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# 04. Electric field of extended objects. Electric flux

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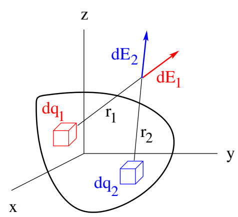
# PHY204 Lecture 4 [rln4]

**Electric Field of Continuous Charge Distribution**

- Divide the charge distribution into infinitesimal blocks.
  - For 3D applications use charge per unit volume: $\rho = \Delta Q/\Delta V$.
  - For 2D applications use charge per unit area: $\sigma = \Delta Q/\Delta A$.
  - For 1D applications use charge per unit length: $\lambda = \Delta Q/\Delta L$.
- Use Coulomb's law to calculate the electric field generated by each block.
- Use the superposition principle to calculate the resultant field from all blocks.
- Use symmetries whenever possible.

$$d\vec{E}_i = k\frac{dq_i}{r_i^2}\hat{r}_i$$

$$\vec{E} = \sum_i d\vec{E}_i \to k\int\frac{dq}{r^2}\hat{r}$$




tsl30

How do we calculate the electric field of electrically charged, extended objects? We begin by pursuing one strategy and then set the stage for a different strategy.

We consider situations where we know up front how the charge is distributed across the object. This is not always the case as we shall see later.

We know the electric field of point-like charged objects. Now we combine this knowledge with the superposition principle in pursuit of our goal.

We divide the charged object into chunks sufficiently small that they can be treated like point charges. Then we calculate the electric field of each chunk at a field point of our choice. What is left to do is to sum up the field contributions vectorially. This easier said than done, for sure. However, we are ready to perform this task for a few simple scenarios.

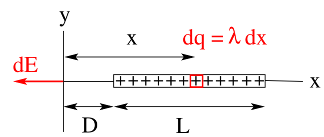
The charge distribution across the object in question is typically given in the form of a charge density. In some cases, the charge density is a constant. In general, it is a function of position across the object.

If the charge is distributed across the volume of a 3-dimensional object, we mean charge per unit volume when we say charge density and use the symbol $\rho$ (rho) to specify it.

If the charge is distributed across the surface of a 3-dimensional object or if the object is thin to be effectively 2-dimensional (e.g. a sheet of paper) then we mean charge per unit area and use the symbol $\sigma$ (sigma). For effectively 1-dimensional objects (rods, wires, threads) we mean charge per unit length and use the symbol $\lambda$ (lambda).

## Electric Field of Charged Rod (1)

- Charge per unit length: $\lambda = Q/L$
- Charge on slice $dx$: $dq = \lambda dx$




- Electric field generated by slice $dx$: $dE = \frac{kdq}{x^2} = \frac{k\lambda dx}{x^2}$
- Electric field generated by charged rod:

$$E = k\lambda \int_D^{D+L} \frac{dx}{x^2} = k\lambda \left[-\frac{1}{x}\right]_D^{D+L} = k\lambda \left[\frac{1}{D} - \frac{1}{D+L}\right] = \frac{kQ}{D(D+L)}$$

- Limiting case of very short rod ($L \ll D$): $E \simeq \frac{kQ}{D^2}$
- Limiting case of very long rod ($L \gg D$): $E \simeq \frac{k\lambda}{D}$

tsl31

In our first application, we consider a uniformly charged, thin rod of length $L$. Our goal is to determine the electric field at a field point a distance $D$ away from one end of the rod.

If the (uniform) charge density is $\lambda$, then the total charge on the rod is $Q = \lambda L$. We assume that the charge is positive, which implies that the direction of the electric field is away from the rod (here to the left).

We begin by placing the rod into a coordinate system as shown. Other ways of positioning the rod work equally well, but a choice has to be made.

The itemized list walks us through the calculation. We pick a generic infinitesimal slice of the rod, a distance $x$ away from the field point. Its width is $dx$ and contains charge $dq = \lambda dx$. It thus contributes the magnitude $dE$ to the field at the field point.

Summing up the contributions from all slices amounts to an integral across the length of the rod from the near end to the far end.

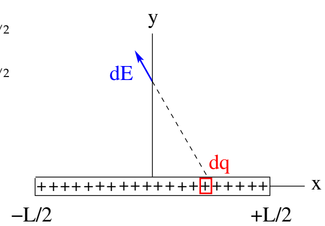
This is a definite integral. The function to be integrated is $1/x^2$, yielding the indefinite integral $-1/x$, which then has to be evaluated at the integration boundaries. The definite integral is the difference between those two values. In the last step we use common denominators and $Q = \lambda L$.

