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05. Electric Potential I

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Abstract

Lecture slides 5 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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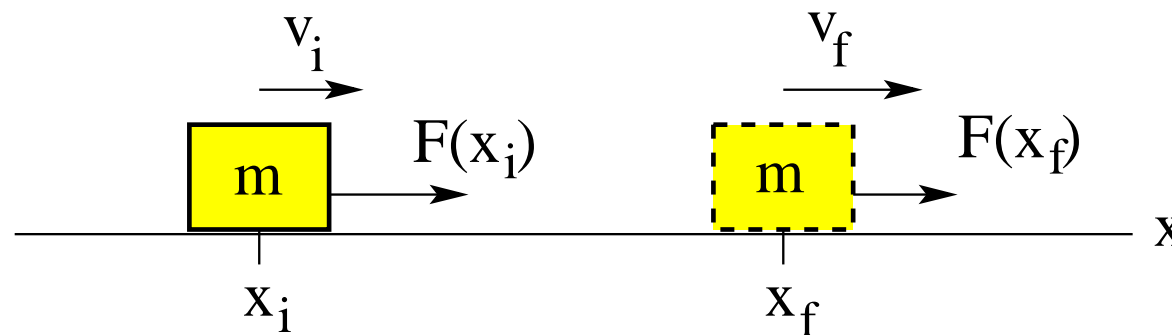
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Work and Energy



Consider a block of mass m moving along the x -axis.

- Conservative force acting on block: $F = F(x)$
- Work done by $F(x)$ on block: $W_{if} = \int_{x_i}^{x_f} F(x)dx$
- Kinetic energy of block: $K = \frac{1}{2}mv^2$
- Potential energy of block: $U(x) = - \int_{x_0}^x F(x)dx \Rightarrow F(x) = -\frac{dU}{dx}$
- Transformation of energy: $\Delta K \equiv K_f - K_i$, $\Delta U \equiv U_f - U_i$
- Total mechanical energy: $E = K + U = \text{const} \Rightarrow \Delta K + \Delta U = 0$
- Work-energy relation: $W_{if} = \Delta K = -\Delta U$





Conservative forces familiar from mechanics:

- Elastic force: $F(x) = -kx \Rightarrow U(x) = -\int_{x_0}^x (-kx)dx = \frac{1}{2}kx^2 \quad (x_0 = 0).$

- Gravitational force (locally): $F(y) = -mg$

$$\Rightarrow U(y) = -\int_{y_0}^y (-mg)dy = mgy \quad (y_0 = 0).$$

- Gravitational force (globally): $F(r) = -G\frac{mm_E}{r^2}$

$$\Rightarrow U(r) = -\int_{r_0}^r \left(-G\frac{mm_E}{r^2}\right) dr = -G\frac{mm_E}{r} \quad (r_0 = \infty).$$

Potential energy depends on integration constant.

Integration constant determines reference position where $U = 0$:

$x = x_0, y = y_0, r = r_0.$

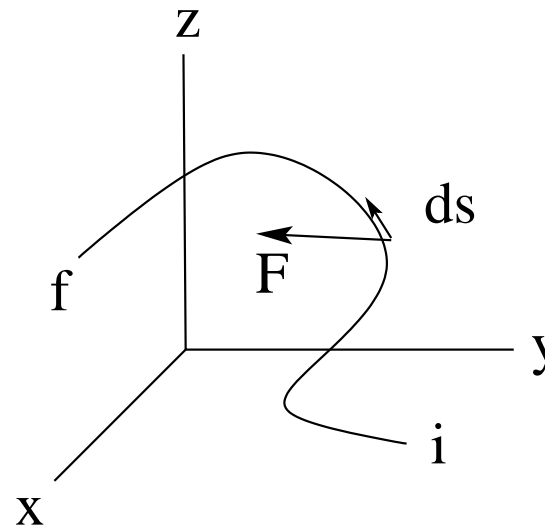
Work and Potential Energy in 3D Space



Consider a particle acted on by a force \vec{F} as it moves along a specific path in 3D space.

- Force: $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$
- Displacement: $d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$
- Work: $W_{if} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{s} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$
- Potential energy: $U(\vec{r}) = - \int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{s} = - \int_{x_0}^x F_x dx - \int_{y_0}^y F_y dy - \int_{z_0}^z F_z dz$

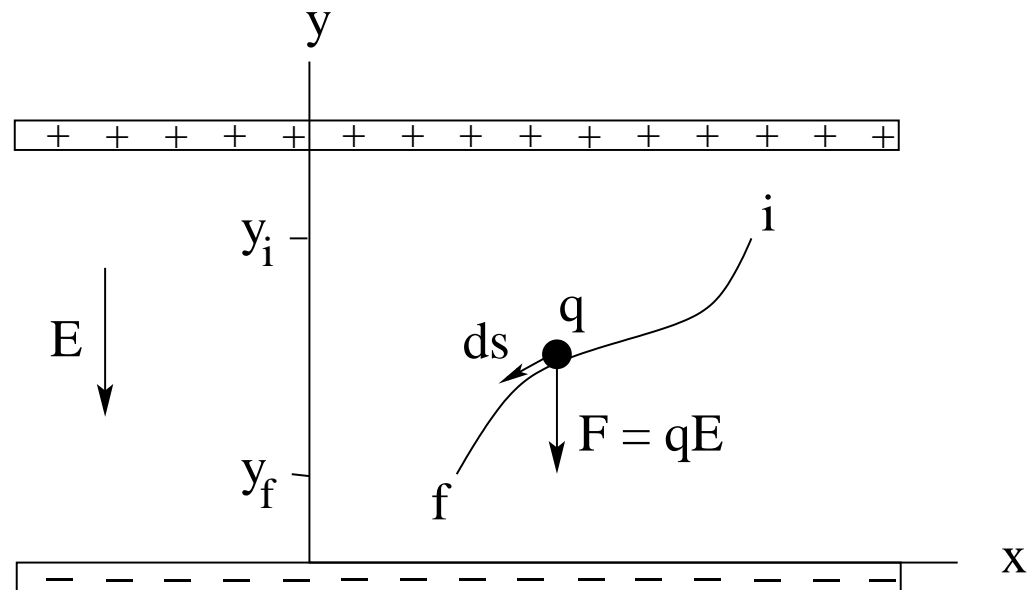
Note: The work done by a conservative force is path-independent.



