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02. Equilibrium Thermodynamics II: Engines

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Abstract

Part two of course materials for Statistical Physics I: PHY525, taught by Gerhard Müller at the University of Rhode Island. Documents will be updated periodically as more entries become presentable.

Contents of this Document [ttc2]

- Circular heat engine I. [tex147]
- Circular heat engine II. [tex148]
- Square heat engine. [tex149]

Carnot engine [tln11]

Second law: Heat flows spontaneously from high to low temperatures.

Thermal contact: Temperature differences disappear without producing work.

Heat engine: Part of the heat flowing from high to low temperatures is converted into work via a cyclic process.

Carnot engine: All wasteful heat flows are eliminated (reversible processes).

The four steps of a Carnot process:

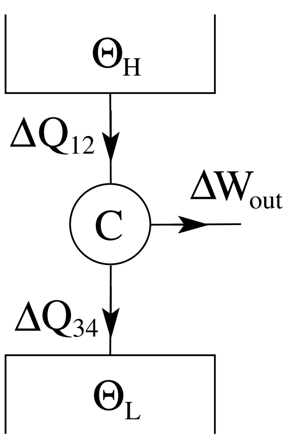
- $1 \to 2$: Isothermal absorption of heat: $\Delta Q_{12} > 0$ at Θ_H.
- $2 \to 3$: Adiabatic cooling: $\Theta_H \to \Theta_L$ with $\Delta Q_{23} = 0$ and $\Delta W_{23} < 0$.
- $3 \to 4$: Isothermal expulsion of heat: $\Delta Q_{34} < 0$ at Θ_L.
- $4 \to 1$: Adiabatic heating: $\Theta_L \to \Theta_H$ with $\Delta Q_{41} = 0$ and $\Delta W_{41} > 0$.

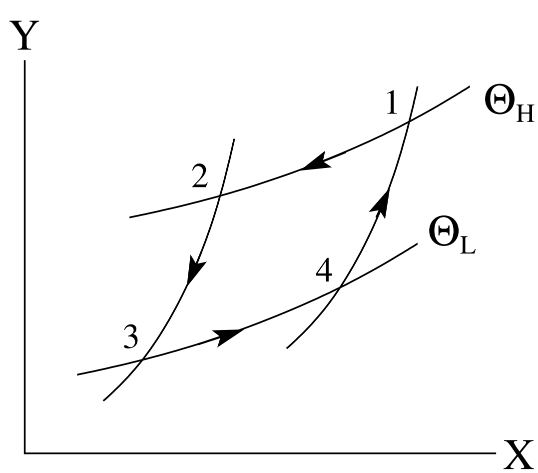
Total heat input: $\Delta Q_{in} = \Delta Q_{12}$.

Use first law: $\Delta U = \Delta Q_{12} + \Delta W_{12} + \Delta W_{23} + \Delta Q_{34} + \Delta W_{34} + \Delta W_{41} = 0$.

Net work output: $\Delta W_{out} \equiv -\Delta W_{12} - \Delta W_{23} - \Delta W_{34} - \Delta W_{41} = \Delta Q_{12} - |\Delta Q_{34}|$

Efficiency: $\eta \equiv \dfrac{\Delta W_{out}}{\Delta Q_{in}} = 1 - \dfrac{|\Delta Q_{34}|}{\Delta Q_{12}}$.

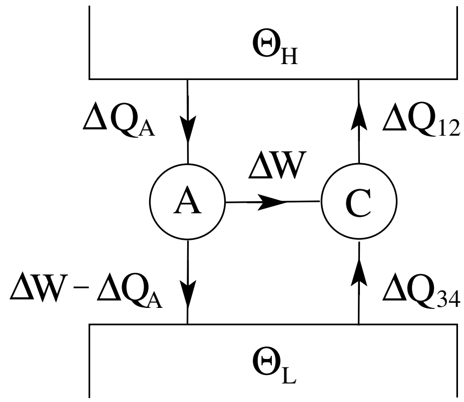




Maximum efficiency [tln12]

Is it possible to construct a heat engine A which is more efficient than the Carnot engine C?

Use engine A to drive engine C in the reverse i.e. as a refrigerator.



Heat transfers: $\Delta Q_A > 0,\ \Delta Q_{12} < 0,\ \Delta Q_{34} > 0.$

Work performance: $\Delta W = \Delta W_{out}^{(A)} = \Delta W_{in}^{(C)} > 0.$

Efficiencies: $\eta_A = \dfrac{\Delta W}{\Delta Q_A}, \quad \eta_C = \dfrac{\Delta W}{|\Delta Q_{12}|}$

Since engine C operates reversibly, η_C is the same in the forward and reverse directions. Note: η_C is not an *efficiency* in the reverse mode.

$\eta_A > \eta_C$ would imply $\Delta Q_A < |\Delta Q_{12}|$.

The two engines combined would then cause heat to flow from low to high temperature without work input, which is a violation of the second law.

Conclusions:

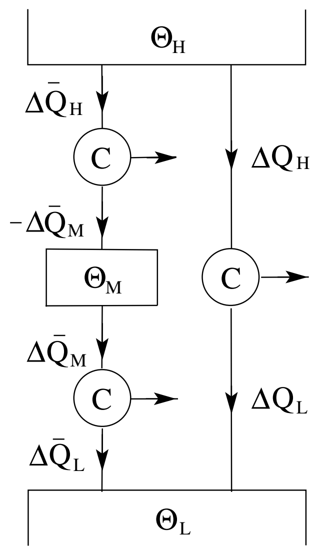
- Engine A cannot be more efficient than engine C.
- All Carnot engines operating between Θ_H and Θ_L must have the same efficiency.

