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07. Capacitors I

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Abstract

Lecture slides 7 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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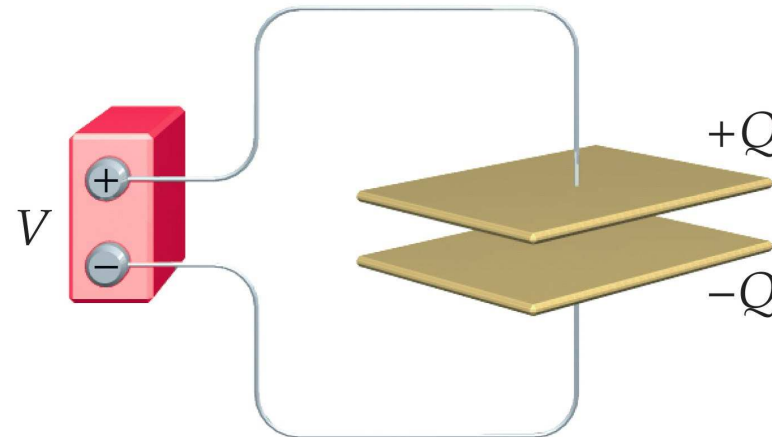


Capacitor (device):

- Two oppositely charged conductors separated by an insulator.
- The charges $+Q$ and $-Q$ on conductors generate an electric field \vec{E} and a potential difference V (voltage).
- Only one conductor may be present. Then the relevant potential difference is between the conductor and a point at infinity.

Capacitance (device property):

- Definition: $C = \frac{Q}{V}$
- SI unit: $1\text{F} = 1\text{C/V}$ (one Farad)



Parallel-Plate Capacitor



- A : area of each plate
- d : distance between plates
- Q : magnitude of charge on inside surface of each plate
- Charge per unit area (magnitude) on each plate: $\sigma = \frac{Q}{A}$
- Uniform electric field between plates:

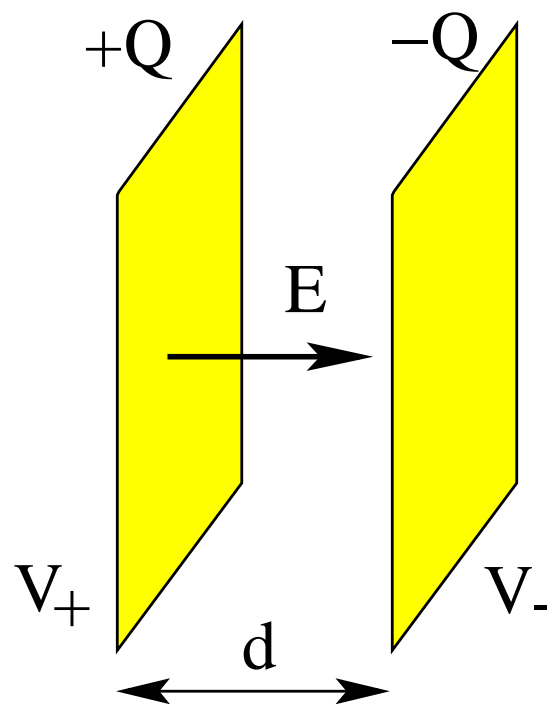
$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

- Voltage between plates:

$$V \equiv V_+ - V_- = Ed = \frac{Qd}{\epsilon_0 A}$$

- Capacitance for parallel-plate geometry:

$$C \equiv \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$



Cylindrical Capacitor



Conducting cylinder of radius a and length L surrounded concentrically by conducting cylindrical shell of inner radius b and equal length.

- Assumption: $L \gg b$.
- λ : charge per unit length (magnitude) on each cylinder
- $Q = \lambda L$: magnitude of charge on each cylinder
- Electric field between cylinders: use Gauss' law

$$E[2\pi rL] = \frac{\lambda L}{\epsilon_0} \Rightarrow E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$$

- Electric potential between cylinders: use $V(a) = 0$

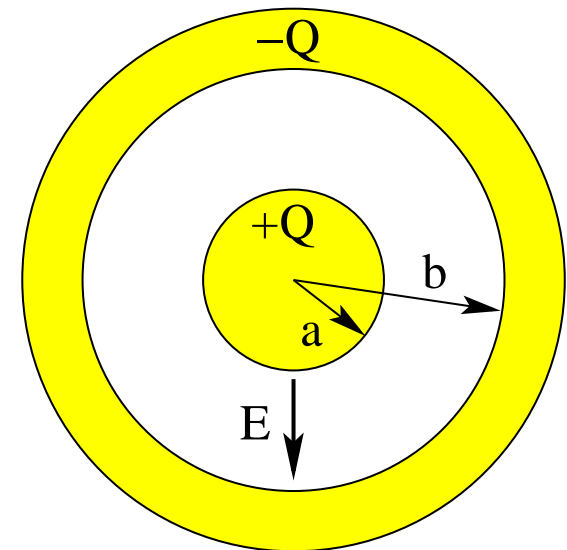
$$V(r) = - \int_a^r E(r)dr = - \frac{\lambda}{2\pi\epsilon_0} \int_a^r \frac{dr}{r} = - \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r}{a}$$

- Voltage between cylinders:

$$V \equiv V_+ - V_- = V(a) - V(b) = \frac{Q}{2\pi\epsilon_0 L} \ln \frac{b}{a}$$

- Capacitance for cylindrical geometry:

$$C \equiv \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$



Spherical Capacitor



Conducting sphere of radius a surrounded concentrically by conducting spherical shell of inner radius b .

