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## 08. Capacitors II

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### Abstract

Lecture slides 8 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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# Capacitor with Dielectric

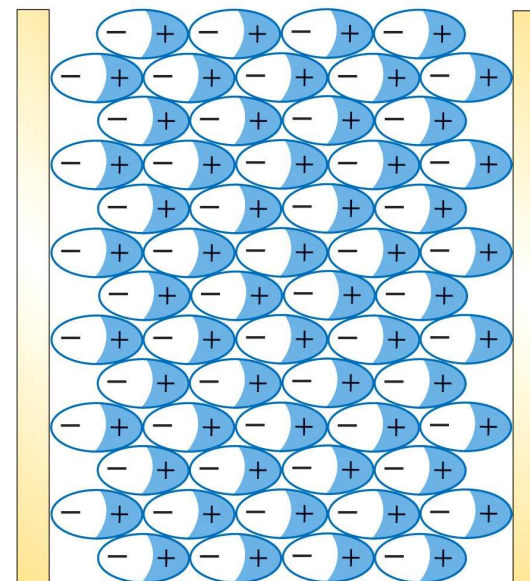
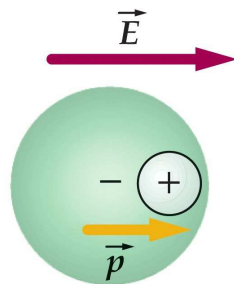
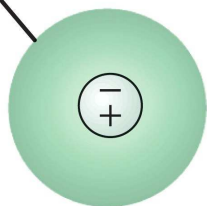


Most capacitors have a dielectric (insulating solid or liquid material) in the space between the conductors. This has several advantages:

- Physical separation of the conductors.
- Prevention of dielectric breakdown.
- Enhancement of capacitance.

The dielectric is polarized by the electric field between the capacitor plates.

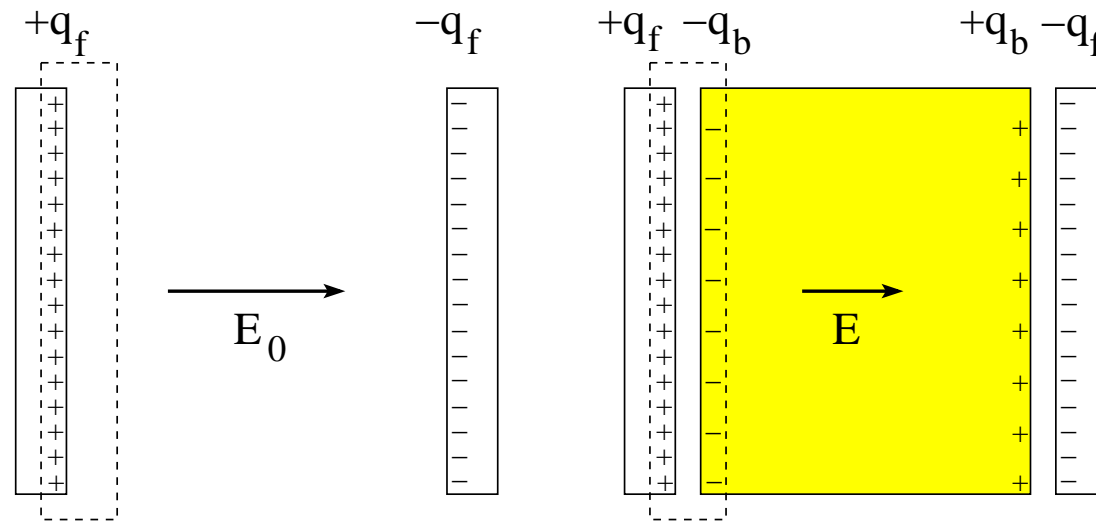
Center of negative charge  
coincides with center of  
positive charge



# Parallel-Plate Capacitor with Dielectric (1)



The polarization produces a bound charge on the surface of the dielectric.



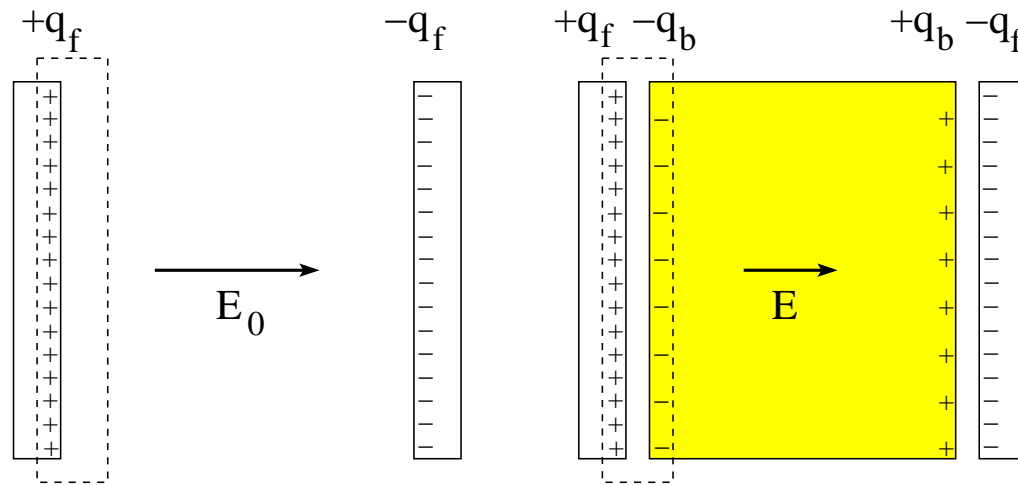
The bound surface charge has the effect of reducing the electric field between the plates from  $\vec{E}_0$  to  $\vec{E}$ .

- $A$ : area of plates
- $d$ : separation between plates
- $\pm q_f$ : free charge on plate
- $\pm q_b$ : bound charge on surface of dielectric
- $\vec{E}_0$ : electric field in vacuum
- $\vec{E}$ : electric field in dielectric

# Parallel-Plate Capacitor with Dielectric (2)



Use Gauss' law to determine the electric fields  $\vec{E}_0$  and  $\vec{E}$ .



