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09. Applications of electric potential and energy conservation

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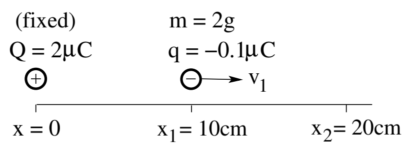
PHY204 Lecture 9 [rln9]

Electric Potential and Potential Energy: Application (1)

Consider a point charge $Q = 2\mu\text{C}$ fixed at position $x = 0$. A particle with mass $m = 2\text{g}$ and charge $q = -0.1\mu\text{C}$ is launched at position $x_1 = 10\text{cm}$ with velocity $v_1 = 12\text{m/s}$.



- Find the velocity v_2 of the particle when it is at position $x_2 = 20\text{cm}$.

tsl73

We begin this lecture with applications of the concepts of electric potential and electric potential energy to charged particles in various situations.

The problem statement of this exercise might remind us of what we discussed in lecture 3: the motion of charged particles in uniform electric fields. Here, however, the electric field is not uniform. Therefore, the electric force on the particle changes with position.

We cannot use the familiar tools for motion with constant acceleration. We must reason differently. We note that the electric force is conservative. The sum of kinetic and potential energies of the particle is constant.

At the initial position of the particle we have,

$$E = K_1 + U_1 = \frac{1}{2}mv_1^2 + \frac{kqQ}{r_1} = 0.144\text{J} - 0.018\text{J} = 0.126\text{J},$$

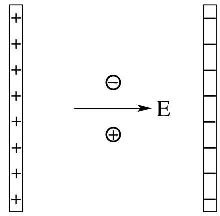
and at the final position,

$$E = K_2 + U_2 = \frac{1}{2}mv_2^2 + \frac{kqQ}{r_2} \quad \Rightarrow \; 0.126\text{J} = \frac{1}{2}mv_2^2 - 0.009\text{J}.$$

Solving the last equation for the unknown final velocity yields the results, $v_2 = \pm 11.6\text{m/s}$. As the particle moves to the right, it slows down to $+11.6\text{m/s}$ when it reaches position x_2, then continues, turns around, and accelerates to velocity -11.6m/s as it revisits position x_2.

Electric Potential and Potential Energy: Application (5)

An electron and a proton are released from rest midway between oppositely charged plates.



(a) Name the particle(s) which move(s) from high to low electric potential.
(b) Name the particle(s) whose electric potential energy decrease(s).
(c) Name the particle(s) which hit(s) the plate in the shortest time.
(d) Name the particle(s) which reach(es) the highest kinetic energy before impact.

tsl77

This little exercise does not require any mathematical analysis but a fair conceptual understanding.

The space between the two oppositely charged plates is filled with a largely uniform electric field in the direction shown. The electron will move to the left and the proton to the right, both in the direction of the force they experience.

(a) The electric potential (an attribute of space) is $V(x) = -Ex +$ const, where we assume the x-direction to be horizontal. High potential is on the left and low potential on the right. Only the proton moves toward low potential.

(b) The electric potential energy of each particle is $U = qV(x)$. Given that q is positive for the proton and negative for the electron, U decreases for both particles as they move right and left, respectively.

(c) Both particles experience a (constant) force of the same magnitude $F = |q|E$ during their motion in opposite directions. The electron with its smaller mass undergoes a larger acceleration, $a = F/m$, than the proton. It will reach the plate first.

(d) The electron will hit the plate first and with higher speed. That does not imply that it has more kinetic energy because it has smaller mass. Energy conservation is in play here. Both particles experience the same drop in potential energy, $\Delta U = -qE\Delta x$. Hence their gain in kinetic energy must be the same too: $\Delta K = -\Delta U$.

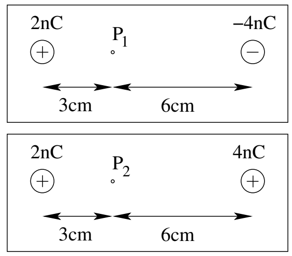
Electric Potential and Potential Energy: Application (8)

(a) Is the electric potential at points P_1, P_2 **positive** or **negative** or **zero**?

(b) Is the potential energy of a negatively charged particle at points P_1, P_2 **positive** or **negative** or **zero**?

(c) Is the electric field at points P_1, P_2 directed **left** or **right** or is it **zero**?

(d) Is the force on a negatively charged particle at points P_1 and P_2 directed **left** or **right** or is it **zero**?



tsl83

Here we have another exercise in the same spirit. Recall that electric potential and electric field field are attributes of space whereas potential energy and force are associated with a charged particle in that space.

The relevant expressions for questions (a) through (d) are

$$V = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2}, \quad U = qV, \quad E = \left| \frac{kQ_1}{r_1^2} + \frac{kQ_2}{r_2^2} \right|, \quad F = |q|E,$$

where Q_1, Q_2 are the fixed charges left and right that generate the potential and the field. The particle to be placed at points P_1 or P_2 has charge q.