- Q : magnitude of charge on each sphere
- Electric field between spheres: use Gauss' law

$$E[4\pi r^2] = \frac{Q}{\epsilon_0} \Rightarrow E(r) = \frac{Q}{4\pi\epsilon_0 r^2}$$

- Electric potential between spheres: use $V(a) = 0$

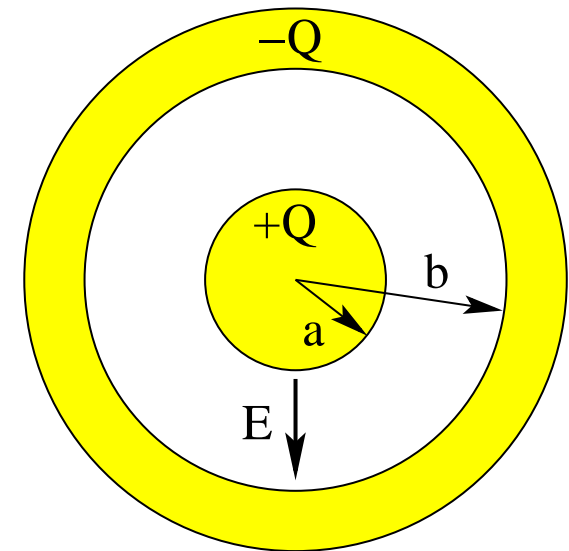
$$V(r) = - \int_a^r E(r) dr = - \frac{Q}{4\pi\epsilon_0} \int_a^r \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{a} \right]$$

- Voltage between spheres:

$$V \equiv V_+ - V_- = V(a) - V(b) = \frac{Q}{4\pi\epsilon_0} \frac{b-a}{ab}$$

- Capacitance for spherical geometry:

$$C \equiv \frac{Q}{V} = 4\pi\epsilon_0 \frac{ab}{b-a}$$



Energy Stored in Capacitor



Charging a capacitor requires work.

The work done is equal to the potential energy stored in the capacitor.

While charging, V increases linearly with q :

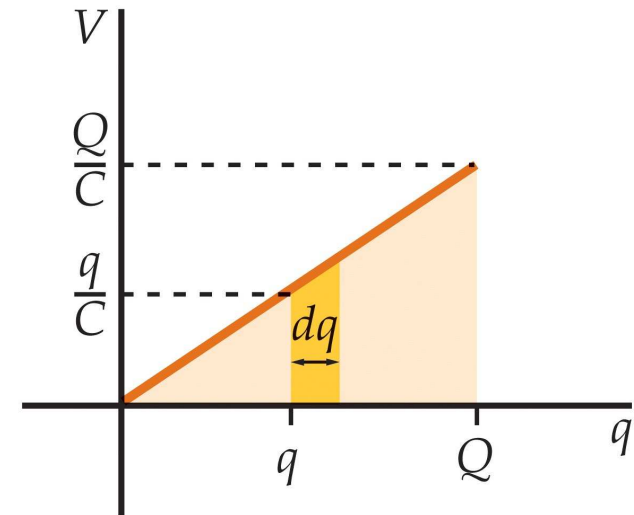
$$V(q) = \frac{q}{C}.$$

Increment of potential energy:

$$dU = V dq = \frac{q}{C} dq.$$

Potential energy of charged capacitor:

$$U = \int_0^Q V dq = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV.$$



Q: where is the potential energy stored?

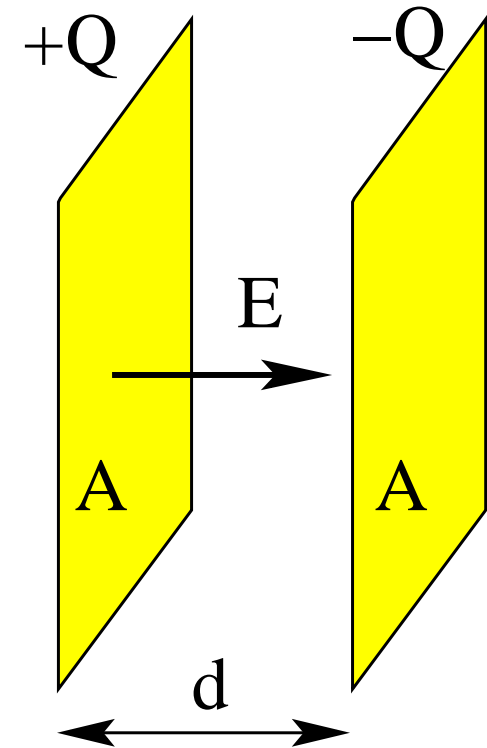
A: in the electric field.

Energy Density Between Parallel Plates



Energy is stored in the electric field between the plates of a capacitor.

