switch (L/R) produces the larger power dissipation in the resistor on the side?



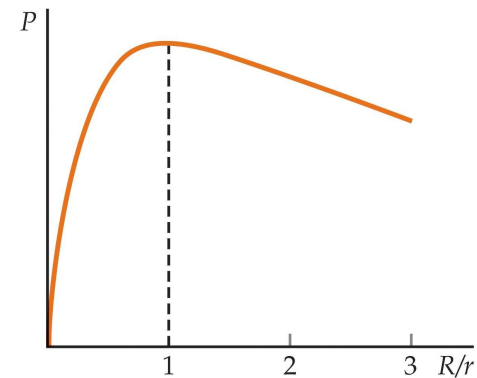
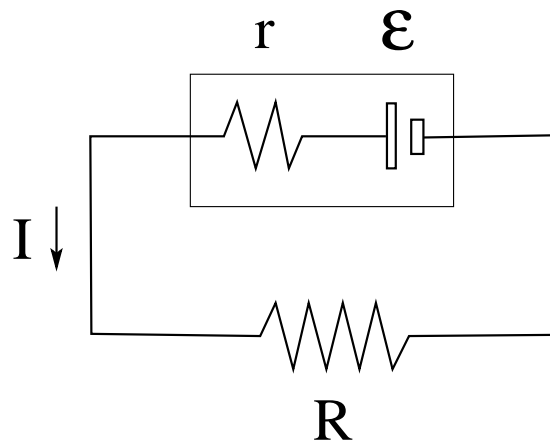
# Impedance Matching



A battery providing an emf  $\mathcal{E}$  with internal resistance  $r$  is connected to an external resistor of resistance  $R$  as shown.

For what value of  $R$  does the battery deliver the maximum power to the external resistor?

- Electric current:  $\mathcal{E} - Ir - IR = 0 \Rightarrow I = \frac{\mathcal{E}}{R + r}$
- Power delivered to external resistor:  $P = I^2 R = \frac{\mathcal{E}^2 R}{(R + r)^2}$
- Condition for maximum power:  $\frac{dP}{dR} = 0 \Rightarrow R = r$



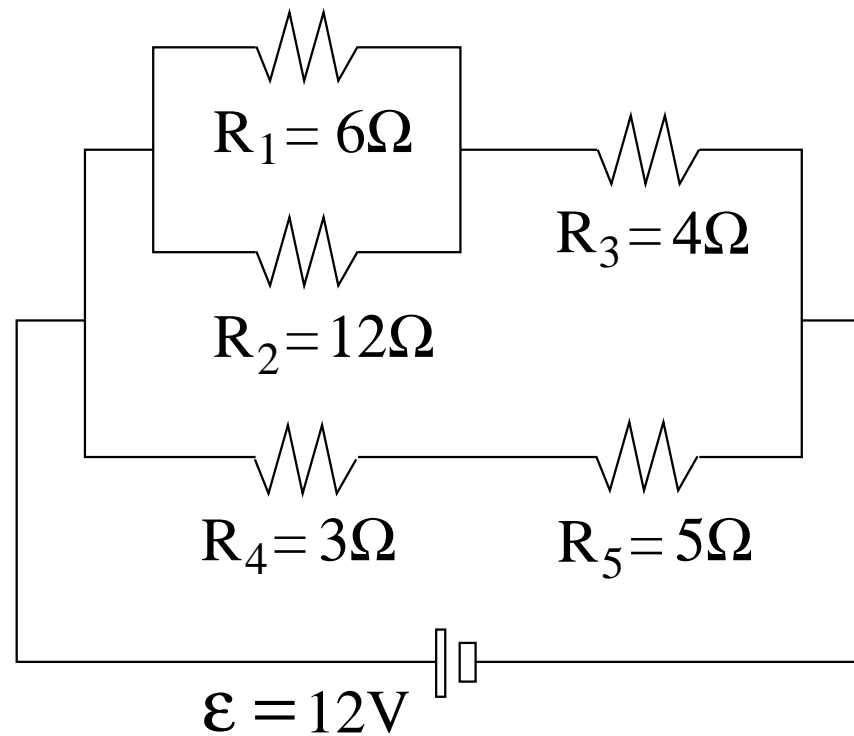


# Resistor Circuit (8)



Consider the circuit of resistors shown.

- Find the equivalent resistance  $R_{eq}$ .
- Find the currents  $I_1, \dots, I_5$  through each resistor and the voltages  $V_1, \dots, V_5$  across each resistor.
- Find the total power  $P$  dissipated in the circuit.





## Loop Rule

- When any closed-circuit loop is traversed, the algebraic sum of the changes in electric potential must be zero.

## Junction Rule

- At any junction in a circuit, the sum of the incoming currents must equal the sum of the outgoing currents.

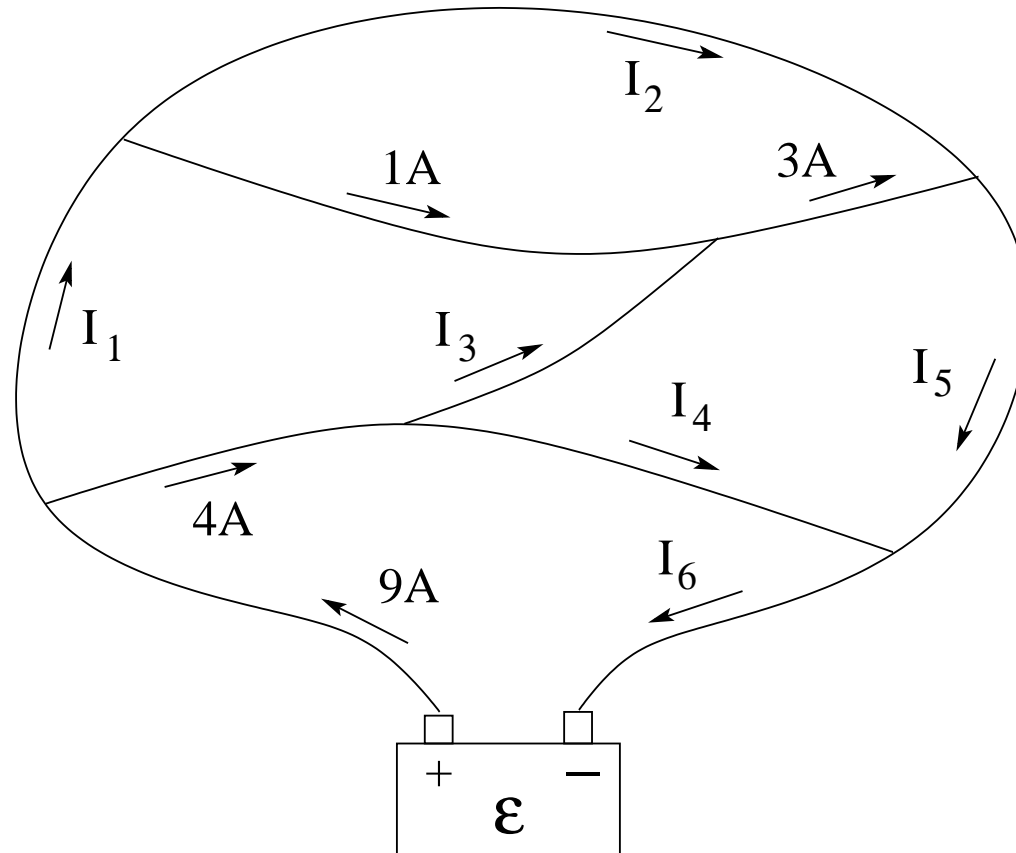
## Strategy

- Use the junction rule to name all independent currents.
- Use the loop rule to determine the independent currents.

# Applying the Junction Rule



In the circuit of steady currents, use the junction rule to find the unknown currents  $I_1, \dots, I_6$ .