The two expressions on the left are scalars, which can be positive, zero, or negative. The two expressions on the right are magnitudes of vectors. They have (non-negative) magnitude and direction (here left/right). The directions are easy enough to determine. We have done it many times before.

Find your own answers before checking them against the tabulated answers below.

	P_1	P_2
(a)	zero	positive
(b)	zero	negative
(c)	right	right
(d)	left	left

Electric Potential and Potential Energy: Application (10)

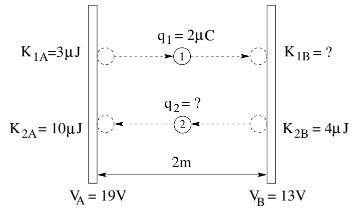
The charged particles 1 and 2 move between the charged conducting plates A and B in opposite directions.

From the information given in the figure...

(a) find the kinetic energy K_{1B} of particle 1,

(b) find the charge q_2 of particle 2,

(c) find the direction and magnitude of the electric field $\vec{E}$ between the plates.



tsl90

The emphasis of this exercise is on energy conservation. The two particles 1 and 2 travel the same distance between the plates but in opposite direction. Each plate is a conductor at given potential, V_A and V_B, respectively.

(a) Particle 1 travels from high potential to low potential. The change in potential between final and initial point is $\Delta V_1 = -6\text{V}$. Therefore, the change in potential energy of particle 1 is $\Delta U_1 = q_1 \Delta V_1 = -12\mu\text{J}$. Energy conservation then implies that the change in kinetic energy is $\Delta K_1 = -\Delta U_1 = +12\mu\text{J}$. Given the initial kinetic energy K_{1A}, the final kinetic energy must be $K_{1B} = K_{1A} + \Delta K_1 = 15\mu\text{J}$.

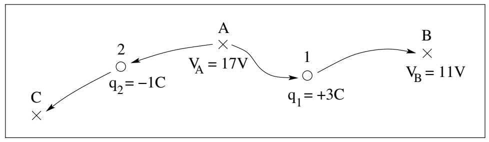
(b) The change in potential between final and initial point in the path of particle 2 is $\Delta V_2 = +6\text{V}$. The change in kinetic energy can be inferred from the given initial and final values: $\Delta K_2 = K_{2A} - K_{2B} = 6\mu\text{J}$. Energy conservation requires that the change in potential energy is $\Delta U_2 = -\Delta K_2 = -6\mu\text{J}$. The unknown charge of particle 2 can now be inferred from the relation $\Delta U_2 = q_2 \Delta V_2$. The answer is $q_2 = -1\mu\text{C}$.

(c) Particle 1 with positive charge travels toward the right because it experiences an electric force $\vec{F}_1 = q_1 \vec{E}$ to the right. Hence $\vec{E}$ must be pointing to the right. This is consistent with the fact that the negatively charged particle 2 is moving to the left. It experiences an electric force $\vec{F}_2 = q_2 \vec{E}$ to the left if the electric field $\vec{E}$ is pointing to the right.

We know from the previous lecture that in a uniform electric field the potential changes linearly with position, downward in the direction of the field, implying that $\Delta V = E \Delta x$. We infer that $E = (19\text{V} - 13\text{V})/(2\text{m}) = 3\text{V/m}$.

Electric Potential and Potential Energy: Application (7)

Consider a region of nonuniform electric field. Charged particles 1 and 2 start moving from rest at point A in opposite directions along the paths shown.



From the information given in the figure...

(a) find the kinetic energy K_1 of particle 1 when it arrives at point B,
(b) find the electric potential V_C at point C if we know that particle 2 arrives there with kinetic energy $K_2 = 8J$.

tsl79

Knowing the electric potential at points of interest brings us a long way toward answering important questions. If we do not know the potential up front, we can often deduce it from other information.

Consider a region of electric field (not shown). If at point A we let go from rest a particle with positive charge $q_1 = +3\text{C}$, we observe that it moves along some curved path that passes through point B.

The electric force that causes the particle to move along that particular curve changes from point to point in both direction and magnitude. We can predict the kinetic energy of the particle upon arrival at B without knowing all that. It suffices to know the potential difference between the initial and final points:

$$K_1 = -\Delta U_1 = -q_1\,\Delta V = -(3\text{C})(11\text{V} - 17\text{V}) = 18\text{J}.$$

If instead we release from rest at point A a particle with negative charge $q_2 = -1\text{C}$, we observe that it moves along a different path that passes through point C. In this case we do not know the potential at the destination but we measure the kinetic energy of the particle upon arrival. Our chain of reasoning is the same but used in reverse:

$$K_2 = -\Delta U_2 = -q_2\,\Delta V \quad \Rightarrow \; 8\text{J} = -(-1\text{C})(V_C - 17\text{V}) \quad \Rightarrow \; V_C = 25\text{V}.$$

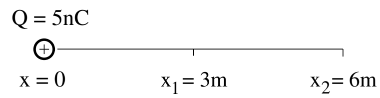
Note that only differences in potential have any physical meaning. If we add any number of volts to all the three potentials V_A, V_B, V_C nothing measurable will change.