- Field in vacuum:  $E_0 A = \frac{q_f}{\epsilon_0} \Rightarrow E_0 = \frac{q_f}{\epsilon_0 A}$
- Field in dielectric:  $E A = \frac{q_f - q_b}{\epsilon_0} \Rightarrow E = \frac{q_f - q_b}{\epsilon_0 A} < E_0$
- Voltage:  $V_0 = E_0 d$  (vacuum),  $V = E d = \frac{V_0}{\kappa} < V_0$  (dielectric)

Dielectric constant:  $\kappa \equiv \frac{E_0}{E} = \frac{q_f}{q_f - q_b} > 1$ . Permittivity of dielectric:  $\epsilon = \kappa \epsilon_0$ .



**TABLE 24-1**

**Dielectric Constants and Dielectric Strengths of Various Materials**

Material	Dielectric Constant $\kappa$	Dielectric Strength, kV/mm
Air	1.00059	3
Bakelite	4.9	24
Glass (Pyrex)	5.6	14
Mica	5.4	10–100
Neoprene	6.9	12
Paper	3.7	16
Paraffin	2.1–2.5	10
Plexiglas	3.4	40
Polystyrene	2.55	24
Porcelain	7	5.7
Transformer oil	2.24	12

- Dielectrics increase the capacitance:  $C/C_0 = \kappa$ .
- The capacitor is discharged spontaneously across the dielectric if the electric field exceeds the value quoted as dielectric strength.

# Impact of Dielectric (1)



What happens when a dielectric is placed into a capacitor with the charge on the capacitor kept constant?

	vacuum	dielectric
charge	$Q_0$	$Q = Q_0$
electric field	$E_0$	$E = \frac{E_0}{\kappa} < E_0$
voltage	$V_0$	$V = \frac{V_0}{\kappa} < V_0$
capacitance	$C_0 = \frac{Q_0}{V_0}$	$C = \frac{Q}{V} = \kappa C_0 > C_0$
potential energy	$U_0 = \frac{Q_0^2}{2C_0}$	$U = \frac{Q^2}{2C} = \frac{U_0}{\kappa} < U_0$
energy density	$u_E^{(0)} = \frac{1}{2} \epsilon_0 E_0^2$	$u_E = \frac{u_E^{(0)}}{\kappa} = \frac{1}{2} \kappa \epsilon_0 E^2 < u_E^{(0)}$

## Impact of Dielectric (2)



What happens when a dielectric is placed into a capacitor with the voltage across the capacitor kept constant?

	vacuum	dielectric
charge	$Q_0$	$Q = \kappa Q_0$
electric field	$E_0$	$E = E_0$
voltage	$V_0$	$V = V_0$
capacitance	$C_0 = \frac{Q_0}{V_0}$	$C = \frac{Q}{V} = \kappa C_0 > C_0$
potential energy	$U_0 = \frac{1}{2} C_0 V_0^2$	$U = \frac{1}{2} C V^2 = \kappa U_0 > U_0$
energy density	$u_E^{(0)} = \frac{1}{2} \epsilon_0 E_0^2$	$u_E = \kappa u_E^{(0)} = \frac{1}{2} \kappa \epsilon_0 E^2 > u_E^{(0)}$

# Stacked Dielectrics



Consider a parallel-plate capacitor with area  $A$  of each plate and spacing  $d$ .

- Capacitance without dielectric:  $C_0 = \frac{\epsilon_0 A}{d}$ .
- Dielectrics stacked in parallel:  $C = C_1 + C_2$

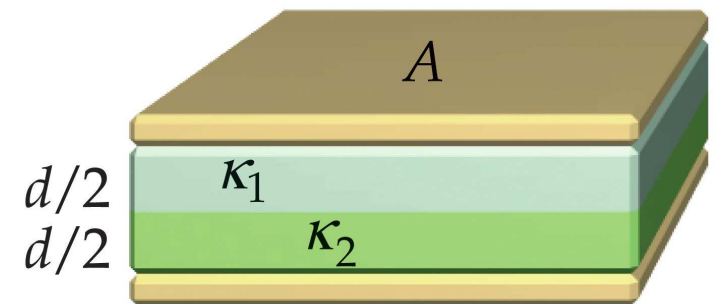
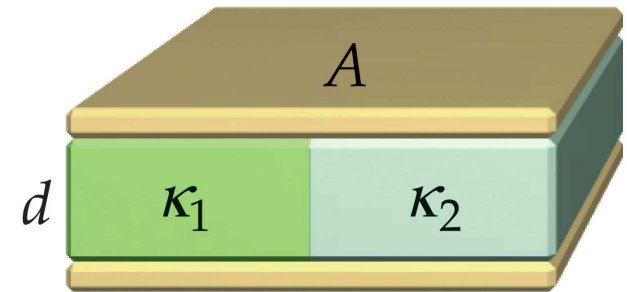
with  $C_1 = \kappa_1 \epsilon_0 \frac{A/2}{d}$ ,  $C_2 = \kappa_2 \epsilon_0 \frac{A/2}{d}$ .

$\Rightarrow C = \frac{1}{2}(\kappa_1 + \kappa_2)C_0$ .

- Dielectrics stacked in series:  $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

with  $C_1 = \kappa_1 \epsilon_0 \frac{A}{d/2}$ ,  $C_2 = \kappa_2 \epsilon_0 \frac{A}{d/2}$

$\Rightarrow C = \frac{2\kappa_1\kappa_2}{\kappa_1 + \kappa_2} C_0$ .