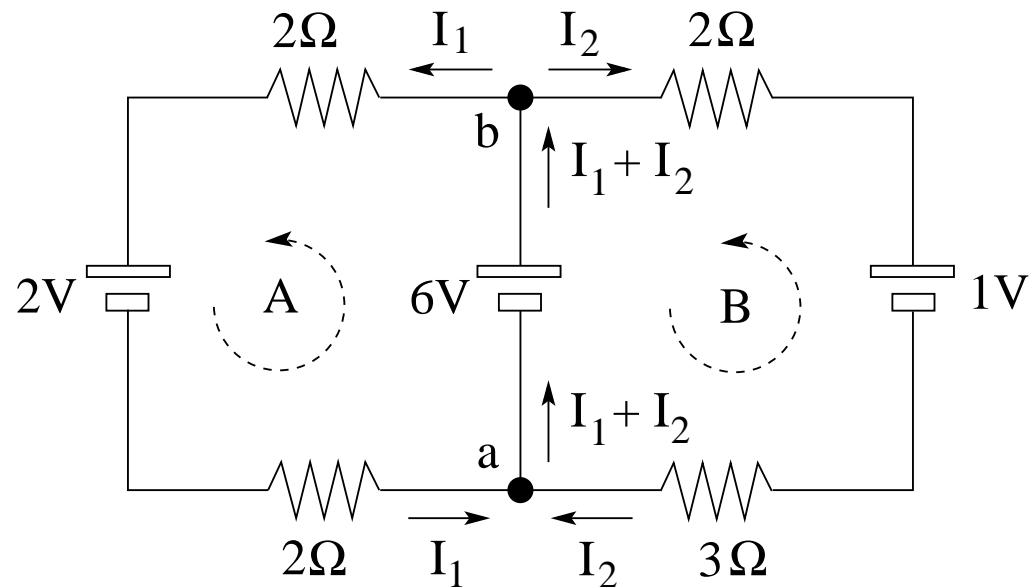


# Applying Kirchhoff's Rules



Consider the circuit shown below.

- Junction  $a$ :  $I_1, I_2$  (in);  $I_1 + I_2$  (out)
- Junction  $b$ :  $I_1 + I_2$  (in);  $I_1, I_2$  (out)
- Two independent currents require the use of two loops.
- Loop  $A$  (ccw):  $6V - (2\Omega)I_1 - 2V - (2\Omega)I_1 = 0$
- Loop  $B$  (ccw):  $(3\Omega)I_2 + 1V + (2\Omega)I_2 - 6V = 0$
- Solution:  $I_1 = 1A, I_2 = 1A$

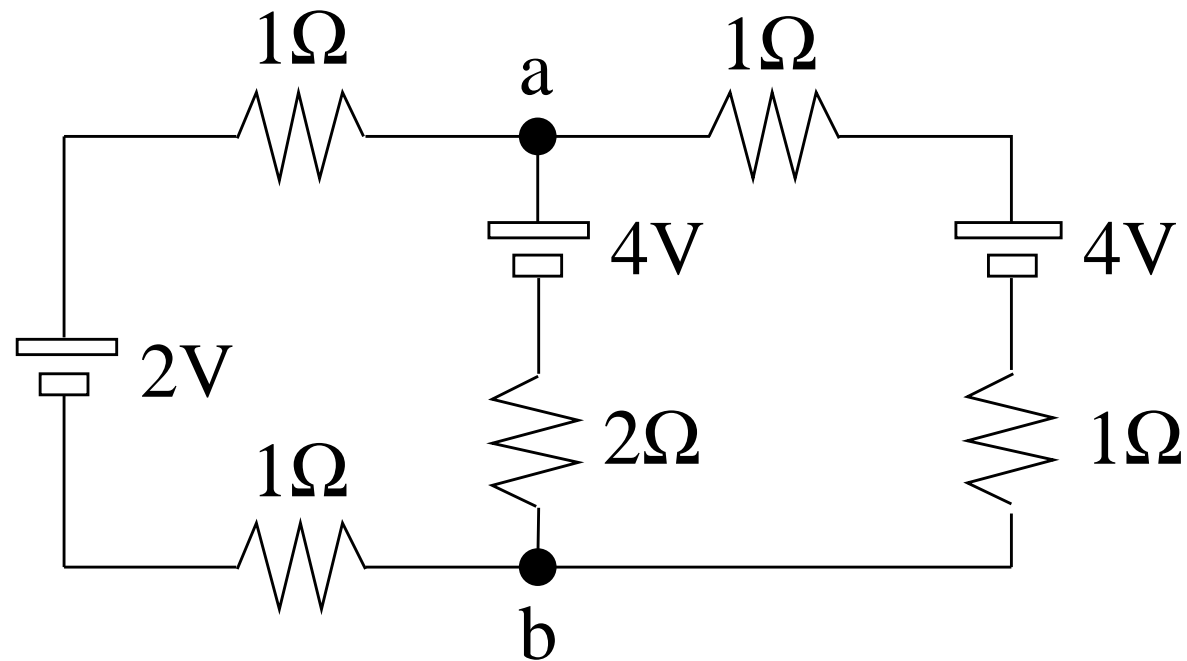


# Resistor Circuit (11)



Consider the electric circuit shown.

- Identify all independent currents via junction rule.
- Determine the independent currents via loop rule.
- Find the Potential difference  $V_{ab} = V_b - V_a$ .

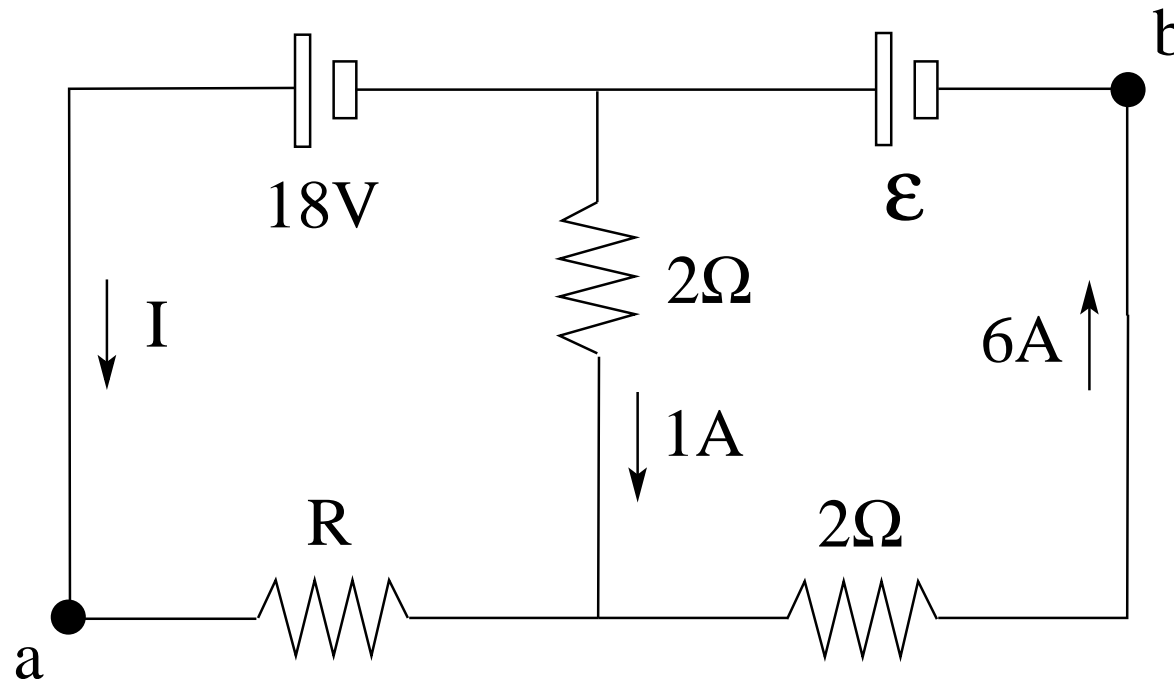


# Resistor Circuit (9)



Use Kirchhoff's rules to find

- (a) the current  $I$ ,
- (b) the resistance  $R$ ,
- (c) the emf  $\mathcal{E}$ ,
- (d) the voltage  $V_{ab} \equiv V_b - V_a$ .

