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## 07. Capacitors I

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### Abstract

Lecture slides 7 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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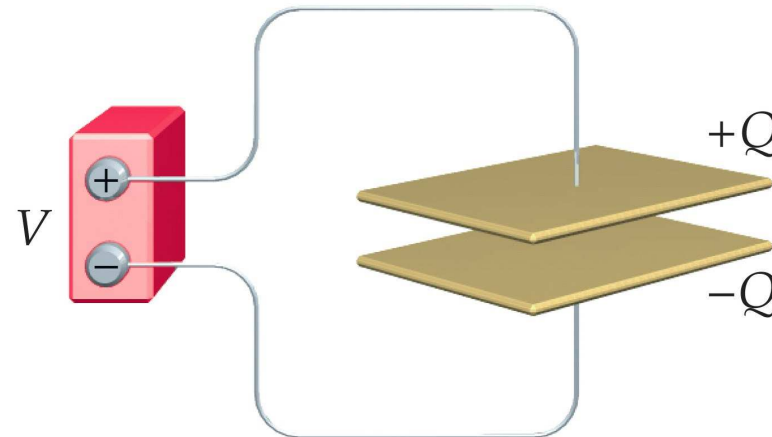


## Capacitor (device):

- Two oppositely charged conductors separated by an insulator.
- The charges  $+Q$  and  $-Q$  on conductors generate an electric field  $\vec{E}$  and a potential difference  $V$  (voltage).
- Only one conductor may be present. Then the relevant potential difference is between the conductor and a point at infinity.

## Capacitance (device property):

- Definition:  $C = \frac{Q}{V}$
- SI unit:  $1\text{F} = 1\text{C/V}$  (one Farad)



# Parallel-Plate Capacitor



- $A$ : area of each plate
- $d$ : distance between plates
- $Q$ : magnitude of charge on inside surface of each plate
- Charge per unit area (magnitude) on each plate:  $\sigma = \frac{Q}{A}$
- Uniform electric field between plates:

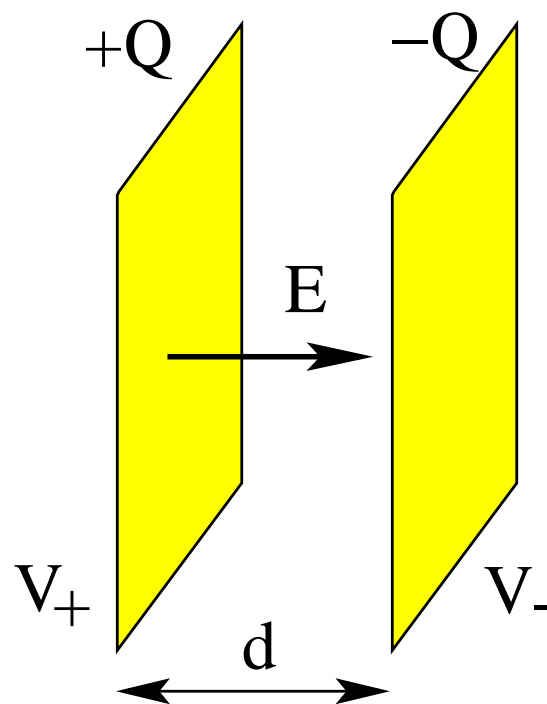
$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

- Voltage between plates:

$$V \equiv V_+ - V_- = Ed = \frac{Qd}{\epsilon_0 A}$$

- Capacitance for parallel-plate geometry:

$$C \equiv \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$



# Cylindrical Capacitor



Conducting cylinder of radius  $a$  and length  $L$  surrounded concentrically by conducting cylindrical shell of inner radius  $b$  and equal length.

- Assumption:  $L \gg b$ .
- $\lambda$ : charge per unit length (magnitude) on each cylinder
- $Q = \lambda L$ : magnitude of charge on each cylinder
- Electric field between cylinders: use Gauss' law

$$E[2\pi rL] = \frac{\lambda L}{\epsilon_0} \Rightarrow E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$$

- Electric potential between cylinders: use  $V(a) = 0$

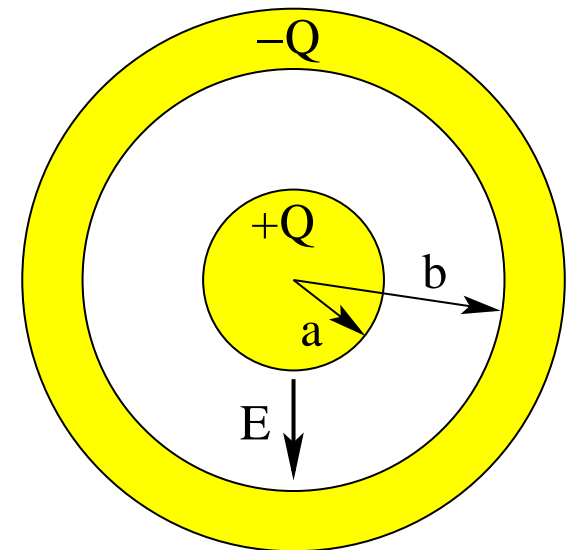
$$V(r) = - \int_a^r E(r)dr = - \frac{\lambda}{2\pi\epsilon_0} \int_a^r \frac{dr}{r} = - \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r}{a}$$

- Voltage between cylinders:

$$V \equiv V_+ - V_- = V(a) - V(b) = \frac{Q}{2\pi\epsilon_0 L} \ln \frac{b}{a}$$

- Capacitance for cylindrical geometry:

$$C \equiv \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$



# Spherical Capacitor



Conducting sphere of radius  $a$  surrounded concentrically by conducting spherical shell of inner radius  $b$ .