Absolute temperature [tln13]

Reservoir temperatures: $\Theta_H, \Theta_M, \Theta_L$.

Efficiency: $\eta = 1 - \dfrac{|\Delta Q_L|}{\Delta Q_H} = 1 - f(\Theta_L, \Theta_H)$.

Likewise: $\dfrac{\Delta\bar{Q}_M}{\Delta\bar{Q}_H} = f(\Theta_M, \Theta_H), \quad \dfrac{|\Delta\bar{Q}_L|}{\Delta\bar{Q}_M} = f(\Theta_L, \Theta_M)$.



Second law implies: If $\Delta\bar{Q}_L = \Delta Q_L$ then $\Delta\bar{Q}_H = \Delta Q_H$.

$$\Rightarrow \frac{|\Delta\bar{Q}_L|}{\Delta\bar{Q}_M}\frac{\Delta\bar{Q}_M}{\Delta\bar{Q}_H} = \frac{|\Delta Q_L|}{\Delta Q_H} \Rightarrow f(\Theta_L, \Theta_M) f(\Theta_M, \Theta_H) = f(\Theta_L, \Theta_H)$$

Functional form: $f(\Theta_L, \Theta_H) = \dfrac{g(\Theta_L)}{g(\Theta_H)} \equiv \dfrac{T_L}{T_H} \Rightarrow \eta = 1 - \dfrac{T_L}{T_H}$.

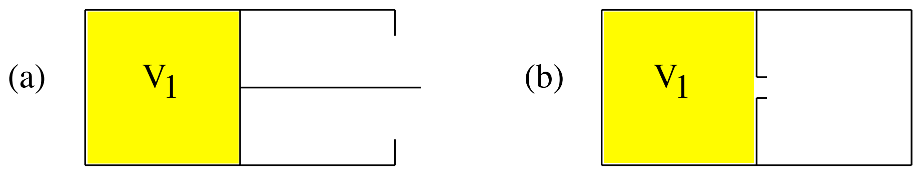
Definition of absolute temperature: $\dfrac{T_L}{T_H} = \dfrac{|\Delta Q_L|}{\Delta Q_H}$.

Kelvin scale is fixed by triple point of water: $T_{trp} = 273.16\text{K}$.

Note: $\eta = 1$ implies $T_L = 0$. However, the third law states $\delta Q = TdS = 0$ at $T = 0$. Hence, all reversible processes at $T = 0$ are adiabatic. Heat cannot be absorbed reversibly at $T = 0$.

[tex1] Entropy change caused by expanding ideal gas

Consider the amount $n = 1\text{mol}$ of a classical ideal gas in a box of volume V_1 with heat-conducting walls. The gas is described by the equation of state $pV = nRT$ and the internal energy $U = C_V T$ with $C_V = \text{const}$. Now we let the gas expand to the volume $V_2 = 2V_1$ via two different processes:
(a) by quasi-static isothermal expansion;
(b) by leakage through a hole in one wall.
Calculate the change in entropy ΔS_G of the gas and ΔS_E of the environment during each process. Express the results in SI units.



Solution:

[tex3] Carnot cycle of the classical ideal gas

Consider the four steps of a Carnot engine with the operating material in the form of a classical ideal gas [$pV = nRT$, $U = C_V T$ with $C_V = \text{const}$].

(a) Determine the heat transfer, ΔQ, the work performance, ΔW, and the change in internal energy, ΔU, for each of the four steps:

$1 \to 2$ *isothermal expansion*: $T = T_H = \text{const}$, $V_2 > V_1$.

$2 \to 3$ *adiabatic expansion*: $S = \text{const}$, $V_3 > V_2$.

$3 \to 4$ *isothermal compression*: $T = T_L = \text{const}$, $V_4 < V_3$.

$4 \to 1$ *adiabatic compression*: $S = \text{const}$, $V_1 < V_4$.

(b) Sketch the Carnot cycle in the (V, p)-plane and in the (U, S)-plane.

(c) Show that the efficiency is $\eta_C = 1 - T_L/T_H$.

Solution:

[tex4] Carnot cycle of an ideal paramagnet

Consider the four steps of a Carnot engine with the operating material in the form of an ideal paramagnet. The equation of state is Curie's law, $M = DH/T$, where H is the magnetic field, T the absolute temperature, and D a constant. The internal energy is a monotonically increasing function, $U(T)$, of temperature.
(a) Determine the heat transfer, ΔQ, the work performance, ΔW, and the change in internal energy, ΔU, for each of the four steps:
$1 \rightarrow 2$ *isothermal demagnetization*: $T = T_H = \text{const}$, $M_2 < M_1$.
$2 \rightarrow 3$ *adiabatic demagnetization*: $S = \text{const}$, $M_3 < M_2$.
$3 \rightarrow 4$ *isothermal magnetization*: $T = T_L = \text{const}$, $M_4 > M_3$.
$4 \rightarrow 1$ *adiabatic magnetization*: $S = \text{const}$, $M_1 > M_4$.
(b) Sketch the Carnot cycle in the (M, H)-plane and in the (U, S)-plane.
(c) Show that the efficiency is $\eta_C = 1 - T_L/T_H$.