Intermediate Exam I: Problem #2 (Spring '05)

Consider a point charge $Q = 5\text{nC}$ fixed at position $x = 0$.

(a) Find the electric potential V_1 at position $x_1 = 3\text{m}$
and the electric potiential V_2 at position $x_2 = 6\text{m}$.

(b) If a charged particle ($q = 4\text{nC}$, $m = 1.5\text{ng}$) is released from rest at x_1,
what are its kinetic energy K_2 and its velocity v_2 when it reaches position x_2?



Solution:

(a) $V_1 = k\frac{Q}{x_1} = 15\text{V}, \quad V_2 = k\frac{Q}{x_2} = 7.5\text{V}.$

(b) $\Delta U = q(V_2 - V_1) = (4\text{nC})(-7.5\text{V}) = -30\text{nJ} \quad \Rightarrow \Delta K = -\Delta U = 30\text{nJ}.$

$\Delta K = K_2 = \frac{1}{2}mv_2^2 \quad \Rightarrow v_2 = \sqrt{\frac{2K_2}{m}} = 200\text{m/s}.$

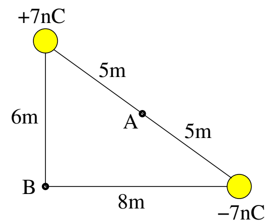
tsl332

In this previous exam problem, I tested the skills acquired on page 1 of this lecture. Understanding the difference between electric potential and electric potential energy is important. Recognizing that energy is conserved during the motion of the particle is the key to the solution.

Unit Exam I: Problem #1 (Fall ’10)

Consider two point charges positioned as shown.

(a) Find the magnitude of the electric field at point A.
(b) Find the electric potential at point A.
(c) Find the magnitude of the electric field at point B.
(d) Find the electric potential at point B.



Solution:

(a) $E_A = 2k\dfrac{|7\text{nC}|}{(5\text{m})^2} = 2(2.52\text{V/m}) = 5.04\text{V/m}.$

(b) $V_A = k\dfrac{(+7\text{nC})}{5\text{m}} + k\dfrac{(-7\text{nC})}{5\text{m}} = 12.6\text{V} - 12.6\text{V} = 0.$

(c) $E_B = \sqrt{\left(k\dfrac{|7\text{nC}|}{(6\text{m})^2}\right)^2 + \left(k\dfrac{|7\text{nC}|}{(8\text{m})^2}\right)^2} \Rightarrow E_B = \sqrt{(1.75\text{V/m})^2 + (0.98\text{V/m})^2} = 2.01\text{V/m}.$

(d) $V_B = k\dfrac{(+7\text{nC})}{6\text{m}} + k\dfrac{(-7\text{nC})}{8\text{m}} = 10.5\text{V} - 7.9\text{V} = 2.6\text{V}.$

tsl398

Here we have another previous exam problem. In this one, I tested the understanding of the difference, physically and mathematically, between electric potential and electric field. The former is a scalar and the latter a vector. The former can be positive, zero, or negative. The latter has magnitude and direction.

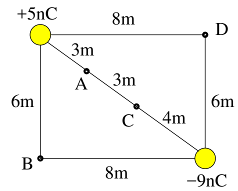
Adding potentials means adding numbers, here a positive number and a negative number.

Adding fields is a more complex task. $\vec{E}_A$ is the sum of two vectors pointing in the same direction, whereas $\vec{E}_B$ is the sum of two vectors pointing in perpendicular directions.

Unit Exam I: Problem #1 (Spring '14)

Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point *A*.
- Find the electric potential at point *B*.
- Find the magnitude of the electric field at point *C*.
- Find the electric potential at point *D*.



Solution:

- $E_A = k\frac{|5\text{nC}|}{(3\text{m})^2} + k\frac{|-9\text{nC}|}{(7\text{m})^2} = 5.00\text{V/m} + 1.65\text{V/m} = 6.65\text{V/m}.$
- $V_B = k\frac{(+5\text{nC})}{6\text{m}} + k\frac{(-9\text{nC})}{8\text{m}} = 7.50\text{V} - 10.13\text{V} = -2.63\text{V}.$
- $E_C = k\frac{|5\text{nC}|}{(6\text{m})^2} + k\frac{|-9\text{nC}|}{(4\text{m})^2} = 1.25\text{V/m} + 5.06\text{V/m} = 6.31\text{V/m}.$
- $V_D = k\frac{(+5\text{nC})}{8\text{m}} + k\frac{(-9\text{nC})}{6\text{m}} = 5.63\text{V} - 13.5\text{V} = -7.87\text{V}.$

tsl469

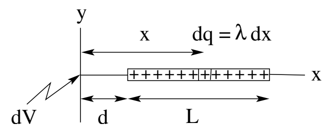
This is a variation of the exam problem on the previous page. The same reasoning leads to the solution. The same set of skills are required.

