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03. Electric Field III and Electric Flux

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Abstract

Lecture slides 3 for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island.

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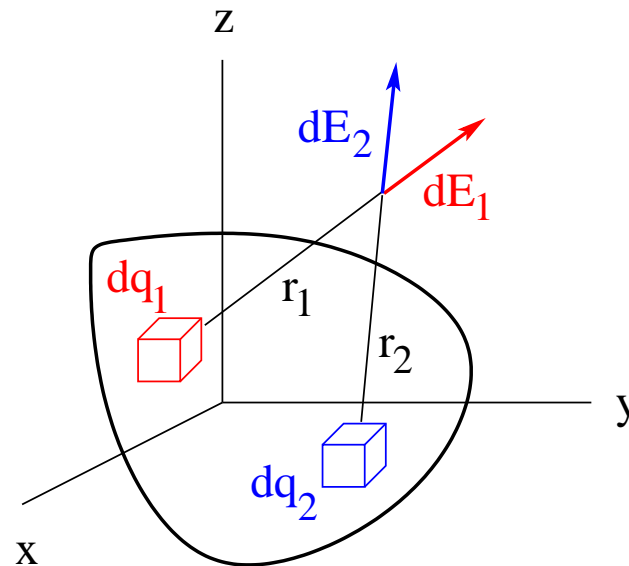
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Electric Field of Continuous Charge Distribution



- Divide the charge distribution into infinitesimal blocks.
 - For 3D applications use charge per unit volume: $\rho = \Delta Q / \Delta V$.
 - For 2D applications use charge per unit area: $\sigma = \Delta Q / \Delta A$.
 - For 1D applications use charge per unit length: $\lambda = \Delta Q / \Delta L$.
- Use Coulomb's law to calculate the electric field generated by each block.
- Use the superposition principle to calculate the resultant field from all blocks.
- Use symmetries whenever possible.

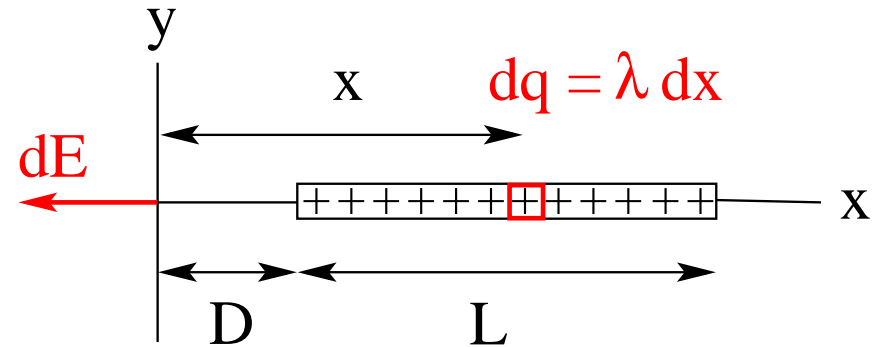
$$d\vec{E}_i = k \frac{dq_i}{r_i^2} \hat{r}_i$$
$$\vec{E} = \sum_i d\vec{E}_i \rightarrow k \int \frac{dq}{r^2} \hat{r}$$



Electric Field of Charged Rod (1)



- Charge per unit length: $\lambda = Q/L$
- Charge on slice dx : $dq = \lambda dx$



- Electric field generated by slice dx : $dE = \frac{k dq}{x^2} = \frac{k \lambda dx}{x^2}$
- Electric field generated by charged rod:

$$E = k\lambda \int_D^{D+L} \frac{dx}{x^2} = k\lambda \left[-\frac{1}{x} \right]_D^{D+L} = k\lambda \left[\frac{1}{D} - \frac{1}{D+L} \right] = \frac{kQ}{D(D+L)}$$

- Limiting case of very short rod ($L \ll D$): $E \simeq \frac{kQ}{D^2}$
- Limiting case of very long rod ($L \gg D$): $E \simeq \frac{k\lambda}{D}$

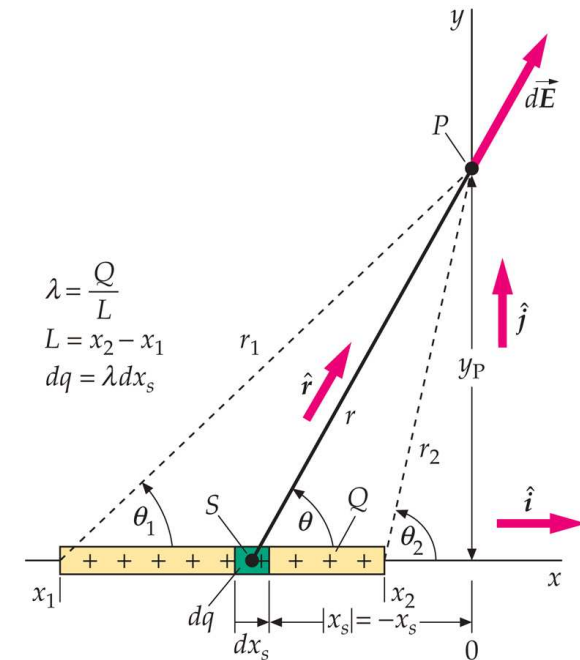
Electric Field of Charged Rod (2)



- Charge per unit length: $\lambda = Q/L$
- Charge on slice dx_s : $dq = \lambda dx_s$
- Trigonometric relations:

$$y_p = r \sin \theta, \quad -x_s = r \cos \theta$$

$$x_s = -y_p \cot \theta, \quad dx_s = \frac{y_p d\theta}{\sin^2 \theta}$$



- $dE = \frac{k\lambda dx_s}{r^2} = \frac{k\lambda dx_s}{y_p^2} \sin^2 \theta = \frac{k\lambda d\theta}{y_p}$

- $dE_y = dE \sin \theta = \frac{k\lambda}{y_p} \sin \theta d\theta \Rightarrow E_y = \frac{k\lambda}{y_p} \int_{\theta_1}^{\theta_2} \sin \theta d\theta = -\frac{k\lambda}{y_p} (\cos \theta_2 - \cos \theta_1)$