Note that the dependence of the field strength on the distance $D$ from the charged rod is more complex than is the case for a point charge. At small distances, when $D \ll L$, the field decreases like $\sim 1/D$. At large distances, on the other hand, the field decreases like $\sim 1/D^2$.

## Electric Field of Charged Rod (4)

Symmetry dictates that the resulting electric field is directed radially (alternative derivation).

- Charge per unit length: $\lambda = Q/L$
- Charge on slice $dx$: $dq = \lambda dx$
- $dE = \dfrac{kdq}{r^2} = \dfrac{k\lambda dx}{x^2+y^2}$
- $dE_y = dE\cos\theta = \dfrac{dEy}{\sqrt{x^2+y^2}} = \dfrac{k\lambda ydx}{(x^2+y^2)^{3/2}}$
- $E_y = \displaystyle\int_{-L/2}^{+L/2} \frac{k\lambda ydx}{(x^2+y^2)^{3/2}} = \left[\frac{k\lambda yx}{y^2\sqrt{x^2+y^2}}\right]_{-L/2}^{+L/2}$
- $E_y = \dfrac{k\lambda L}{y\sqrt{(L/2)^2+y^2}} = \dfrac{kQ}{y\sqrt{(L/2)^2+y^2}}$
- Large distance ($y \gg L$): $E_y \simeq \dfrac{kQ}{y^2}$
- Small distances ($y \ll L$): $E_y \simeq \dfrac{2k\lambda}{y}$

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For our next application, we consider the same rod, but move the field point to the side of the rod halfway between the ends. For convenience, we place the rod symmetrically on the $x$-axis as shown. The field point thus comes to lie on the $y$-axis.

The itemized list again walks us through the calculation. The first two items are the same as on the previous page.

The third item shows how to calculate the field contribution $dE$ of a generic slice at the field point. The relevant distance now is $\sqrt{x^2+y^2}$, the length of the dashed line.

One complication in this case is that the field contributions of different slices have different directions. We can solve this by splitting each contribution into a vertical and a horizontal component. The symmetrical placement of the rod relative to the field point ensures that the horizontal field contributions add up to zero.

The fourth item shows how to calculate the vertical component $dE_y$. The angle of the vector $d\vec{E}$ away from the vertical is the same as that of the dashed line. Thus we can use $\cos\theta = y/\sqrt{x^2+y^2}$.

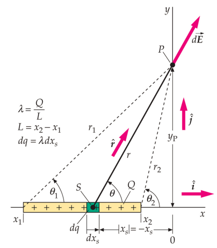
Now we are ready to carry out the integral, which is done in the next two items. The result depends on the length $L$ of the rod, the total charge $Q$ on it, and the distance $y$ of the field point from the rod.

The last two items tell us how the field varies with distance very far away and very close to the rod. From a sufficiently large distance, all charged objects look (or better, feel) like point charges.

## Electric Field of Charged Rod (2)

- Charge per unit length: $\lambda = Q/L$
- Charge on slice $dx_s$: $dq = \lambda dx_s$
- Trigonometric relations:

$$y_p = r\sin\theta, \quad -x_s = r\cos\theta$$
$$x_s = -y_p\cot\theta, \quad dx_s = \frac{y_p d\theta}{\sin^2\theta}$$




- $dE = \frac{k\lambda dx_s}{r^2} = \frac{k\lambda dx_s}{y_p^2}\sin^2\theta = \frac{k\lambda d\theta}{y_p}$
- $dE_y = dE\sin\theta = \frac{k\lambda}{y_p}\sin\theta d\theta \;\Rightarrow\; E_y = \frac{k\lambda}{y_p}\int_{\theta_1}^{\theta_2}\sin\theta d\theta = -\frac{k\lambda}{y_p}(\cos\theta_2 - \cos\theta_1)$
- $dE_x = dE\cos\theta = \frac{k\lambda}{y_p}\cos\theta d\theta \;\Rightarrow\; E_x = \frac{k\lambda}{y_p}\int_{\theta_1}^{\theta_2}\cos\theta d\theta = \frac{k\lambda}{y_p}(\sin\theta_2 - \sin\theta_1)$

tsl32

Here we extend the calculation of the previous page and calculate the electric field of a uniformly charged rod with length $L$ and line charge density $\lambda$ at a field point in arbitrary position relative to the rod.