Potential Energy of Charged Particle in Uniform Electric Field



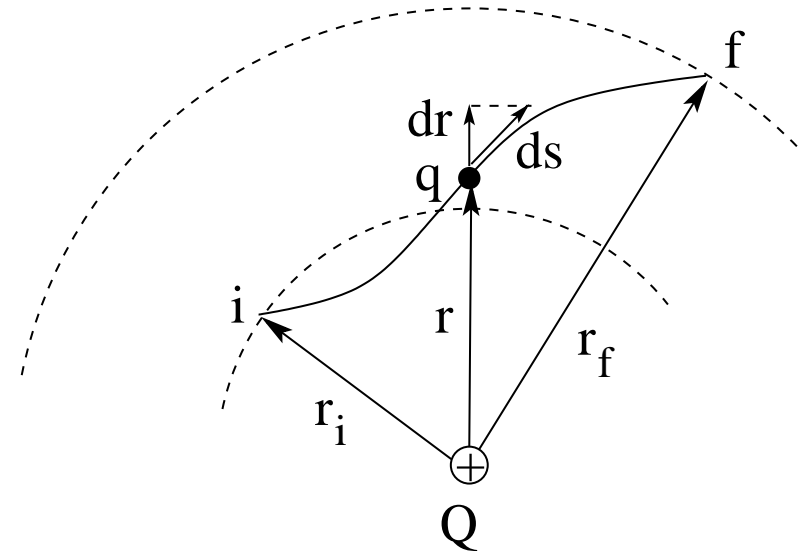
- Electrostatic force: $\vec{F} = -qE\hat{j}$ (conservative)
- Displacement: $d\vec{s} = dx\hat{i} + dy\hat{j}$
- Work: $W_{if} = \int_i^f \vec{F} \cdot d\vec{s} = \int_{y_i}^{y_f} (-qE)dy = -qE(y_f - y_i)$
- Potential energy: $U = -\int_0^y (-qE)dy = qEy$
- Electric potential: $V(y) = Ey$



Potential Energy of Charged Particle in Coulomb Field



- Electrostatic force: $\vec{F} = \frac{kqQ}{r^2} \hat{r}$ (conservative)
- Displacement: $d\vec{s} = d\vec{r} + d\vec{s}_\perp$, $d\vec{r} = dr\hat{r}$
- Work: $W_{if} = \int_i^f \vec{F} \cdot d\vec{s} = kqQ \int_i^f \frac{\hat{r} \cdot d\vec{s}}{r^2} = kqQ \int_{r_i}^{r_f} \frac{dr}{r^2}$
 $= kqQ \left[-\frac{1}{r} \right]_{r_i}^{r_f} = -kqQ \left[\frac{1}{r_f} - \frac{1}{r_i} \right]$
- Potential energy: $U = - \int_\infty^r F dr = -kqQ \int_\infty^r \frac{dr}{r^2} = k \frac{qQ}{r}$
- Electric potential: $V(r) = \frac{kQ}{r}$



Attributes of Space and of Charged Particles



	planar source	point source	SI unit
electric field	$\vec{E} = E_x \hat{i}$	$\vec{E} = \frac{kQ}{r^2} \hat{r}$	[N/C]=[V/m]
electric potential	$V = -E_x x$	$V = \frac{kQ}{r}$	[V]=[J/C]
electric force	$\vec{F} = q\vec{E} = qE_x \hat{i}$	$\vec{F} = q\vec{E} = \frac{kQq}{r^2} \hat{r}$	[N]
electric potential energy	$U = qV = -qE_x x$	$U = qV = \frac{kQq}{r}$	[J]

Electric field \vec{E} is present at points in space.

Points in space are at electric potential V .

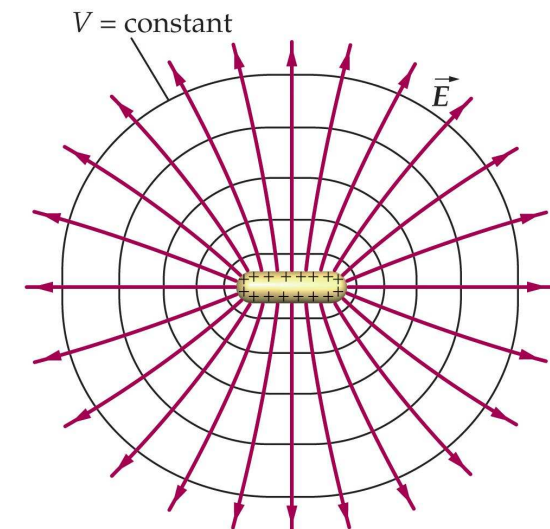
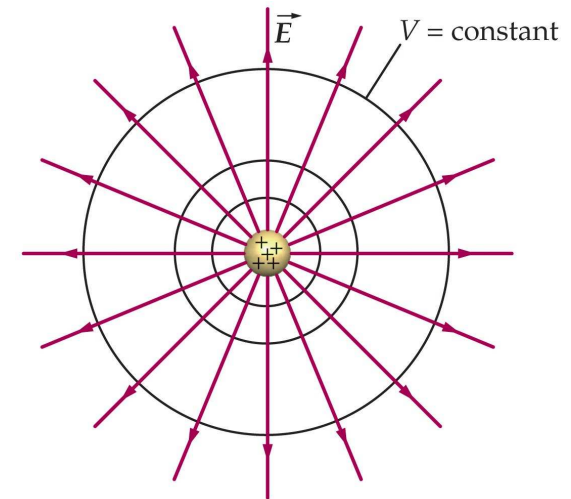
Charged particles experience electric force $\vec{F} = q\vec{E}$.

Charged particles have electric potential energy $U = qV$.