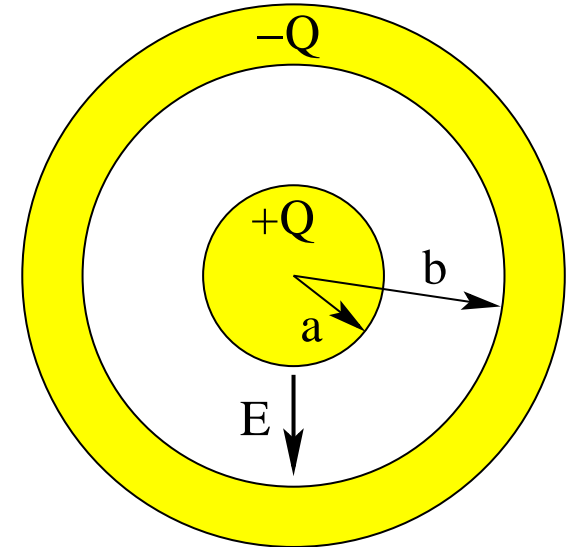
- Capacitance: $C = \frac{\epsilon_0 A}{d}$.
- Voltage: $V = Ed$.
- Potential energy: $U = \frac{1}{2}CV^2 = \frac{1}{2}\epsilon_0 E^2(Ad)$.
- Volume between the plates: Ad .
- Energy density of the electric field: $u_E = \frac{U}{Ad} = \frac{1}{2}\epsilon_0 E^2$



Integrating Energy Density in Spherical Capacitor



- Electric field: $E(r) = \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2}$
- Voltage: $V = \frac{Q}{4\pi\epsilon_0} \frac{b-a}{ab} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right]$
- Energy density: $u_E(r) = \frac{1}{2} \epsilon_0 E^2(r)$



- Energy stored in capacitor: $U = \int_a^b u_E(r) (4\pi r^2) dr$
- $\Rightarrow U = \int_a^b \frac{1}{2} \epsilon_0 \frac{Q^2}{(4\pi\epsilon_0)^2} \frac{1}{r^4} (4\pi r^2) dr$
- $\Rightarrow U = \frac{1}{2} \frac{Q^2}{4\pi\epsilon_0} \int_a^b \frac{1}{r^2} dr = \frac{1}{2} \frac{Q^2}{4\pi\epsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right] = \frac{1}{2} QV$

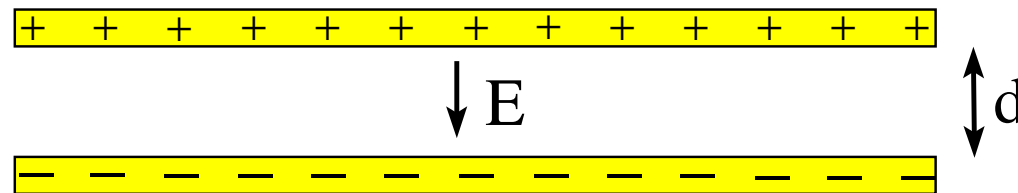
Capacitor Problem (1)



Consider two oppositely charged parallel plates separated by a very small distance d .

What happens when the plates are pulled apart a fraction of d ? Will the quantities listed below increase or decrease in magnitude or stay unchanged?

- (a) Electric field \vec{E} between the plates.
- (b) Voltage V across the plates.
- (c) Capacitance C of the device.
- (d) Energy U stored in the device.



Capacitors Connected in Parallel

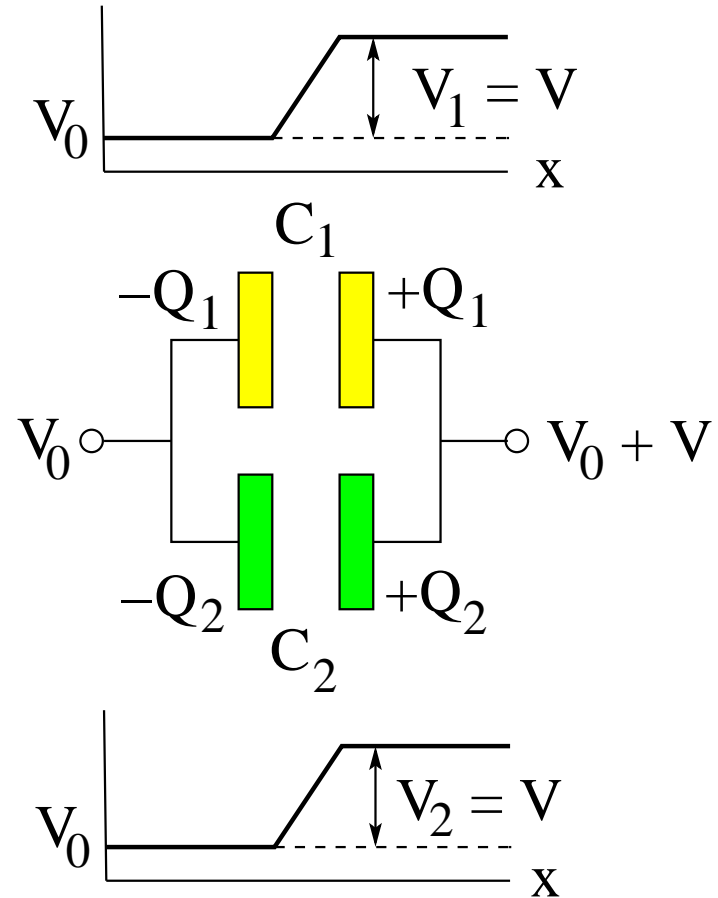


Find the equivalent capacitance of two capacitors connected in parallel:

- Charge on capacitors: $Q_1 + Q_2 = Q$
- Voltage across capacitors: $V_1 = V_2 = V$
- Equivalent capacitance:

$$C \equiv \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V_1} + \frac{Q_2}{V_2}$$

- $\Rightarrow C = C_1 + C_2$

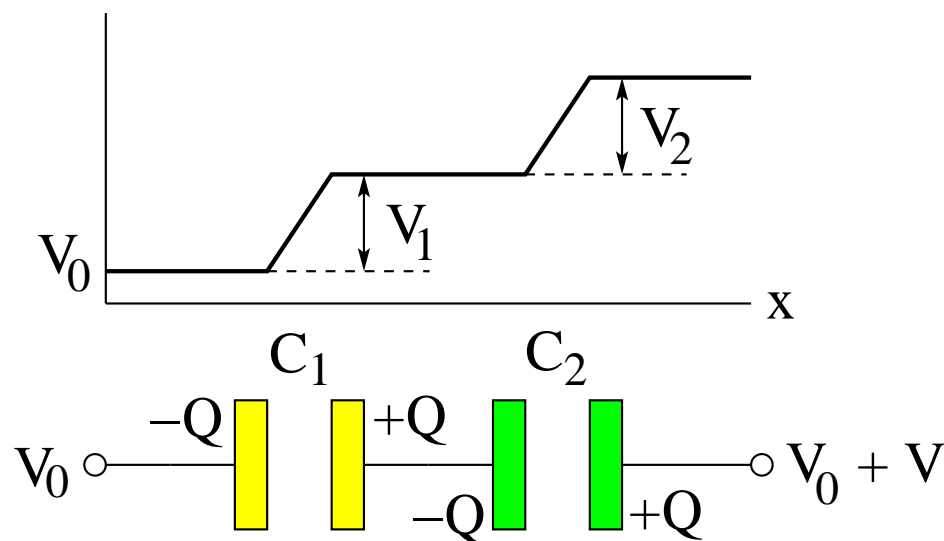


Capacitors Connected in Series



Find the equivalent capacitance of two capacitors connected in series:

- Charge on capacitors: $Q_1 = Q_2 = Q$
- Voltage across capacitors: $V_1 + V_2 = V$
- Equivalent capacitance: $\frac{1}{C} \equiv \frac{V}{Q} = \frac{V_1 + V_2}{Q} = \frac{V_1}{Q_1} + \frac{V_2}{Q_2}$
- $\Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

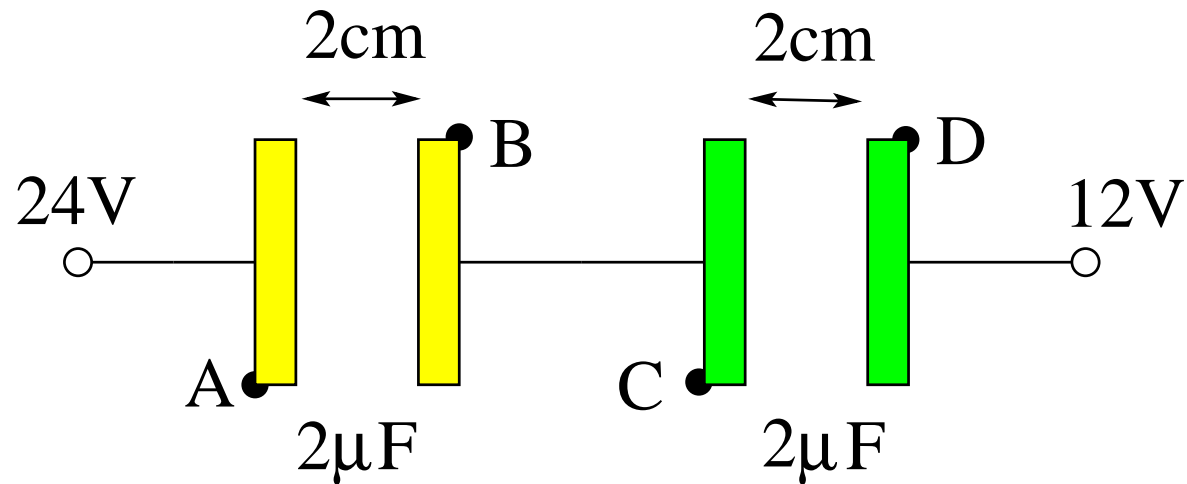


Capacitor Problem (2)



Consider two equal capacitors connected in series.

