05. Electric Potential I

Gerhard Müller
University of Rhode Island, gmuller@uri.edu

Follow this and additional works at: https://digitalcommons.uri.edu/elementary_physics_2

Abstract
Lecture slides 5 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

Some of the slides contain figures from the textbook, Paul A. Tipler and Gene Mosca. Physics for Scientists and Engineers, 5th/6th editions. The copyright to these figures is owned by W.H. Freeman. We acknowledge permission from W.H. Freeman to use them on this course web page. The textbook figures are not to be used or copied for any purpose outside this class without direct permission from W.H. Freeman.

Recommended Citation
https://digitalcommons.uri.edu/elementary_physics_2/21

This Course Material is brought to you for free and open access by the Physics Open Educational Resources at DigitalCommons@URI. It has been accepted for inclusion in PHY 204: Elementary Physics II (2015) by an authorized administrator of DigitalCommons@URI. For more information, please contact digitalcommons@etal.uri.edu.
Consider a block of mass \( m \) moving along the \( x \)-axis.

- Conservative force acting on block: \( F = F(x) \)
- Work done by \( F(x) \) on block: \( W_{if} = \int_{x_i}^{x_f} F(x) \, dx \)
- Kinetic energy of block: \( K = \frac{1}{2}mv^2 \)
- Potential energy of block: \( U(x) = -\int_{x_0}^{x} F(x) \, dx \Rightarrow F(x) = -\frac{dU}{dx} \)
- Transformation of energy: \( \Delta K \equiv K_f - K_i, \Delta U \equiv U_f - U_i \)
- Total mechanical energy: \( E = K + U = \text{const} \Rightarrow \Delta K + \Delta U = 0 \)
- Work-energy relation: \( W_{if} = \Delta K = -\Delta U \)
Conservative Forces in Mechanics

Conservative forces familiar from mechanics:

- Elastic force: \( F(x) = -kx \) \( \Rightarrow \) \( U(x) = -\int_{x_0}^{x} (-kx)dx = \frac{1}{2}kx^2 \) \( (x_0 = 0) \).

- Gravitational force (locally): \( F(y) = -mg \)

  \( \Rightarrow \) \( U(y) = -\int_{y_0}^{y} (-mg)dy = mgy \) \( (y_0 = 0) \).

- Gravitational force (globally): \( F(r) = -G\frac{mm_E}{r^2} \)

  \( \Rightarrow \) \( U(r) = -\int_{r_0}^{r} \left(-G\frac{mm_E}{r^2}\right)dr = -G\frac{mm_E}{r} \) \( (r_0 = \infty) \).

Potential energy depends on integration constant.

Integration constant determines reference position where \( U = 0 \):

\( x = x_0, \ y = y_0, \ r = r_0 \).
Consider a particle acted on by a force $\vec{F}$ as it moves along a specific path in 3D space.

- **Force:** $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$

- **Displacement:** $d\vec{s} = dx \hat{i} + dy \hat{j} + dz \hat{k}$

- **Work:** $W_{i\rightarrow f} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{s} = \int_{x_i}^{x_f} F_x \, dx + \int_{y_i}^{y_f} F_y \, dy + \int_{z_i}^{z_f} F_z \, dz$

- **Potential energy:**
  
  $U(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{s} = -\int_{x_0}^{x} F_x \, dx - \int_{y_0}^{y} F_y \, dy - \int_{z_0}^{z} F_z \, dz$

Note: The work done by a conservative force is path-independent.
Potential Energy of Charged Particle in Uniform Electric Field

- Electrostatic force: $\vec{F} = -qE\hat{j}$ (conservative)
- Displacement: $d\vec{s} = dx\hat{i} + dy\hat{j}$
- Work: $W_{if} = \int_{i}^{f} \vec{F} \cdot d\vec{s} = \int_{y_{i}}^{y_{f}} (-qE)dy = -qE(y_{f} - y_{i})$
- Potential energy: $U = -\int_{0}^{y} (-qE)dy = qEy$
- Electric potential: $V(y) = Ey$
Potential Energy of Charged Particle in Coulomb Field