# Lateral Force on Dielectric

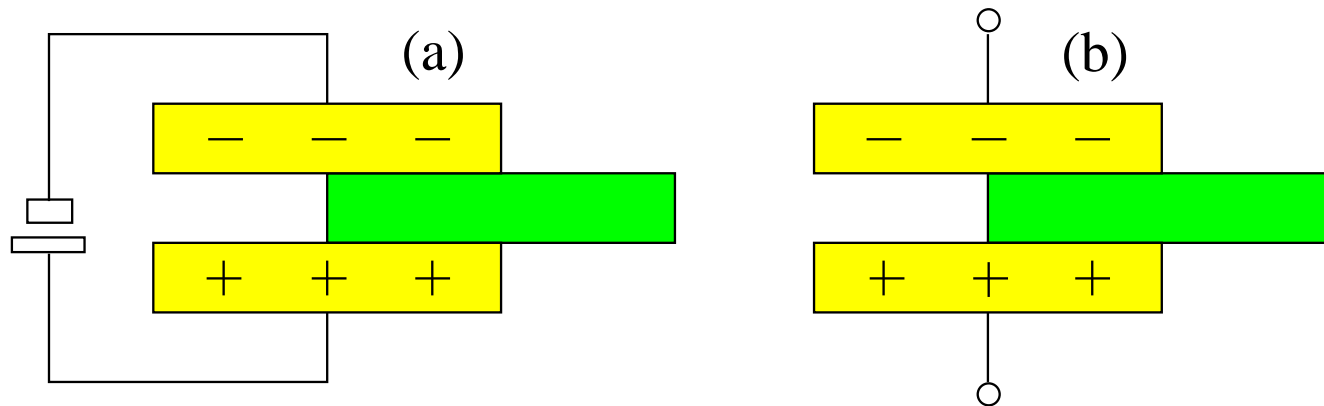


Consider two charged capacitors with dielectrics only halfway between the plates.

In configuration (a) any lateral motion of the dielectric takes place at **constant voltage** across the plates.

In configuration (b) any lateral motion of the dielectric takes place at **constant charge** on the plates.

Determine in each case the direction (left/zero/right) of the lateral force experienced by the dielectric.

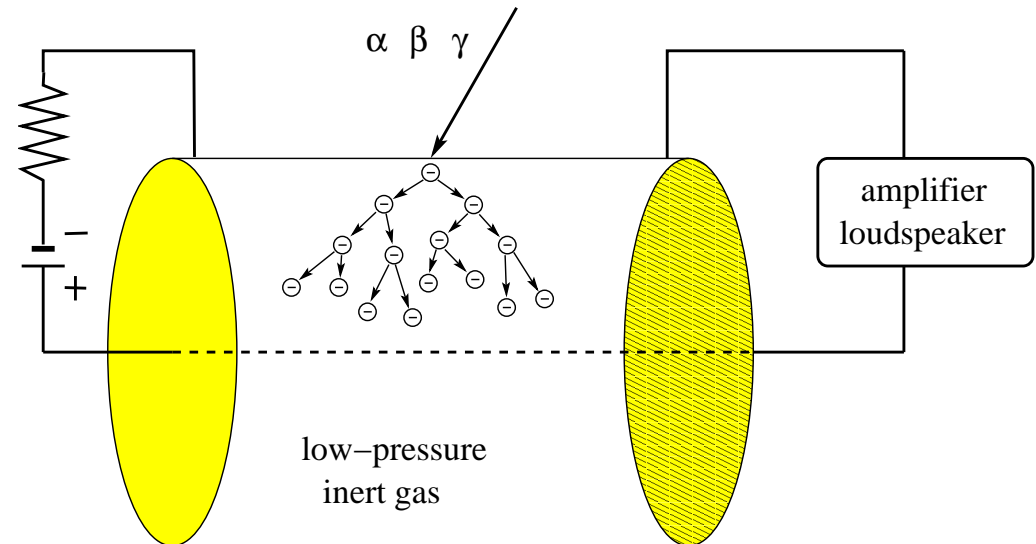


# Geiger Counter



Radioactive atomic nuclei produce high-energy particles of three different kinds:

- $\alpha$ -particles are  ${}^4\text{He}$  nuclei.
- $\beta$ -particles are electrons or positrons.
- $\gamma$ -particles are high-energy photons.



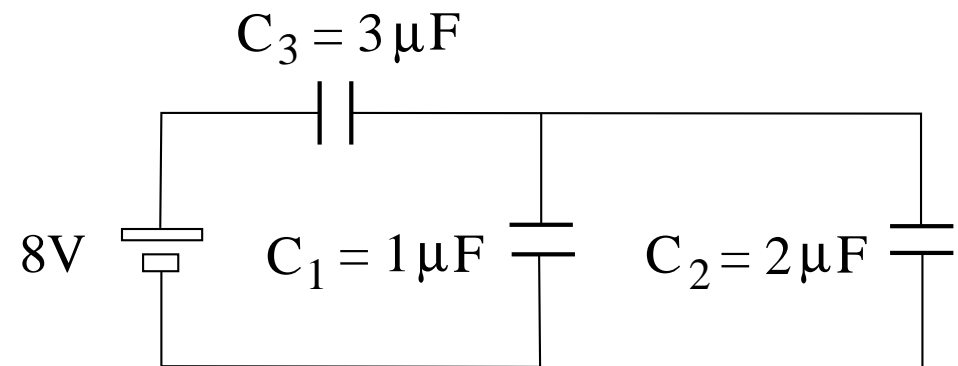
- Free electrons produced by ionizing radiation are strongly accelerated toward the central wire.
- Collisions with gas atoms produce further free electrons, which are accelerated in the same direction.
- An avalanche of electrons reaching the wire produces a current pulse in the circuit.

## Intermediate Exam II: Problem #1 (Spring '05)



The circuit of capacitors connected to a battery is at equilibrium.

- (a) Find the equivalent capacitance  $C_{eq}$ .
- (b) Find the voltage  $V_3$  across capacitor  $C_3$ .
- (c) Find the the charge  $Q_2$  on capacitor  $C_2$ .