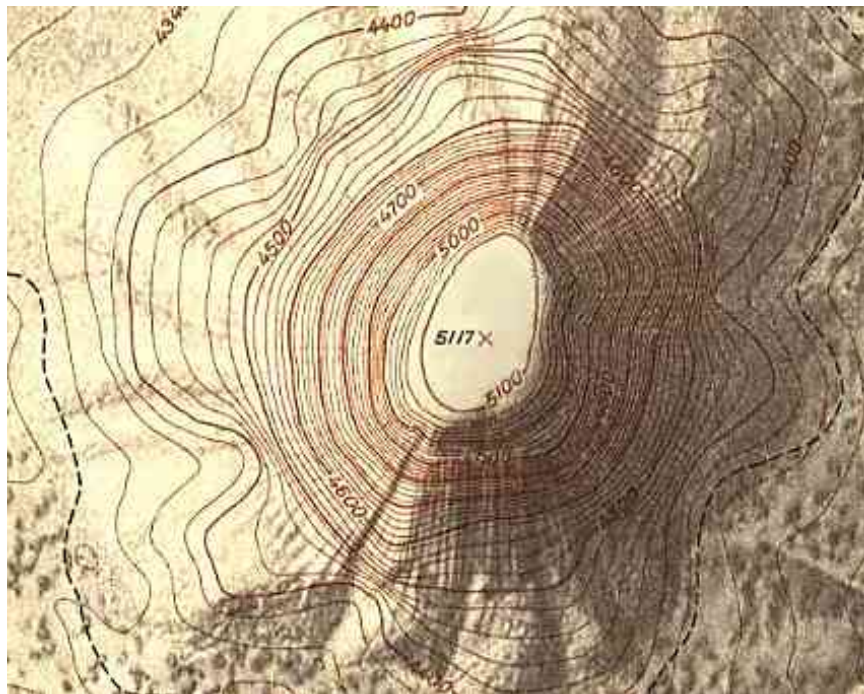
Equipotential Surfaces and Field Lines



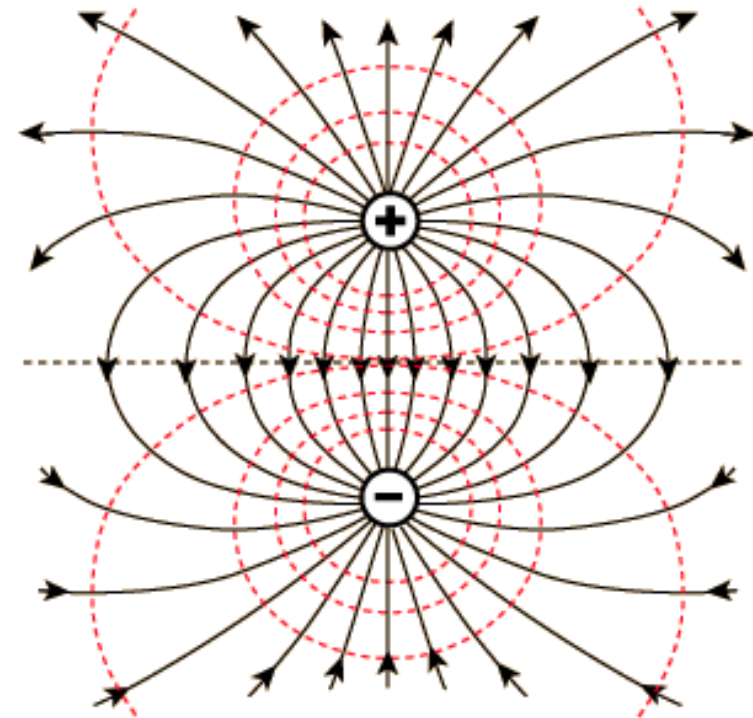
- Definition: $V(\vec{r}) = \text{const}$ on equipotential surface.
- Potential energy $U(\vec{r}) = \text{const}$ for point charge q on equipotential surface.
- The surface of a conductor at equilibrium is an equipotential surface.
- Electric field vectors $\vec{E}(\vec{r})$ (tangents to field lines) are perpendicular to equipotential surface.
- Electrostatic force $\vec{F} = q\vec{E}(\vec{r})$ does zero work on point charge q moving on equipotential surface.
- The electric field $\vec{E}(\vec{r})$ exerts a force on a positive (negative) point charge q in the direction of steepest potential drop (rise).
- When a positive (negative) point charge q moves from a region of high potential to a region of low potential, the electric field does positive (negative) work on it. In the process, the potential energy decreases (increases).



Gravitation



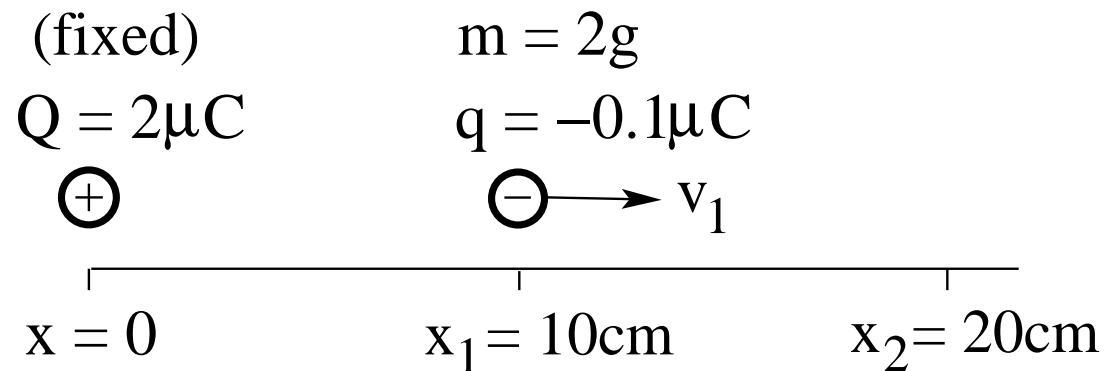
Electricity



Electric Potential and Potential Energy: Application (1)



Consider a point charge $Q = 2\mu\text{C}$ fixed at position $x = 0$. A particle with mass $m = 2\text{g}$ and charge $q = -0.1\mu\text{C}$ is launched at position $x_1 = 10\text{cm}$ with velocity $v_1 = 12\text{m/s}$.

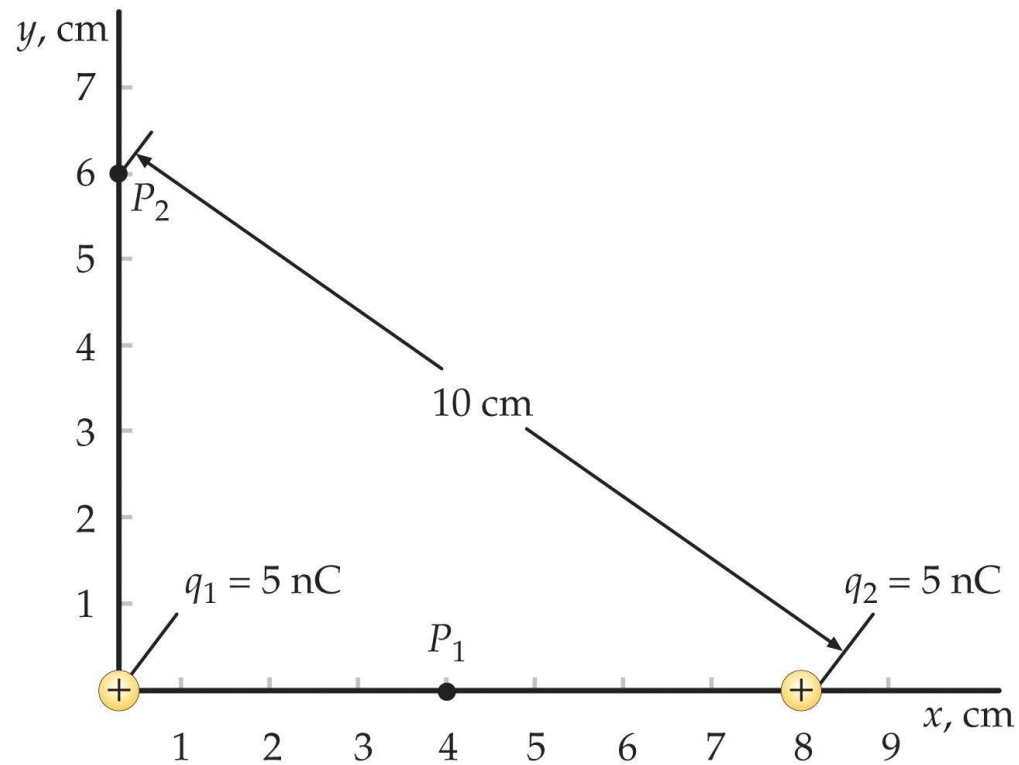


- Find the velocity v_2 of the particle when it is at position $x_2 = 20\text{cm}$.

Electric Potential and Potential Energy: Application (2)



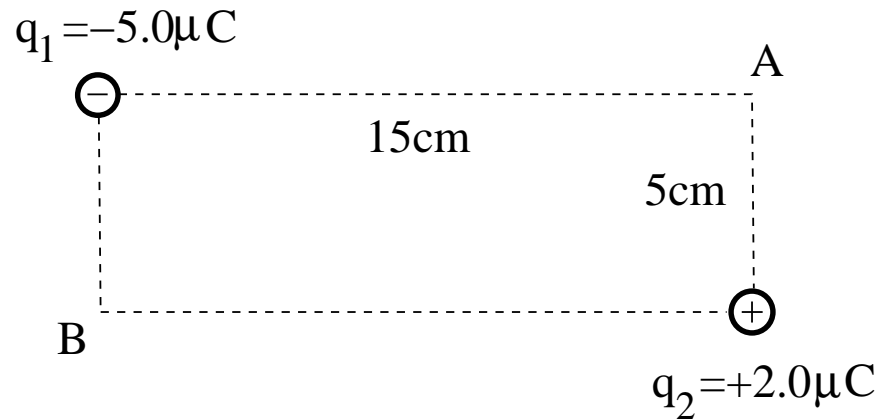
- Electric potential at point P_1 : $V = \frac{kq_1}{0.04\text{m}} + \frac{kq_2}{0.04\text{m}} = 1125\text{V} + 1125\text{V} = 2250\text{V}$.
- Electric potential at point P_2 : $V = \frac{kq_1}{0.06\text{m}} + \frac{kq_2}{0.10\text{m}} = 750\text{V} + 450\text{V} = 1200\text{V}$.