If the field point lies on the line that extends the rod, we are back to the situation analyzed on page 2. If that is not the case then the rod an the field point define a plane and the electric field points in some direction in the same plane.

Without loss of generality, we can place the rod on the $x$-axis and the field point on the positive $y$-axis of a Cartesian coordinate system. The electric field then has an $x$-component $E_x$ and a $y$-component $E_y$ but no $z$-component.

The rod is again split into infinitesimal slices. We again use the Coulomb field for each slice then sum up the field components. What is different from the calculation on the previous page is that we change the integration variable from an element of length to an element of angle.

Mathematically, this is called a substitution. We replace $x_s$ by $\theta$ and $dx_s$ by $d\theta$, using the derivative $dx_s/d\theta = y_P/\sin^2\theta$ of the function $x_s = -y_P\cot\theta$. The rest of the analysis then only involves integrations over sines and cosines.

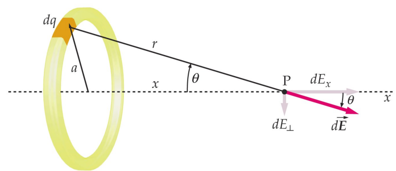
Note that the range of both angles is $0 \leq \theta_i \leq \pi$. If the rod extends over the entire $x$-axis, we have $\theta_1 = 0$ and $\theta_2 = \pi$.

The electric field of an infinitely long rod would then be directed straight up and have the magnitude,

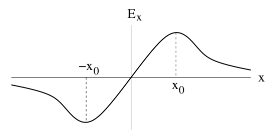
$$E_y = \frac{2k\lambda}{y_P}.$$

## Electric Field of Charged Ring

- Total charge on ring: $Q$
- Charge per unit length: $\lambda = Q/2\pi a$
- Charge on arc: $dq$




- $dE = \frac{kdq}{r^2} = \frac{kdq}{x^2+a^2}$
- $dE_x = dE\cos\theta = dE\,\frac{x}{\sqrt{x^2+a^2}} = \frac{kxdq}{(x^2+a^2)^{3/2}}$
- $E_x = \frac{kx}{(x^2+a^2)^{3/2}}\int dq \;\Rightarrow\; E_x = \frac{kQx}{(x^2+a^2)^{3/2}}$
- $|x| \ll a:\; E_x \simeq \frac{kQx}{a^3}, \qquad x \gg a:\; E_x \simeq \frac{kQ}{x^2}$
- $(dE_x/dx)_{x=x_0} = 0 \quad\Rightarrow\; x_0 = \pm a/\sqrt{2}$

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Calculating the electric field generated by a uniformly charged ring of radius $a$ looks like a more difficult task. With the expertise gained in the previous two applications, we can perform the task with little effort if we limit the choice of field point to positions on the dashed line (the axis of the ring), a distance $x$ from the center of the ring.

We split the ring into infinitesimal segments of charge $dq$ (assumed positive). Each segment has the same distance, $\sqrt{x^2+a^2}$, from the field point, thus producing a field $dE$ of the same magnitude (fourth item).

Next we split the $dE$ into a component $dE_x$ parallel to the axis and a vector $d\vec{E}_\perp$ perpendicular to it. We note that the contributions to $\vec{E}_\perp$ from all segments add up to zero for reasons of symmetry. The resultant field points in $x$-direction. The expression for $dE_x$ is calculated in the fifth item, using a familiar bit of trigonometry.

What remains to be done is to add up the contributions from all segments, which amounts to summing up the charge of all segments. The result is given in the sixth item. A graphical representation is also shown on the slide.

How do we interpret that curve? For positive $x$, $E_x$ is positive and for negative $x$, $E_x$ is negative. This means that to the right of the ring, the field is pointing to the right and to the left of the ring it is pointing left, always away from the (positively charged) ring.

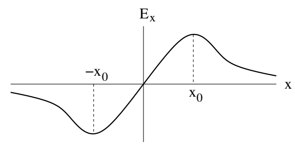
At the center of the ring, the field is zero. Very close to the ring, the field strength varies linearly with distance from the center. Far away from the ring, the field is indistinguishable from the field of a point charge, as expected.