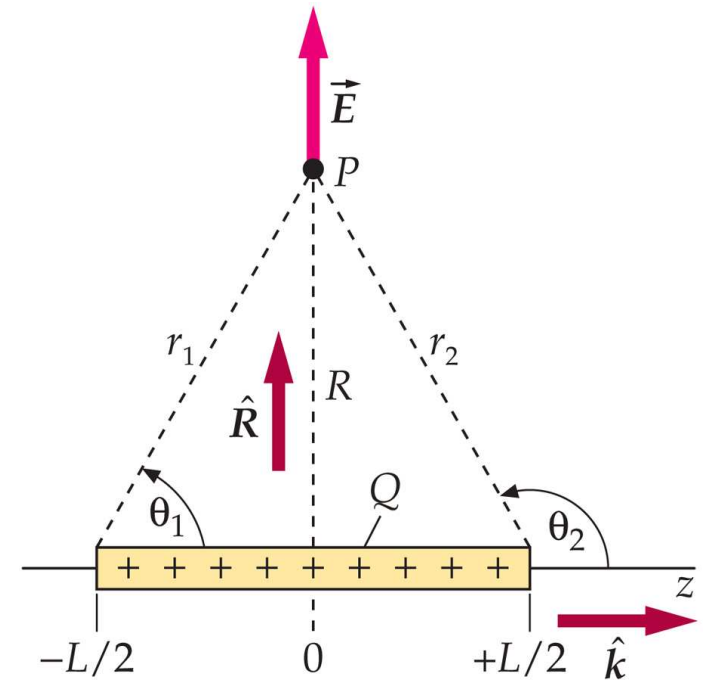
- $dE_x = dE \cos \theta = \frac{k\lambda}{y_p} \cos \theta d\theta \Rightarrow E_x = \frac{k\lambda}{y_p} \int_{\theta_1}^{\theta_2} \cos \theta d\theta = \frac{k\lambda}{y_p} (\sin \theta_2 - \sin \theta_1)$

Electric Field of Charged Rod (3)



Symmetry dictates that the resulting electric field is directed radially.

- $\theta_2 = \pi - \theta_1, \Rightarrow \sin \theta_2 = \sin \theta_1, \quad \cos \theta_2 = -\cos \theta_1.$
- $\cos \theta_1 = \frac{L/2}{\sqrt{L^2/4 + R^2}}.$
- $E_R = -\frac{k\lambda}{R} (\cos \theta_2 - \cos \theta_1) = \frac{k\lambda}{R} \frac{L}{\sqrt{L^2/4 + R^2}}.$
- $E_z = \frac{k\lambda}{R} (\sin \theta_2 - \sin \theta_1) = 0.$
- Large distance ($R \gg L$): $E_R \simeq \frac{kQ}{R^2}.$
- Small distances ($R \ll L$): $E_R \simeq \frac{2k\lambda}{R}$
- Rod of infinite length: $\vec{E} = \frac{2k\lambda}{R} \hat{R}.$

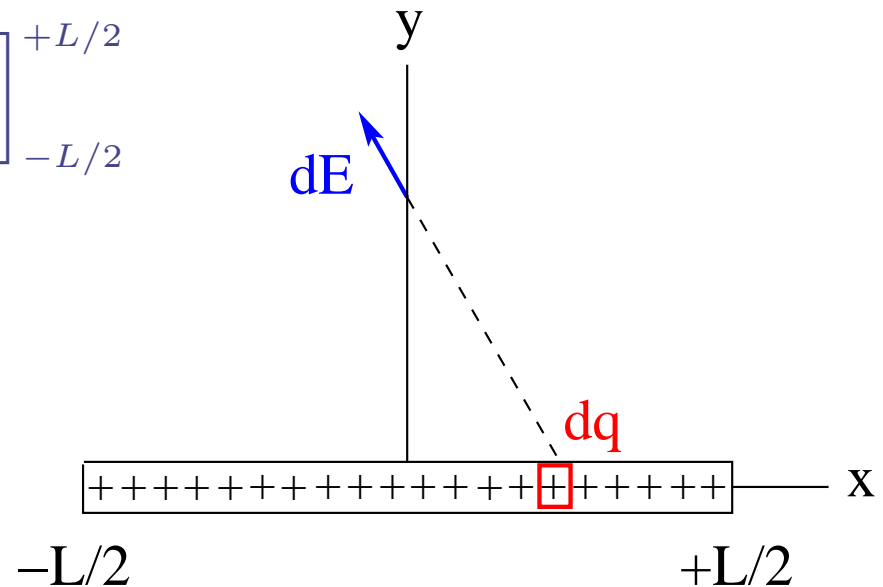


Electric Field of Charged Rod (4)



Symmetry dictates that the resulting electric field is directed radially.

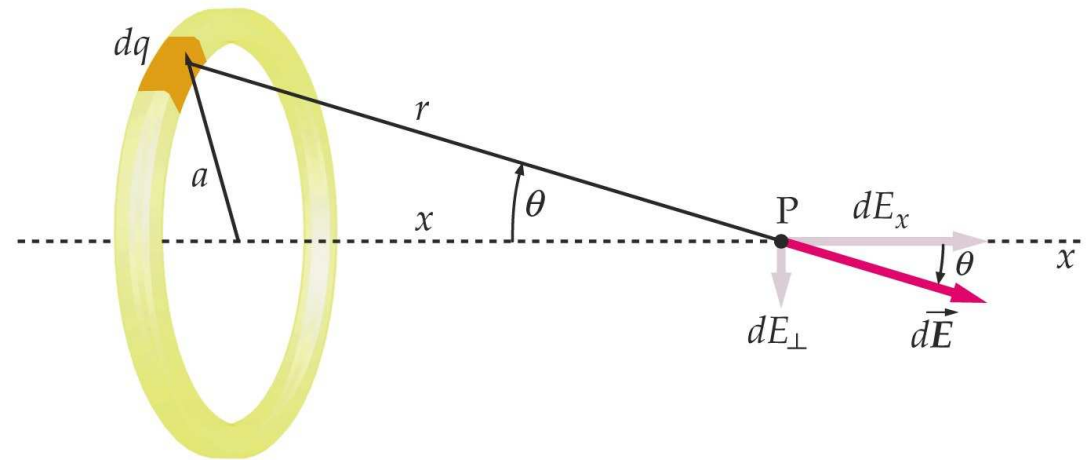
- Charge per unit length: $\lambda = Q/L$
- Charge on slice dx : $dq = \lambda dx$
- $dE = \frac{k dq}{r^2} = \frac{k \lambda dx}{x^2 + y^2}$
- $dE_y = dE \cos \theta = \frac{dE y}{\sqrt{x^2 + y^2}} = \frac{k \lambda y dx}{(x^2 + y^2)^{3/2}}$
- $E_y = \int_{-L/2}^{+L/2} \frac{k \lambda y dx}{(x^2 + y^2)^{3/2}} = \left[\frac{k \lambda y x}{y^2 \sqrt{x^2 + y^2}} \right]_{-L/2}^{+L/2}$
- $E_y = \frac{k \lambda L}{y \sqrt{(L/2)^2 + y^2}} = \frac{k Q}{y \sqrt{(L/2)^2 + y^2}}$
- Large distance ($y \gg L$): $E_y \simeq \frac{k Q}{y^2}$
- Small distances ($y \ll L$): $E_y \simeq \frac{2k \lambda}{y}$