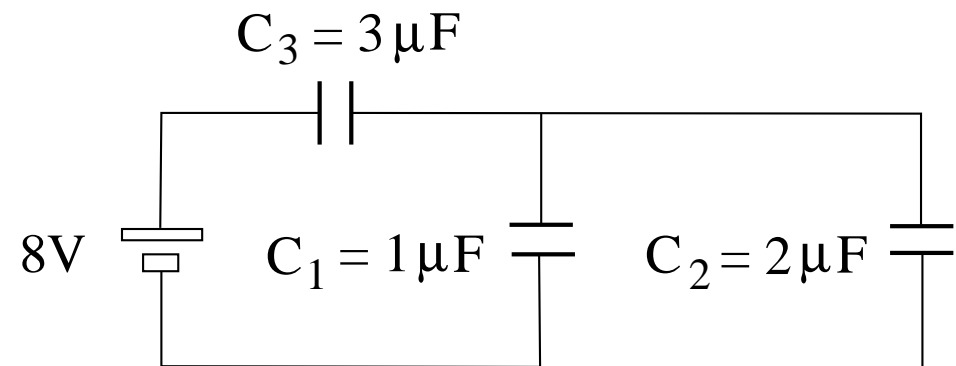


## Intermediate Exam II: Problem #1 (Spring '05)



The circuit of capacitors connected to a battery is at equilibrium.

- (a) Find the equivalent capacitance  $C_{eq}$ .
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- (c) Find the charge  $Q_2$  on capacitor  $C_2$ .



**Solution:**

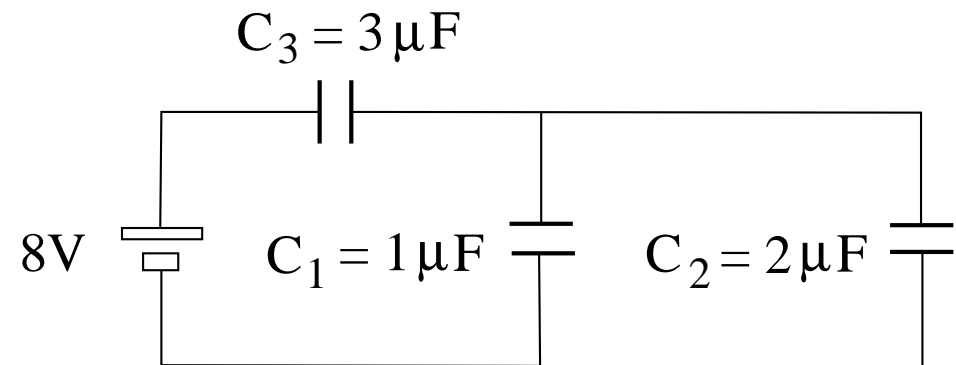
(a)  $C_{12} = C_1 + C_2 = 3 \mu\text{F}$ ,  $C_{eq} = \left( \frac{1}{C_{12}} + \frac{1}{C_3} \right)^{-1} = 1.5 \mu\text{F}$ .

# Intermediate Exam II: Problem #1 (Spring '05)



The circuit of capacitors connected to a battery is at equilibrium.

- (a) Find the equivalent capacitance  $C_{eq}$ .
- (b) Find the voltage  $V_3$  across capacitor  $C_3$ .
- (c) Find the charge  $Q_2$  on capacitor  $C_2$ .



**Solution:**

$$(a) \quad C_{12} = C_1 + C_2 = 3\mu\text{F}, \quad C_{eq} = \left( \frac{1}{C_{12}} + \frac{1}{C_3} \right)^{-1} = 1.5\mu\text{F}.$$

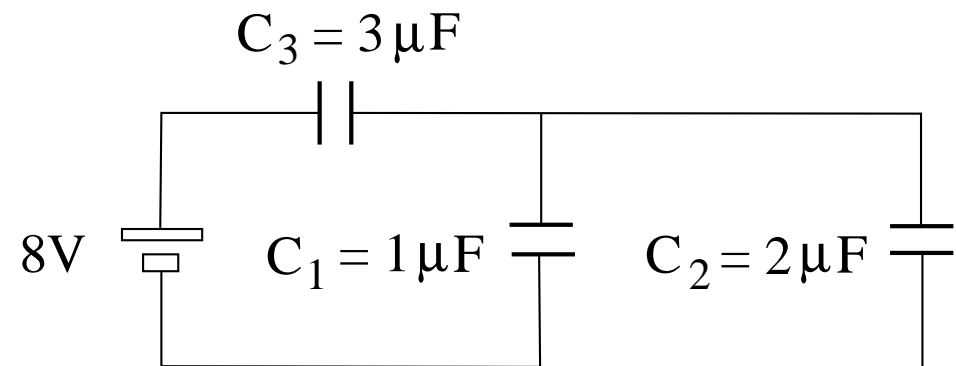
$$(b) \quad Q_3 = Q_{12} = Q_{eq} = C_{eq}(8\text{V}) = 12\mu\text{C}$$
$$\Rightarrow V_3 = \frac{Q_3}{C_3} = \frac{12\mu\text{C}}{3\mu\text{F}} = 4\text{V}.$$

## Intermediate Exam II: Problem #1 (Spring '05)



The circuit of capacitors connected to a battery is at equilibrium.

- (a) Find the equivalent capacitance  $C_{eq}$ .
- (b) Find the voltage  $V_3$  across capacitor  $C_3$ .
- (c) Find the the charge  $Q_2$  on capacitor  $C_2$ .



**Solution:**

$$(a) \quad C_{12} = C_1 + C_2 = 3\mu\text{F}, \quad C_{eq} = \left( \frac{1}{C_{12}} + \frac{1}{C_3} \right)^{-1} = 1.5\mu\text{F}.$$

$$(b) \quad Q_3 = Q_{12} = Q_{eq} = C_{eq}(8\text{V}) = 12\mu\text{C}$$
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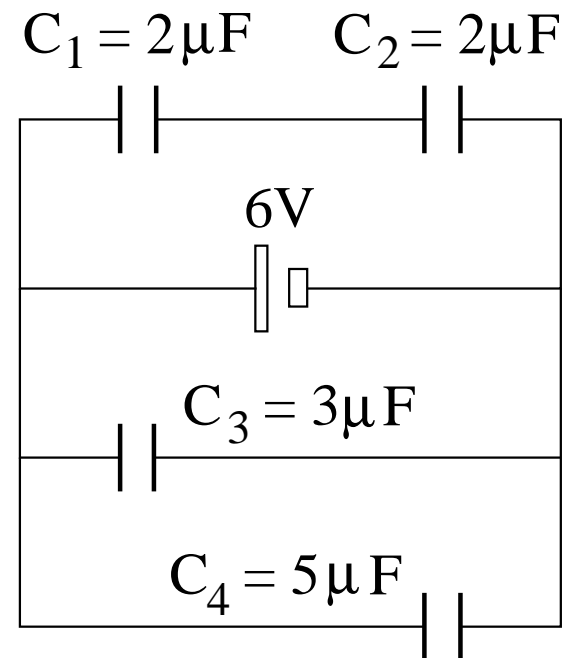
$$(c) \quad Q_2 = V_2 C_2 = 8\mu\text{C}.$$

## Unit Exam II: Problem #1 (Spring '07)



Consider the configuration of two point charges as shown.