At the positions, $x = \pm a/\sqrt{2}$, the field strength has a maximum. Here the curve has zero slope.

## Charged Bead Moving Along Axis of Charged Ring

Consider a negatively charged bead (mass $m$, charge $-q$) constrained to move without friction along the axis of a positively charged ring.

- Place bead on $x$-axis near center of ring: $|x| \ll a$ : $E_x \simeq \frac{kQx}{a^3}$
- Restoring force: $F = -qE_x = -k_s x$ with $k_s = \frac{kQq}{a^3}$
- Acceleration: $a = \frac{F}{m} = -\frac{k_s}{m}x$
- Equation of motion: $\frac{d^2x}{dt^2} = -\frac{k_s}{m}x$
- Harmonic oscillation: $x(t) = A\cos(\omega t + \phi)$
- Angular frequency: $\omega = \sqrt{\frac{k_s}{m}} = \sqrt{\frac{kQq}{ma^3}}$

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If we make the $x$-axis a thin wire which restricts a negatively charged bead to motion in $\pm x$-direction, then that bead experiences a linear restoring force, as if it were attached to a spring with an effective stiffness $k_s$ as shown.

It is straightforward then to solve the equation of motion for the bead. The solution is a harmonic oscillation with an angular frequency as stated.

Note that this solution only holds for small amplitudes. The maximum displacement should remain well within the range where the electric field depends linearly on position $x$.

At larger distances from the center of the ring, the restoring force gets weaker, which makes the period of oscillation longer. It is, in fact, possible to give the bead a kick strong enough that it will keep going forever and not return.

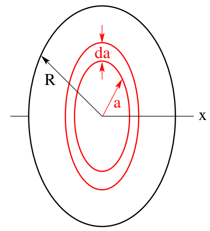
If, in an actual experiment, we watch the bead oscillate back and forth harmonically, we will notice that the motion will gradually taper off. It does so for two reasons.

One reason is a combination of friction between wire and bead and air resistance. Kinetic and potential energy of the bead are gradually converted into heat (a different form of energy).

The second reason is that the bead emits an electromagnetic wave. Electric charge under acceleration, quite generally, radiates. All radiation transports energy away from the source. We are not yet ready to discuss this further.

Nevertheless, we should always keep in mind when we discuss moving charged particles that our description is incomplete.

## Electric Field of Charged Disk




- Charge per unit area: $\sigma = \dfrac{Q}{\pi R^2}$
- Area of ring: $dA = 2\pi a da$
- Charge on ring: $dq = 2\pi\sigma a da$

- $dE_x = \dfrac{kxdq}{(x^2+a^2)^{3/2}} = \dfrac{2\pi\sigma kxada}{(x^2+a^2)^{3/2}}$
- $E_x = 2\pi\sigma kx \displaystyle\int_0^R \frac{ada}{(x^2+a^2)^{3/2}} = 2\pi\sigma kx \left[\frac{-1}{\sqrt{x^2+a^2}}\right]_0^R$
- $E_x = 2\pi\sigma k \left[1 - \dfrac{x}{\sqrt{x^2+R^2}}\right]$ for $x > 0$
- $x \ll R: \; E_x \simeq 2\pi\sigma k = \dfrac{\sigma}{2\epsilon_0}$
- Infinite sheet of charge produces uniform electric field perpendicular to plane.

tsl36

Returning to electrostatics, our goal here is to calculate the electric field generated by a uniformly charged disk. We limit our effort to field points along the axis of the disk, a distance $x$ from it.

The disk is effectively two-dimensional. For that reason, we work with the quantity $\sigma$, representing charge per unit area.

In solving this task, we take advantage of the fact that we already know the electric field generated by a ring. Our strategy is to split the disk into concentric rings of infinitesimal width and then add up the field contributions from all rings. The directions are the same: parallel to the axis.

A generic ring such as shown has radius $a$ and width $da$. The first three items on the slide show how to calculate the charge $dq$ on that ring.

We then take the final results from page 4 and replace $Q$ by $dQ$, as done in the fourth item here. What remains to be done is the summation over concentric rings, converted into an integration from the center to the perimeter of the disk, as carried out in the next two items.