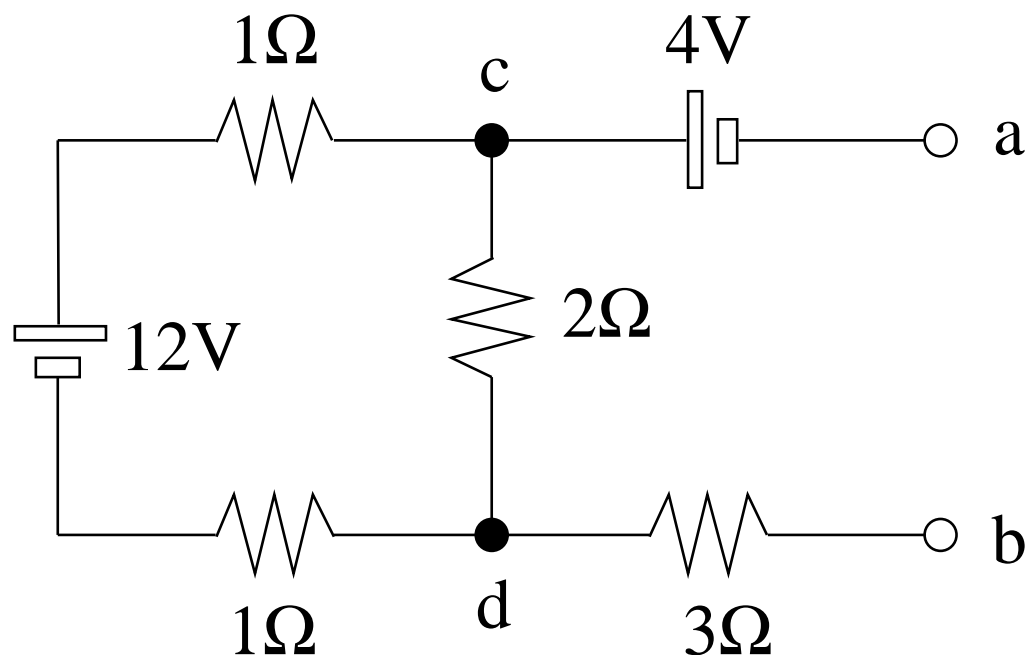


# Resistor Circuit (10)



Consider the electric circuit shown.

- (a) Find the current through the 12V battery.
- (b) Find the current through the  $2\Omega$  resistor.
- (c) Find the total power dissipated.
- (d) Find the voltage  $V_{cd} \equiv V_d - V_c$ .
- (e) Find the voltage  $V_{ab} \equiv V_b - V_a$ .

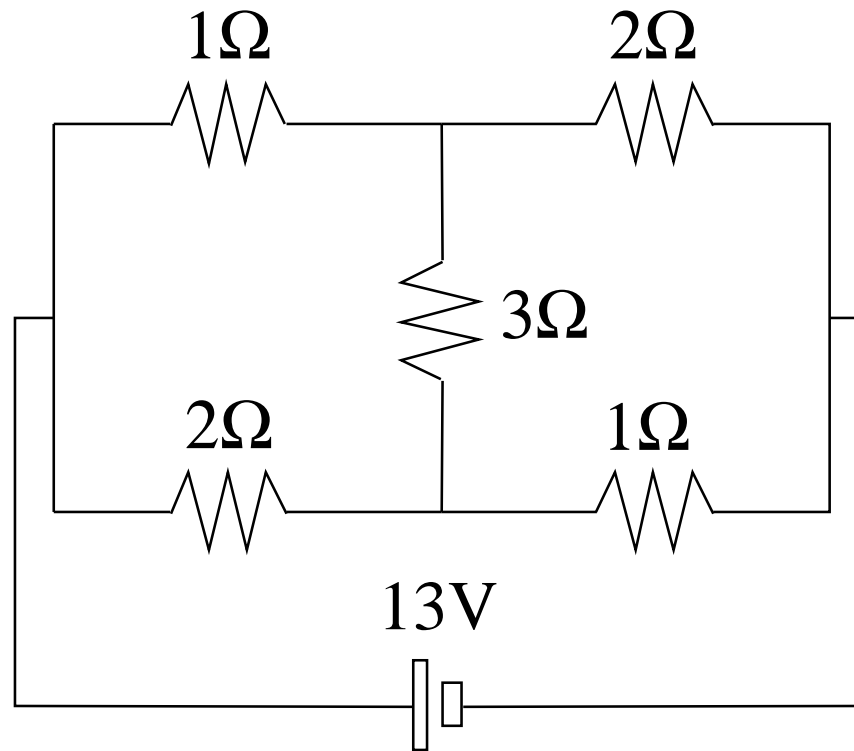


## Resistor Circuit (12)



Consider the electric circuit shown.

- Find the equivalent resistance  $R_{eq}$  of the circuit.
- Find the total power  $P$  dissipated in the circuit.





# More Complex Capacitor Circuit



No two capacitors are in parallel or in series.  
Solution requires different strategy:

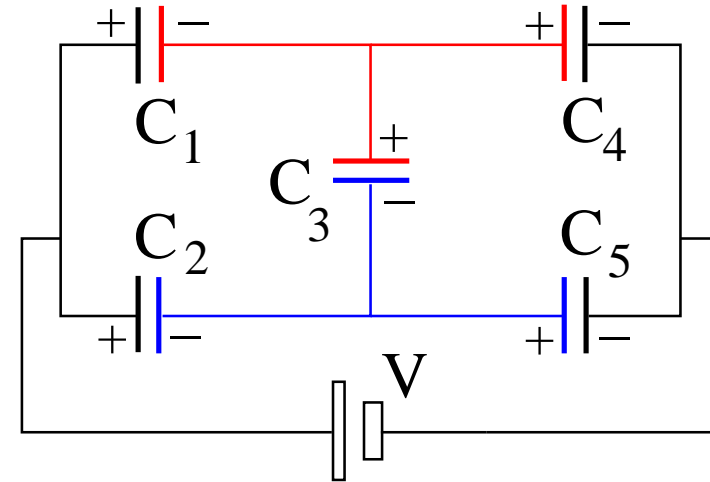
- zero charge on each conductor (here color coded),
- zero voltage around any closed loop.

Specifications:  $C_1, \dots, Q_5, V$ .

Five equations for unknowns  $Q_1, \dots, Q_5$ :

- $Q_1 + Q_2 - Q_4 - Q_5 = 0$
- $Q_3 + Q_4 - Q_1 = 0$
- $\frac{Q_5}{C_5} + \frac{Q_3}{C_3} - \frac{Q_4}{C_4} = 0$
- $\frac{Q_2}{C_2} - \frac{Q_1}{C_1} - \frac{Q_3}{C_3} = 0$
- $V - \frac{Q_1}{C_1} - \frac{Q_4}{C_4} = 0$

Equivalent capacitance:  $C_{eq} = \frac{Q_1 + Q_2}{V}$



(a)  $C_m = 1\text{pF}, m = 1, \dots, 5$  and  $V = 1\text{V}$ :

$$C_{eq} = 1\text{pF}, Q_3 = 0,$$

$$Q_1 = Q_2 = Q_4 = Q_5 = \frac{1}{2}\text{pC}.$$

(b)  $C_m = m\text{pF}, m = 1, \dots, 5$  and  $V = 1\text{V}$ :

$$C_{eq} = \frac{159}{71}\text{pF}, Q_1 = \frac{55}{71}\text{pC}, Q_2 = \frac{104}{71}\text{pC},$$

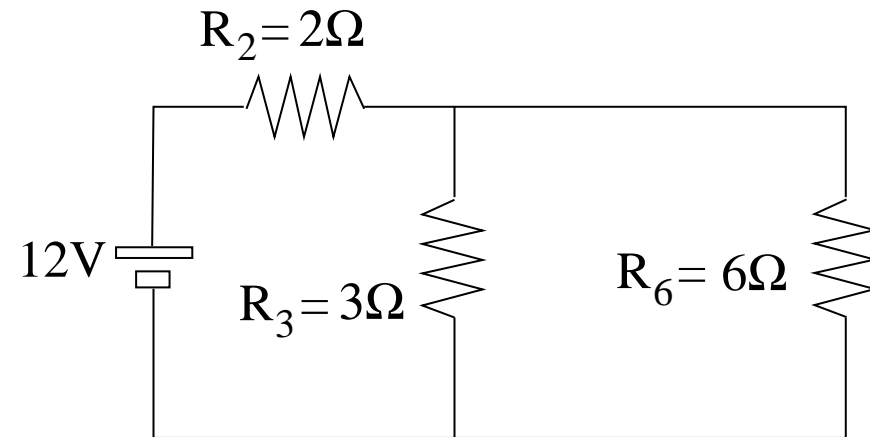
$$Q_3 = -\frac{9}{71}\text{pC}, Q_4 = \frac{64}{71}\text{pC}, Q_5 = \frac{95}{71}\text{pC}.$$

## Intermediate Exam II: Problem #2 (Spring '05)



Consider the electrical circuit shown.