- Electrostatic force: $\vec{F} = \frac{kqQ}{r^2} \hat{r}$ (conservative)
- Displacement: $d\vec{s} = d\vec{r} + d\vec{s}_\perp$, $d\vec{r} = dr \hat{r}$
- Work: $W_{if} = \int_i^f \vec{F} \cdot d\vec{s} = kqQ \int_i^f \frac{\hat{r} \cdot d\vec{s}}{r^2} = kqQ \int_{r_i}^{r_f} \frac{dr}{r^2}$
  
\[= kqQ \left[ -\frac{1}{r} \right]_{r_i}^{r_f} = -kqQ \left[ \frac{1}{r_f} - \frac{1}{r_i} \right] \]
- Potential energy: $U = -\int_{\infty}^r F \, dr = -kqQ \int_{\infty}^r \frac{dr}{r^2} = \frac{kqQ}{r}$
- Electric potential: $V(r) = \frac{kQ}{r}$
<table>
<thead>
<tr>
<th>Attributes of Space and of Charged Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>planar source</td>
</tr>
<tr>
<td>electric field</td>
</tr>
<tr>
<td>electric potential</td>
</tr>
<tr>
<td>electric force</td>
</tr>
<tr>
<td>electric potential energy</td>
</tr>
<tr>
<td>point source</td>
</tr>
<tr>
<td>electric potential</td>
</tr>
<tr>
<td>electric force</td>
</tr>
<tr>
<td>electric potential energy</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SI unit</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[$N/C$] = [$V/m$]</td>
<td></td>
</tr>
<tr>
<td>[V] = [J/C]</td>
<td></td>
</tr>
<tr>
<td>[N]</td>
<td></td>
</tr>
<tr>
<td>[J]</td>
<td></td>
</tr>
</tbody>
</table>

Electric field $\vec{E}$ is present at points in space. Points in space are at electric potential $V$.

Charged particles experience electric force $\vec{F} = q \vec{E}$. Charged particles have electric potential energy $U = qV$. 
- Definition: $V(\vec{r}) = \text{const}$ on equipotential surface.
- Potential energy $U(\vec{r}) = \text{const}$ for point charge $q$ on equipotential surface.
- The surface of a conductor at equilibrium is an equipotential surface.
- Electric field vectors $\vec{E}(\vec{r})$ (tangents to field lines) are perpendicular to equipotential surface.
- Electrostatic force $\vec{F} = q\vec{E}(\vec{r})$ does zero work on point charge $q$ moving on equipotential surface.
- The electric field $\vec{E}(\vec{r})$ exerts a force on a positive (negative) point charge $q$ in the direction of steepest potential drop (rise).
- When a positive (negative) point charge $q$ moves from a region of high potential to a region of low potential, the electric field does positive (negative) work on it. In the process, the potential energy decreases (increases).
Consider a point charge $Q = 2\mu C$ fixed at position $x = 0$. A particle with mass $m = 2g$ and charge $q = -0.1\mu C$ is launched at position $x_1 = 10\text{cm}$ with velocity $v_1 = 12\text{m/s}$.

- Find the velocity $v_2$ of the particle when it is at position $x_2 = 20\text{cm}$. 

\[
\begin{align*}
(Q = 2\mu C) & \quad m = 2g \\
\oplus & \quad q = -0.1\mu C \\
x = 0 & \quad x_1 = 10\text{cm} \\
\quad v_1 & \quad x_2 = 20\text{cm}
\end{align*}
\]
Electric Potential and Potential Energy: Application (2)

- Electric potential at point $P_1$: $V = \frac{kq_1}{0.04\text{m}} + \frac{kq_2}{0.04\text{m}} = 1125V + 1125V = 2250V$.