Notice that if the field point approaches the disk, the field strength does not diverge. It approaches the value,

$$E_x = 2\pi\sigma k = \frac{\sigma}{2\epsilon_0}.$$

The same value is being approached if we keep the field point at an arbitrary distance $x$ from the disk and increase the radius $R$ of the disk to values much larger than $x$. In the limit $R \to \infty$ the electric field remains exactly the same in both magnitude and direction when we move the field point.

We now have found a way to realize a uniform electric field such as we have used in the previous lecture to describe the motion of charged particles.

## Electric Field of Charged Semicircle

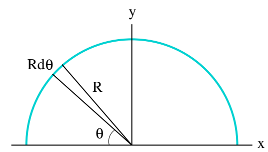
Consider a uniformly charged thin rod bent into a semicircle of radius $R$.

Find the electric field generated at the origin of the coordinate system.

- Charge per unit length: $\lambda = Q/\pi R$
- Charge on slice: $dq = \lambda R d\theta$ (assumed positive)
- Electric field generated by slice: $dE = k\dfrac{|dq|}{R^2} = \dfrac{k|\lambda|}{R}d\theta$ directed radially (inward for $\lambda > 0$)
- Components of $d\vec{E}$: $dE_x = dE\cos\theta, \quad dE_y = -dE\sin\theta$
- Electric field from all slices added up:

$$E_x = \frac{k\lambda}{R}\int_0^{\pi}\cos\theta\,d\theta = \frac{k\lambda}{R}\Big[\sin\theta\Big]_0^{\pi} = 0$$

$$E_y = -\frac{k\lambda}{R}\int_0^{\pi}\sin\theta\,d\theta = \frac{k\lambda}{R}\Big[\cos\theta\Big]_0^{\pi} = -\frac{2k\lambda}{R}$$

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Here we use the same method one more time, namely to calculate the electric field generated by a uniformly charged rod bent into a semicircle. We place the object into the coordinate system as shown and only ask for the electric field at its origin.

The charged object is effectively one-dimensional. Therefore we go back to $\lambda$ (charge per unit length) as a convenient way to express the charge content of an infinitesimal slice (see first two items).

If we assume that the charge on the semicircle is positive, then the field at the origin of the slice identified is pointing down and to the right. It has a positive $x$-component and an negative $y$-componenent (see items 3 and 4).

The summation over all slices is carried out in the last item, which amounts to separate integrals for the field differentials $dE_x$ and $dE_y$. We use the angle $\theta$ as a convenient integration variable.

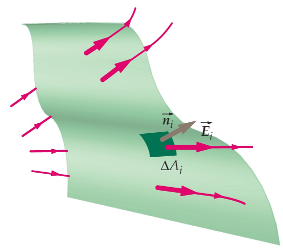
Unsurprisingly, the resultant field is pointing straight down.

**Electric Flux: Definition**

Consider a surface $S$ of arbitrary shape in the presence of an electric field $\vec{E}$.

Prescription for the calculation of the electric flux through $S$:

- Divide $S$ into small tiles of area $\Delta A_i$.
- Introduce vector $\Delta\vec{A}_i = \hat{n}_i \Delta A_i$ perpendicular to tile.
  - If $S$ is open choose consistently one of two possible directions for $\Delta\vec{A}_i$.
  - If $S$ is closed choose $\Delta\vec{A}_i$ to be directed outward.
- Electric field at position of tile $i$: $\vec{E}_i$.
- Electric flux through tile $i$: $\Delta\Phi_i^{(E)} = \vec{E}_i \cdot \Delta\vec{A}_i = E_i \Delta A_i \cos\theta_i$.
- Electric flux through $S$: $\Phi_E = \sum_i \vec{E}_i \cdot \Delta\vec{A}_i$.
- Limit of infinitesimal tiles: $\Phi_E = \int \vec{E} \cdot d\vec{A}$.
- Electric flux is a scalar.
- The SI unit of electric flux is $\mathrm{Nm}^2/\mathrm{C}$.

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For what comes next we need to introduce electric flux, a quantity associated with a surface of any shape or size placed into a region of electric field.

We divide the surface into infinitesimal tiles and associate with each tile an area vector $\Delta\vec{A}$ such that its magnitude is equal to the area of the tile and its direction perpendicular to the surface at the position of the tile.