- (a) Find the equivalent resistance  $R_{eq}$ .
- (b) Find the current  $I_3$  through resistor  $R_3$ .

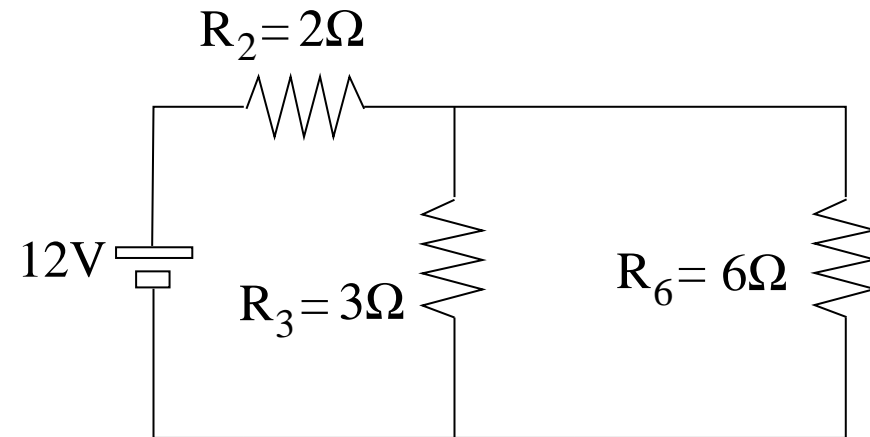


## Intermediate Exam II: Problem #2 (Spring '05)



Consider the electrical circuit shown.

- (a) Find the equivalent resistance  $R_{eq}$ .
- (b) Find the current  $I_3$  through resistor  $R_3$ .



**Solution:**

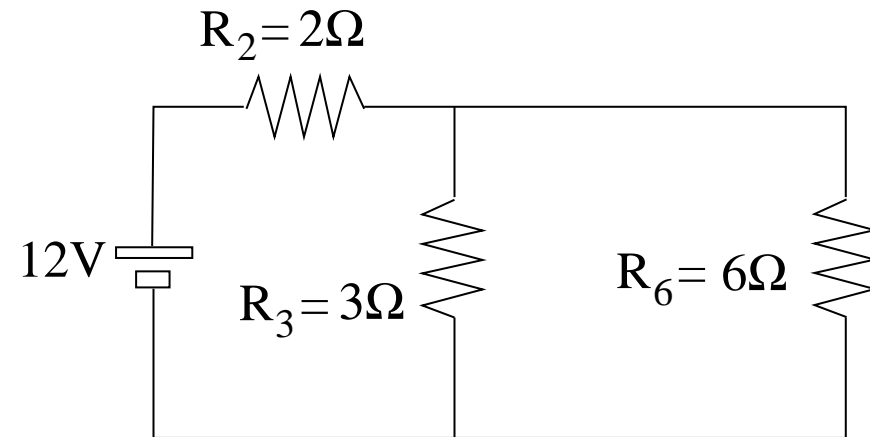
$$(a) \quad R_{36} = \left( \frac{1}{R_3} + \frac{1}{R_6} \right)^{-1} = 2\Omega, \quad R_{eq} = R_2 + R_{36} = 4\Omega.$$

## Intermediate Exam II: Problem #2 (Spring '05)



Consider the electrical circuit shown.

- (a) Find the equivalent resistance  $R_{eq}$ .
- (b) Find the current  $I_3$  through resistor  $R_3$ .



**Solution:**

$$(a) \quad R_{36} = \left( \frac{1}{R_3} + \frac{1}{R_6} \right)^{-1} = 2\Omega, \quad R_{eq} = R_2 + R_{36} = 4\Omega.$$

$$(b) \quad I_2 = I_{36} = \frac{12V}{R_{eq}} = 3A$$

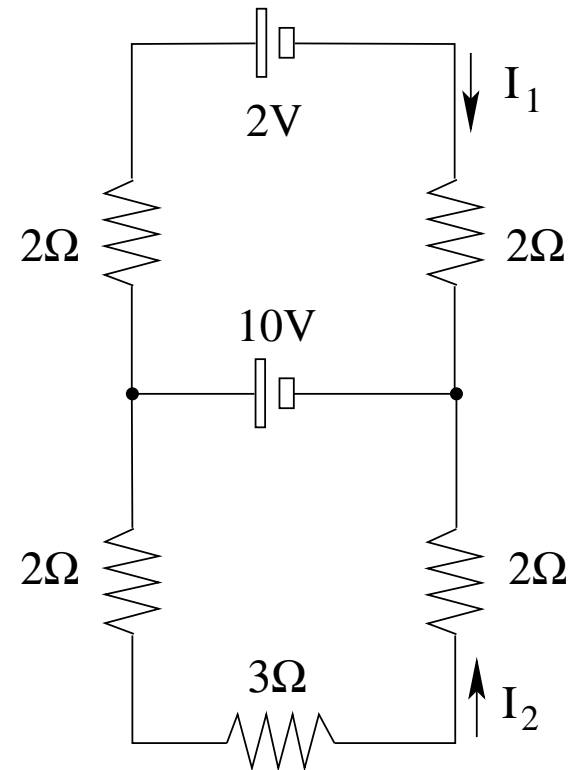
$$\Rightarrow V_3 = V_{36} = I_{36}R_{36} = 6V \quad \Rightarrow I_3 = \frac{V_3}{R_3} = 2A.$$

## Intermediate Exam II: Problem #2 (Spring '06)



Consider the two-loop circuit shown.

- (a) Find the current  $I_1$ .
- (b) Find the current  $I_2$ .

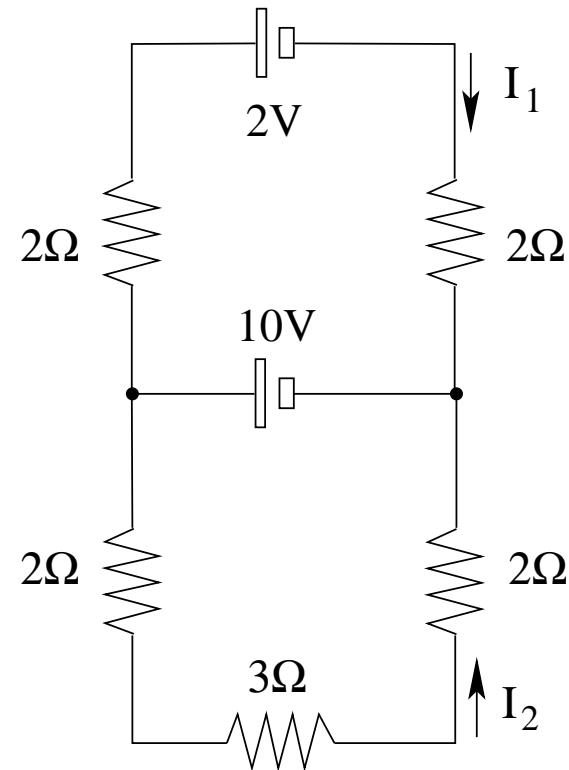


## Intermediate Exam II: Problem #2 (Spring '06)



Consider the two-loop circuit shown.