Solution:

Reversible processes in fluid system [tln15]

Isothermal process: $T = \text{const.}$ $\quad \delta Q \neq 0$ in general.

Isochoric process: $V = \text{const.}$ $\quad \delta Q = C_V dT,\ dU = C_V dT.$

Isobaric process: $p = \text{const.}$ $\quad \delta Q = C_p dT.$

Isentropic (adiabatic) process: $S = \text{const.}$ $\quad \delta Q = 0.$

Internal energy: $dU = \delta Q + \delta W = TdS - pdV.$

- $V = \text{const.} \;\Rightarrow\; \delta W = 0 \;\Rightarrow\; dU = \delta Q$ (no work performed).
- $S = \text{const.} \;\Rightarrow\; \delta Q = 0 \;\Rightarrow\; dU = \delta W$ (no heat transferred).

Classical ideal gas:

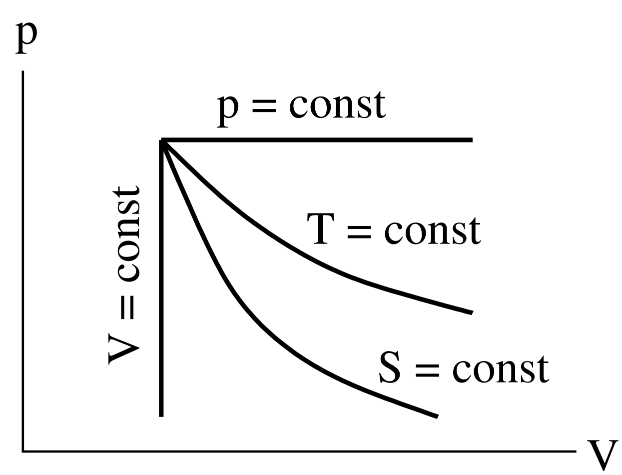
Equation of state: $pV = nRT.$

Internal energy: $U = C_V T,\ C_V = \alpha nR = \text{const.}$

Isotherm: $T = \text{const.} \;\Rightarrow\; pV = \text{const.}$

Adiabate: $S = \text{const.} \;\Rightarrow\; pV^\gamma = \text{const.},\ \gamma = 1 + 1/\alpha$

- monatomic gas: $\alpha = \frac{3}{2},\ \gamma = \frac{5}{3}.$
- diatomic gas: $\alpha = \frac{5}{2},\ \gamma = \frac{7}{5}.$
- polyatomic gas: $\alpha = 3,\ \gamma = \frac{4}{3}.$



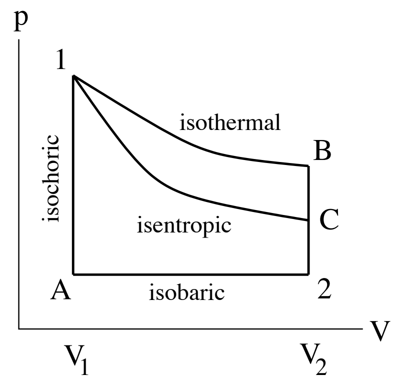
[tex7] Adiabates of the classical ideal gas

The classical ideal gas is specified by the thermodynamic equation of state $pV = nRT$ and by the internal energy (caloric equation of state) $U = C_V T$ with $C_V = \alpha nR = \text{const}$ [$\alpha = \frac{3}{2}$ (monatomic), $\alpha = \frac{5}{2}$ (diatomic), $\alpha = 3$ (polyatomic)]. A reversible process with $S = \text{const}$ is called *isentropic* or *adiabatic* and is characterized by the curve $pV^\gamma = \text{const}$. No heat is exchanged in an adiabatic process: $dU = \delta W$, $\delta Q = 0$. Find γ as a function of α.

Solution:

[tex25] Roads from 1 to 2: isothermal, isentropic, isochoric, isobaric

The amount $n = 1\text{mol}$ of an ideal gas undergoes three different quasistatic processes (see Figure) from the initial state (p_1, V_1, T_1) to the final state (p_2, V_2, T_2):
(i) $1 \rightarrow A \rightarrow 2$; (ii) $1 \rightarrow B \rightarrow 2$; (iii) $1 \rightarrow C \rightarrow 2$.
Find the work ΔW done on the system and the heat ΔQ added to the system in each process. Express all results in terms of (T_1, V_1, T_2, V_2).



Solution:

[tex13] Room heater: Electric radiator versus heat pump

A room is to be kept at temperature $T_H = 294\text{K}, (21°\text{C})$. The outdoor temperature is T_L. Heat, which leaks through the windows and doors at the rate $\bar{Q}_{leak} = \gamma\Delta T$, must be replaced by a room heater at the same rate. The electric radiator converts electric power $\bar{W}_{el}$ into heat with 100% efficiency. The electric heat pump uses the amount $\bar{W}_{sup}$ of electric power to drive a Carnot cycle in the reverse, which extracts heat $\bar{Q}_L$ at temperature T_L from the exterior and converts it (reversibly) into heat $\bar{Q}_H = \bar{Q}_L + \bar{W}_{hp}$ at temperature T_H. In the relation $\bar{W}_{hp} = (1-\lambda)\bar{W}_{sup},\ \lambda$ represents the energy loss in the gears of the heat pump. Quantitites with overbars denote energy transfers per time unit.

(a) Find $\bar{W}_{el}$ as a function of γ, T_H, T_L, and $\bar{W}_{sup}$ as a function of $\gamma, \lambda, T_H, T_L$.