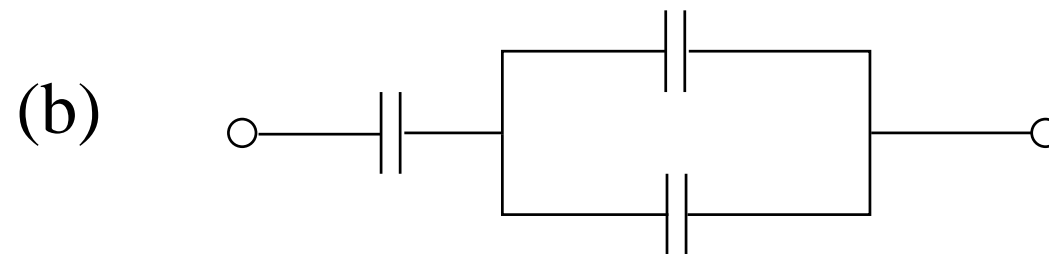
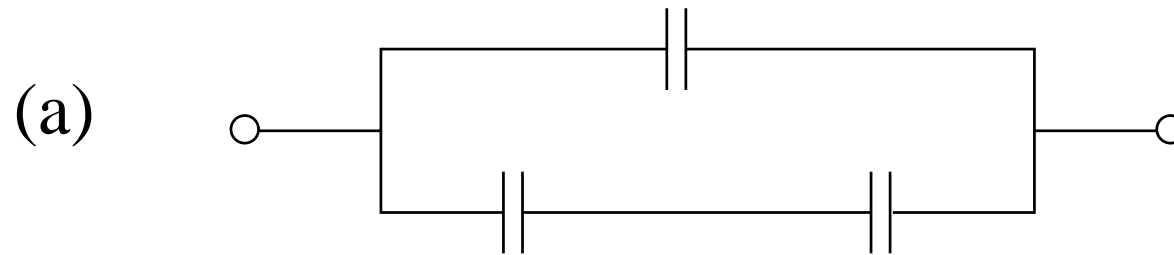
- (a) Find the voltages $V_A - V_B$, $V_B - V_C$, $V_A - V_D$.
- (b) Find the charge Q_A on plate A .
- (c) Find the electric field E between plates C and D .



Capacitor Circuit (1)



Find the equivalent capacitances of the two capacitor networks.
All capacitors have a capacitance of $1\mu F$.

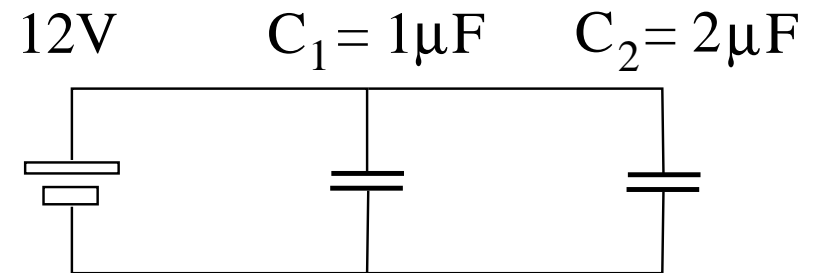


Capacitor Circuit (2)



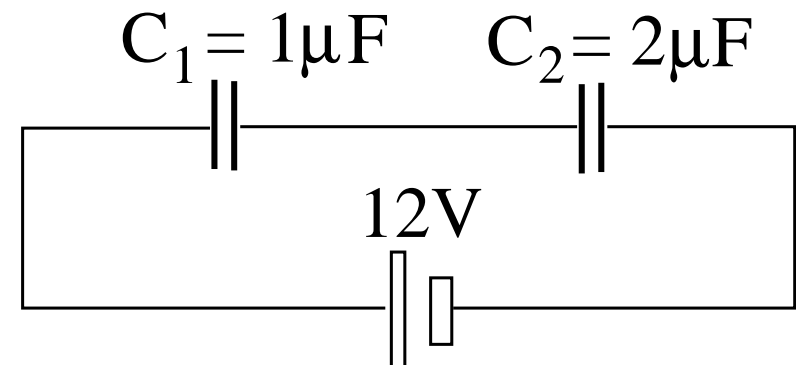
Consider the two capacitors connected in parallel.

- (a) Which capacitor has the higher voltage?
- (b) Which capacitor has more charge?
- (c) Which capacitor has more energy?



Consider the two capacitors connected in series.

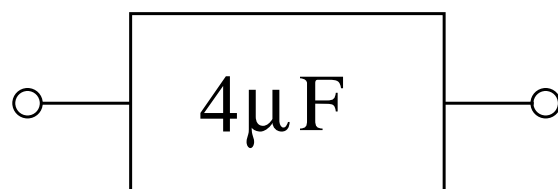
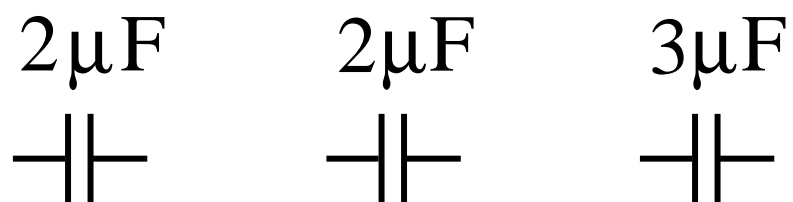
- (d) Which capacitor has the higher voltage?
- (e) Which capacitor has more charge?
- (f) Which capacitor has more energy?