- (a) Find the current  $I_1$ .
- (b) Find the current  $I_2$ .



**Solution:**

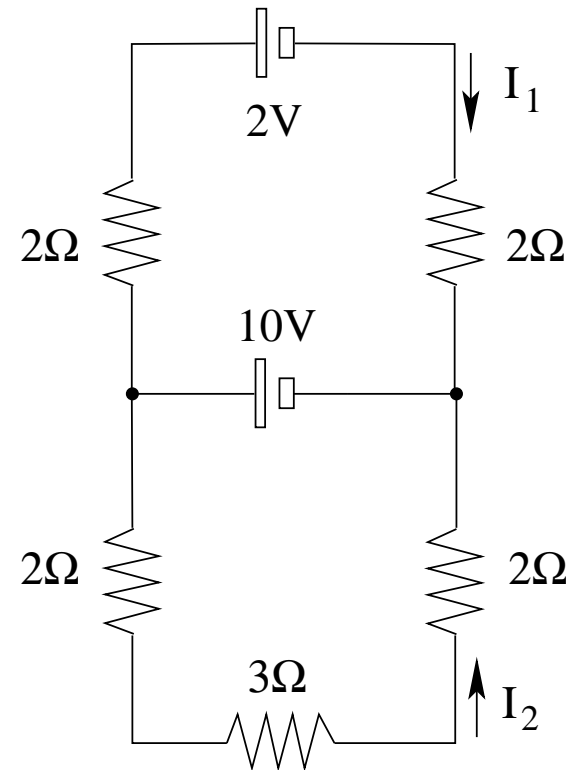
$$(a) \quad -(2\Omega)(I_1) + 10V - (2\Omega)(I_1) - 2V = 0 \quad \Rightarrow \quad I_1 = \frac{8V}{4\Omega} = 2A.$$

## Intermediate Exam II: Problem #2 (Spring '06)



Consider the two-loop circuit shown.

- (a) Find the current  $I_1$ .
- (b) Find the current  $I_2$ .



**Solution:**

$$(a) \quad -(2\Omega)(I_1) + 10V - (2\Omega)(I_1) - 2V = 0 \quad \Rightarrow \quad I_1 = \frac{8V}{4\Omega} = 2A.$$

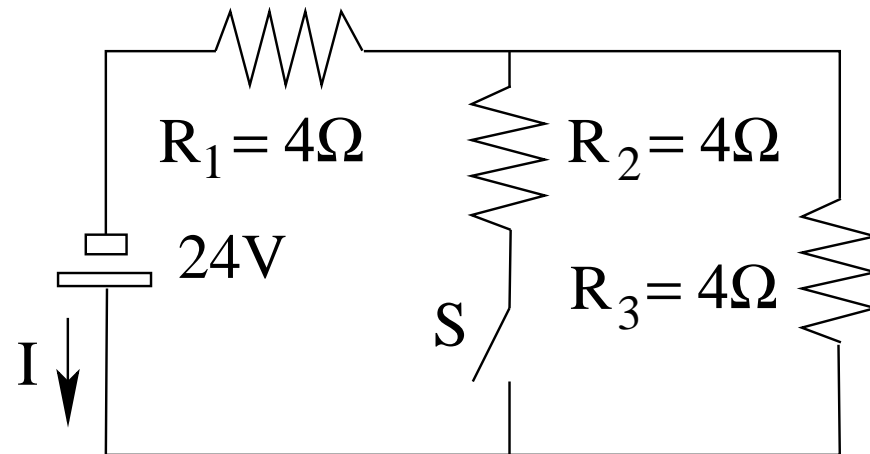
$$(b) \quad -(2\Omega)(I_2) + 10V - (2\Omega)(I_2) - (3\Omega)(I_2) = 0 \quad \Rightarrow \quad I_2 = \frac{10V}{7\Omega} = 1.43A.$$

## Unit Exam II: Problem #2 (Spring '07)



Consider the electric circuit shown.

- (a) Find the current  $I$  when the switch  $S$  is open.
- (b) Find the power  $P_3$  dissipated in resistor  $R_3$  when the switch is open.
- (c) Find the current  $I$  when the switch  $S$  is closed.
- (d) Find the power  $P_3$  dissipated in resistor  $R_3$  when the switch is closed.





## Unit Exam II: Problem #2 (Spring '07)

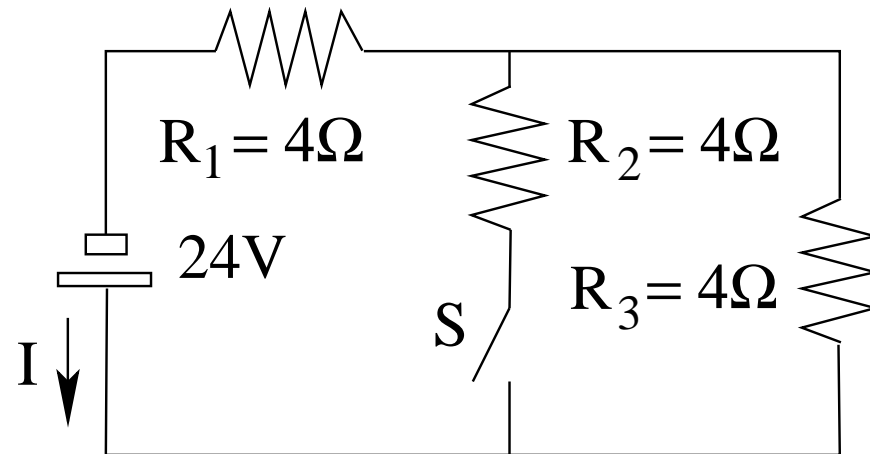


Consider the electric circuit shown.

- (a) Find the current  $I$  when the switch  $S$  is open.
- (b) Find the power  $P_3$  dissipated in resistor  $R_3$  when the switch is open.
- (c) Find the current  $I$  when the switch  $S$  is closed.
- (d) Find the power  $P_3$  dissipated in resistor  $R_3$  when the switch is closed.

**Solution:**

(a) 
$$I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$$



## Unit Exam II: Problem #2 (Spring '07)



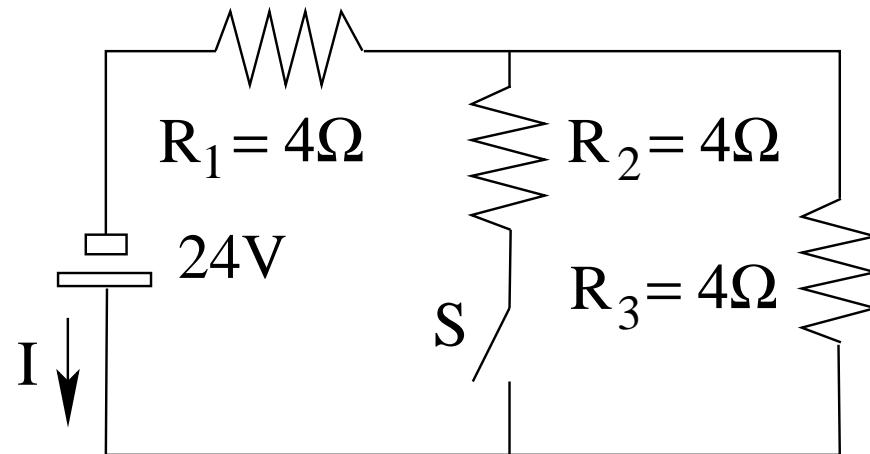
Consider the electric circuit shown.

- (a) Find the current  $I$  when the switch  $S$  is open.
- (b) Find the power  $P_3$  dissipated in resistor  $R_3$  when the switch is open.
- (c) Find the current  $I$  when the switch  $S$  is closed.
- (d) Find the power  $P_3$  dissipated in resistor  $R_3$  when the switch is closed.

**Solution:**

$$(a) \quad I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$$

$$(b) \quad P_3 = (3\text{A})^2(4\Omega) = 36\text{W}.$$



## Unit Exam II: Problem #2 (Spring '07)



Consider the electric circuit shown.