There are countless variations of almost every theme. Learning physics is not about collecting results for as many variations as possible. Learning physics means acquiring the skills needed to solve any conceivable variation of a theme.

The problems we need to solve in our professional lives as scientists and engineers are almost always new in at least some aspects. Taking care of those new aspects requires creativity, imagination, and skills. We can google knowledge but we cannot google creativity, imagination, and skills.

Electric Potential of Charged Rod

- Charge per unit length: $\lambda = Q/L$
- Charge on slice dx: $dq = \lambda dx$



- Electric potential generated by slice dx: $dV = \frac{kdq}{x} = \frac{k\lambda dx}{x}$
- Electric potential generated by charged rod:

$$V = k\lambda \int_d^{d+L} \frac{dx}{x} = k\lambda \Big[\ln x\Big]_d^{d+L} = k\lambda\,[\ln(d+L) - \ln d] = k\lambda \ln \frac{d+L}{d}$$

- Limiting case of very short rod $(L \ll d)$: $V = k\lambda \ln\left(1 + \frac{L}{d}\right) \simeq k\lambda \frac{L}{d} = \frac{kQ}{d}$

tsl330

We now shift gears and begin a discussion that we will continue in the next lecture. The theme is the calculation of the electric potential generated by charged objects. This endeavor parallels that in lecture 4, where we calculated the electric field generated by some of the same objects.

We use the same strategy. We divide the object into parts for which we already know how to determine the potential. Then we assemble the object from such parts and add up the contributions of all parts to the potential.

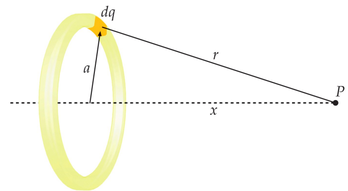
We begin with a uniformly charged rod positioned in a coordinate system as shown. We divide the rod into thin slices. The electric potential of each slice at the point indicated is, effectively, that of a point charge, which we already know. Adding up all contributions then amounts to performing a definite integral.

The result depends on the distance d between the near end of the rod and the point at which we have determined the potential. The distance d is a parameter in the definite integral. We now reinterpret it as a variable and say that we have determined the potential as a function of d.

In the last item we take a closer look at the behavior of that function for the case of large distance or the case of a short rod. In both cases we have $L/d \ll 1$ and can expand the natural-log function. We note that the potential generated by the rod looks, unsurprisingly, like the potential of a point charge.

Electric Potential of Charged Ring

- Total charge on ring: Q
- Charge per unit length: $\lambda = Q/2\pi a$
- Charge on arc: dq



Find the electric potential at point P on the axis of the ring.

- $dV = k\frac{dq}{r} = \frac{kdq}{\sqrt{x^2+a^2}}$
- $V(x) = k\int \frac{dq}{\sqrt{x^2+a^2}} = \frac{k}{\sqrt{x^2+a^2}}\int dq = \frac{kQ}{\sqrt{x^2+a^2}}$

tsl81

Determining the electric potential generated by a charged ring at a point located on its axis, a distance x from its center, is even simpler. All the slices of charge now have the same distance from the point at which we intend to find the potential. We are left with an integral over the charge dq on the slice, which is the total charge Q on the ring.

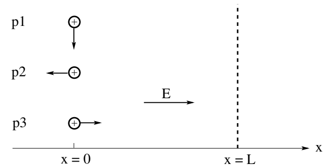
The position x of point P, a parameter in that integral, becomes the variable of the function $V(x)$, the electric potential of the ring at an arbitrary point on the x-axis.

It is common practice to refer to the point P as the "field point" even though we actually calculate the potential, not the field. "Potential point" sounds awkward and its meaning is easily misunderstood.

We shall see in the next lecture that the name "field point" is appropriate because once we have the potential as a function of position, that function also encodes information about the electric field.

Electric Potential and Potential Energy: Application (6)

Three protons are projected from $x = 0$ with equal initial speed v_0 in different directions. They all experience the force of a uniform horizontal electric field $\vec{E}$. Ultimately, they all hit the vertical screen at $x = L$. Ignore gravity.



(a) Which proton travels the longest time?

(b) Which proton travels the longest path?

(c) Which particle has the highest speed when it hits the screen?

Two of the questions are easy, one is hard.

tsl78

This is the quiz for lecture 9.

Identify the hard question and answer the other two.