Capacitor Circuit (3)



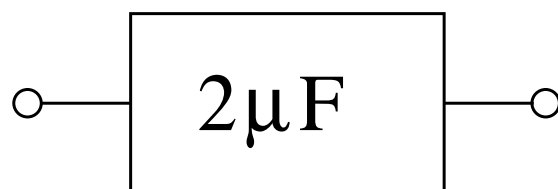
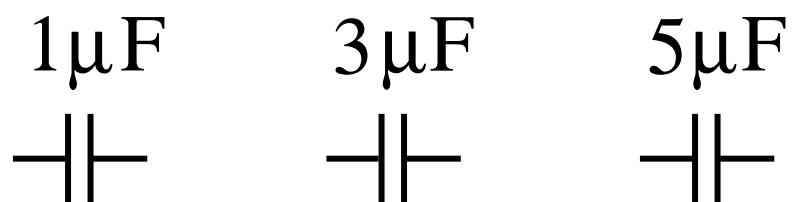
Connect the three capacitors in such a way that the equivalent capacitance is $C_{eq} = 4\mu\text{F}$. Draw the circuit diagram.



Capacitor Circuit (4)



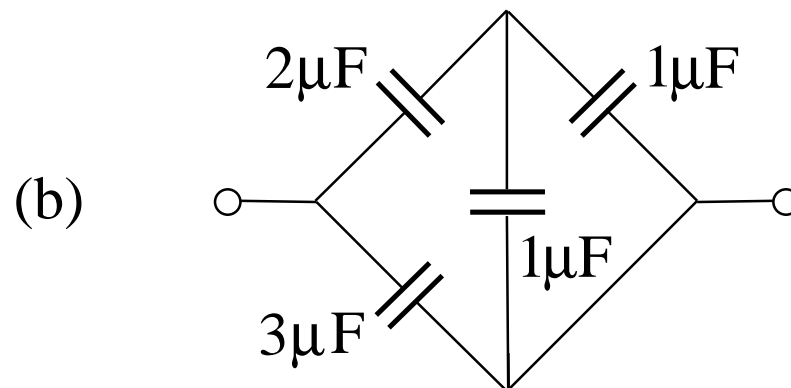
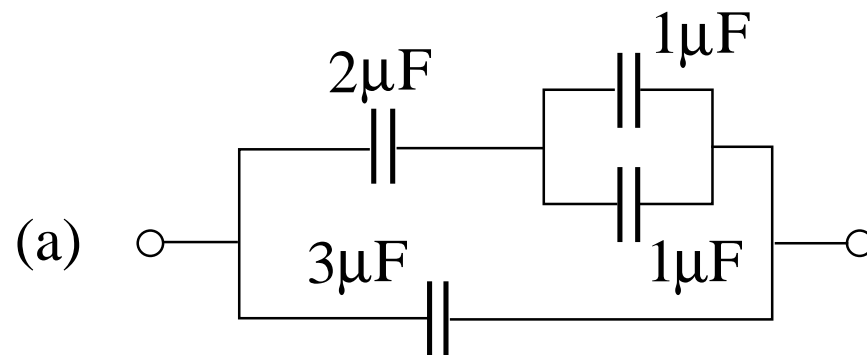
Connect the three capacitors in such a way that the equivalent capacitance is $C_{eq} = 2\mu\text{F}$. Draw the circuit diagram.



Capacitor Circuit (5)



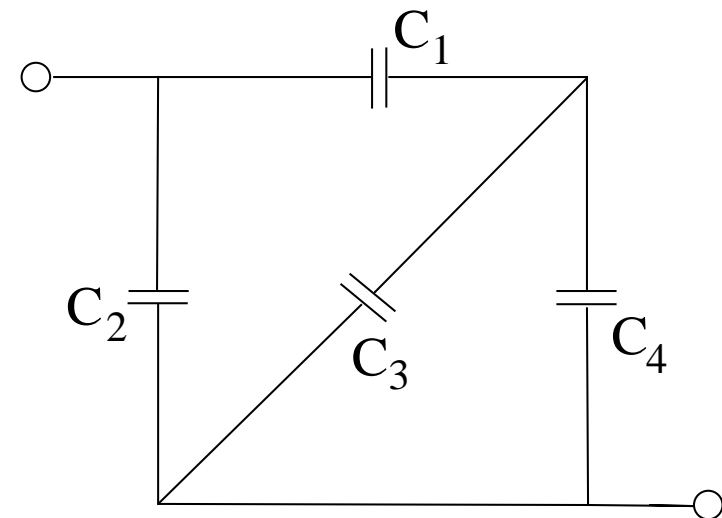
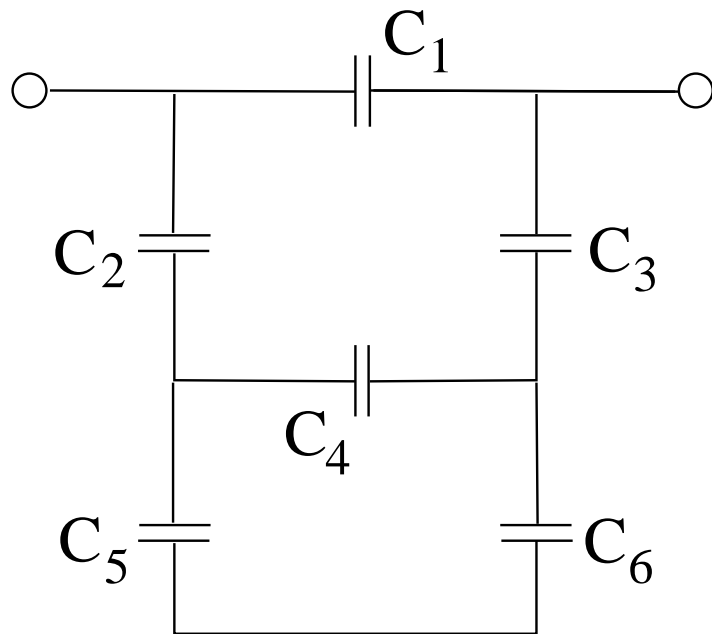
Find the equivalent capacitances of the following circuits.