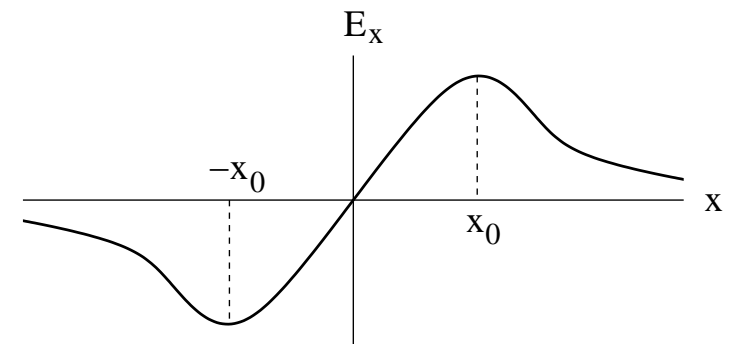
Electric Field of Charged Ring



- Total charge on ring: Q
- Charge per unit length: $\lambda = Q/2\pi a$
- Charge on arc: dq



- $dE = \frac{kdq}{r^2} = \frac{kdq}{x^2 + a^2}$
- $dE_x = dE \cos \theta = dE \frac{x}{\sqrt{x^2 + a^2}} = \frac{kxdq}{(x^2 + a^2)^{3/2}}$
- $E_x = \frac{kx}{(x^2 + a^2)^{3/2}} \int dq \Rightarrow E_x = \frac{kQx}{(x^2 + a^2)^{3/2}}$
- $|x| \ll a : E_x \simeq \frac{kQx}{a^3}, \quad x \gg a : E_x \simeq \frac{kQ}{x^2}$
- $(dE_x/dx)_{x=x_0} = 0 \Rightarrow x_0 = \pm a/\sqrt{2}$

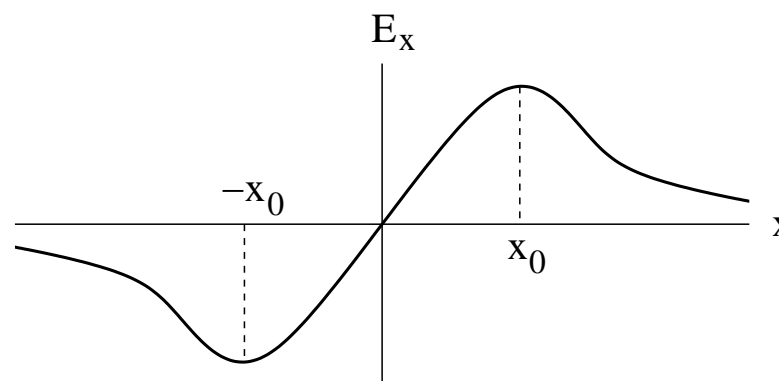


Charged Bead Moving Along Axis of Charged Ring



Consider a negatively charged bead (mass m , charge $-q$) constrained to move without friction along the axis of a positively charged ring.

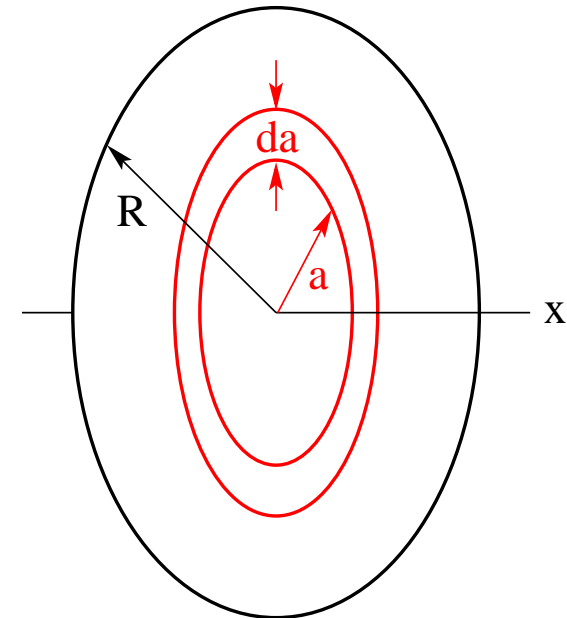
- Place bead on x -axis near center of ring: $|x| \ll a$: $E_x \simeq \frac{kQx}{a^3}$
- Restoring force: $F = -qE_x = -k_s x$ with $k_s = \frac{kQq}{a^3}$
- Harmonic oscillation: $x(t) = A \cos(\omega t + \phi)$
- Angular frequency: $\omega = \sqrt{\frac{k_s}{m}} = \sqrt{\frac{kQq}{ma^3}}$



Electric Field of Charged Disk



- Charge per unit area: $\sigma = \frac{Q}{\pi R^2}$
- Area of ring: $dA = 2\pi a da$
- Charge on ring: $dq = 2\pi\sigma a da$



- $$dE_x = \frac{kxdq}{(x^2 + a^2)^{3/2}} = \frac{2\pi\sigma kxada}{(x^2 + a^2)^{3/2}}$$

- $$E_x = 2\pi\sigma kx \int_0^R \frac{ada}{(x^2 + a^2)^{3/2}} = 2\pi\sigma kx \left[\frac{-1}{\sqrt{x^2 + a^2}} \right]_0^R$$

- $$E_x = 2\pi\sigma k \left[1 - \frac{x}{\sqrt{x^2 + R^2}} \right] \text{ for } x > 0$$

- $x \ll R : E_x \simeq 2\pi\sigma k$

- Infinite sheet of charge produces uniform electric field perpendicular to plane.