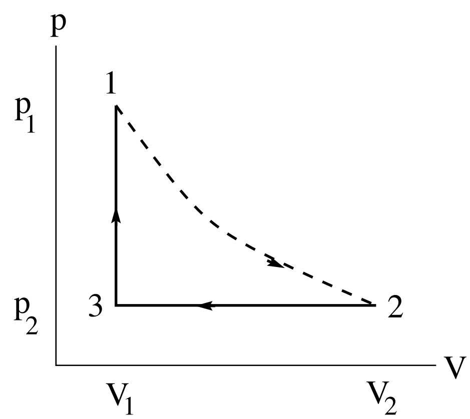
(b) Plot $\bar{W}_{el}/\gamma$ and $\bar{W}_{sup}/\gamma$ versus $t_L \equiv T_L - 273\text{K}$ (measured in °C) for fixed $T_H = 294\text{K}$ and $\lambda = 0.8$ (20% efficiency).

(c) Determine the range of T_L where the heat pump is more economical than the radiator.

Solution:

[tex12] Mayer's relation for the heat capacities of the classical ideal gas

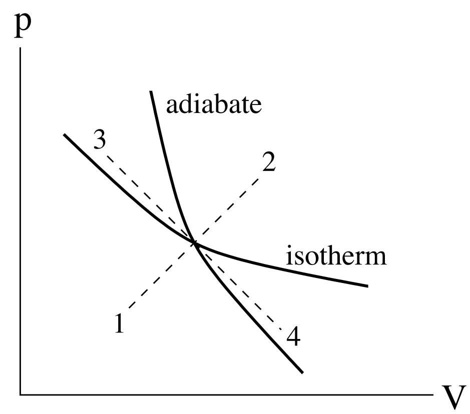
The amount $n = 1\text{mol}$ of a classical ideal gas [$pV = nRT$, $U = C_V T$ with $C_V = \text{const}$] is initially confined to a volume V_1 at pressure p_1. In step $1 \to 2$ of Mayer's cycle, the gas undergoes free expansion to volume V_2 while it is thermally isolated ($\delta Q = 0, \delta W = 0$) The pressure decreases from p_1 to p_2 during this step. In step $2 \to 3$ the gas is quasi-statically compressed back to volume V_1, while the pressure is maintained at p_2. With the temperature decreasing during this step, heat is expelled. In step $3 \to 1$ the gas is heated up quasi-statically at constant volume V_1 until the pressure returns to p_1. Use the first law to derive Mayer's relation, $C_p - C_V = R$, between the heat capacities of the classical ideal gas.



Solution:

[tex26] Positive and negative heat capacities

The $p-V$ diagram shows an *isotherm* and an *adiabate* for the classical ideal gas. Show that a quasistatic process of the type $1 \to 2$ is characterized by a *positive* heat capacity and a process of the type $3 \to 4$ by a *negative* heat capacity.



Solution:

[tex9] Work extracted from finite heat reservoir in infinite environment

A (finite) heat reservoir with heat capacity $C = \text{const}$ is initially at temperature T_H and the (infinite) environment at the lower temperature T_0. Now the reservoir is connected to the environment by a heat engine, which absorbs an infinitesimal amount of heat δQ per cycle, converts part of it into work δW, and dumps the rest into the environment. During each cycle the temperature of the reservoir decreases infinitesimally: $\delta Q = -CdT$. Determine the maximum amount of work ΔW that can be extracted from the reservoir before its temperature has dropped to that of the environment. The fraction of the excess internal energy $U_{ex} = C(T_H - T_0)$ that can be converted into work is characterized by the quantity $\Delta W/U_{ex}$. Plot this quantity versus the reduced temperature $(T_H - T_0)/T_0$ for $T_0 < T_H < 3T_0$. Set $T_H/T_0 = 1 + \epsilon$ with $\epsilon \ll 1$ and find the dependence of $\Delta W/U_{ex}$ on ϵ to leading order.

Solution:

[tex10] Work extracted from finite heat reservoir in finite environment

A (finite) heat reservoir with heat capacity $C_H = \text{const}$ is initially at temperature T_H and the (finite) environment with heat capacity C_L at the lower temperature T_L.
(a) When heat is allowed to flow from the reservoir to the environment, both will end up at the temperature $T_f = (C_H T_H + C_L T_L)/(C_H + C_L)$ (arithmetic mean). Verify this and determine the total amount of heat ΔQ that has been transferred.
(b) When the reservoir is connected to the environment by a Carnot engine which absorbs an infinitesimal amount of heat δQ per cycle, converts part of it into work δW, and dumps the rest into the environment, the final common temperature of the reservoir and the environment will be $T_f = T_H^{C_H/(C_H+C_L)} T_L^{C_L/(C_H+C_L)}$ (geometric mean). Verify this and determine the total amout of work ΔW that has been extracted from the system. The fraction of the excess internal energy $U_{ex} = C_H(T_H - T_L)$ that can be converted into work is characterized by the quantity $\Delta W/U_{ex}$. Plot this quantity versus the reduced temperature $(T_H - T_L)/T_L$ for $C_H = C_L$ and $T_L < T_H < 3T_L$. Discuss the properties of this function in the limit $T_H \to T_L$.