- $Q$ : magnitude of charge on each sphere
- Electric field between spheres: use Gauss' law

$$E[4\pi r^2] = \frac{Q}{\epsilon_0} \Rightarrow E(r) = \frac{Q}{4\pi\epsilon_0 r^2}$$

- Electric potential between spheres: use  $V(a) = 0$

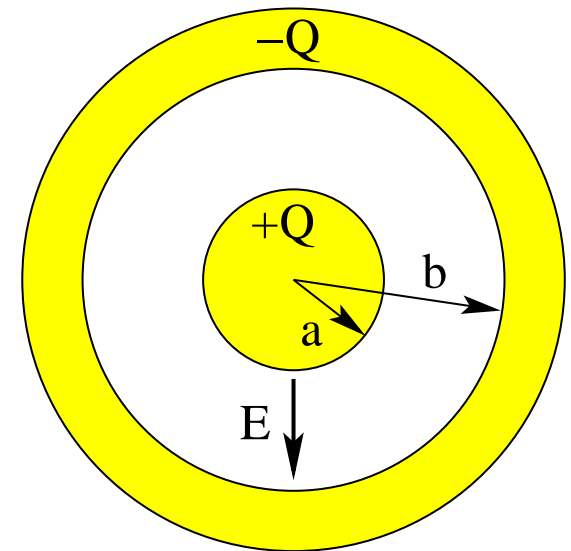
$$V(r) = - \int_a^r E(r) dr = - \frac{Q}{4\pi\epsilon_0} \int_a^r \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{r} - \frac{1}{a} \right]$$

- Voltage between spheres:

$$V \equiv V_+ - V_- = V(a) - V(b) = \frac{Q}{4\pi\epsilon_0} \frac{b-a}{ab}$$

- Capacitance for spherical geometry:

$$C \equiv \frac{Q}{V} = 4\pi\epsilon_0 \frac{ab}{b-a}$$



# Energy Stored in Capacitor



Charging a capacitor requires work.

The work done is equal to the potential energy stored in the capacitor.

While charging,  $V$  increases linearly with  $q$ :

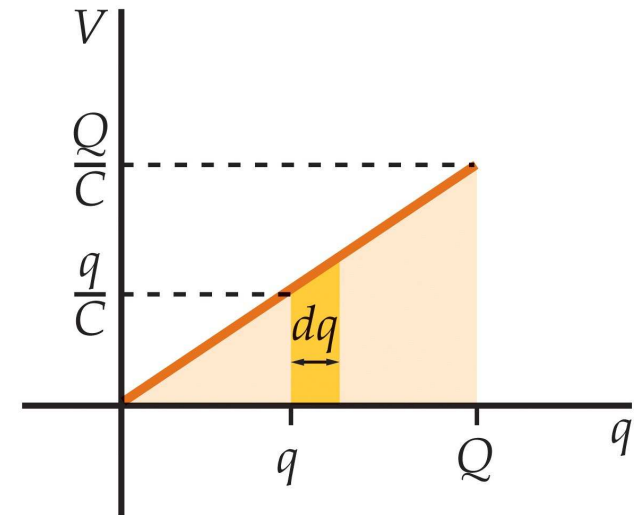
$$V(q) = \frac{q}{C}.$$

Increment of potential energy:

$$dU = V dq = \frac{q}{C} dq.$$

Potential energy of charged capacitor:

$$U = \int_0^Q V dq = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV.$$



Q: where is the potential energy stored?

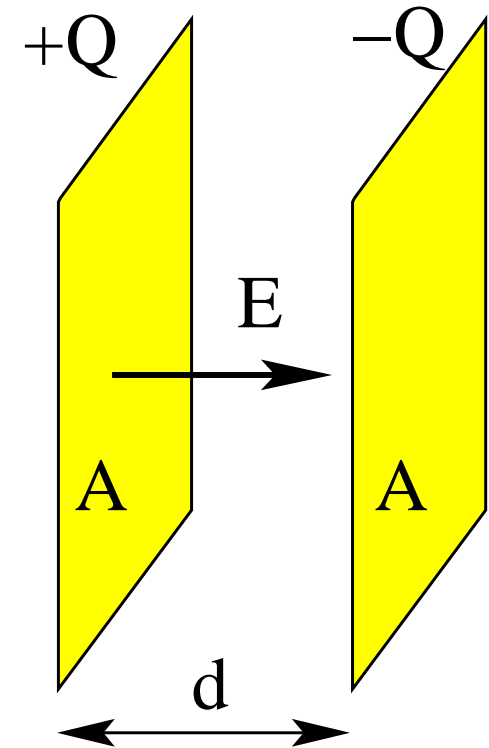
A: in the electric field.

# Energy Density Between Parallel Plates



Energy is stored in the electric field between the plates of a capacitor.