If the surface is open (as the one shown), then one of two options must be chosen for the direction of $\Delta\vec{A}$. If the surface is closed, then the convention is to have $\Delta\vec{A}$ point toward the outside.

At any point on the surface, the electric field $\vec{E}$ has a particular magnitude and direction. The tiles must be made sufficiently small, such that the field does not vary significantly across any tile.

The electric flux through a given tile $i$ is then the dot product of two local vectors, the electric field $\vec{E}_i$ at the position of tile $i$ and the area vector $d\vec{A}_i$ of that tile,

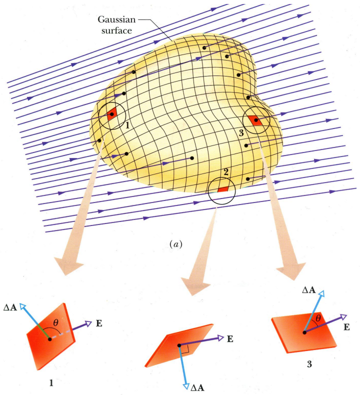
$$\Delta\Phi_i^{(E)} = \vec{E}_i \cdot \Delta\vec{A}_i = E_i \Delta A_i \cos\theta_i, \quad i = 1, 2, \ldots,$$

where $\theta_i$ is the angle between the two vectors. The flux contribution $\Delta\Phi_i^{(E)}$ is positive if $\theta_i < \pi/2$, negative if $\theta_i > \pi/2$, and zero if the two vectors are perpendicular to each other ($\theta_i = \pi/2$).

Summing up the contributions of all tiles amounts to summing up positive and negative numbers. Electric flux, unlike the electric field, is not a vector. Its SI unit is $[\mathrm{Nm}^2/\mathrm{C}]$. In the infinitesimal limit, the sum turns into an integral, symbolically represented by the expression,

$$\Phi_E = \int \vec{E} \cdot d\vec{A}.$$

## Electric Flux: Illustration

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The task of carrying out the integral in the expression shown at the end of the previous page is not simple, in general. Developing the skills to carry out this task for situations of increasing complexity, requires some effort. Opportunities for building up this skill will present themselves.

On the slide we see a heart-shaped closed surface positioned in a region of electric field that roughly points from the lower left toward the upper right. The field lines indicate that the field is somewhat stronger on the left than on the right.

The surface is divided into tiles. Each tile has an area vector associated with it, directed perpendicular to its plane toward the outside, as is the convention. For three tiles, details about the directions of the local electric field $\vec{E}$ and the area vector $\Delta\vec{A}$ are shown.

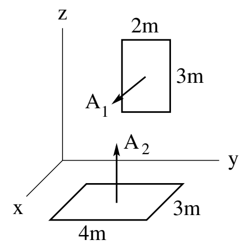
In the case of tile 1, the angle between the two vectors is larger than 90°, which implies that the flux contribution is negative. In the case of tile 2, the field just grazes the surface, implying that there is zero flux contribution. In the case of tile 3, the angle is smaller than 90° and the flux contribution is positive.

Wherever, the electric field points from the outside toward the inside of a closed surface, negative flux is produced. Positive flux is always associated with regions where the electric field points from the inside toward the outside. For the surface shown on the slide, negative flux is produced on the left and positive flux on the right.

Consider two plane surfaces with area vectors $\vec{A}_1$ (pointing in positive $x$-direction) and $\vec{A}_2$ (pointing in positive $z$-direction). The region is filled with a uniform electric field $\vec{E} = (2\hat{\imath} + 7\hat{\jmath} - 3\hat{k})$N/C.

(a) Find the electric flux $\Phi_E^{(1)}$ through area $A_1$.

(b) Find the electric flux $\Phi_E^{(2)}$ through area $A_2$.




Solution:

(a) $\vec{A}_1 = 6\hat{\imath}\,\mathrm{m}^2$,
$\Phi_E^{(1)} = \vec{E} \cdot \vec{A}_1 = (2\mathrm{N/C})(6\mathrm{m}^2) = 12\mathrm{Nm}^2/\mathrm{C}$.

(b) $\vec{A}_2 = 12\hat{k}\,\mathrm{m}^2$,
$\Phi_E^{(2)} = \vec{E} \cdot \vec{A}_2 = (-3\mathrm{N/C})(12\mathrm{m}^2) = -36\mathrm{Nm}^2/\mathrm{C}$.

tsl333

When the electric field is uniform, then a flat surface does not have to be divided into tiles because the electric field would change neither direction nor magnitude between tiles. The entire surface can be treated as a single tile.