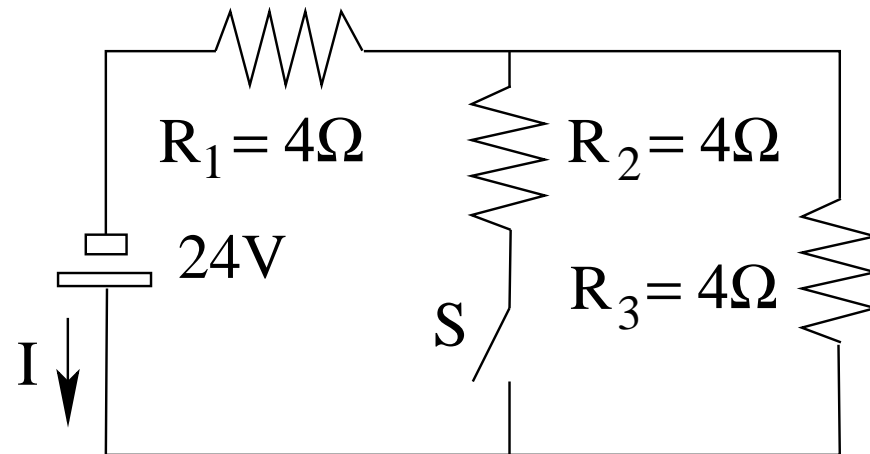
- (a) Find the current  $I$  when the switch  $S$  is open.
- (b) Find the power  $P_3$  dissipated in resistor  $R_3$  when the switch is open.
- (c) Find the current  $I$  when the switch  $S$  is closed.
- (d) Find the power  $P_3$  dissipated in resistor  $R_3$  when the switch is closed.

**Solution:**

$$(a) \quad I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$$

$$(b) \quad P_3 = (3\text{A})^2(4\Omega) = 36\text{W}.$$

$$(c) \quad I = \frac{24\text{V}}{6\Omega} = 4\text{A}.$$



## Unit Exam II: Problem #2 (Spring '07)



Consider the electric circuit shown.

- (a) Find the current  $I$  when the switch  $S$  is open.
- (b) Find the power  $P_3$  dissipated in resistor  $R_3$  when the switch is open.
- (c) Find the current  $I$  when the switch  $S$  is closed.
- (d) Find the power  $P_3$  dissipated in resistor  $R_3$  when the switch is closed.

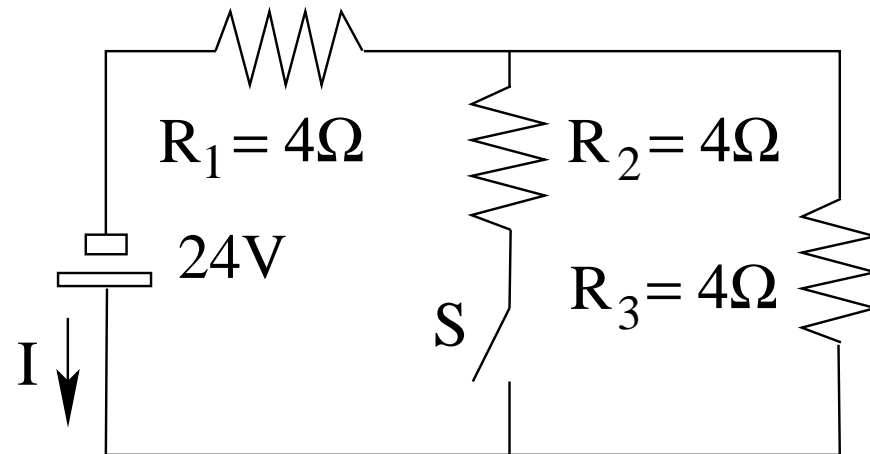
**Solution:**

$$(a) \quad I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$$

$$(b) \quad P_3 = (3\text{A})^2(4\Omega) = 36\text{W}.$$

$$(c) \quad I = \frac{24\text{V}}{6\Omega} = 4\text{A}.$$

$$(d) \quad P_3 = (2\text{A})^2(4\Omega) = 16\text{W}.$$

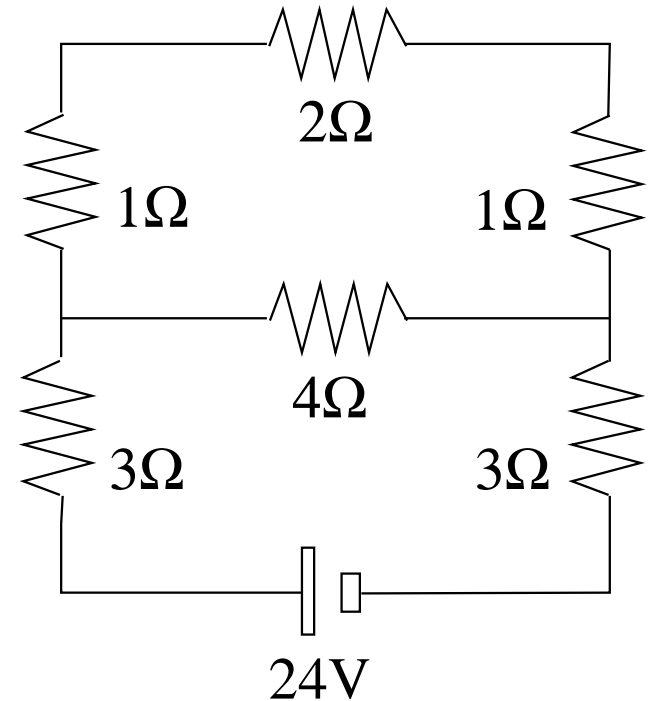


## Unit Exam II: Problem #2 (Spring '09)



Consider the resistor circuit shown.

- (a) Find the equivalent resistance  $R_{eq}$ .
- (b) Find the power  $P$  supplied by the battery.
- (c) Find the current  $I_4$  through the  $4\Omega$ -resistor.
- (d) Find the voltage  $V_2$  across the  $2\Omega$ -resistor.



## Unit Exam II: Problem #2 (Spring '09)

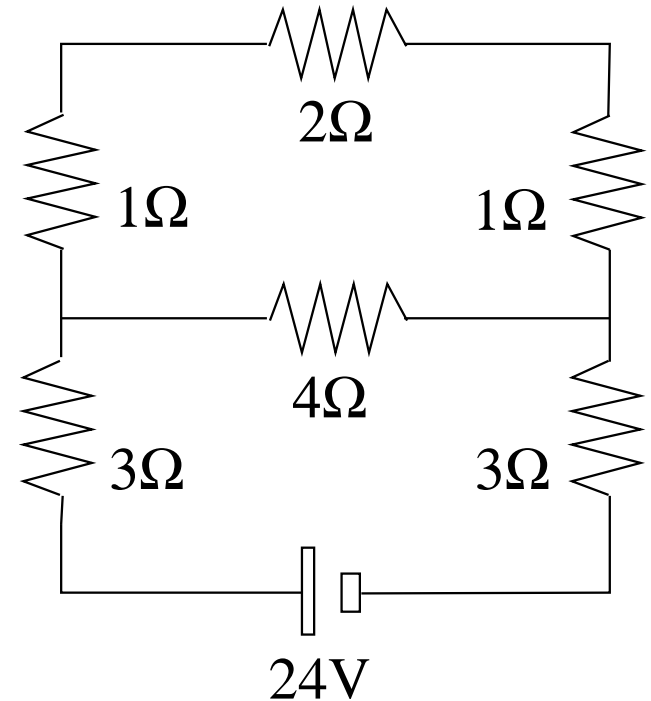


Consider the resistor circuit shown.

- (a) Find the equivalent resistance  $R_{eq}$ .
- (b) Find the power  $P$  supplied by the battery.
- (c) Find the current  $I_4$  through the  $4\Omega$ -resistor.
- (d) Find the voltage  $V_2$  across the  $2\Omega$ -resistor.

**Solution:**

(a)  $R_{eq} = 8\Omega$ .