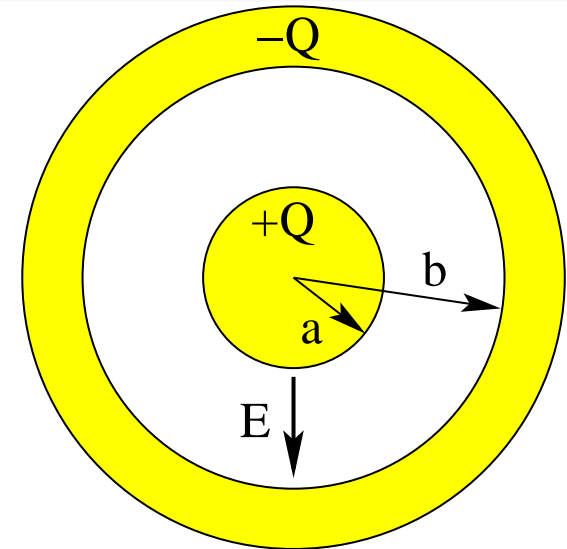
- Capacitance:  $C = \frac{\epsilon_0 A}{d}$ .
- Voltage:  $V = Ed$ .
- Potential energy:  $U = \frac{1}{2}CV^2 = \frac{1}{2}\epsilon_0 E^2(Ad)$ .
- Volume between the plates:  $Ad$ .
- Energy density of the electric field:  $u_E = \frac{U}{Ad} = \frac{1}{2}\epsilon_0 E^2$



# Integrating Energy Density in Spherical Capacitor



- Electric field:  $E(r) = \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2}$
- Voltage:  $V = \frac{Q}{4\pi\epsilon_0} \frac{b-a}{ab} = \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{a} - \frac{1}{b} \right]$
- Energy density:  $u_E(r) = \frac{1}{2} \epsilon_0 E^2(r)$



- Energy stored in capacitor:  $U = \int_a^b u_E(r) (4\pi r^2) dr$
- $\Rightarrow U = \int_a^b \frac{1}{2} \epsilon_0 \frac{Q^2}{(4\pi\epsilon_0)^2} \frac{1}{r^4} (4\pi r^2) dr$
- $\Rightarrow U = \frac{1}{2} \frac{Q^2}{4\pi\epsilon_0} \int_a^b \frac{1}{r^2} dr = \frac{1}{2} \frac{Q^2}{4\pi\epsilon_0} \left[ \frac{1}{a} - \frac{1}{b} \right] = \frac{1}{2} QV$



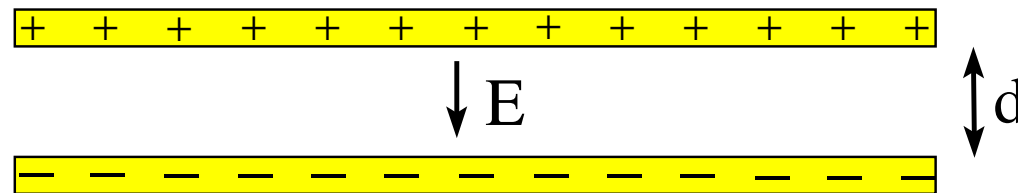
# Capacitor Problem (1)



Consider two oppositely charged parallel plates separated by a very small distance  $d$ .

What happens when the plates are pulled apart a fraction of  $d$ ? Will the quantities listed below increase or decrease in magnitude or stay unchanged?

- (a) Electric field  $\vec{E}$  between the plates.
- (b) Voltage  $V$  across the plates.
- (c) Capacitance  $C$  of the device.
- (d) Energy  $U$  stored in the device.



# Capacitors Connected in Parallel

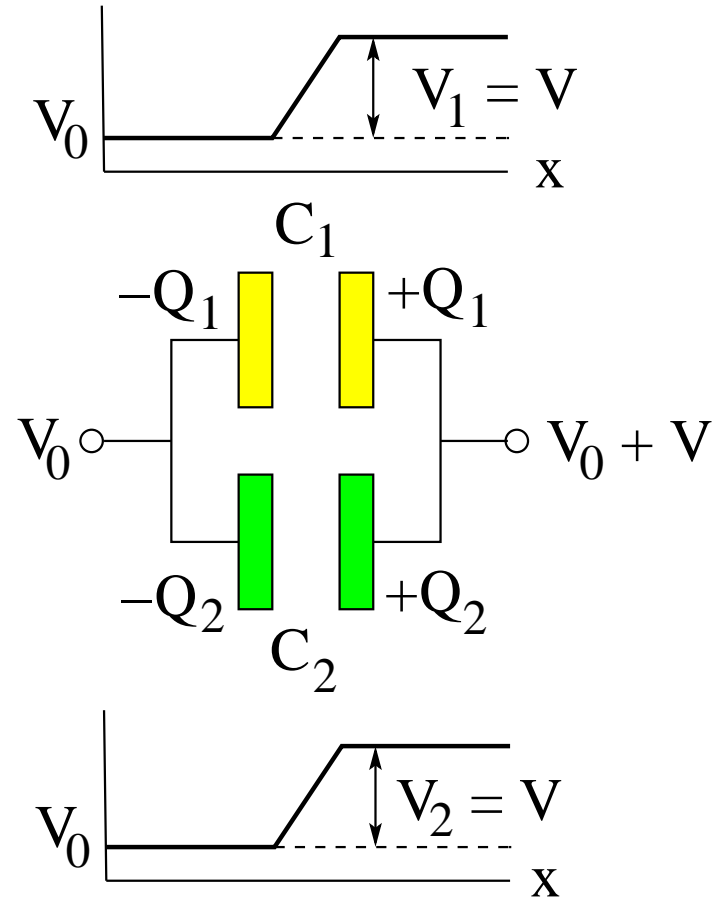


Find the equivalent capacitance of two capacitors connected in parallel:

- Charge on capacitors:  $Q_1 + Q_2 = Q$
- Voltage across capacitors:  $V_1 = V_2 = V$
- Equivalent capacitance:

$$C \equiv \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V_1} + \frac{Q_2}{V_2}$$

- $\Rightarrow C = C_1 + C_2$

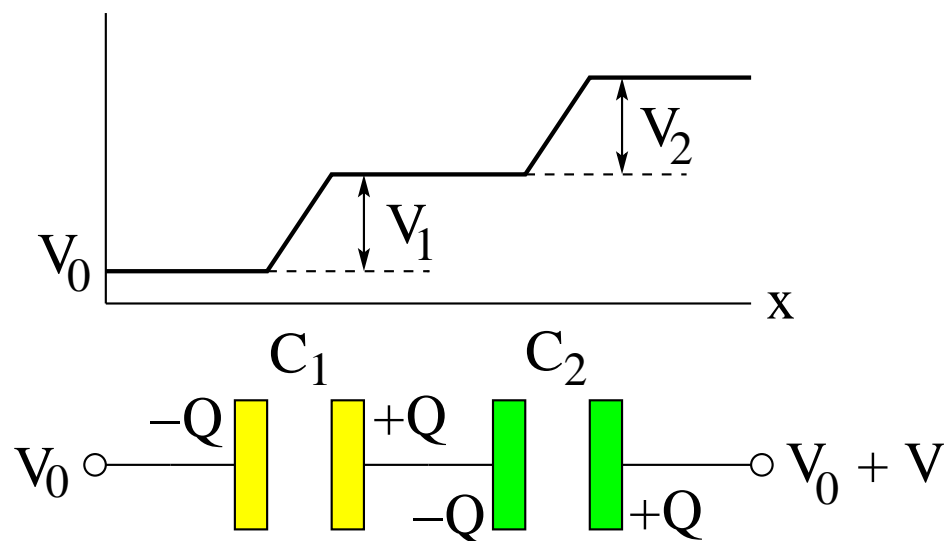


# Capacitors Connected in Series



Find the equivalent capacitance of two capacitors connected in series:

- Charge on capacitors:  $Q_1 = Q_2 = Q$
- Voltage across capacitors:  $V_1 + V_2 = V$
- Equivalent capacitance:  $\frac{1}{C} \equiv \frac{V}{Q} = \frac{V_1 + V_2}{Q} = \frac{V_1}{Q_1} + \frac{V_2}{Q_2}$
- $\Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

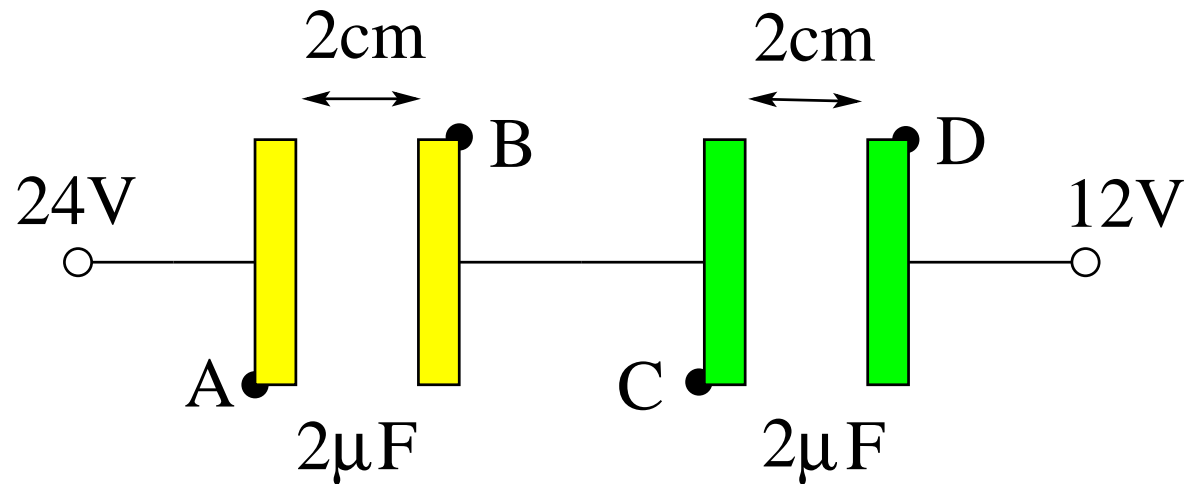


## Capacitor Problem (2)



Consider two equal capacitors connected in series.