Electric Potential and Potential Energy: Application (3)



Point charges $q_1 = -5.0\mu\text{C}$ and $q_2 = +2.0\mu\text{C}$ are positioned at two corners of a rectangle as shown.

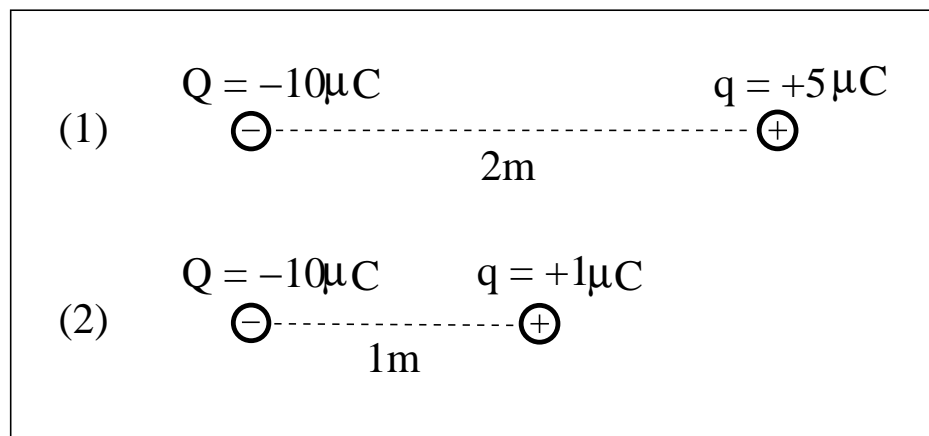


- Find the electric potential at the corners A and B .
- Find the electric field at point B .
- How much work is required to move a point charge $q_3 = +3\mu\text{C}$ from B to A ?

Electric Potential and Potential Energy: Application (4)



A positive point charge q is positioned in the electric field of a negative point charge Q .

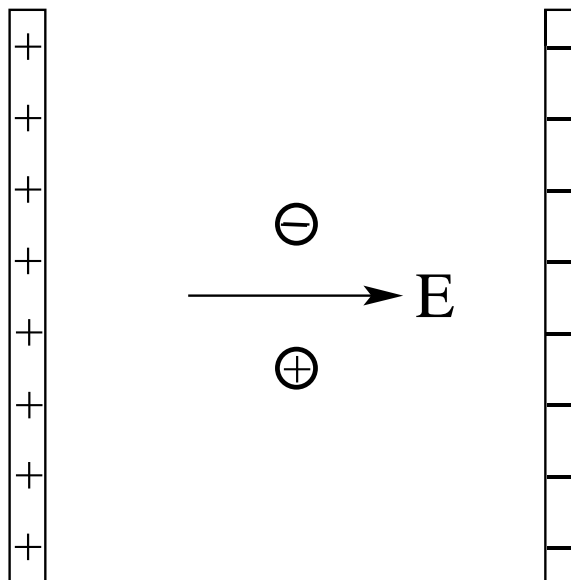


- (a) In which configuration is the charge q positioned in the stronger electric field?
- (b) In which configuration does the charge q experience the stronger force?
- (c) In which configuration is the charge q positioned at the higher electric potential?
- (d) In which configuration does the charge q have the higher potential energy?

Electric Potential and Potential Energy: Application (5)



An electron and a proton are released from rest midway between oppositely charged plates.

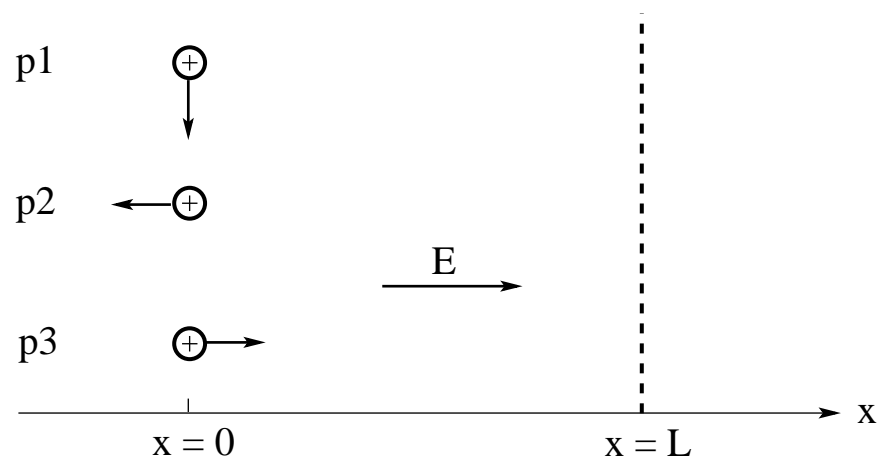


- Name the particle(s) which move(s) from high to low electric potential.
- Name the particle(s) whose electric potential energy decrease(s).
- Name the particle(s) which hit(s) the plate in the shortest time.
- Name the particle(s) which reach(es) the highest kinetic energy before impact.

Electric Potential and Potential Energy: Application (6)



Three protons are projected from $x = 0$ with equal initial speed v_0 in different directions. They all experience the force of a uniform horizontal electric field \vec{E} . Ultimately, they all hit the vertical screen at $x = L$.



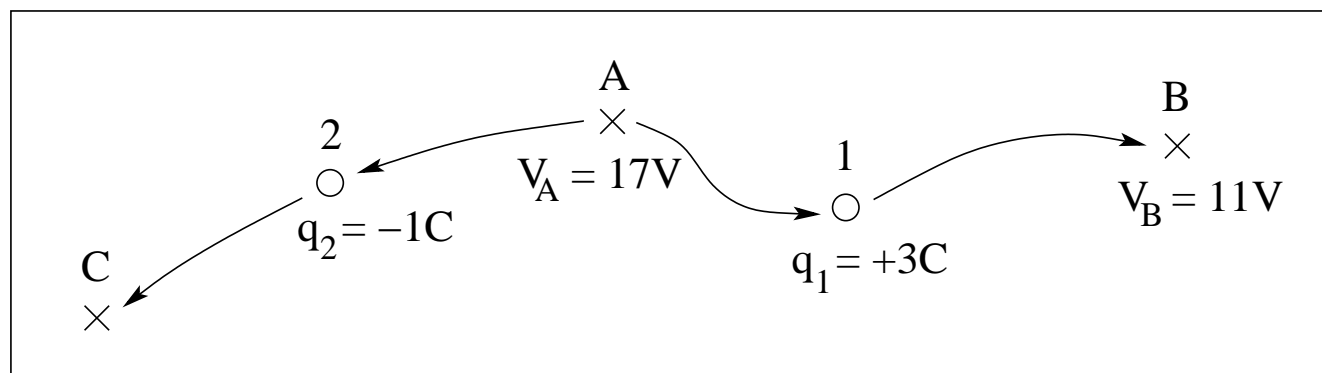
- (a) Which proton travels the longest time?
- (b) Which proton travels the longest path?
- (c) Which particle has the highest speed when it hits the screen?