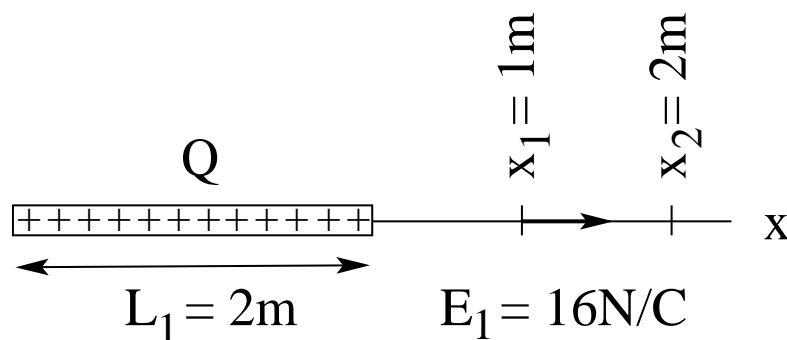
Electric Field of Charged Rubber Band



The electric field at position x along the line of a charged rubber band is

$$E = \frac{kQ}{x(x + L)}$$

The value of E at $x_1 = 1\text{m}$ is $E_1 = 16\text{N/C}$.

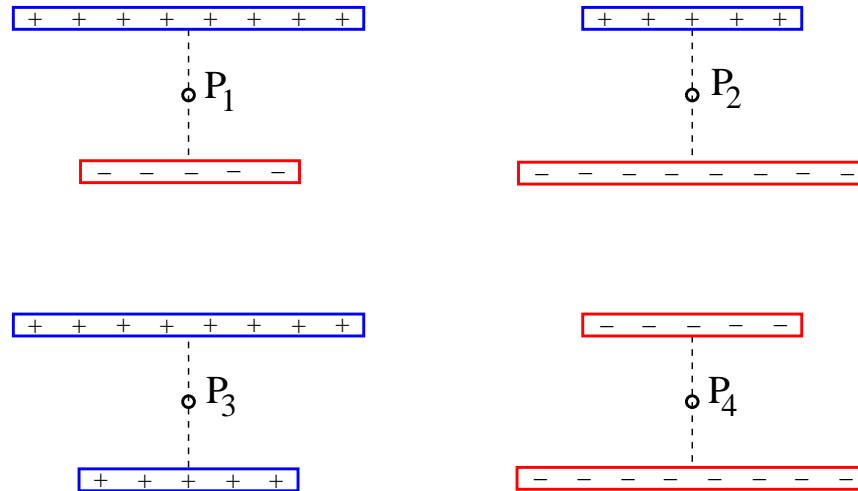


- (a) What is the electric field E_2 at a distance $x_2 = 2\text{m}$ from the edge of the band?
- (b) To what length L_2 must the band be stretched (toward the left) such that it generates the field $E_2 = 8\text{N/C}$ at $x_1 = 1\text{m}$?

Electric Field Between Charged Rods



Consider four configurations of two charged rods with equal amounts of charge per unit length $|\lambda|$ on them.



- Determine the direction of the electric field at points P_1, P_2, P_3, P_4 .
- Rank the electric field at the four points according to strength.

Electric Field of Charged Semicircle



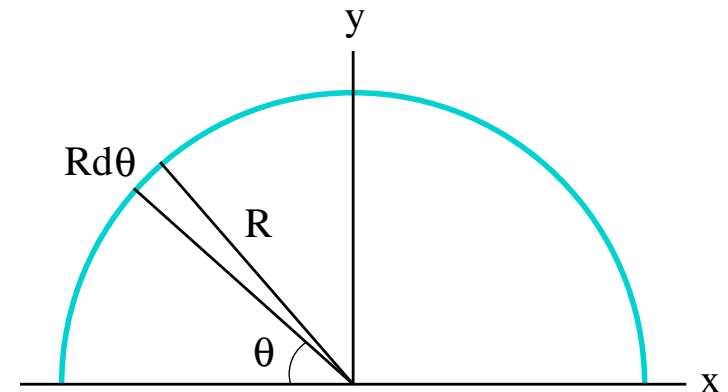
Consider a uniformly charged thin rod bent into a semicircle of radius R .

Find the electric field generated at the origin of the coordinate system.

- Charge per unit length: $\lambda = Q/\pi R$
- Charge on slice: $dq = \lambda R d\theta$ (assumed positive)
- Electric field generated by slice: $dE = k \frac{|dq|}{R^2} = \frac{k|\lambda|}{R} d\theta$
directed radially (inward for $\lambda > 0$)
- Components of $d\vec{E}$: $dE_x = dE \cos \theta$, $dE_y = -dE \sin \theta$
- Electric field from all slices added up:

$$E_x = \frac{k\lambda}{R} \int_0^\pi \cos \theta d\theta = \frac{k\lambda}{R} [\sin \theta]_0^\pi = 0$$

$$E_y = -\frac{k\lambda}{R} \int_0^\pi \sin \theta d\theta = \frac{k\lambda}{R} [\cos \theta]_0^\pi = -\frac{2k\lambda}{R}$$