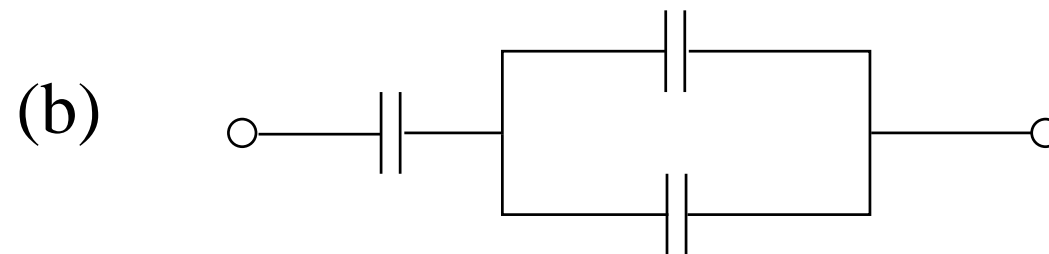
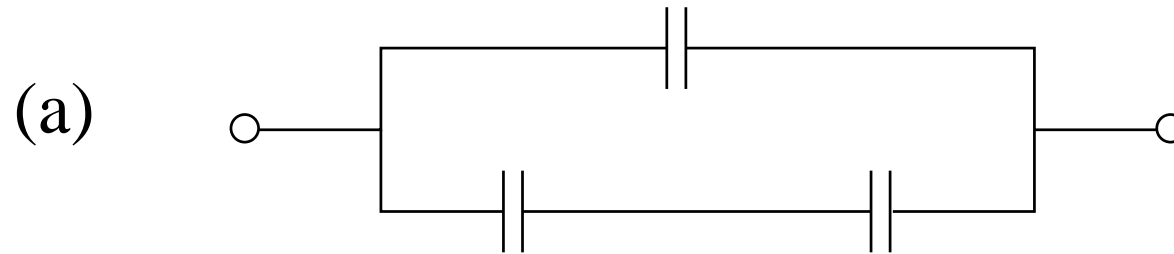
- (a) Find the voltages  $V_A - V_B$ ,  $V_B - V_C$ ,  $V_A - V_D$ .
- (b) Find the charge  $Q_A$  on plate  $A$ .
- (c) Find the electric field  $E$  between plates  $C$  and  $D$ .



# Capacitor Circuit (1)



Find the equivalent capacitances of the two capacitor networks.  
All capacitors have a capacitance of  $1\mu F$ .

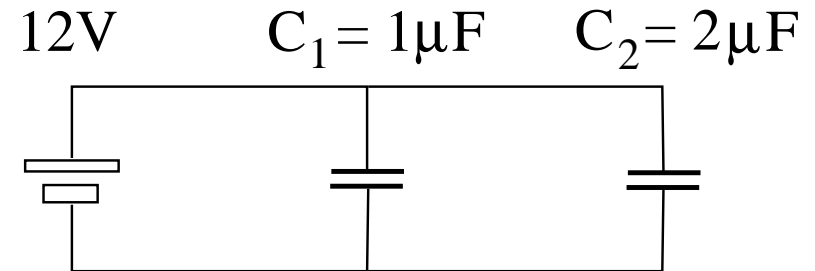


## Capacitor Circuit (2)



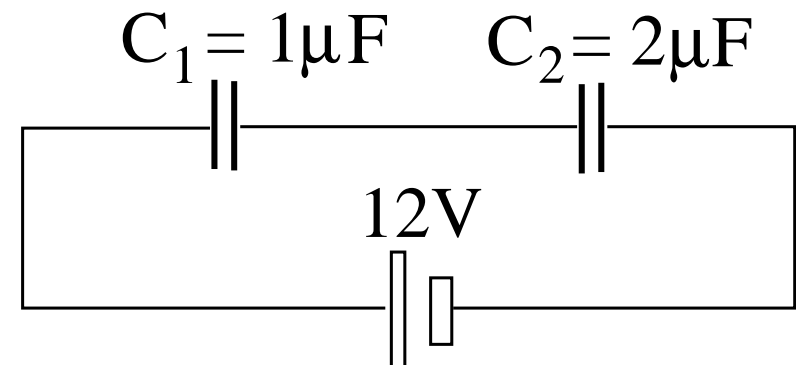
Consider the two capacitors connected in parallel.

- (a) Which capacitor has the higher voltage?
- (b) Which capacitor has more charge?
- (c) Which capacitor has more energy?



Consider the two capacitors connected in series.

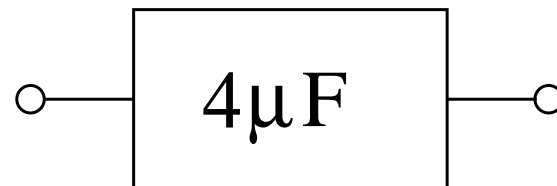
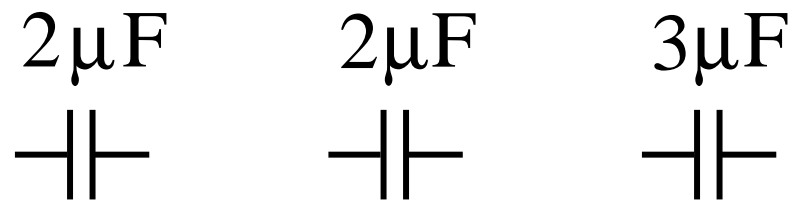
- (d) Which capacitor has the higher voltage?
- (e) Which capacitor has more charge?
- (f) Which capacitor has more energy?



## Capacitor Circuit (3)



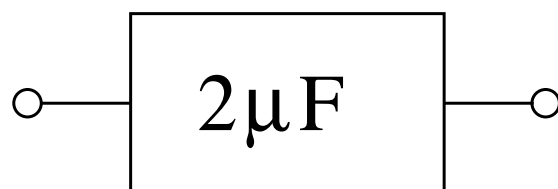
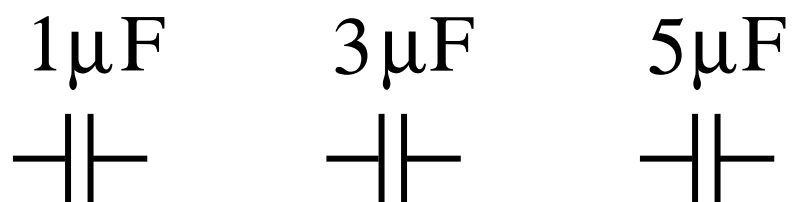
Connect the three capacitors in such a way that the equivalent capacitance is  $C_{eq} = 4\mu\text{F}$ . Draw the circuit diagram.



## Capacitor Circuit (4)



Connect the three capacitors in such a way that the equivalent capacitance is  $C_{eq} = 2\mu\text{F}$ . Draw the circuit diagram.

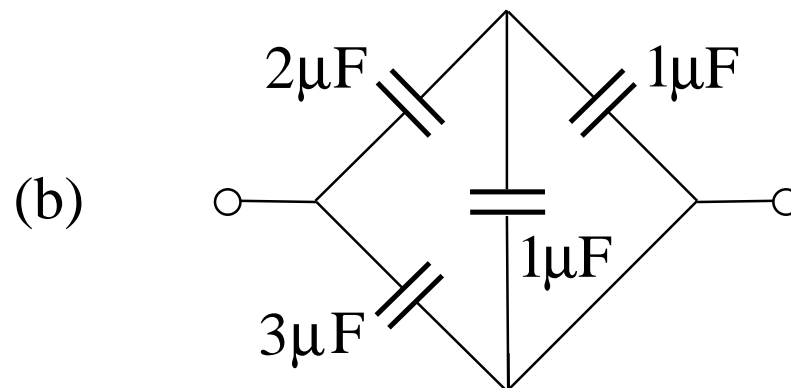
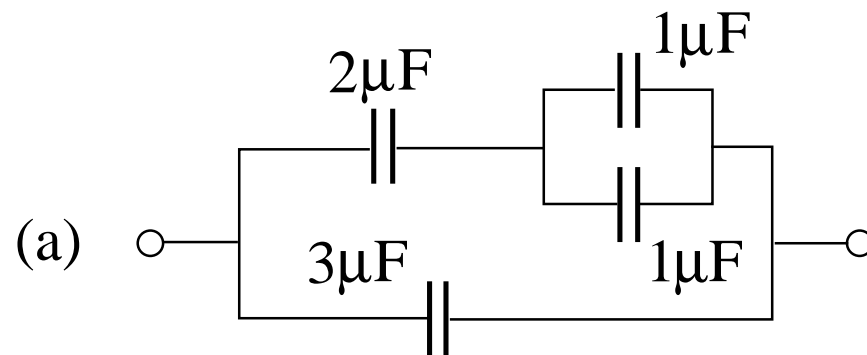




# Capacitor Circuit (5)



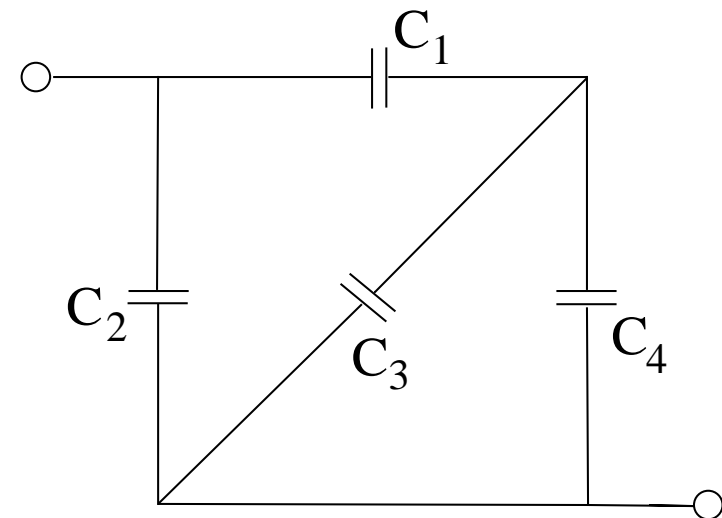
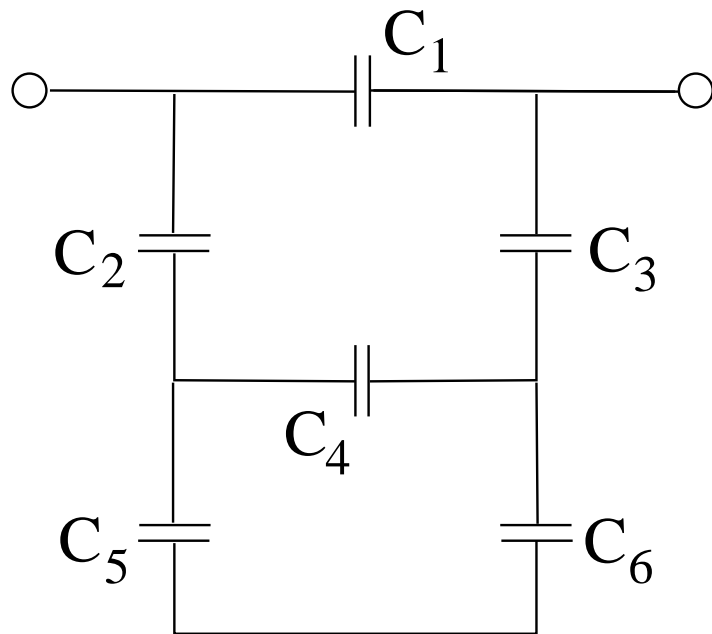
Find the equivalent capacitances of the following circuits.



# Capacitor Circuit (6)



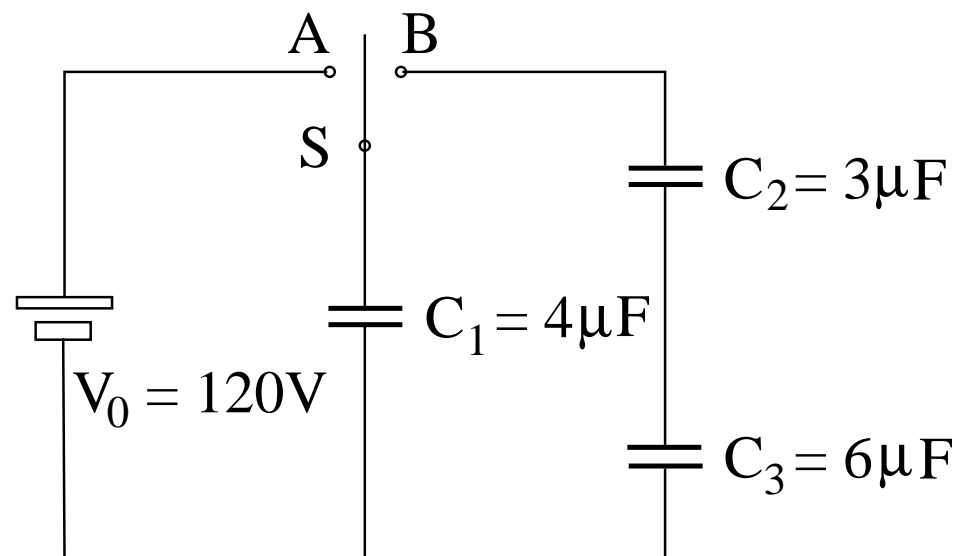
- (a) Name two capacitors from the circuit on the **left** that are connected in **series**.
- (b) Name two capacitors from the circuit on the **right** that are connected in **parallel**.



# Capacitor Circuit (7)



- (a) In the circuit shown the switch is first thrown to  $A$ . Find the charge  $Q_0$  and the energy  $U_A$  on the capacitor  $C_1$  once it is charged up.
- (b) Then the switch is thrown to  $B$ , which charges up the capacitors  $C_2$  and  $C_3$ . The capacitor  $C_1$  is partially discharged in the process. Find the charges  $Q_1, Q_2, Q_3$  on all three capacitors and the voltages  $V_1, V_2, V_3$  across each capacitor once equilibrium has been reached again. What is the energy  $U_B$  now stored in the circuit?

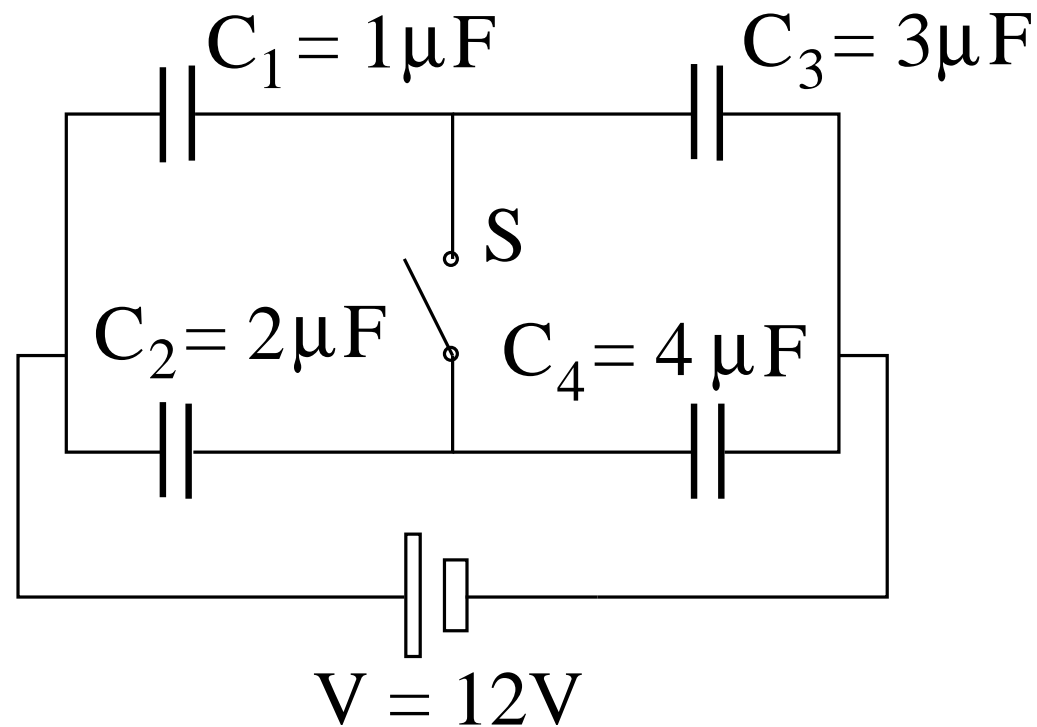


## Capacitor Circuit (8)



In the circuit shown find the charges  $Q_1, Q_2, Q_3, Q_4$  on each capacitor and the voltages  $V_1, V_2, V_3, V_4$  across each capacitor

- (a) when the switch  $S$  is open,
- (b) when the switch  $S$  is closed.

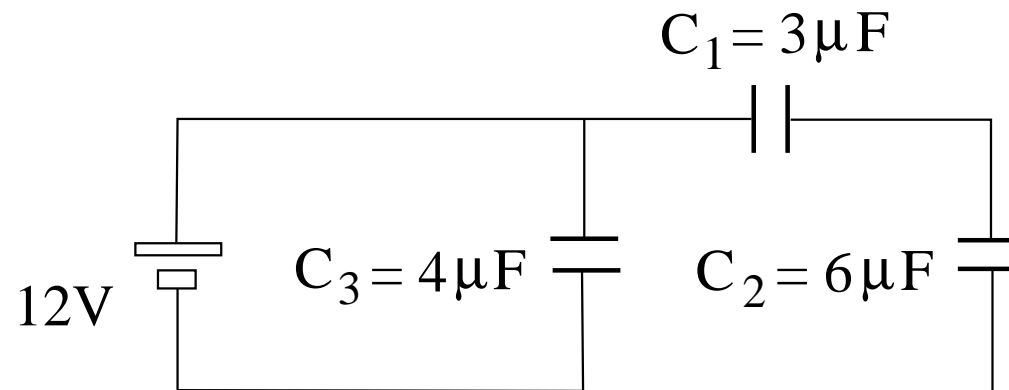


# Capacitor Circuit (9)



The circuit of capacitors connected to a battery is at equilibrium.

- (a) Find the equivalent capacitance  $C_{eq}$ .
- (b) Find the total energy  $U$  stored in the circuit (excluding the battery).
- (c) Find the charge  $Q_3$  on capacitor  $C_3$ .
- (d) Find the voltage  $V_2$  across capacitor  $C_2$ .



# More Complex Capacitor Circuit



No two capacitors are in parallel or in series.  
Solution requires different strategy:

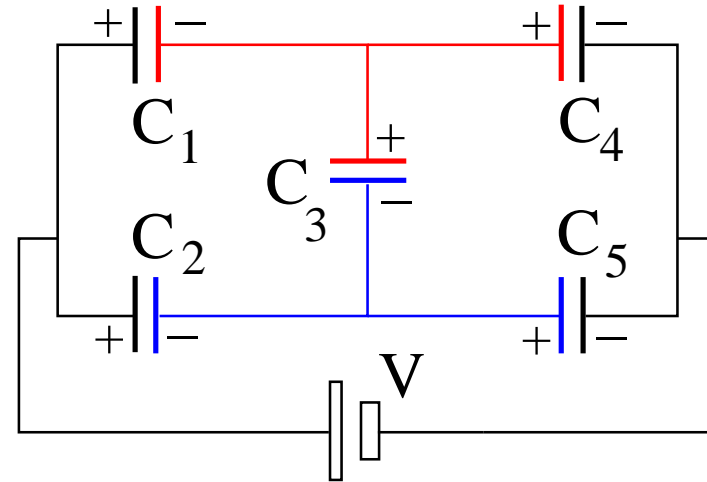
- zero charge on each conductor (here color coded),
- zero voltage around any closed loop.

Specifications:  $C_1, \dots, Q_5, V$ .

Five equations for unknowns  $Q_1, \dots, Q_5$ :

- $Q_1 + Q_2 - Q_4 - Q_5 = 0$
- $Q_3 + Q_4 - Q_1 = 0$
- $\frac{Q_5}{C_5} + \frac{Q_3}{C_3} - \frac{Q_4}{C_4} = 0$
- $\frac{Q_2}{C_2} - \frac{Q_1}{C_1} - \frac{Q_3}{C_3} = 0$
- $V - \frac{Q_1}{C_1} - \frac{Q_4}{C_4} = 0$

Equivalent capacitance:  $C_{eq} = \frac{Q_1 + Q_2}{V}$



(a)  $C_m = 1\text{pF}, m = 1, \dots, 5$  and  $V = 1\text{V}$ :

$$C_{eq} = 1\text{pF}, Q_3 = 0,$$

$$Q_1 = Q_2 = Q_4 = Q_5 = \frac{1}{2}\text{pC}.$$

(b)  $C_m = m\text{pF}, m = 1, \dots, 5$  and  $V = 1\text{V}$ :

$$C_{eq} = \frac{159}{71}\text{pF}, Q_1 = \frac{55}{71}\text{pC}, Q_2 = \frac{104}{71}\text{pC},$$

$$Q_3 = -\frac{9}{71}\text{pC}, Q_4 = \frac{64}{71}\text{pC}, Q_5 = \frac{95}{71}\text{pC}.$$