- (a) Find the energy  $U_3$  stored on capacitor  $C_3$ .
- (b) Find the voltage  $V_4$  across capacitor  $C_4$ .
- (c) Find the voltage  $V_2$  across capacitor  $C_2$ .
- (d) Find the charge  $Q_1$  on capacitor  $C_1$ .



## Unit Exam II: Problem #1 (Spring '07)

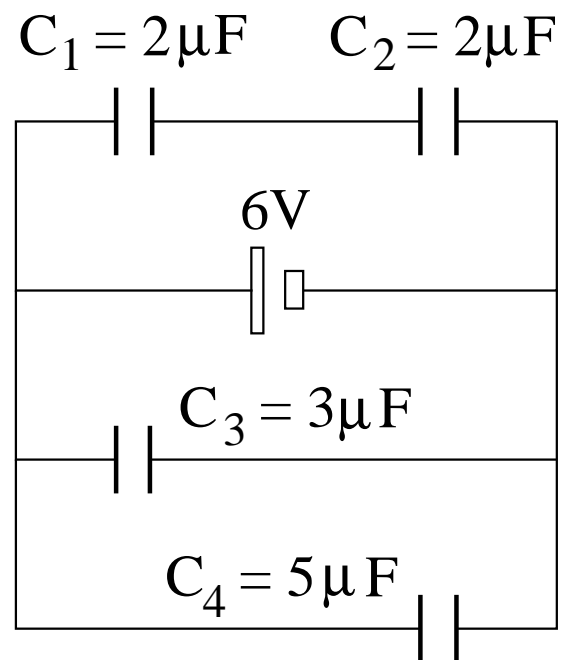


Consider the configuration of two point charges as shown.

- (a) Find the energy  $U_3$  stored on capacitor  $C_3$ .
- (b) Find the voltage  $V_4$  across capacitor  $C_4$ .
- (c) Find the voltage  $V_2$  across capacitor  $C_2$ .
- (d) Find the charge  $Q_1$  on capacitor  $C_1$ .

**Solution:**

(a)  $U_3 = \frac{1}{2}(3\mu\text{F})(6\text{V})^2 = 54\mu\text{J}.$





## Unit Exam II: Problem #1 (Spring '07)



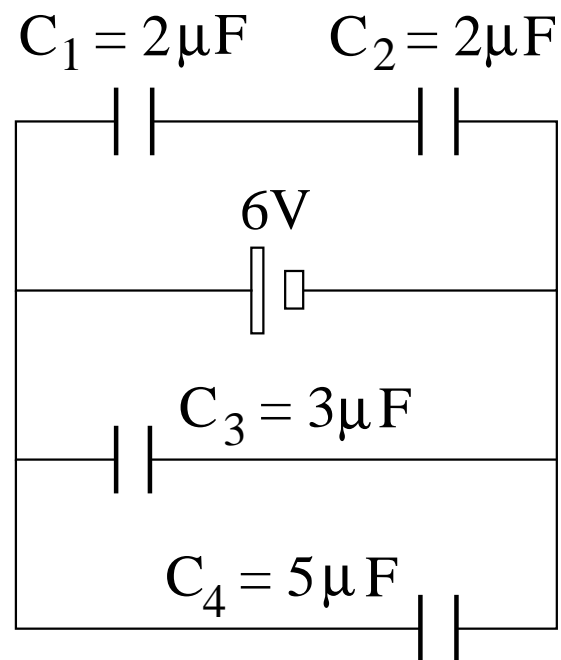
Consider the configuration of two point charges as shown.

- (a) Find the energy  $U_3$  stored on capacitor  $C_3$ .
- (b) Find the voltage  $V_4$  across capacitor  $C_4$ .
- (c) Find the voltage  $V_2$  across capacitor  $C_2$ .
- (d) Find the charge  $Q_1$  on capacitor  $C_1$ .

**Solution:**

(a)  $U_3 = \frac{1}{2}(3\mu\text{F})(6\text{V})^2 = 54\mu\text{J}.$

(b)  $V_4 = 6\text{V}.$



## Unit Exam II: Problem #1 (Spring '07)



Consider the configuration of two point charges as shown.

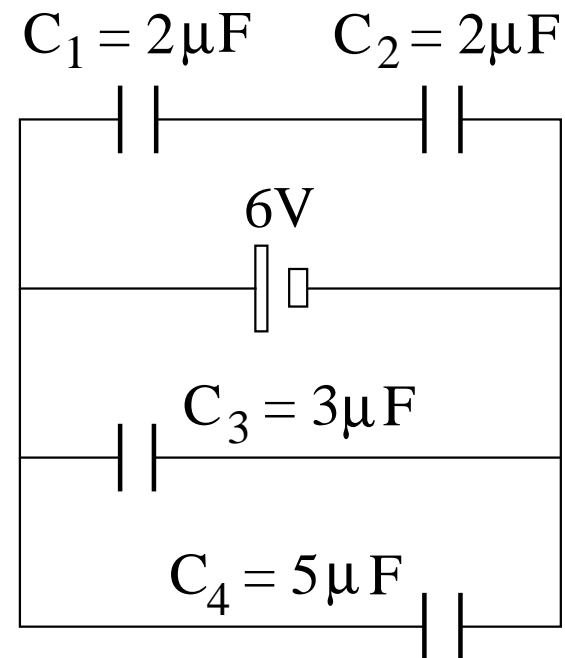
- (a) Find the energy  $U_3$  stored on capacitor  $C_3$ .
- (b) Find the voltage  $V_4$  across capacitor  $C_4$ .
- (c) Find the voltage  $V_2$  across capacitor  $C_2$ .
- (d) Find the charge  $Q_1$  on capacitor  $C_1$ .