Capacitor Circuit (6)



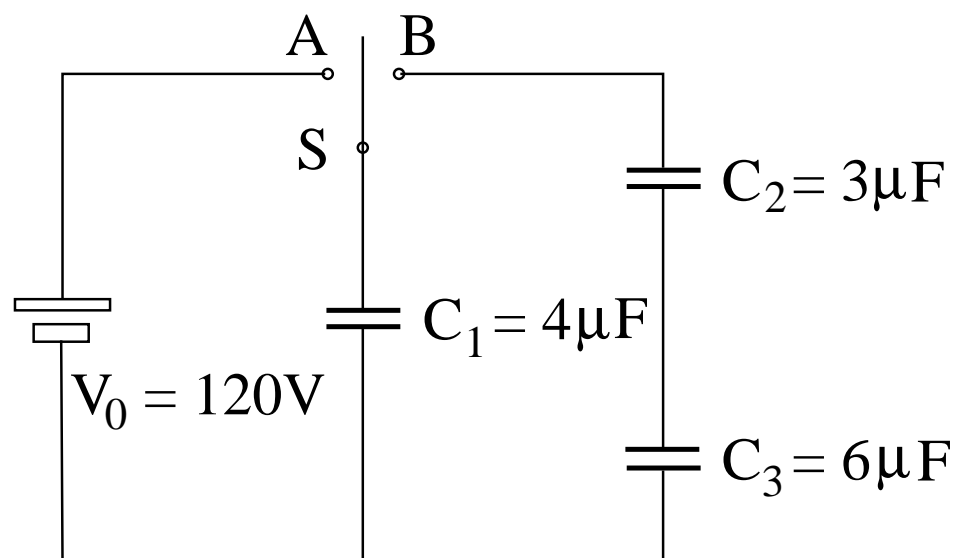
- (a) Name two capacitors from the circuit on the **left** that are connected in **series**.
- (b) Name two capacitors from the circuit on the **right** that are connected in **parallel**.



Capacitor Circuit (7)



- (a) In the circuit shown the switch is first thrown to A . Find the charge Q_0 and the energy U_A on the capacitor C_1 once it is charged up.
- (b) Then the switch is thrown to B , which charges up the capacitors C_2 and C_3 . The capacitor C_1 is partially discharged in the process. Find the charges Q_1, Q_2, Q_3 on all three capacitors and the voltages V_1, V_2, V_3 across each capacitor once equilibrium has been reached again. What is the energy U_B now stored in the circuit?

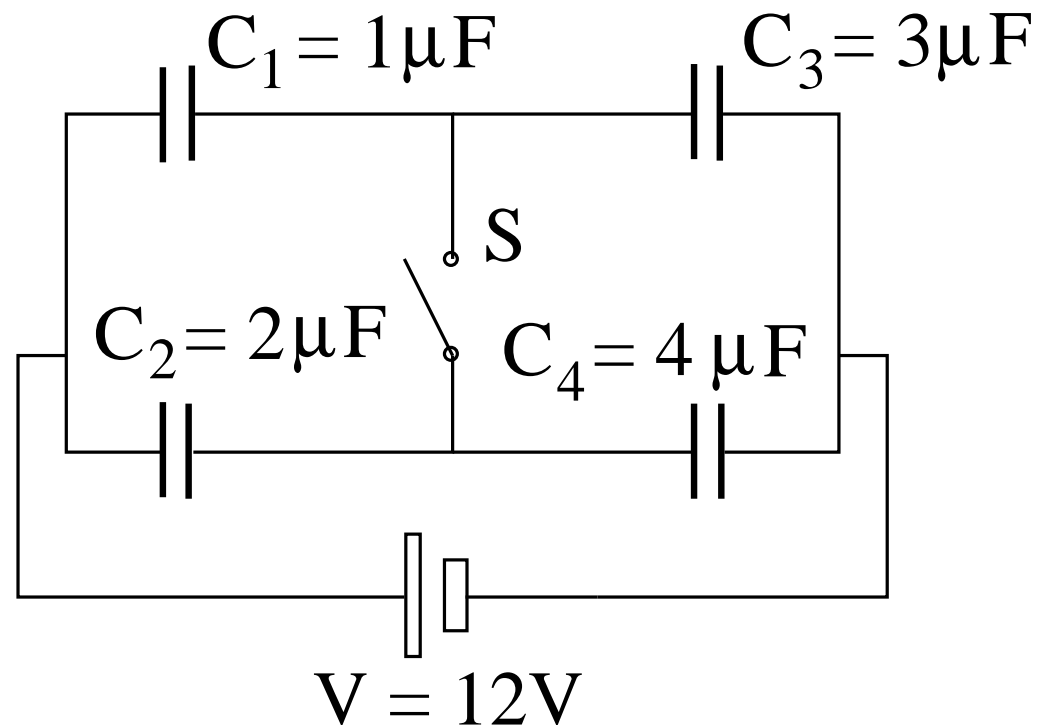


Capacitor Circuit (8)



In the circuit shown find the charges Q_1, Q_2, Q_3, Q_4 on each capacitor and the voltages V_1, V_2, V_3, V_4 across each capacitor

- (a) when the switch S is open,
- (b) when the switch S is closed.