## Unit Exam II: Problem #2 (Spring '09)



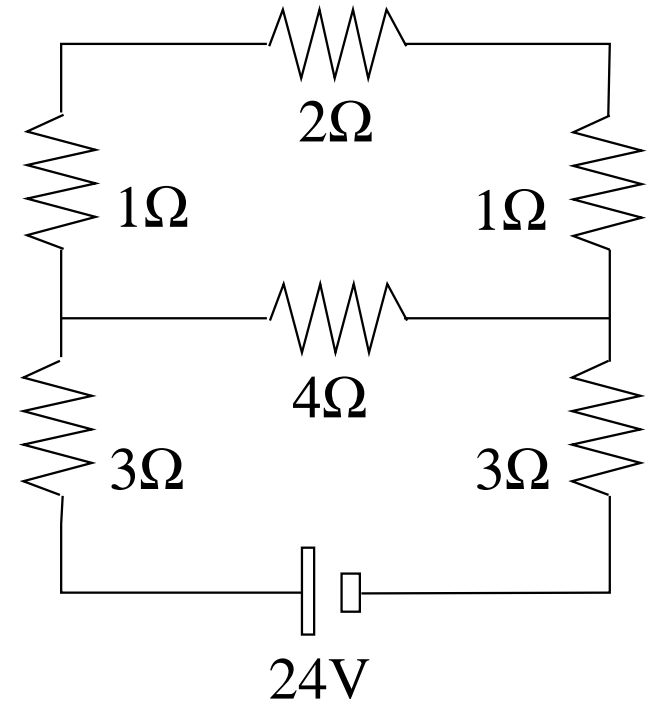
Consider the resistor circuit shown.

- (a) Find the equivalent resistance  $R_{eq}$ .
- (b) Find the power  $P$  supplied by the battery.
- (c) Find the current  $I_4$  through the  $4\Omega$ -resistor.
- (d) Find the voltage  $V_2$  across the  $2\Omega$ -resistor.

**Solution:**

(a)  $R_{eq} = 8\Omega$ .

(b)  $P = \frac{(24V)^2}{8\Omega} = 72W$ .



## Unit Exam II: Problem #2 (Spring '09)



Consider the resistor circuit shown.

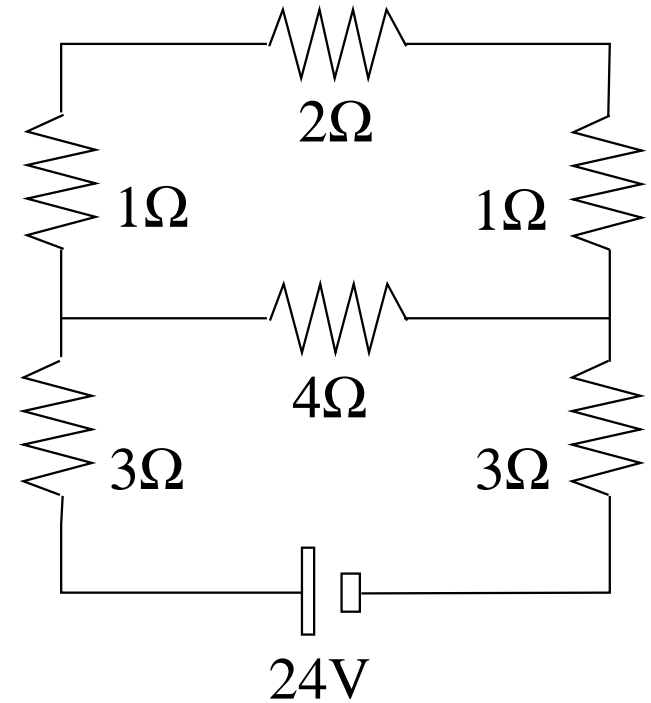
- (a) Find the equivalent resistance  $R_{eq}$ .
- (b) Find the power  $P$  supplied by the battery.
- (c) Find the current  $I_4$  through the  $4\Omega$ -resistor.
- (d) Find the voltage  $V_2$  across the  $2\Omega$ -resistor.

**Solution:**

(a)  $R_{eq} = 8\Omega$ .

(b)  $P = \frac{(24V)^2}{8\Omega} = 72W$ .

(c)  $I_4 = \frac{1}{2} \frac{24V}{8\Omega} = 1.5A$ .





## Unit Exam II: Problem #2 (Spring '09)

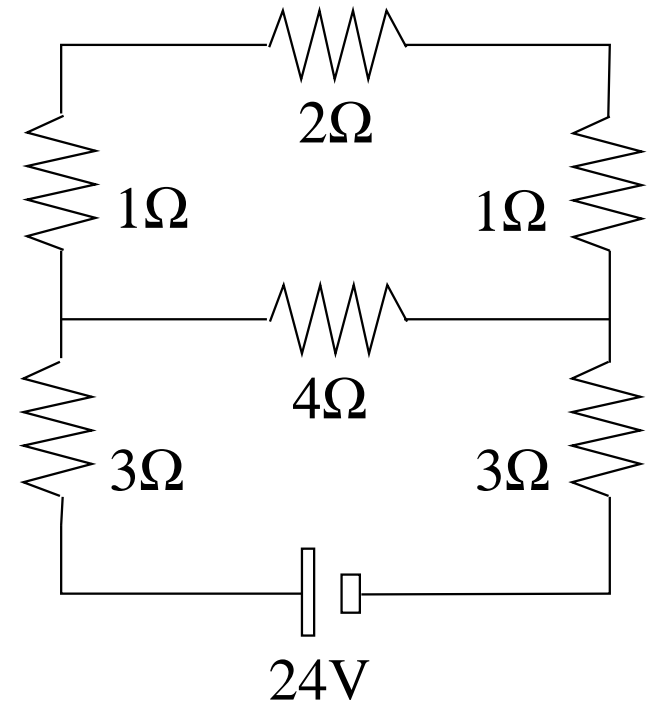


Consider the resistor circuit shown.

- (a) Find the equivalent resistance  $R_{eq}$ .
- (b) Find the power  $P$  supplied by the battery.
- (c) Find the current  $I_4$  through the  $4\Omega$ -resistor.
- (d) Find the voltage  $V_2$  across the  $2\Omega$ -resistor.

**Solution:**

- (a)  $R_{eq} = 8\Omega$ .
- (b)  $P = \frac{(24V)^2}{8\Omega} = 72W$ .
- (c)  $I_4 = \frac{1}{2} \frac{24V}{8\Omega} = 1.5A$ .
- (d)  $V_2 = (1.5A)(2\Omega) = 3V$ .

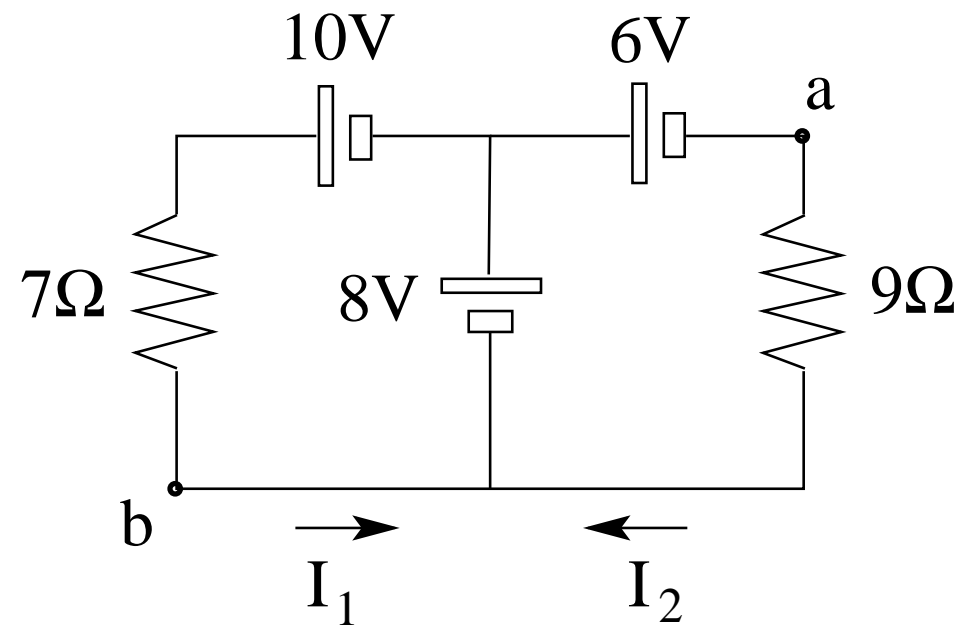


## Unit Exam II: Problem #3 (Spring '07)



Consider the two-loop circuit shown.