**Solution:**

(a)  $U_3 = \frac{1}{2}(3\mu\text{F})(6\text{V})^2 = 54\mu\text{J}.$

(b)  $V_4 = 6\text{V}.$

(c)  $V_2 = \frac{1}{2}6\text{V} = 3\text{V}.$



## Unit Exam II: Problem #1 (Spring '07)



Consider the configuration of two point charges as shown.

- (a) Find the energy  $U_3$  stored on capacitor  $C_3$ .
- (b) Find the voltage  $V_4$  across capacitor  $C_4$ .
- (c) Find the voltage  $V_2$  across capacitor  $C_2$ .
- (d) Find the charge  $Q_1$  on capacitor  $C_1$ .

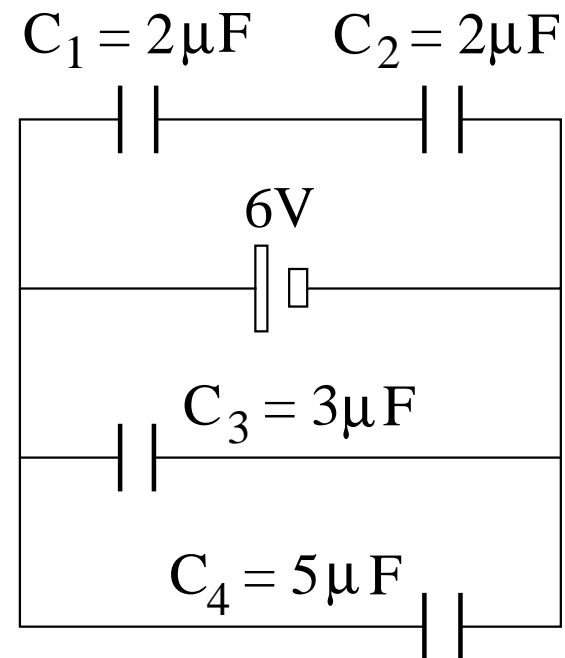
**Solution:**

(a)  $U_3 = \frac{1}{2}(3\mu\text{F})(6\text{V})^2 = 54\mu\text{J}.$

(b)  $V_4 = 6\text{V}.$

(c)  $V_2 = \frac{1}{2}6\text{V} = 3\text{V}.$

(d)  $Q_1 = (2\mu\text{F})(3\text{V}) = 6\mu\text{C}.$

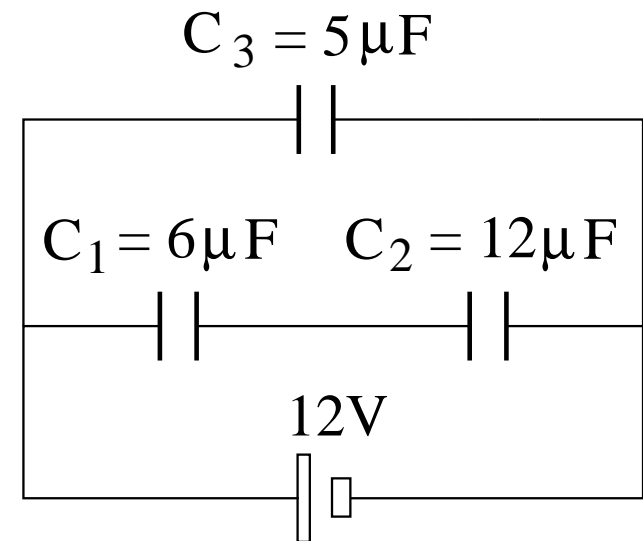


## Unit Exam II: Problem #1 (Spring '08)



The circuit of capacitors is at equilibrium.

- Find the charge  $Q_1$  on capacitor 1 and the charge  $Q_2$  on capacitor 2.
- Find the voltage  $V_1$  across capacitor 1 and the voltage  $V_2$  across capacitor 2.
- Find the charge  $Q_3$  and the energy  $U_3$  on capacitor 3.



## Unit Exam II: Problem #1 (Spring '08)

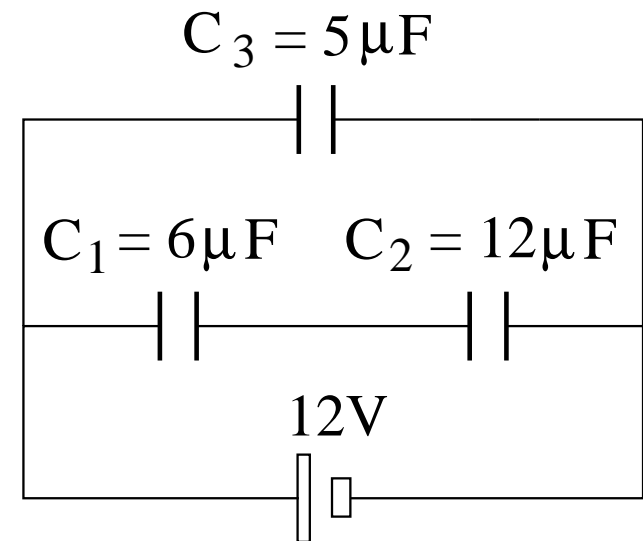


The circuit of capacitors is at equilibrium.

- Find the charge  $Q_1$  on capacitor 1 and the charge  $Q_2$  on capacitor 2.
- Find the voltage  $V_1$  across capacitor 1 and the voltage  $V_2$  across capacitor 2.
- Find the charge  $Q_3$  and the energy  $U_3$  on capacitor 3.

**Solution:**

$$(a) \quad C_{12} = \left( \frac{1}{6\mu\text{F}} + \frac{1}{12\mu\text{F}} \right)^{-1} = 4\mu\text{F},$$
$$Q_1 = Q_2 = Q_{12} = (4\mu\text{F})(12\text{V}) = 48\mu\text{C}.$$



## Unit Exam II: Problem #1 (Spring '08)



The circuit of capacitors is at equilibrium.