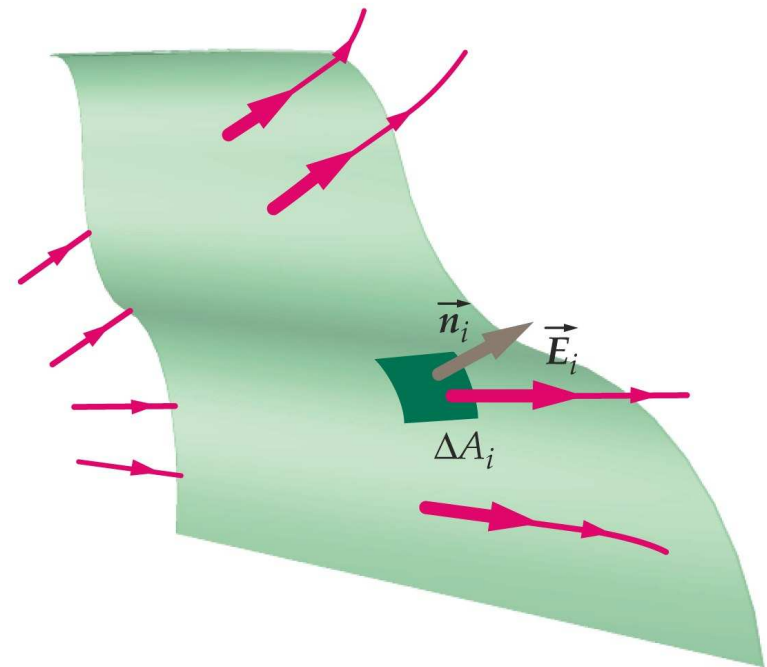


Electric Flux: Definition

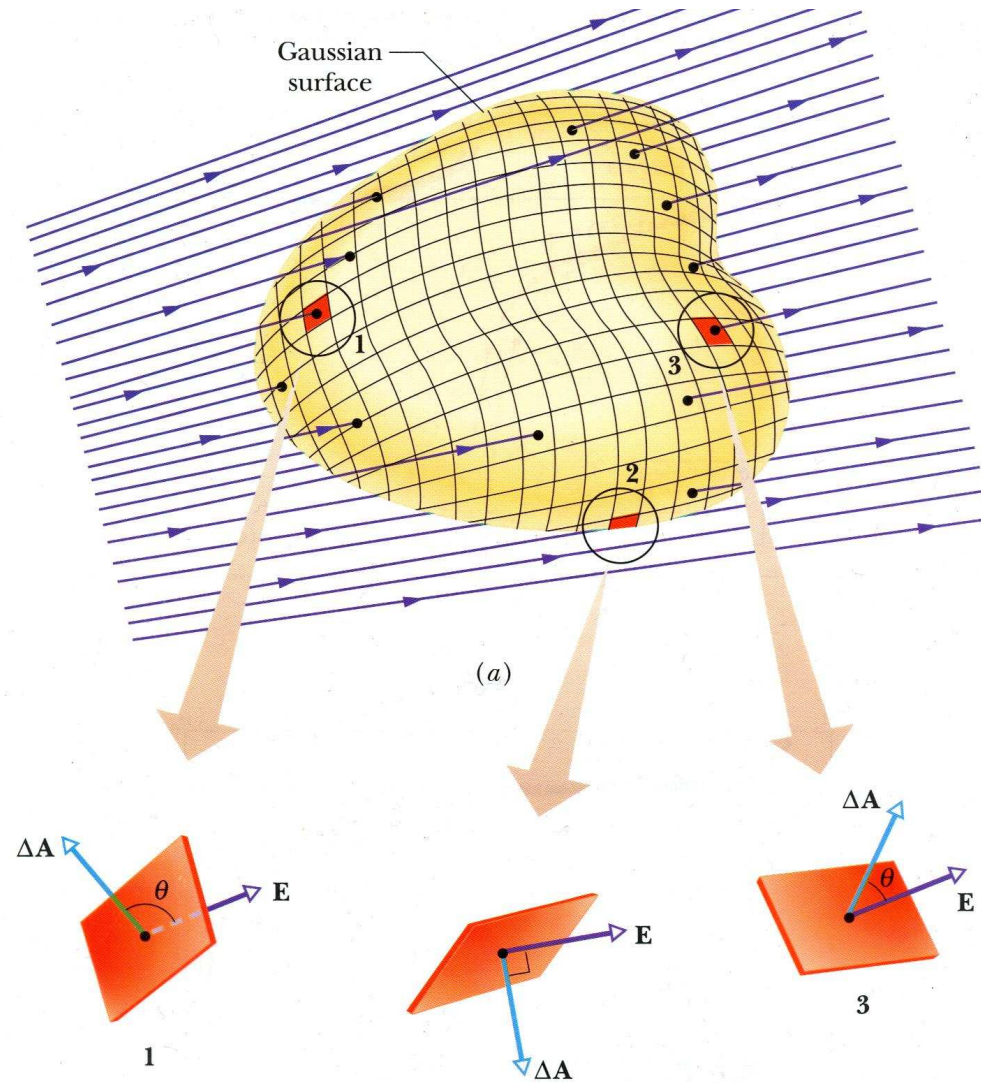


Consider a surface S of arbitrary shape in the presence of an electric field \vec{E} .
Prescription for the calculation of the electric flux through S :

- Divide S into small tiles of area ΔA_i .
- Introduce vector $\Delta \vec{A}_i = \hat{n}_i \Delta A_i$ perpendicular to tile.
 - If S is open choose consistently one of two possible directions for $\Delta \vec{A}_i$.
 - If S is closed choose $\Delta \vec{A}_i$ to be directed outward.
- Electric field at position of tile i : \vec{E}_i .
- Electric flux through tile i :
$$\Delta \Phi_i^{(E)} = \vec{E}_i \cdot \Delta \vec{A}_i = E_i \Delta A_i \cos \theta_i.$$
- Electric flux through S : $\Phi_E = \sum_i \vec{E}_i \cdot \Delta \vec{A}_i$.
- Limit of infinitesimal tiles: $\Phi_E = \int \vec{E} \cdot d\vec{A}$.
- Electric flux is a scalar.
- The SI unit of electric flux is Nm^2/C .