Two of the questions are easy, one is hard.

Electric Potential and Potential Energy: Application (7)



Consider a region of nonuniform electric field. Charged particles 1 and 2 start moving from rest at point A in opposite directions along the paths shown.



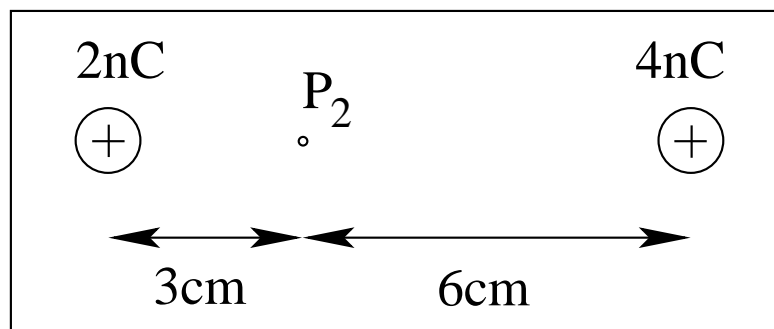
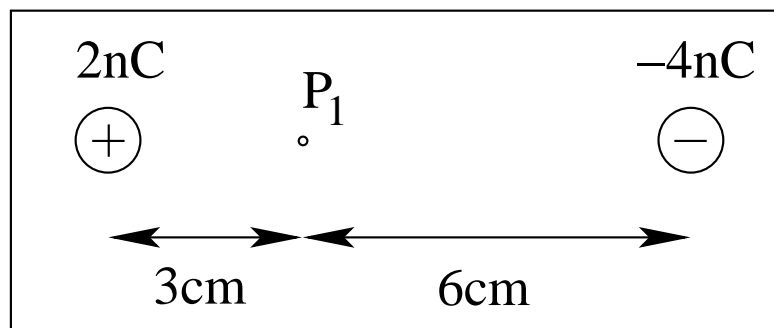
From the information given in the figure...

- find the kinetic energy K_1 of particle 1 when it arrives at point B ,
- find the electric potential V_C at point C if we know that particle 2 arrives there with kinetic energy $K_2 = 8J$.

Electric Potential and Potential Energy: Application (8)



- (a) Is the electric potential at points P_1, P_2 **positive** or **negative** or **zero**?
- (b) Is the potential energy of a negatively charged particle at points P_1, P_2 **positive** or **negative** or **zero**?
- (c) Is the electric field at points P_1, P_2 directed **left** or **right** or is it **zero**?
- (d) Is the force on a negatively charged particle at points P_1 and P_2 directed **left** or **right** or is it **zero**?

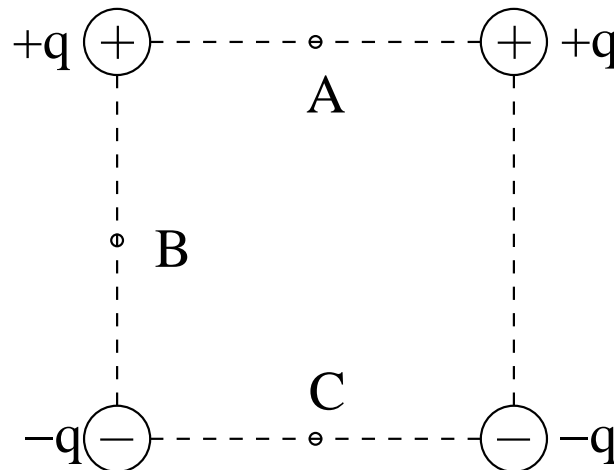


Electric Potential and Potential Energy: Application (9)



Consider four point charges of equal magnitude positioned at the corners of a square as shown. Answer the following questions for points A , B , C .

- (1) Which point is at the highest electric potential?
- (2) Which point is at the lowest electric potential?
- (3) At which point is the electric field the strongest?
- (4) At which point is the electric field the weakest?



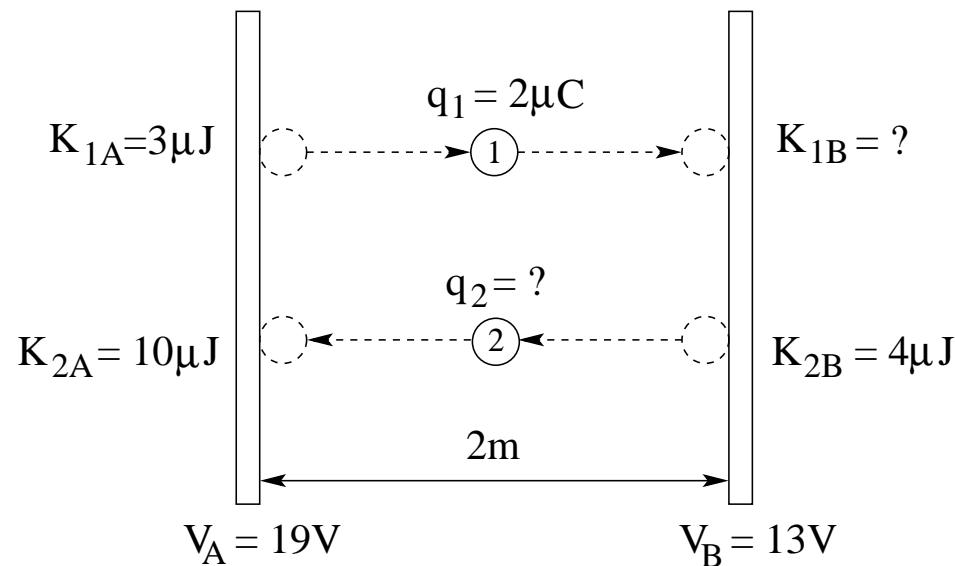
Electric Potential and Potential Energy: Application (10)



The charged particles 1 and 2 move between the charged conducting plates A and B in opposite directions.

From the information given in the figure...

- (a) find the kinetic energy K_{1B} of particle 1,
- (b) find the charge q_2 of particle 2,
- (c) find the direction and magnitude of the electric field \vec{E} between the plates.

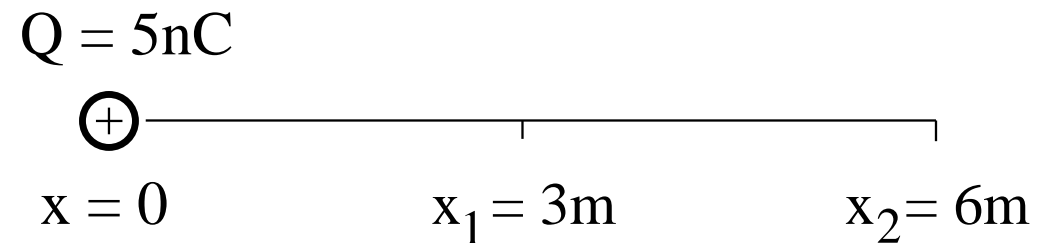


Intermediate Exam I: Problem #2 (Spring '05)



Consider a point charge $Q = 5\text{nC}$ fixed at position $x = 0$.