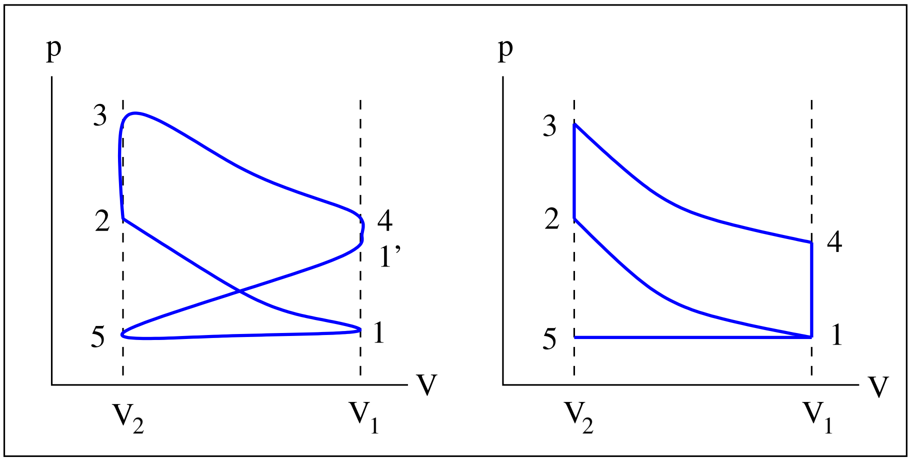
Solution:

[tex2] Heating the air in a room

Calculate the amount of energy ΔQ that must be supplied to heat the air in a room from 0°C to 20°C under three different circumstances. For each case, calculate also the change in internal energy ΔU of the air in the room. Mass density of air at STP (0°C and 1atm): $\rho = 0.00129\text{g/cm}^3$. Specific heats of air: $c_V = 0.169\text{cal/gK}$, $c_p/c_V \equiv \gamma = 1.41$. Express all results in SI units.
(a) The room has rigid, insulating walls. The volume is 27m^3. The initial pressure is 1atm.
(b) The room has insulating walls. One wall is mobile. The process takes place at constant pressure (1atm). The initial volume is 27m^3.
(c) The room has rigid, insulating walls. The volume is 27m^3. One wall has a small hole through which air leaks out slowly. The process takes place at constant pressure (1atm).

Solution:

Gasoline engine (Otto cycle) [tln65]



Four-stroke Otto cycle (left)

1-2: compression stroke

2-3-4: power stroke (spark plug ignites at 2)

4-1'-5: exhaust stroke (exhaust valve opens at 4)

5-1: intake stroke (intake valve opens at 5)

Idealized Otto cycle (right)

1-2: adiabatic compression of air-fuel mixture ($S = \text{const}$)

2-3: explosion of air-fuel mixture ($V = \text{const}$)

3-4: adiabatic expansion of exhaust gas ($S = \text{const}$)

4-1: isochoric release of exhaust gas ($V = \text{const}$).

1-5-1: intake stroke (thermodynamically ignored)

Parameter: $K \doteq V_1/V_2$ (compression ratio).

The compression ratio K must not be chosen too large to prevent the air-fuel mixture from igniting spontaneously, i.e. prematurely.

[tex8] Idealized Otto cycle

Consider the four steps of the idealized Otto cycle for a classical ideal gas [$pV = nRT$, $U = C_V T$ with $C_V = \alpha n R$].

(a) Determine the heat transfer, ΔQ, the work performance, ΔW, and the change in internal energy, ΔU, for each of the four steps:

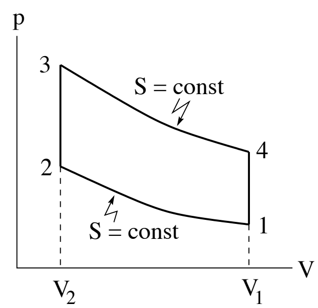
$1 \to 2$ adiabatic compression of air-fuel mixture: $S = \text{const.}$

$2 \to 3$ explosion of air-fuel mixture: $V = \text{const.}$

$3 \to 4$ adiabatic expansion of exhaust gas: $S = \text{const.}$

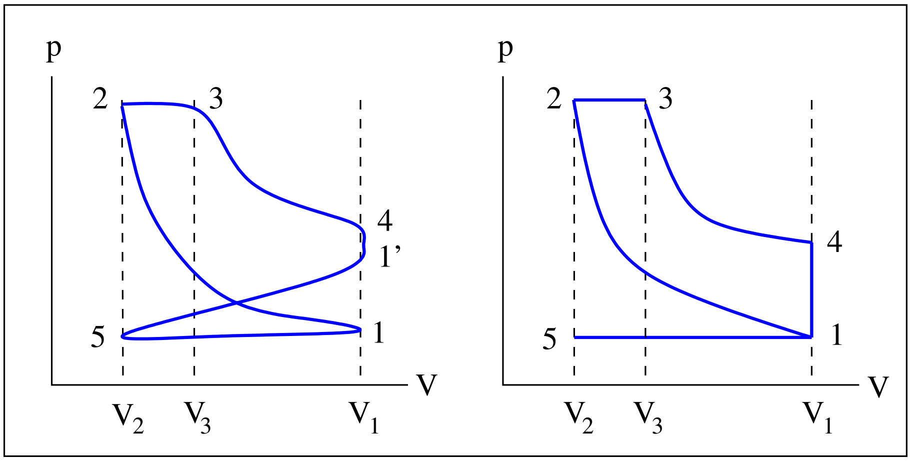
$4 \to 1$ isochoric release of exhaust gas: $V = \text{const.}$

(c) Calculate the efficiency η and express it as a function of the compression ratio $K \equiv V_1/V_2$.