- Find the charge  $Q_1$  on capacitor 1 and the charge  $Q_2$  on capacitor 2.
- Find the voltage  $V_1$  across capacitor 1 and the voltage  $V_2$  across capacitor 2.
- Find the charge  $Q_3$  and the energy  $U_3$  on capacitor 3.

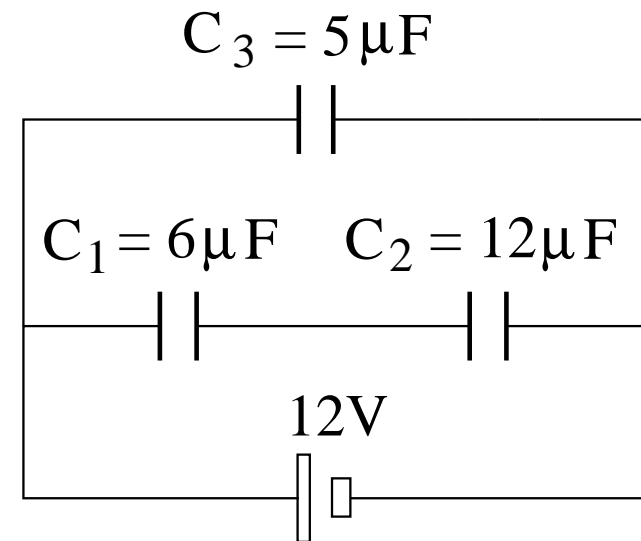
**Solution:**

$$(a) \quad C_{12} = \left( \frac{1}{6\mu\text{F}} + \frac{1}{12\mu\text{F}} \right)^{-1} = 4\mu\text{F},$$

$$Q_1 = Q_2 = Q_{12} = (4\mu\text{F})(12\text{V}) = 48\mu\text{C}.$$

$$(b) \quad V_1 = \frac{Q_1}{C_1} = \frac{48\mu\text{C}}{6\mu\text{F}} = 8\text{V},$$

$$V_2 = \frac{Q_2}{C_2} = \frac{48\mu\text{C}}{12\mu\text{F}} = 4\text{V}.$$



## Unit Exam II: Problem #1 (Spring '08)



The circuit of capacitors is at equilibrium.

- Find the charge  $Q_1$  on capacitor 1 and the charge  $Q_2$  on capacitor 2.
- Find the voltage  $V_1$  across capacitor 1 and the voltage  $V_2$  across capacitor 2.
- Find the charge  $Q_3$  and the energy  $U_3$  on capacitor 3.

**Solution:**

$$(a) \quad C_{12} = \left( \frac{1}{6\mu\text{F}} + \frac{1}{12\mu\text{F}} \right)^{-1} = 4\mu\text{F},$$

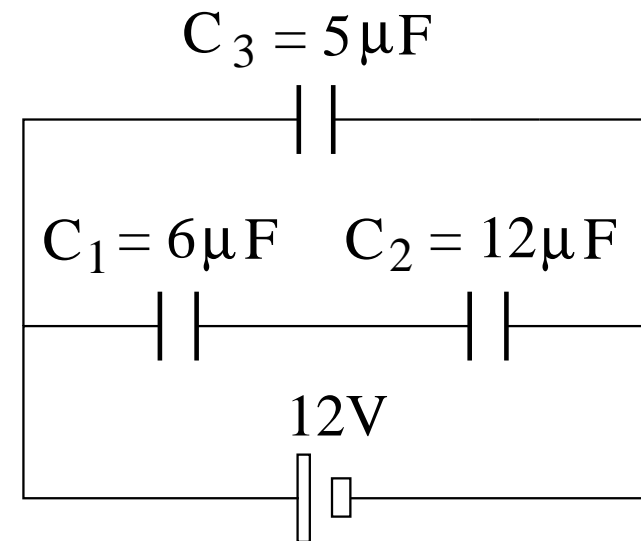
$$Q_1 = Q_2 = Q_{12} = (4\mu\text{F})(12\text{V}) = 48\mu\text{C}.$$

$$(b) \quad V_1 = \frac{Q_1}{C_1} = \frac{48\mu\text{C}}{6\mu\text{F}} = 8\text{V},$$

$$V_2 = \frac{Q_2}{C_2} = \frac{48\mu\text{C}}{12\mu\text{F}} = 4\text{V}.$$

$$(c) \quad Q_3 = (5\mu\text{F})(12\text{V}) = 60\mu\text{C},$$

$$U_3 = \frac{1}{2}(5\mu\text{F})(12\text{V})^2 = 360\mu\text{J}.$$

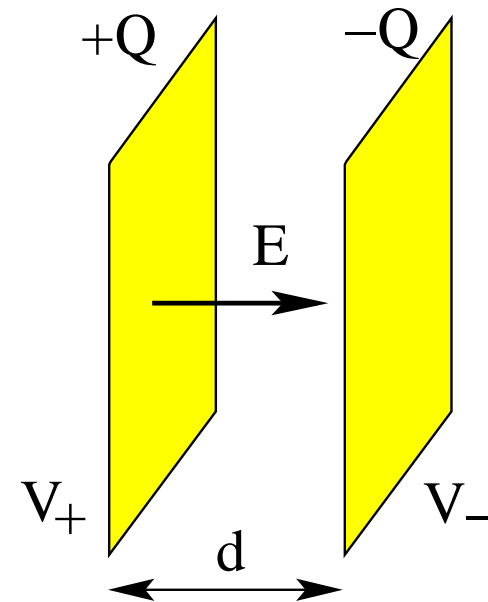


## Unit Exam II: Problem #2 (Spring '12)



Consider a parallel-plate capacitor of capacitance  $C = 6\text{pF}$  with plates separated a distance  $d = 1\text{mm}$  and a potential difference  $V = V_+ - V_- = 3\text{V}$  between them.

- Find the magnitude  $E$  of the electric field between the plates.
- Find the amount  $Q$  of charge on each plate.
- Find the energy  $U$  stored on the capacitor.
- Find the area  $A$  of each plate.