- (a) Find the electric potential V_1 at position $x_1 = 3\text{m}$ and the electric potential V_2 at position $x_2 = 6\text{m}$.
- (b) If a charged particle ($q = 4\text{nC}$, $m = 1.5\text{ng}$) is released from rest at x_1 , what are its kinetic energy K_2 and its velocity v_2 when it reaches position x_2 ?

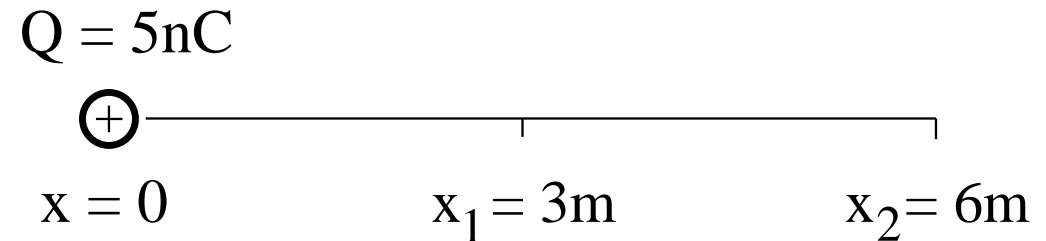


Intermediate Exam I: Problem #2 (Spring '05)



Consider a point charge $Q = 5\text{nC}$ fixed at position $x = 0$.

- (a) Find the electric potential V_1 at position $x_1 = 3\text{m}$ and the electric potential V_2 at position $x_2 = 6\text{m}$.
- (b) If a charged particle ($q = 4\text{nC}$, $m = 1.5\text{ng}$) is released from rest at x_1 , what are its kinetic energy K_2 and its velocity v_2 when it reaches position x_2 ?



Solution:

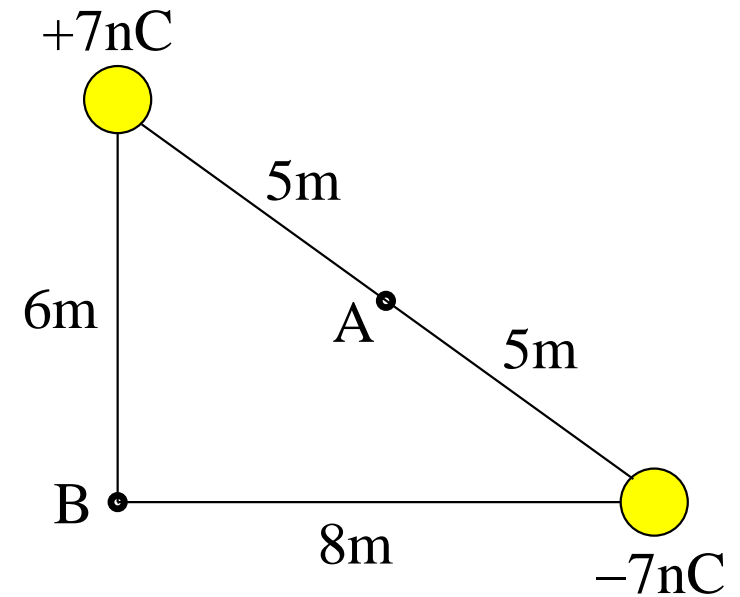
(a) $V_1 = k \frac{Q}{x_1} = 15\text{V}$, $V_2 = k \frac{Q}{x_2} = 7.5\text{V}$.

Unit Exam I: Problem #1 (Fall '10)



Consider two point charges positioned as shown.

- (a) Find the magnitude of the electric field at point A .
- (b) Find the electric potential at point A .
- (c) Find the magnitude of the electric field at point B .
- (d) Find the electric potential at point B .



Unit Exam I: Problem #1 (Fall '10)

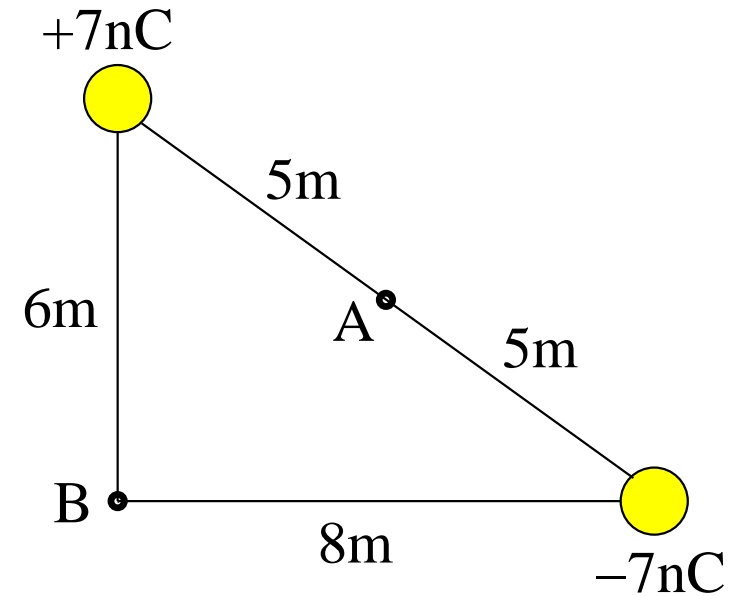


Consider two point charges positioned as shown.

- (a) Find the magnitude of the electric field at point A .
- (b) Find the electric potential at point A .
- (c) Find the magnitude of the electric field at point B .
- (d) Find the electric potential at point B .

Solution:

(a) $E_A = 2k \frac{|7\text{nC}|}{(5\text{m})^2} = 2(2.52\text{V/m}) = 5.04\text{V/m}.$



Unit Exam I: Problem #1 (Fall '10)



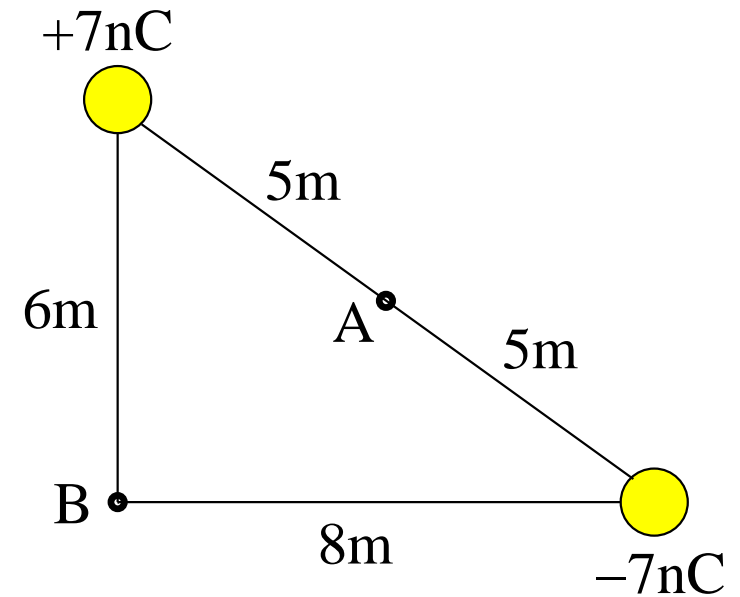
Consider two point charges positioned as shown.