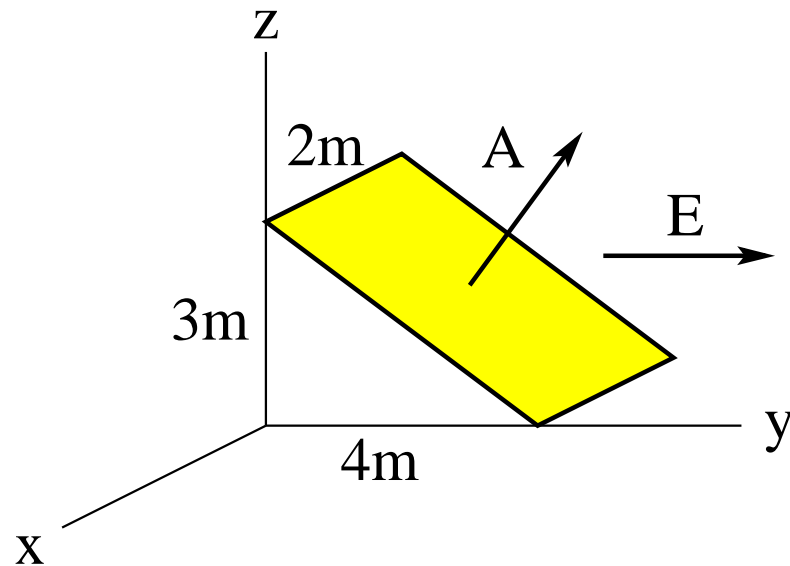
Electric Flux: Illustration



Electric Flux: Application (1)



Consider a rectangular sheet oriented perpendicular to the yz plane as shown and positioned in a uniform electric field $\vec{E} = (2\hat{j})\text{N/C}$.

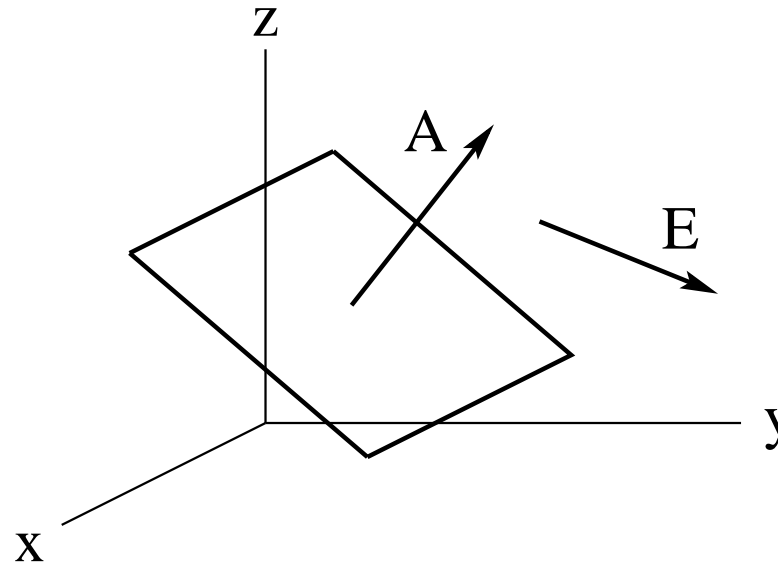


- (a) Find the area A of the sheet.
- (b) Find the angle between \vec{A} and \vec{E} .
- (c) Find the electric flux through the sheet.

Electric Flux: Application (2)



Consider a plane sheet of paper whose orientation in space is described by the area vector $\vec{A} = (3\hat{j} + 4\hat{k})\text{m}^2$ positioned in a region of uniform electric field $\vec{E} = (1\hat{i} + 5\hat{j} - 2\hat{k})\text{N/C}$.

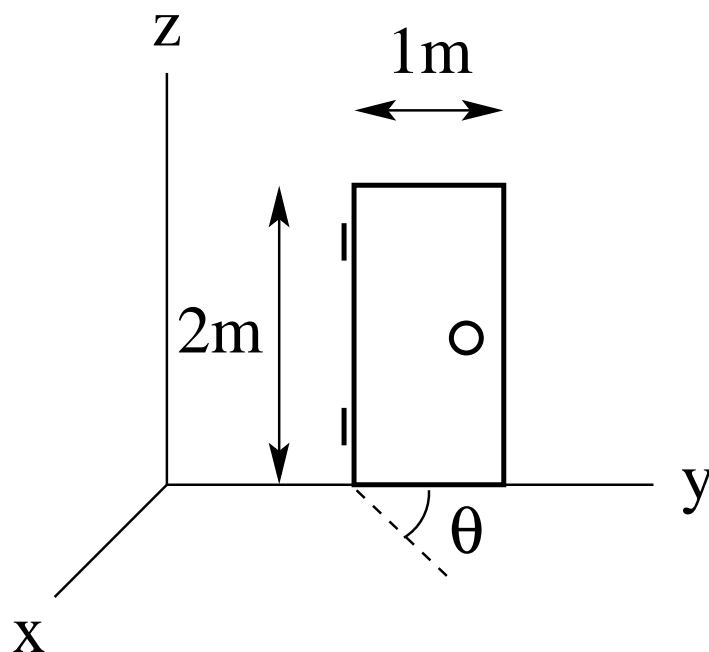


- Find the area of the sheet.
- Find the magnitude of the electric field.
- Find the electric flux through the sheet.
- Find the angle between \vec{A} and \vec{E} .

Electric Flux: Application (3)



The room shown below is positioned in an electric field $\vec{E} = (3\hat{i} + 2\hat{j} + 5\hat{k})\text{N/C}$.



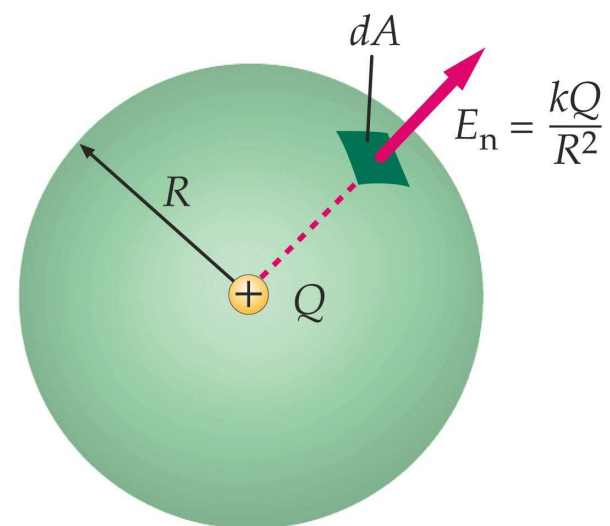
- What is the electric flux Φ_E through the closed door?
- What is the electric flux Φ_E through the door opened at $\theta = 90^\circ$?
- At what angle θ_1 is the electric flux through the door zero?
- At what angle θ_2 is the electric flux through the door a maximum?