Solution:

Diesel engine [tln66]



Four-stroke Diesel cycle (left)

1-2: compression stroke (fuel injected and spontaneously ignited at 2)
2-3-4: power stroke (Diesel fuel burns more slowly than gasoline)
4-1'-5: exhaust stroke (exhaust valve opens at 4)
5-1: intake stroke (intake valve opens at 5)

Idealized Diesel cycle (right)

1-2: adiabatic compression of air ($S = \text{const}$)
2-3: isobaric expansion as fuel explodes ($p = \text{const}$)
3-4: adiabatic expansion of exhaust gas ($S = \text{const}$)
4-1: isochoric release of exhaust gas ($V = \text{const}$).
1-5-1: intake stroke (thermodynamically ignored)

Parameter: $K \doteq V_1/V_2$ (compression ratio), $L \doteq V_3/V_2$

[tex16] Idealized Diesel cycle

Consider the four steps of the Diesel cycle for a classical ideal gas [$pV = nRT$, $U \equiv C_V T$ with $C_V = \alpha nR$].

(a) Determine the heat transfer, ΔQ, the work performance, ΔW, and the change in internal energy, ΔU, for each of the four steps:

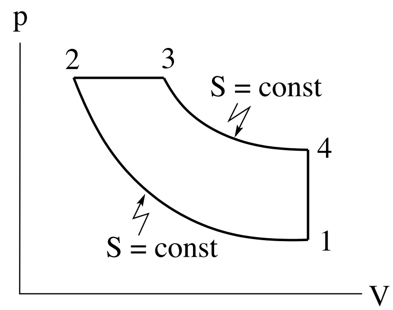
$1 \to 2$ adiabatic compression of air: $S = \text{const.}$

$2 \to 3$ isobaric expansion during explosion: $p = \text{const.}$

$3 \to 4$ adiabatic expansion after explosion: $S = \text{const.}$

$4 \to 1$ isochoric release of exhaust gas: $V = \text{const.}$

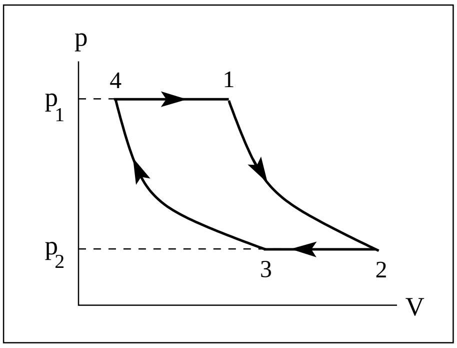
Calculate the efficiency η and express it as a function of the two parameters $K \equiv V_1/V_2$ and $L \equiv V_3/V_2$.



Solution:

Escher-Wyss gas turbine [tln75]

A gas flows in a closed system from the boiler via the turbine to the radiator and then via the compressor back into the boiler. As the beam of hot gas hits the blades of the turbine during the power stroke it expands with little heat transfer. The compression of the cooled gas is also roughly adiabatic. The gas is heated up inside the boiler and cooled down inside the radiator at different but roughly constant pressures.



Idealized process (Joule cycle)

1-2: Adiabatic expansion of the hot gas after ejection from the boiler as it drives the turbine ($S = \text{const}$).

2-3: Isobaric contraction as the gas flows through the radiator and cools down further in the process ($p = \text{const}$).

3-4: Adiabatic compression of the cooled gas for injection into the boiler ($S = \text{const}$).

4-1: Isobaric expansion of the gas as it heats up inside the boiler ($p = \text{const}$).

Notes:

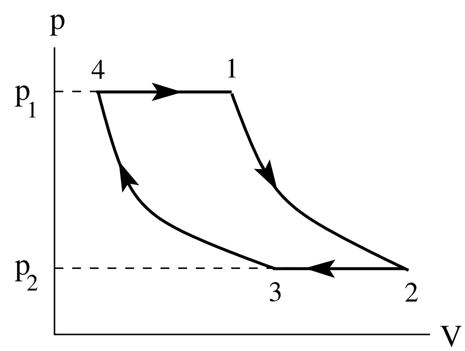
- The pressure inside the boiler is regulated by the rates of gas injection and ejection and the rate of heat transfer from the energy source to the gas.
- The injection and ejection rates are the same in mass units but the ejection rate is larger than the injection rate in volume units. This accounts for the expansion of the gas inside the boiler as described in step 4-1.

[tex108] Joule cycle

Consider the four steps of the Joule cycle for the classical ideal gas [$pV = Nk_BT, C_V = \alpha Nk_B, \gamma \doteq C_p/C_V = (\alpha+1)/\alpha$]. It represents an idealized version of the Escher-Wyss gas turbine.
(a) Calculate the work performance, ΔW, the heat transfer, ΔQ, and the change in internal energy, ΔU, for each step.