- (a) Find the magnitude of the electric field at point A .
- (b) Find the electric potential at point A .
- (c) Find the magnitude of the electric field at point B .
- (d) Find the electric potential at point B .

Solution:

$$(a) E_A = 2k \frac{|7\text{nC}|}{(5\text{m})^2} = 2(2.52\text{V/m}) = 5.04\text{V/m}.$$

$$(b) V_A = k \frac{(+7\text{nC})}{5\text{m}} + k \frac{(-7\text{nC})}{5\text{m}} = 12.6\text{V} - 12.6\text{V} = 0.$$



Unit Exam I: Problem #1 (Fall '10)



Consider two point charges positioned as shown.

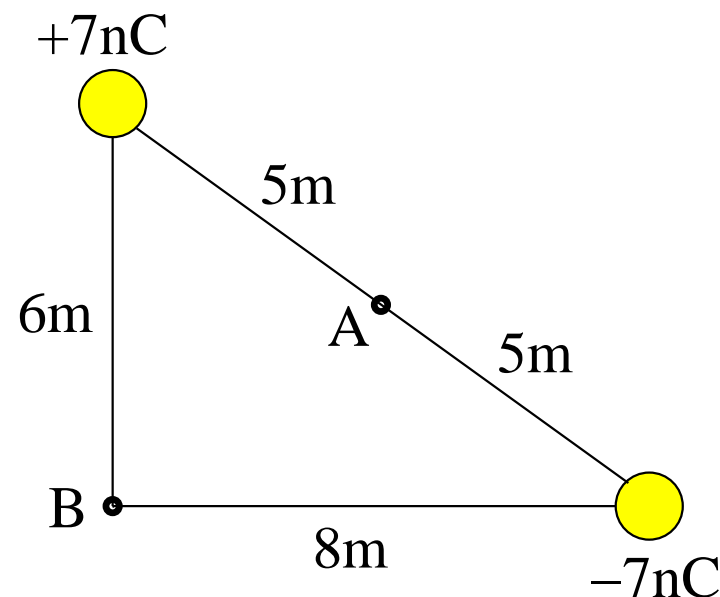
- (a) Find the magnitude of the electric field at point A .
- (b) Find the electric potential at point A .
- (c) Find the magnitude of the electric field at point B .
- (d) Find the electric potential at point B .

Solution:

$$(a) E_A = 2k \frac{|7\text{nC}|}{(5\text{m})^2} = 2(2.52\text{V/m}) = 5.04\text{V/m}.$$

$$(b) V_A = k \frac{(+7\text{nC})}{5\text{m}} + k \frac{(-7\text{nC})}{5\text{m}} = 12.6\text{V} - 12.6\text{V} = 0.$$

$$(c) E_B = \sqrt{\left(k \frac{|7\text{nC}|}{(6\text{m})^2}\right)^2 + \left(k \frac{|7\text{nC}|}{(8\text{m})^2}\right)^2} \Rightarrow E_B = \sqrt{(1.75\text{V/m})^2 + (0.98\text{V/m})^2} = 2.01\text{V/m}.$$



Unit Exam I: Problem #1 (Fall '10)



Consider two point charges positioned as shown.

- (a) Find the magnitude of the electric field at point A .
- (b) Find the electric potential at point A .
- (c) Find the magnitude of the electric field at point B .
- (d) Find the electric potential at point B .

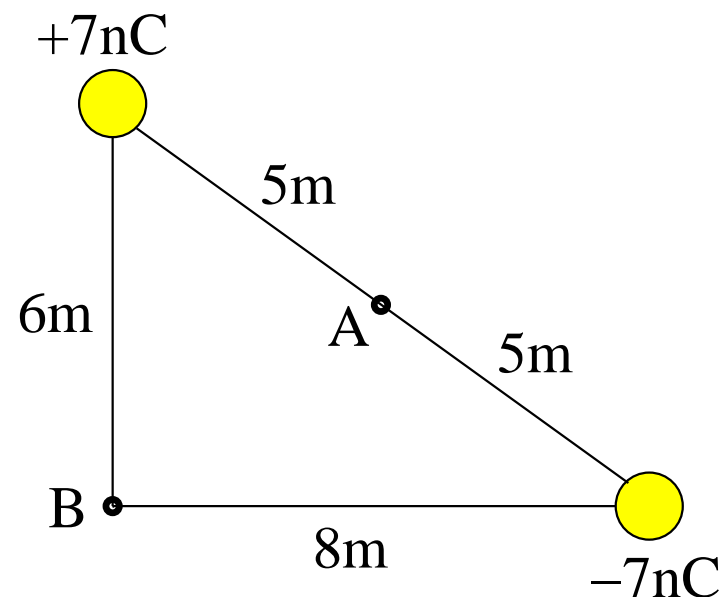
Solution:

$$(a) E_A = 2k \frac{|7\text{nC}|}{(5\text{m})^2} = 2(2.52\text{V/m}) = 5.04\text{V/m}.$$

$$(b) V_A = k \frac{(+7\text{nC})}{5\text{m}} + k \frac{(-7\text{nC})}{5\text{m}} = 12.6\text{V} - 12.6\text{V} = 0.$$

$$(c) E_B = \sqrt{\left(k \frac{|7\text{nC}|}{(6\text{m})^2}\right)^2 + \left(k \frac{|7\text{nC}|}{(8\text{m})^2}\right)^2} \Rightarrow E_B = \sqrt{(1.75\text{V/m})^2 + (0.98\text{V/m})^2} = 2.01\text{V/m}.$$

$$(d) V_B = k \frac{(+7\text{nC})}{6\text{m}} + k \frac{(-7\text{nC})}{8\text{m}} = 10.5\text{V} - 7.9\text{V} = 2.6\text{V}.$$

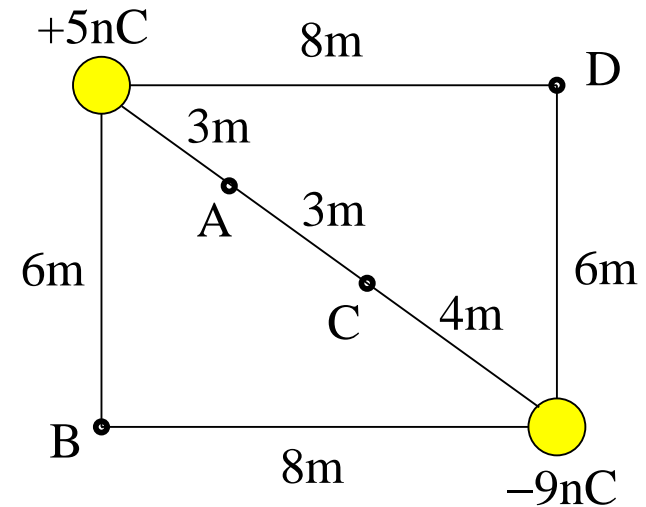


Unit Exam I: Problem #1 (Spring '14)



Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point A .
- Find the electric potential at point B .
- Find the magnitude of the electric field at point C .
- Find the electric potential at point D .