Electric Flux: Application (4)



Consider a positive point charge Q at the center of a spherical surface of radius R . Calculate the electric flux through the surface.

- \vec{E} is directed radially outward. Hence \vec{E} is parallel to $d\vec{A}$ everywhere on the surface.
- \vec{E} has the same magnitude, $E = kQ/R^2$, everywhere on the surface.
- The area of the spherical surface is $A = 4\pi R^2$.
- Hence the electric flux is $\Phi_E = EA = 4\pi kQ$.
- Note that Φ_E is independent of R .

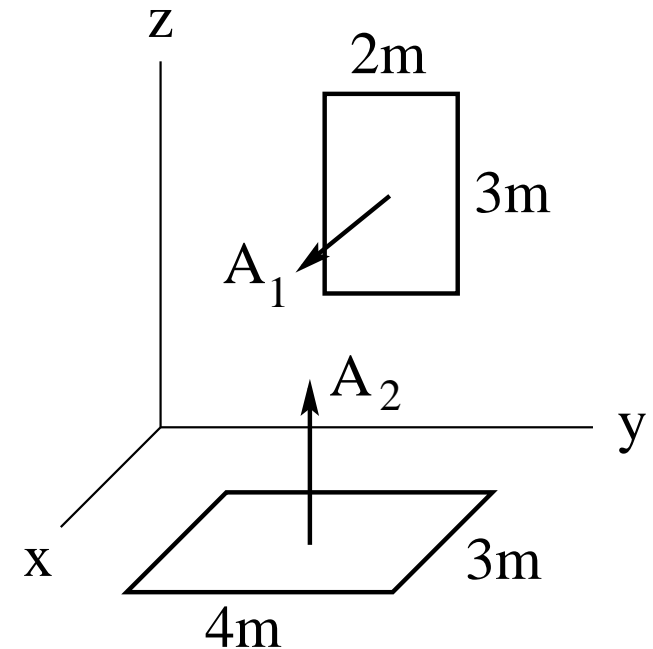


Intermediate Exam I: Problem #3 (Spring '05)



Consider two plane surfaces with area vectors \vec{A}_1 (pointing in positive x -direction) and \vec{A}_2 (pointing in positive z -direction). The region is filled with a uniform electric field $\vec{E} = (2\hat{i} + 7\hat{j} - 3\hat{k})\text{N/C}$.

- (a) Find the electric flux $\Phi_E^{(1)}$ through area A_1 .
- (b) Find the electric flux $\Phi_E^{(2)}$ through area A_2 .



Intermediate Exam I: Problem #3 (Spring '05)

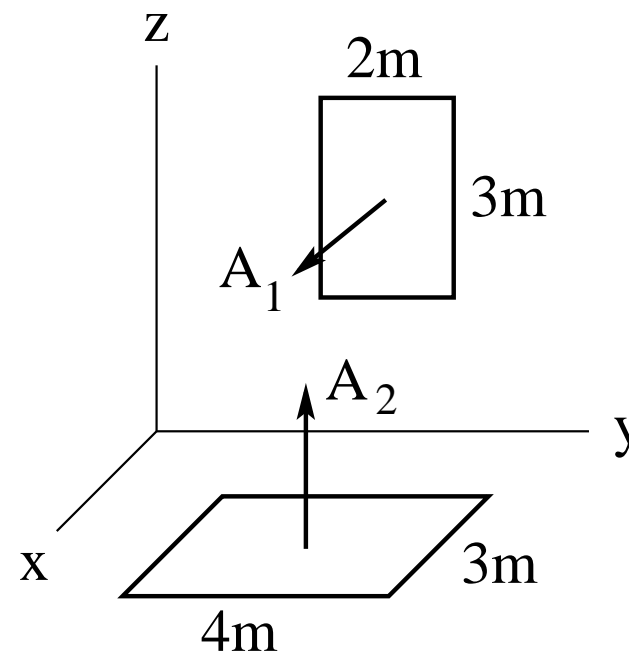


Consider two plane surfaces with area vectors \vec{A}_1 (pointing in positive x -direction) and \vec{A}_2 (pointing in positive z -direction). The region is filled with a uniform electric field $\vec{E} = (2\hat{i} + 7\hat{j} - 3\hat{k})\text{N/C}$.

- (a) Find the electric flux $\Phi_E^{(1)}$ through area A_1 .
- (b) Find the electric flux $\Phi_E^{(2)}$ through area A_2 .

Solution:

- (a) $\vec{A}_1 = 6\hat{i} \text{ m}^2$,
 $\Phi_E^{(1)} = \vec{E} \cdot \vec{A}_1 = (2\text{N/C})(6\text{m}^2) = 12\text{Nm}^2/\text{C}$.



Intermediate Exam I: Problem #3 (Spring '05)

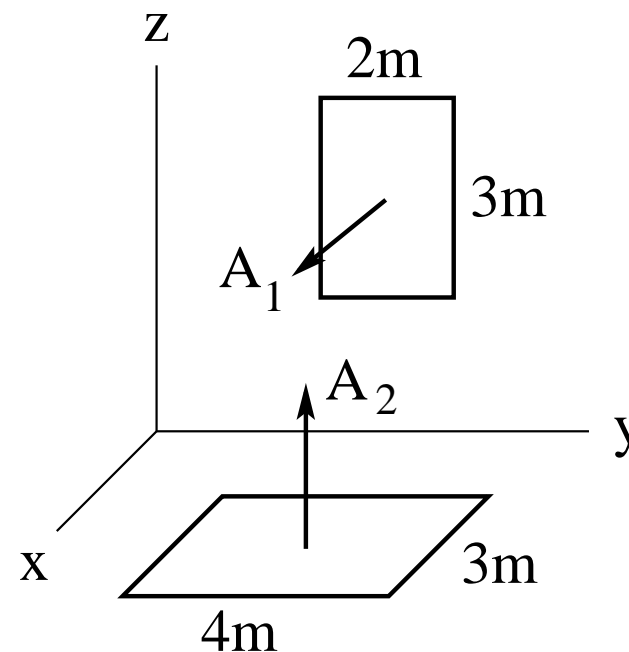


Consider two plane surfaces with area vectors \vec{A}_1 (pointing in positive x -direction) and \vec{A}_2 (pointing in positive z -direction). The region is filled with a uniform electric field $\vec{E} = (2\hat{i} + 7\hat{j} - 3\hat{k})\text{N/C}$.