## Unit Exam II: Problem #2 (Spring '12)

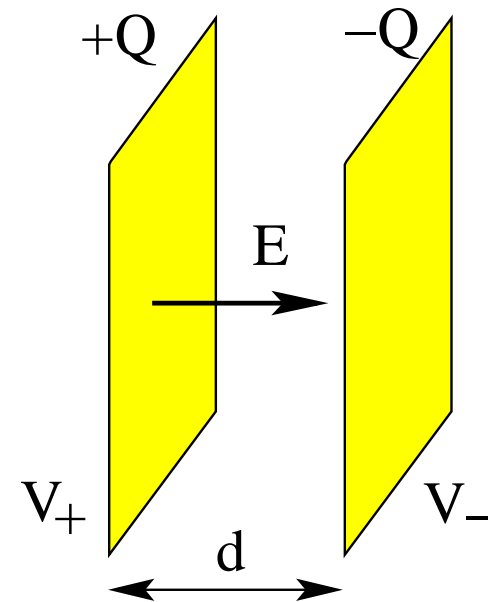


Consider a parallel-plate capacitor of capacitance  $C = 6\text{pF}$  with plates separated a distance  $d = 1\text{mm}$  and a potential difference  $V = V_+ - V_- = 3\text{V}$  between them.

- Find the magnitude  $E$  of the electric field between the plates.
- Find the amount  $Q$  of charge on each plate.
- Find the energy  $U$  stored on the capacitor.
- Find the area  $A$  of each plate.

**Solution:**

$$(a) \quad E = \frac{V}{d} = \frac{3\text{V}}{1\text{mm}} = 3000\text{V/m}.$$



## Unit Exam II: Problem #2 (Spring '12)



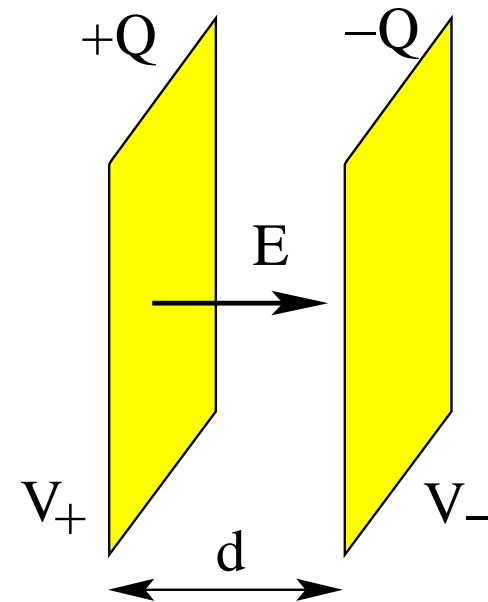
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- (a) Find the magnitude  $E$  of the electric field between the plates.
- (b) Find the amount  $Q$  of charge on each plate.
- (c) Find the energy  $U$  stored on the capacitor.
- (d) Find the area  $A$  of each plate.

**Solution:**

$$(a) \quad E = \frac{V}{d} = \frac{3\text{V}}{1\text{mm}} = 3000\text{V/m.}$$

$$(b) \quad Q = CV = (6\text{pF})(3\text{V}) = 18\text{pC.}$$



## Unit Exam II: Problem #2 (Spring '12)



Consider a parallel-plate capacitor of capacitance  $C = 6\text{pF}$  with plates separated a distance  $d = 1\text{mm}$  and a potential difference  $V = V_+ - V_- = 3\text{V}$  between them.

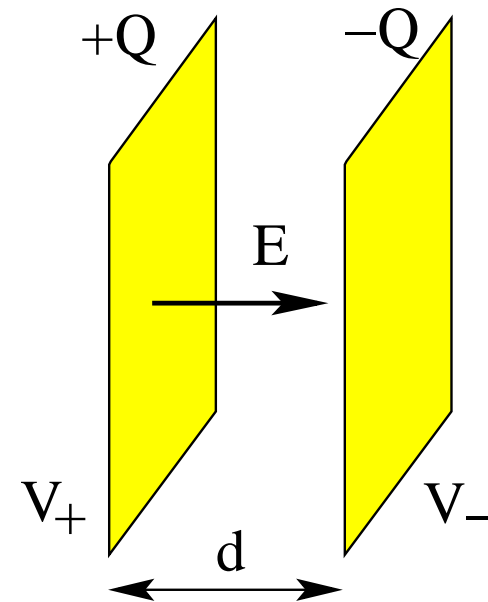
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- Find the amount  $Q$  of charge on each plate.
- Find the energy  $U$  stored on the capacitor.
- Find the area  $A$  of each plate.

**Solution:**

$$(a) \quad E = \frac{V}{d} = \frac{3\text{V}}{1\text{mm}} = 3000\text{V/m}.$$

$$(b) \quad Q = CV = (6\text{pF})(3\text{V}) = 18\text{pC}.$$

$$(c) \quad U = \frac{1}{2}QV = 0.5(18\text{pC})(3\text{V}) = 27\text{pJ}.$$



## Unit Exam II: Problem #2 (Spring '12)



Consider a parallel-plate capacitor of capacitance  $C = 6\text{pF}$  with plates separated a distance  $d = 1\text{mm}$  and a potential difference  $V = V_+ - V_- = 3\text{V}$  between them.

- Find the magnitude  $E$  of the electric field between the plates.
- Find the amount  $Q$  of charge on each plate.
- Find the energy  $U$  stored on the capacitor.
- Find the area  $A$  of each plate.

**Solution:**

$$(a) \quad E = \frac{V}{d} = \frac{3\text{V}}{1\text{mm}} = 3000\text{V/m}.$$

$$(b) \quad Q = CV = (6\text{pF})(3\text{V}) = 18\text{pC}.$$

$$(c) \quad U = \frac{1}{2}QV = 0.5(18\text{pC})(3\text{V}) = 27\text{pJ}.$$

$$(d) \quad A = \frac{Cd}{\epsilon_0} = \frac{(6\text{pF})(1\text{mm})}{8.85 \times 10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2}} = 6.78 \times 10^{-4}\text{m}^2.$$