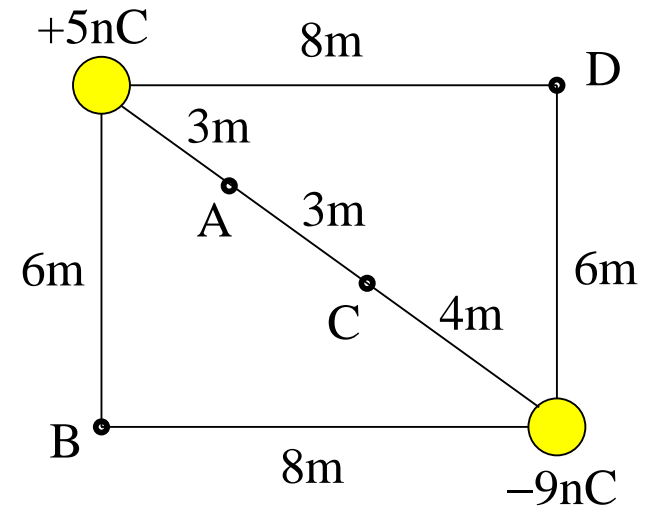


Unit Exam I: Problem #1 (Spring '14)



Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point *A*.
- Find the electric potential at point *B*.
- Find the magnitude of the electric field at point *C*.
- Find the electric potential at point *D*.



Solution:

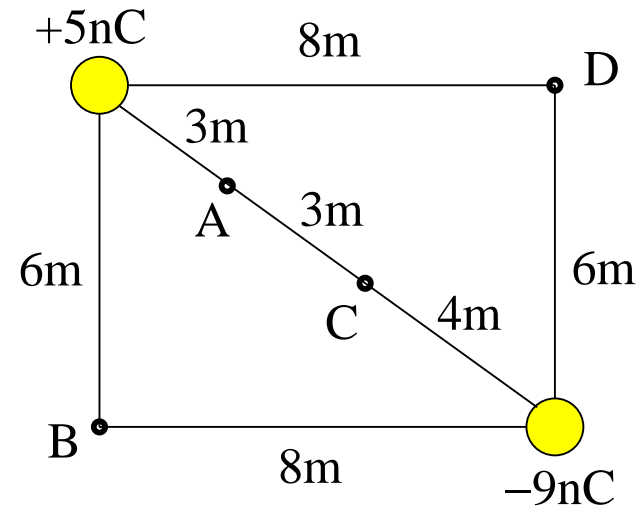
- $$E_A = k \frac{|5\text{nC}|}{(3\text{m})^2} + k \frac{|-9\text{nC}|}{(7\text{m})^2} = 5.00\text{V/m} + 1.65\text{V/m} = 6.65\text{V/m}.$$

Unit Exam I: Problem #1 (Spring '14)



Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point *A*.
- Find the electric potential at point *B*.
- Find the magnitude of the electric field at point *C*.
- Find the electric potential at point *D*.



Solution:

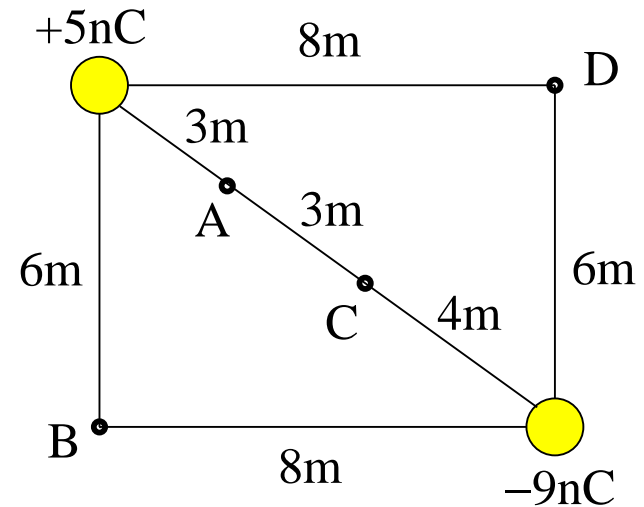
- $E_A = k \frac{|5\text{nC}|}{(3\text{m})^2} + k \frac{|-9\text{nC}|}{(7\text{m})^2} = 5.00\text{V/m} + 1.65\text{V/m} = 6.65\text{V/m}.$
- $V_B = k \frac{(+5\text{nC})}{6\text{m}} + k \frac{(-9\text{nC})}{8\text{m}} = 7.50\text{V} - 10.13\text{V} = -2.63\text{V}.$

Unit Exam I: Problem #1 (Spring '14)



Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point *A*.
- Find the electric potential at point *B*.
- Find the magnitude of the electric field at point *C*.
- Find the electric potential at point *D*.



Solution:

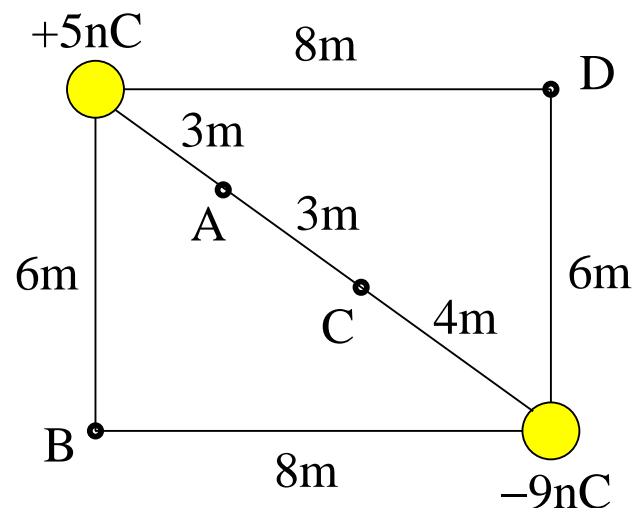
- $E_A = k \frac{|5\text{nC}|}{(3\text{m})^2} + k \frac{|-9\text{nC}|}{(7\text{m})^2} = 5.00\text{V/m} + 1.65\text{V/m} = 6.65\text{V/m}.$
- $V_B = k \frac{(+5\text{nC})}{6\text{m}} + k \frac{(-9\text{nC})}{8\text{m}} = 7.50\text{V} - 10.13\text{V} = -2.63\text{V}.$
- $E_C = k \frac{|5\text{nC}|}{(6\text{m})^2} + k \frac{|-9\text{nC}|}{(4\text{m})^2} = 1.25\text{V/m} + 5.06\text{V/m} = 6.31\text{V/m}.$

Unit Exam I: Problem #1 (Spring '14)



Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point *A*.
- Find the electric potential at point *B*.
- Find the magnitude of the electric field at point *C*.
- Find the electric potential at point *D*.



Solution:

- $E_A = k \frac{|5\text{nC}|}{(3\text{m})^2} + k \frac{|-9\text{nC}|}{(7\text{m})^2} = 5.00\text{V/m} + 1.65\text{V/m} = 6.65\text{V/m}.$
- $V_B = k \frac{(+5\text{nC})}{6\text{m}} + k \frac{(-9\text{nC})}{8\text{m}} = 7.50\text{V} - 10.13\text{V} = -2.63\text{V}.$
- $E_C = k \frac{|5\text{nC}|}{(6\text{m})^2} + k \frac{|-9\text{nC}|}{(4\text{m})^2} = 1.25\text{V/m} + 5.06\text{V/m} = 6.31\text{V/m}.$
- $V_D = k \frac{(+5\text{nC})}{8\text{m}} + k \frac{(-9\text{nC})}{6\text{m}} = 5.63\text{V} - 13.5\text{V} = -7.87\text{V}.$