- (a) Find the electric flux $\Phi_E^{(1)}$ through area A_1 .
- (b) Find the electric flux $\Phi_E^{(2)}$ through area A_2 .

Solution:

- (a) $\vec{A}_1 = 6\hat{i} \text{ m}^2$,
 $\Phi_E^{(1)} = \vec{E} \cdot \vec{A}_1 = (2\text{N/C})(6\text{m}^2) = 12\text{Nm}^2/\text{C}$.
- (b) $\vec{A}_2 = 12\hat{k} \text{ m}^2$,
 $\Phi_E^{(2)} = \vec{E} \cdot \vec{A}_2 = (-3\text{N/C})(12\text{m}^2) = -36\text{Nm}^2/\text{C}$.

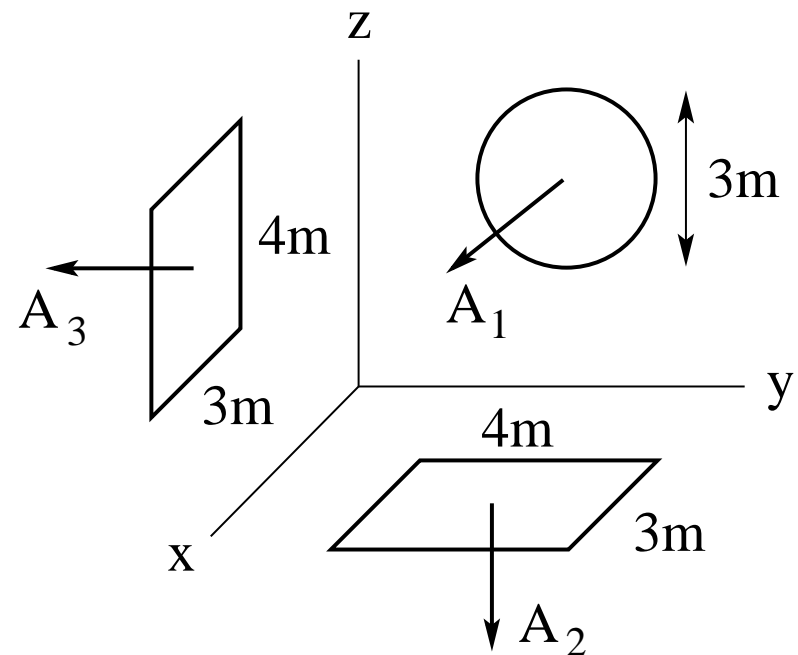


Unit Exam I: Problem #2 (Spring '13)



Consider three plane surfaces (one circle and two rectangles) with area vectors \vec{A}_1 (pointing in positive x -direction), \vec{A}_2 (pointing in negative z -direction), and \vec{A}_3 (pointing in negative y -direction) as shown. The region is filled with a uniform electric field $\vec{E} = (-3\hat{i} + 9\hat{j} - 4\hat{k})\text{N/C}$ or $\vec{E} = (2\hat{i} - 6\hat{j} + 5\hat{k})\text{N/C}$.

- Find the electric flux $\Phi_E^{(1)}$ through surface 1.
- Find the electric flux $\Phi_E^{(2)}$ through surface 2.
- Find the electric flux $\Phi_E^{(3)}$ through surface 3.



Unit Exam I: Problem #2 (Spring '13)



Solution:

$$(a) \vec{A}_1 = \pi(1.5\text{m})^2\hat{i} = 7.07\text{m}^2\hat{i}, \quad \Phi_E^{(1)} = \vec{E} \cdot \vec{A}_1 = (-3\text{N/C})(7.07\text{m}^2) = -21.2\text{Nm}^2/\text{C}.$$

$$\vec{A}_1 = \pi(1.5\text{m})^2\hat{i} = 7.07\text{m}^2\hat{i}, \quad \Phi_E^{(1)} = \vec{E} \cdot \vec{A}_1 = (2\text{N/C})(7.07\text{m}^2) = 14.1\text{Nm}^2/\text{C}.$$

$$(b) \vec{A}_2 = (3\text{m})(4\text{m})(-\hat{k}) = -12\text{m}^2\hat{k}, \quad \Phi_E^{(2)} = \vec{E} \cdot \vec{A}_2 = (-4\text{N/C})(-12\text{m}^2) = 48\text{Nm}^2/\text{C}.$$

$$\vec{A}_2 = (3\text{m})(4\text{m})(-\hat{k}) = -12\text{m}^2\hat{k}, \quad \Phi_E^{(2)} = \vec{E} \cdot \vec{A}_2 = (5\text{N/C})(-12\text{m}^2) = -60\text{Nm}^2/\text{C}.$$

$$(b) \vec{A}_3 = (3\text{m})(4\text{m})(-\hat{j}) = -12\text{m}^2\hat{j}, \quad \Phi_E^{(3)} = \vec{E} \cdot \vec{A}_3 = (9\text{N/C})(-12\text{m}^2) = -108\text{Nm}^2/\text{C}.$$

$$\vec{A}_3 = (3\text{m})(4\text{m})(-\hat{j}) = -12\text{m}^2\hat{j}, \quad \Phi_E^{(3)} = \vec{E} \cdot \vec{A}_3 = (-6\text{N/C})(-12\text{m}^2) = 72\text{Nm}^2/\text{C}.$$