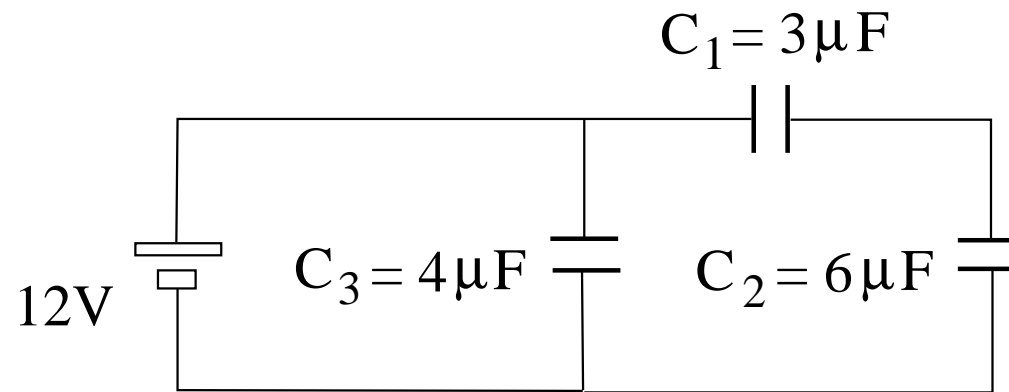


Capacitor Circuit (9)



The circuit of capacitors connected to a battery is at equilibrium.

- (a) Find the equivalent capacitance C_{eq} .
- (b) Find the total energy U stored in the circuit (excluding the battery).
- (c) Find the charge Q_3 on capacitor C_3 .
- (d) Find the voltage V_2 across capacitor C_2 .



More Complex Capacitor Circuit



No two capacitors are in parallel or in series.
Solution requires different strategy:

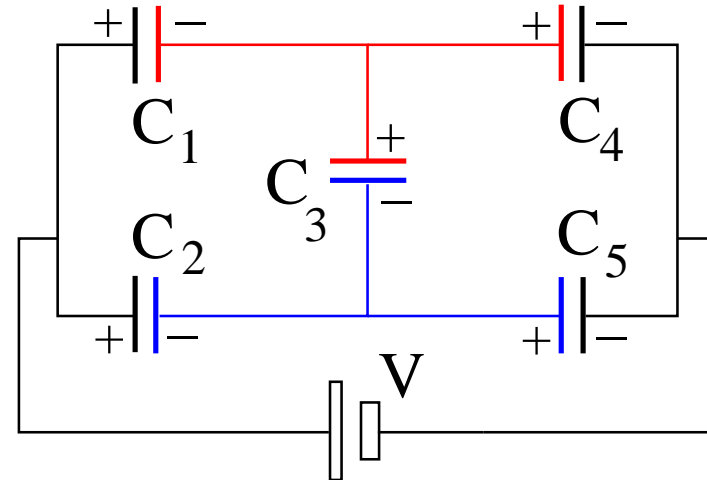
- zero charge on each conductor (here color coded),
- zero voltage around any closed loop.

Specifications: C_1, \dots, Q_5, V .

Five equations for unknowns Q_1, \dots, Q_5 :

- $Q_1 + Q_2 - Q_4 - Q_5 = 0$
- $Q_3 + Q_4 - Q_1 = 0$
- $\frac{Q_5}{C_5} + \frac{Q_3}{C_3} - \frac{Q_4}{C_4} = 0$
- $\frac{Q_2}{C_2} - \frac{Q_1}{C_1} - \frac{Q_3}{C_3} = 0$
- $V - \frac{Q_1}{C_1} - \frac{Q_4}{C_4} = 0$

Equivalent capacitance: $C_{eq} = \frac{Q_1 + Q_2}{V}$



(a) $C_m = 1\text{pF}, m = 1, \dots, 5$ and $V = 1\text{V}$:

$$C_{eq} = 1\text{pF}, Q_3 = 0,$$

$$Q_1 = Q_2 = Q_4 = Q_5 = \frac{1}{2}\text{pC}.$$

(b) $C_m = m\text{pF}, m = 1, \dots, 5$ and $V = 1\text{V}$:

$$C_{eq} = \frac{159}{71}\text{pF}, Q_1 = \frac{55}{71}\text{pC}, Q_2 = \frac{104}{71}\text{pC},$$

$$Q_3 = -\frac{9}{71}\text{pC}, Q_4 = \frac{64}{71}\text{pC}, Q_5 = \frac{95}{71}\text{pC}.$$