Here we are dealing with two rectangular surfaces, whose dimensions, positions, and orientations are shown graphically in a particular coordinate system. For open surfaces, one of two choices for the direction of the area vector must be made. The choices made are shown graphically.

The (uniform) electric field present in the region is stated in components. This fact prompts us to express the two area vectors in components as well, which will simplify the calculation of electric flux.

The area vector $\vec{A}_1$ only has an $x$-component, which we indicate by a unit vector $\hat{i}$, whereas the area vector $\vec{A}_2$ only has a $z$-component, which we indicate by a unit vector $\hat{k}$.

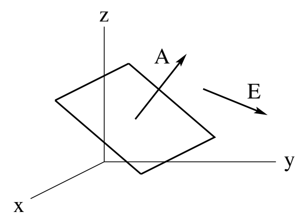
Next we recall how the dot product of two arbitrary vectors $\vec{a}$ and $\vec{b}$, given in components, is worked out:

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}, \quad \vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k} \quad \Rightarrow \quad \vec{a} \cdot \vec{b} = a_xb_x + a_yb_y + a_zb_z.$$

We thus find that the electric flux through tile is positive and the flux through tile 2 negative.

## Electric Flux: Application (2)

Consider a plane sheet of paper whose orientation in space is described by the area vector $\vec{A} = (3\hat{j} + 4\hat{k})\mathrm{m}^2$ positioned in a region of uniform electric field $\vec{E} = (1\hat{i} + 5\hat{j} - 2\hat{k})\mathrm{N/C}$.




(a) Find the area $A$ of the sheet.

(b) Find the magnitude $E$ of the electric field $\vec{E}$.

(c) Find the electric flux $\Phi_E$ through the sheet.

(d) Find the angle $\theta$ between vectors $\vec{A}$ and $\vec{E}$.

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In this application, we again have a rectangular surface positioned in a uniform electric field. The electric field is declared to be uniform and is given in components. The coordinate system in use is indicated graphically.

For the rectangular surface, the area vector is given, also in components. The area vector specifies the area (square-footage) of the rectangle and its orientation, but not its position.

The electric flux through the rectangle remains the same when its position is changed in a uniform electric field. However, the flux does change when the rectangle is reoriented.

For parts (a) and (b) we recall how to calculate the magnitude of a vector given in components:

$$A = \sqrt{\vec{A} \cdot \vec{A}} = \sqrt{9 + 16}\mathrm{m}^2 = 5\mathrm{m}^2.$$

$$E = \sqrt{\vec{E} \cdot \vec{E}} = \sqrt{1 + 25 + 4}\mathrm{N/C} = 5.48\mathrm{N/C}.$$

For part (c) we simply evaluate a dot product:

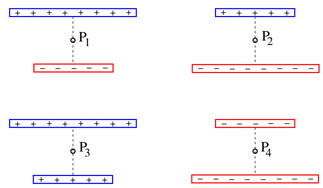
$$\Phi_E = \vec{A} \cdot \vec{E} = (0 + 15 - 8)\mathrm{Nm}^2/\mathrm{C} = 7\mathrm{Nm}^2/\mathrm{C}.$$

For part (d) we recall how a dot product is calculated from the magnitudes of the two vectors and the angle between them:

$$\vec{A} \cdot \vec{E} = AE\cos\theta \quad \Rightarrow \quad \cos\theta = \frac{\vec{A} \cdot \vec{E}}{AE} = \frac{7\mathrm{Nm}^2/\mathrm{C}}{(5\mathrm{m}^2)(5.48\mathrm{N/C})} = 0.255 \quad \Rightarrow \quad \theta = 75.2^\circ.$$

## Electric Field Between Charged Rods

Consider four configurations of two charged rods with equal amounts of charge per unit length $|\lambda|$ on them.




(a) Determine the direction of the electric field at points $P_1, P_2, P_3, P_4$.

(b) Rank the electric field at the four points according to strength.

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This is the quiz for lecture 3.

We return to charged rods and recall that the electric field points away from a positively charged rod and toward a negatively charged rod. We also note that the electric field of a longer rod is stronger than the electric field of a shorter rod at the same distance.