$1 \to 2$ adiabatic expansion: $S = \text{const.}$
$2 \to 3$ isobaric contraction: $p = \text{const.}$
$3 \to 4$ adiabatic compression: $S = \text{const.}$
$4 \to 1$ isobaric expansion: $p = \text{const.}$

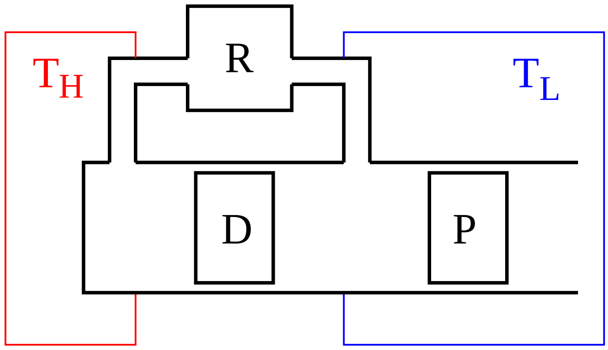
(b) Calculate the efficiency η and express it as a function of the pressure ratio p_2/p_1.
(c) Sketch the Joule cycle in the (U, S)-plane.

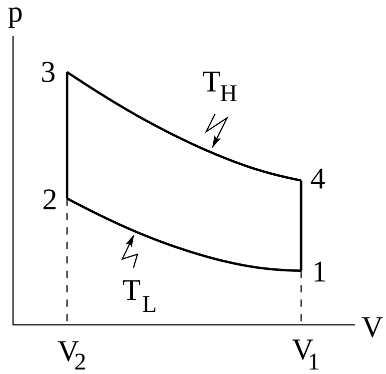


Solution:

Stirling engine [tln76]

The Stirling engine is an external combustion engine. It isolates the working fluid from the heat source. Combustion is better controlled than in internal combustion engines.





Piston P expands gas at high temperature T_H and compresses gas at low temperature T_L.

Displacer D moves gas between regions of high temperature T_H and low temperature T_L through the regenerator.

Regenerator R acts as a heat exchanger. It stores heat when hot gas flows from left to right and releases heat when colder gas flows from right to left.

Idealized Stirling cycle

1-2: Isothermal compression at temperature T_L.
Displacer stationary at left. Piston moving left.

2-3: Isochoric heating up at volume V_2.
Piston stationary at left. Displacer moving right.

3-4: Isothermal expansion at temperature T_H.
Displacer and piston moving right.

4-1: Isochoric cooling down at volume V_1.
Piston stationary at right. Displacer moving left.

Note: Some of the heat is recycled in the regenerator. This amount should not be counted in the expression $\eta = \Delta W_{out}/\Delta Q_{in}$ of the efficiency.

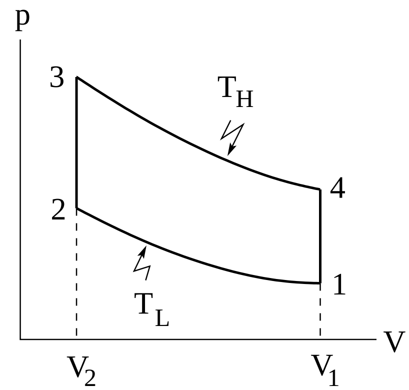
[tex131] Idealized Stirling cycle

Consider the four steps of the idealized Stirling cycle for the classical ideal gas [$pV = Nk_BT, C_V = \alpha Nk_B, \gamma \doteq C_p/C_V = (\alpha+1)/\alpha$].

(a) Calculate the work performance, ΔW, the heat transfer, ΔQ, and the change in internal energy, ΔU, for each step.

$1 \to 2$ isothermal compression: $T = T_L$,
$2 \to 3$ isochoric heating up: $V = V_2$,
$3 \to 4$ isothermal expansion: $T = T_H$,
$4 \to 1$ isochoric cooling down: $V = V_1$.

(b) Calculate the efficiency η and express it as a function of T_H and T_L.



Solution:

[tex106] Ideal-gas engine with two-step cycle I

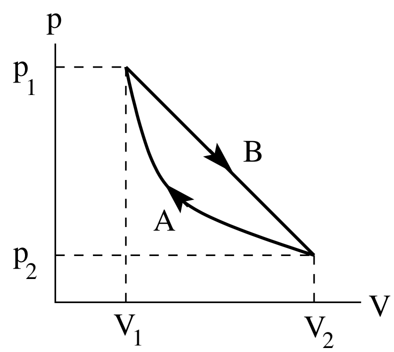
Consider the two-step cycle for a classical ideal gas [$pV = Nk_BT, C_V = \alpha Nk_B, \gamma \doteq C_p/C_V = (\alpha+1)/\alpha$] as shown. The first step (A) is an adiabatic compression and the second step (B) an expansion along a straight line segment in the (V,p)-plane.
(a) Show that the difference in internal energy $\Delta U \doteq U_1 - U_2$ is determined by the expression

$$\frac{\Delta U}{p_1V_1} = \alpha\left[1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}\right].$$

(b) Show that the heat transfer δQ between system and environment during a volume increase from V to $V+\delta V$ along the straight line segment is given by the expression

$$\frac{\delta Q}{p_1V_1} = \left[(1+\alpha)(1+\sigma) - (1+2\alpha)\sigma\,\frac{V}{V_1}\right]\frac{dV}{V_1}, \quad \sigma = \frac{1-(V_1/V_2)^\gamma}{V_2/V_1 - 1}.$$

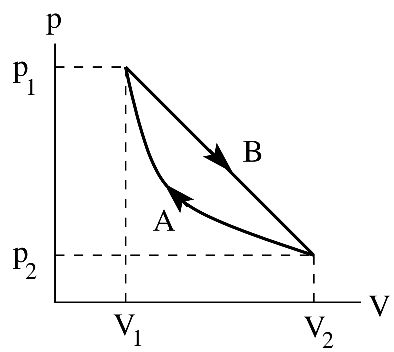
(c) Show that along the straight-line segment the system absorbs heat if $V_1 < V < V_c$ and expells heat if $V_c < V < V_2$, where $V_c/V_1 = [(1+\alpha)(1+\sigma)]/[(1+2\alpha)\sigma]$.