- (a) Find the current  $I_1$ .
- (b) Find the current  $I_2$ .
- (c) Find the potential difference  $V_a - V_b$ .



## Unit Exam II: Problem #3 (Spring '07)

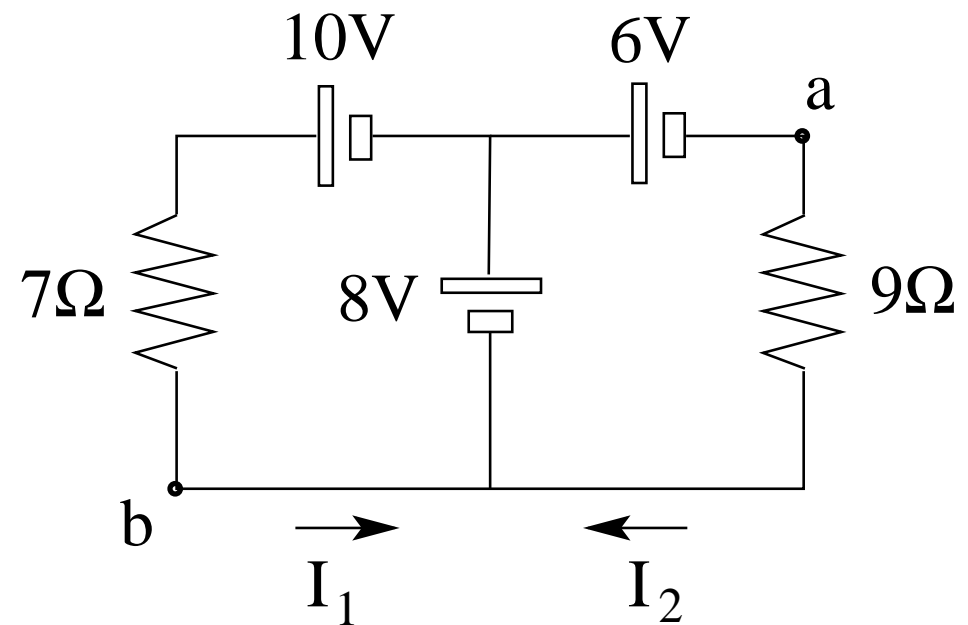


Consider the two-loop circuit shown.

- (a) Find the current  $I_1$ .
- (b) Find the current  $I_2$ .
- (c) Find the potential difference  $V_a - V_b$ .

**Solution:**

(a) 
$$I_1 = \frac{8V + 10V}{7\Omega} = 2.57A.$$



## Unit Exam II: Problem #3 (Spring '07)



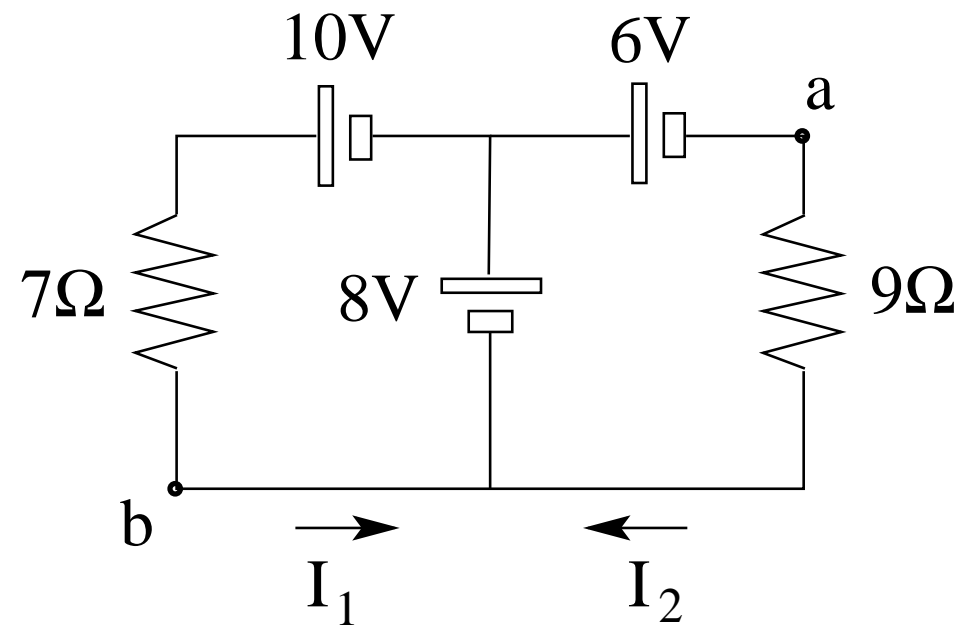
Consider the two-loop circuit shown.

- (a) Find the current  $I_1$ .
- (b) Find the current  $I_2$ .
- (c) Find the potential difference  $V_a - V_b$ .

**Solution:**

(a) 
$$I_1 = \frac{8V + 10V}{7\Omega} = 2.57A.$$

(b) 
$$I_2 = \frac{8V - 6V}{9\Omega} = 0.22A.$$

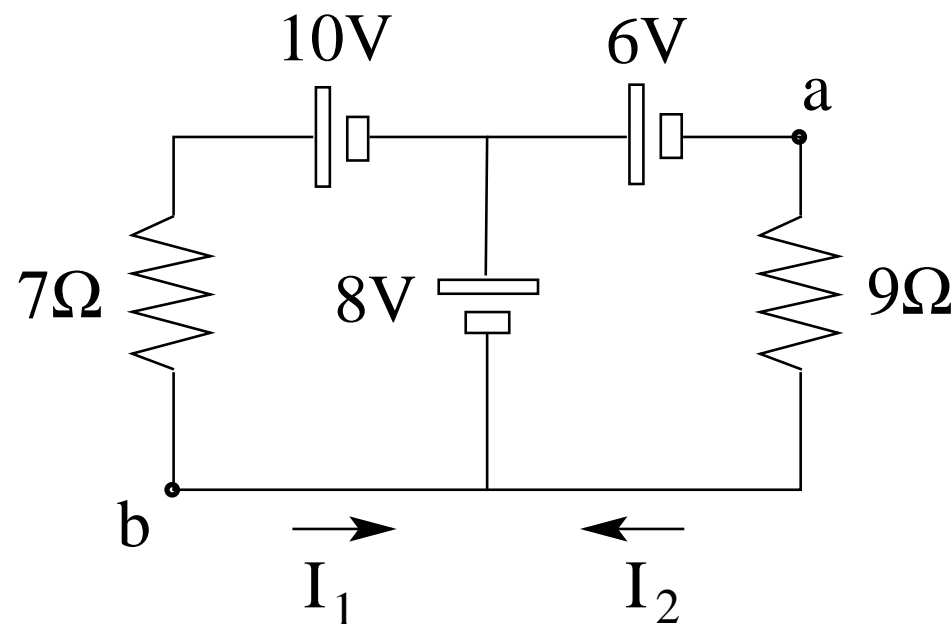


## Unit Exam II: Problem #3 (Spring '07)



Consider the two-loop circuit shown.

- (a) Find the current  $I_1$ .
- (b) Find the current  $I_2$ .
- (c) Find the potential difference  $V_a - V_b$ .



**Solution:**

- (a)  $I_1 = \frac{8V + 10V}{7\Omega} = 2.57A.$
- (b)  $I_2 = \frac{8V - 6V}{9\Omega} = 0.22A.$
- (c)  $V_a - V_b = 8V - 6V = 2V.$