- Electric potential at point $P_2$: $V = \frac{kq_1}{0.06\text{m}} + \frac{kq_2}{0.10\text{m}} = 750V + 450V = 1200V$. 

![Diagram showing electric potentials at P1 and P2 with distances and charges labeled.](image-url)
Point charges \( q_1 = -5.0 \mu \text{C} \) and \( q_2 = +2.0 \mu \text{C} \) are positioned at two corners of a rectangle as shown.

(a) Find the electric potential at the corners A and B.

(b) Find the electric field at point B.

(c) How much work is required to move a point charge \( q_3 = +3 \mu \text{C} \) from B to A?
A positive point charge $q$ is positioned in the electric field of a negative point charge $Q$.

(a) In which configuration is the charge $q$ positioned in the stronger electric field?

(b) In which configuration does the charge $q$ experience the stronger force?

(c) In which configuration is the charge $q$ positioned at the higher electric potential?

(d) In which configuration does the charge $q$ have the higher potential energy?
An electron and a proton are released from rest midway between oppositely charged plates.

(a) Name the particle(s) which move(s) from high to low electric potential.
(b) Name the particle(s) whose electric potential energy decrease(s).
(c) Name the particle(s) which hit(s) the plate in the shortest time.
(d) Name the particle(s) which reach(es) the highest kinetic energy before impact.
Three protons are projected from $x = 0$ with equal initial speed $v_0$ in different directions. They all experience the force of a uniform horizontal electric field $\vec{E}$. Ultimately, they all hit the vertical screen at $x = L$.

(a) Which proton travels the longest time?
(b) Which proton travels the longest path?
(c) Which particle has the highest speed when it hits the screen?

Two of the questions are easy, one is hard.
Consider a region of nonuniform electric field. Charged particles 1 and 2 start moving from rest at point $A$ in opposite directions along the paths shown.

From the information given in the figure...

(a) find the kinetic energy $K_1$ of particle 1 when it arrives at point $B$,

(b) find the electric potential $V_C$ at point $C$ if we know that particle 2 arrives there with kinetic energy $K_2 = 8 \text{ J}$.
(a) Is the electric potential at points $P_1, P_2$ positive or negative or zero?

(b) Is the potential energy of a negatively charged particle at points $P_1, P_2$ positive or negative or zero?

(c) Is the electric field at points $P_1, P_2$ directed left or right or is it zero?

(d) Is the force on a negatively charged particle at points $P_1$ and $P_2$ directed left or right or is it zero?
Consider four point charges of equal magnitude positioned at the corners of a square as shown. Answer the following questions for points $A$, $B$, $C$.

(1) Which point is at the highest electric potential?
(2) Which point is at the lowest electric potential?
(3) At which point is the electric field the strongest?
(4) At which point is the electric field the weakest?
The charged particles 1 and 2 move between the charged conducting plates $A$ and $B$ in opposite directions.

From the information given in the figure...

(a) find the kinetic energy $K_{1B}$ of particle 1,

(b) find the charge $q_2$ of particle 2,

(c) find the direction and magnitude of the electric field $\vec{E}$ between the plates.
Consider a point charge $Q = 5 \text{nC}$ fixed at position $x = 0$.

(a) Find the electric potential $V_1$ at position $x_1 = 3 \text{m}$ and the electric potential $V_2$ at position $x_2 = 6 \text{m}$.

(b) If a charged particle ($q = 4 \text{nC}, m = 1.5 \text{ng}$) is released from rest at $x_1$, what are its kinetic energy $K_2$ and its velocity $v_2$ when it reaches position $x_2$?
Consider a point charge $Q = 5 \text{nC}$ fixed at position $x = 0$.

(a) Find the electric potential $V_1$ at position $x_1 = 3 \text{ m}$ and the electric potential $V_2$ at position $x_2 = 6 \text{ m}$.

(b) If a charged particle ($q = 4 \text{nC}$, $m = 1.5 \text{ ng}$) is released from rest at $x_1$, what are its kinetic energy $K_2$ and its velocity $v_2$ when it reaches position $x_2$?

**Solution:**

(a) $V_1 = k \frac{Q}{x_1} = 15 \text{ V}$, $V_2 = k \frac{Q}{x_2} = 7.5 \text{ V}$. 

![Diagram](image.png)
Consider a point charge $Q = 5 \text{nC}$ fixed at position $x = 0$.

(a) Find the electric potential $V_1$ at position $x_1 = 3 \text{m}$ and the electric potential $V_2$ at position $x_2 = 6 \text{m}$.

(b) If a charged particle ($q = 4 \text{nC}$, $m = 1.5 \text{ng}$) is released from rest at $x_1$, what are its kinetic energy $K_2$ and its velocity $v_2$ when it reaches position $x_2$?

\[
V_1 = k \frac{Q}{x_1} = 15 \text{V}, \quad V_2 = k \frac{Q}{x_2} = 7.5 \text{V}.
\]

\[
\Delta U = q(V_2 - V_1) = (4 \text{nC})(-7.5 \text{V}) = -30 \text{nJ} \quad \Rightarrow \quad \Delta K = -\Delta U = 30 \text{nJ}.
\]

\[
\Delta K = K_2 = \frac{1}{2}mv_2^2 \quad \Rightarrow \quad v_2 = \sqrt{\frac{2K_2}{m}} = 200 \text{m/s}.
\]
Consider two point charges positioned as shown.

(a) Find the magnitude of the electric field at point $A$.
(b) Find the electric potential at point $A$.
(c) Find the magnitude of the electric field at point $B$.
(d) Find the electric potential at point $B$. 
Consider two point charges positioned as shown.

(a) Find the magnitude of the electric field at point \( A \).
(b) Find the electric potential at point \( A \).
(c) Find the magnitude of the electric field at point \( B \).
(d) Find the electric potential at point \( B \).

Solution:

(a) \[ E_A = 2k \frac{|7 \text{nC}|}{(5 \text{m})^2} = 2(2.52 \text{V/m}) = 5.04 \text{V/m}. \]
Consider two point charges positioned as shown.

(a) Find the magnitude of the electric field at point $A$.
(b) Find the electric potential at point $A$.
(c) Find the magnitude of the electric field at point $B$.
(d) Find the electric potential at point $B$.

Solution:

(a) \[ E_A = 2k \frac{|7 \text{nC}|}{(5 \text{m})^2} = 2(2.52 \text{V/m}) = 5.04 \text{V/m}. \]

(b) \[ V_A = k \frac{(+7 \text{nC})}{5 \text{m}} + k \frac{(-7 \text{nC})}{5 \text{m}} = 12.6 \text{V} - 12.6 \text{V} = 0. \]
Consider two point charges positioned as shown.

(a) Find the magnitude of the electric field at point $A$.
(b) Find the electric potential at point $A$.
(c) Find the magnitude of the electric field at point $B$.
(d) Find the electric potential at point $B$.

Solution:

(a) $E_A = 2k \frac{|7 \text{nC}|}{(5\text{m})^2} = 2(2.52\text{V/m}) = 5.04\text{V/m}$.

(b) $V_A = k \frac{(+7 \text{nC})}{5\text{m}} + k \frac{(-7 \text{nC})}{5\text{m}} = 12.6\text{V} - 12.6\text{V} = 0$.

(c) $E_B = \sqrt{\left(k \frac{|7 \text{nC}|}{(6\text{m})^2}\right)^2 + \left(k \frac{|7 \text{nC}|}{(8\text{m})^2}\right)^2} \Rightarrow E_B = \sqrt{(1.75\text{V/m})^2 + (0.98\text{V/m})^2} = 2.01\text{V/m}$.
Consider two point charges positioned as shown.

(a) Find the magnitude of the electric field at point $A$.
(b) Find the electric potential at point $A$.
(c) Find the magnitude of the electric field at point $B$.
(d) Find the electric potential at point $B$.

Solution:

(a) $E_A = 2k \left( \frac{|7nC|}{5m} \right)^2 = 2(2.52V/m) = 5.04V/m$.

(b) $V_A = k \left( \frac{(+7nC)}{5m} \right) + k \left( \frac{(-7nC)}{5m} \right) = 12.6V - 12.6V = 0$.

(c) $E_B = \sqrt{\left( k \frac{|7nC|}{(6m)^2} \right)^2 + \left( k \frac{|7nC|}{(8m)^2} \right)^2} \Rightarrow E_B = \sqrt{(1.75V/m)^2 + (0.98V/m)^2} = 2.01V/m$.

(d) $V_B = k \left( \frac{(+7nC)}{6m} \right) + k \left( \frac{(-7nC)}{8m} \right) = 10.5V - 7.9V = 2.6V$. 
Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point $A$.
- Find the electric potential at point $B$.
- Find the magnitude of the electric field at point $C$.
- Find the electric potential at point $D$. 
Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point $A$.
- Find the electric potential at point $B$.
- Find the magnitude of the electric field at point $C$.
- Find the electric potential at point $D$.

Solution:

$$E_A = k \frac{|5\text{nC}|}{(3\text{m})^2} + k \frac{|-9\text{nC}|}{(7\text{m})^2} = 5.00\text{V/m} + 1.65\text{V/m} = 6.65\text{V/m}.$$
Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point A.
- Find the electric potential at point B.
- Find the magnitude of the electric field at point C.
- Find the electric potential at point D.

Solution:

\[ E_A = k \frac{|5nC|}{(3m)^2} + k \frac{|-9nC|}{(7m)^2} = 5.00V/m + 1.65V/m = 6.65V/m. \]

\[ V_B = k \frac{(+5nC)}{6m} + k \frac{(-9nC)}{8m} = 7.50V - 10.13V = -2.63V. \]
Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point $A$.
- Find the electric potential at point $B$.
- Find the magnitude of the electric field at point $C$.
- Find the electric potential at point $D$.

**Solution:**

- $E_A = k \frac{|5\text{nC}|}{(3\text{m})^2} + k \frac{|-9\text{nC}|}{(7\text{m})^2} = 5.00\text{V/m} + 1.65\text{V/m} = 6.65\text{V/m}$.

- $V_B = k \frac{(+5\text{nC})}{6\text{m}} + k \frac{(-9\text{nC})}{8\text{m}} = 7.50\text{V} - 10.13\text{V} = -2.63\text{V}$.

- $E_C = k \frac{|5\text{nC}|}{(6\text{m})^2} + k \frac{|-9\text{nC}|}{(4\text{m})^2} = 1.25\text{V/m} + 5.06\text{V/m} = 6.31\text{V/m}$.
Unit Exam I: Problem #1 (Spring ’14)

Consider two point charges positioned as shown.

- Find the magnitude of the electric field at point $A$.
- Find the electric potential at point $B$.
- Find the magnitude of the electric field at point $C$.
- Find the electric potential at point $D$.

Solution:

- $E_A = k \frac{|5\text{nC}|}{(3\text{m})^2} + k \frac{|-9\text{nC}|}{(7\text{m})^2} = 5.00\text{V/m} + 1.65\text{V/m} = 6.65\text{V/m}$.

- $V_B = k \frac{(+5\text{nC})}{6\text{m}} + k \frac{(-9\text{nC})}{8\text{m}} = 7.50\text{V} - 10.13\text{V} = -2.63\text{V}$.

- $E_C = k \frac{|5\text{nC}|}{(6\text{m})^2} + k \frac{|-9\text{nC}|}{(4\text{m})^2} = 1.25\text{V/m} + 5.06\text{V/m} = 6.31\text{V/m}$.

- $V_D = k \frac{(+5\text{nC})}{8\text{m}} + k \frac{(-9\text{nC})}{6\text{m}} = 5.63\text{V} - 13.5\text{V} = -7.87\text{V}$.