Solution:

[tex107] Ideal-gas engine with two-step cycle II

Consider the two-step cycle for a classical ideal gas [$pV = Nk_BT, C_V = \alpha Nk_B, \gamma \doteq C_p/C_V = (\alpha+1)/\alpha$] as previously discussed in [tex106]. For the following we assume that the compression ratio is $V_2/V_1 = 2$ and that the gas is monatomic ($\alpha = \frac{3}{2}$).
(a) Show that the net work output along the adiabate and along the straight line segment are $\Delta W_A/p_1V_1 \simeq 0.55506$ and $\Delta W_B/p_1V_1 \simeq -0.65749$.
(b) Show that the total heat absorbed during the cycle is $\Delta Q_{in}/p_1V_1 \simeq 0.39564$.
(c) Determine the efficiency η_2 of the two-step cycle and compare it with the efficiency η_C of the Carnot engine operating between the same two temperatures.



Solution:

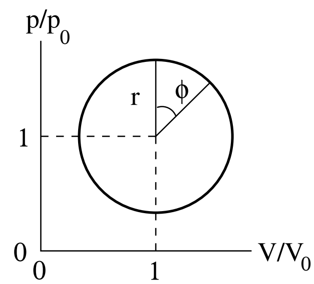
[tex147] Circular heat engine I

Consider 1 mol of a classical ideal gas [$pV = RT$] confined to a cylinder by a piston. The cylinder is in thermal contact with a heat bath of adjustable temperature. As the piston moves back and forth between volume $V = V_0(1-r)$ and $V = V_0(1+r)$ quasistatically, the temperature of the gas is being adjusted via thermal contact such that the cycle becomes circular in the (V,p)-plane and proceeds in clockwise direction (ϕ from 0 to 2π).

(a) Calculate the net work output ΔW_{out} during one cycle.

(b) Set $r = 0.5$ and identify the segments along the circle where the temperature of the gas rises and the segments where it falls.

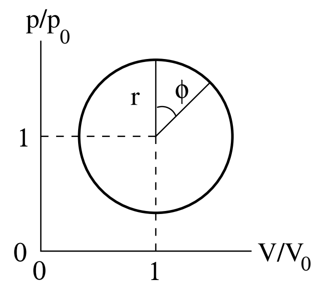
(c) Repeat the previous part for $r = 0.9$. Note that there now are more segments.



Solution:

[tex148] Circular heat engine II

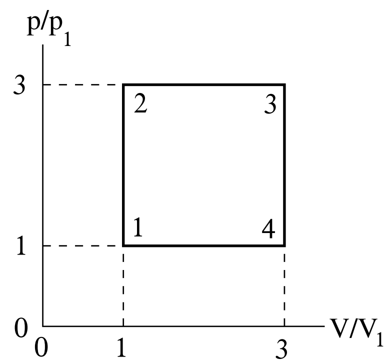
Consider 1 mol of a monatomic classical ideal gas [$pV = RT$, $U = \frac{3}{2}RT$] confined to a cylinder by a piston. The cylinder is in thermal contact with a heat bath of adjustable temperature. As the piston moves back and forth between volume $V = V_0(1 - r)$ and $V = V_0(1 + r)$ quasistatically, the temperature of the gas is being adjusted via thermal contact such that the cycle becomes circular in the (V, p)-plane and proceeds in clockwise direction (ϕ from 0 to 2π).
(a) Calculate the rate $dW/d\phi$ at which work is being performed, the rate $dU/d\phi$ at which the internal energy changes, and the rate $dQ/d\phi$ at which heat is being transferred.
(b) Set $r = 0.5$ and identify the segments along the circle where each rate is positive or negative.
(c) Repeat the previous part for $r = 0.9$.
(d) Plot all three rates as functions of ϕ/π for $r = 0.5$ in one graph and then for $r = 0.9$ in a second graph.



Solution:

[tex149] Square heat engine

Consider 1 mol of a monatomic classical ideal gas [$pV = RT$, $U = \frac{3}{2}RT$] confined to a cylinder by a piston. The cylinder is in thermal contact with a heat bath of adjustable temperature. The gas undergoes a quasistatic, cyclic process $1 \to 2 \to 3 \to 4 \to 1$ as shown. Use p_1, V_1, T_1 as units for pressure, volume, and temperature, respectively.
(a) Find p_2, V_2, T_2, p_3, V_3, T_3, and p_4, V_4, T_4 in these units.
(b) Find the work performance, $\Delta W_{12}, \Delta W_{23}, \Delta W_{34}, \Delta W_{41}$, the change in internal energy, ΔU_{12}, ΔU_{23}, ΔU_{34}, ΔU_{41}, and the heat transfer, $\Delta Q_{12}, \Delta Q_{23}, \Delta Q_{34}, \Delta Q_{41}$, along the legs of the cycle. Express these quantities in units of RT_1.
(c) Find the net work ΔW_{net} performed during the cycle. Find also the heat ΔQ_{in} absorbed and the heat ΔQ_{out} expelled by the gas during the cycle.
(d) Find the efficiency η_S of this cycle in the role of heat engine.



Solution: