2015

10. Resistors II

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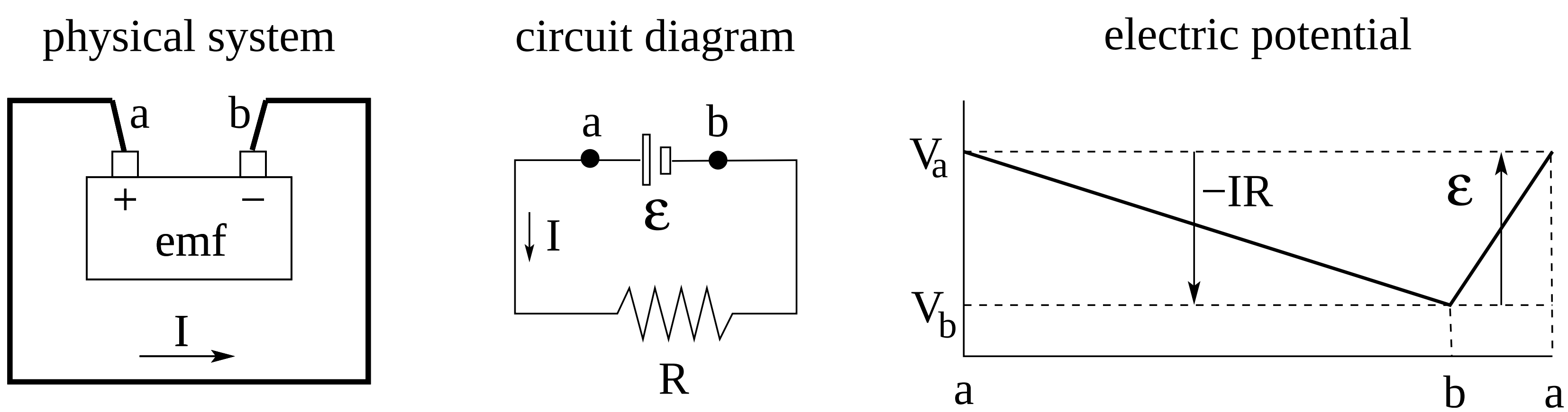
Abstract
Part ten of course materials for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island. Documents will be updated periodically as more entries become presentable.
Some of the slides contain figures from the textbook, Paul A. Tipler and Gene Mosca. *Physics for Scientists and Engineers*, $5^{th}/6^{th}$ editions. The copyright to these figures is owned by W.H. Freeman. We acknowledge permission from W.H. Freeman to use them on this course web page. The textbook figures are not to be used or copied for any purpose outside this class without direct permission from W.H. Freeman.

M. C. Escher: Waterfall

Direct Current Circuit

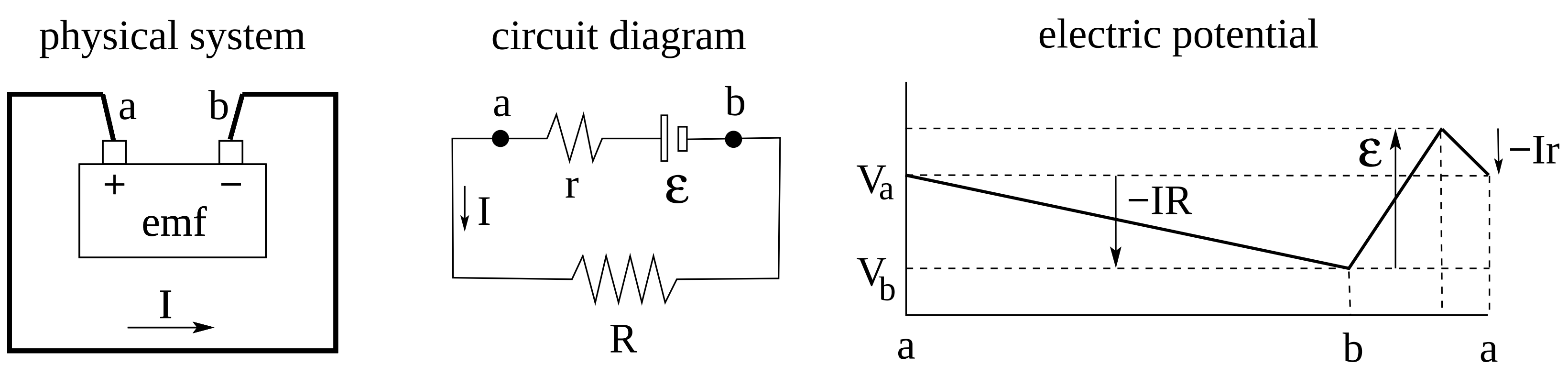
Consider a wire with resistance $R = \rho\ell/A$ connected to a battery.

- **Resistor rule**: In the direction of I across a resistor with resistance R, the electric potential drops: $\Delta V = -IR$.
- **EMF rule**: From the $(-)$ terminal to the $(+)$ terminal in an ideal source of emf, the potential rises: $\Delta V = \mathcal{E}$.
- **Loop rule**: The algebraic sum of the changes in potential encountered in a complete traversal of any loop in a circuit must be zero: $\sum \Delta V_i = 0$.

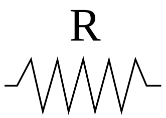


Battery with Internal Resistance

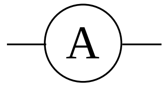
- Real batteries have an internal resistance r.
- The terminal voltage $V_{ba} \equiv V_a - V_b$ is smaller than the emf $\mathcal{E}$ written on the label if a current flows through the battery.
- Usage of the battery increases its internal resistance.
- Current from loop rule: $\mathcal{E} - Ir - IR = 0 \quad \Rightarrow \; I = \dfrac{\mathcal{E}}{R+r}$
- Current from terminal voltage: $V_{ba} = \mathcal{E} - Ir = IR \quad \Rightarrow \; I = \dfrac{V_{ba}}{R}$



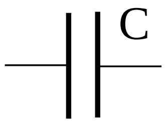
Symbols Used in Cicuit Diagrams



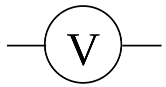
resistor



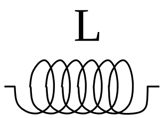
ammeter (connect in series)



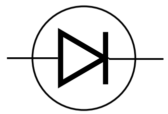
capacitor



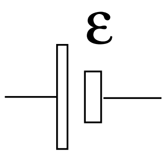
voltmeter (connect in parallel)



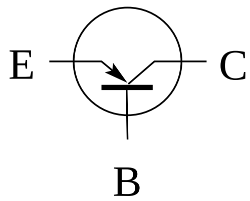
inductor



diode



emf source



transistor

Resistor Circuit (4)

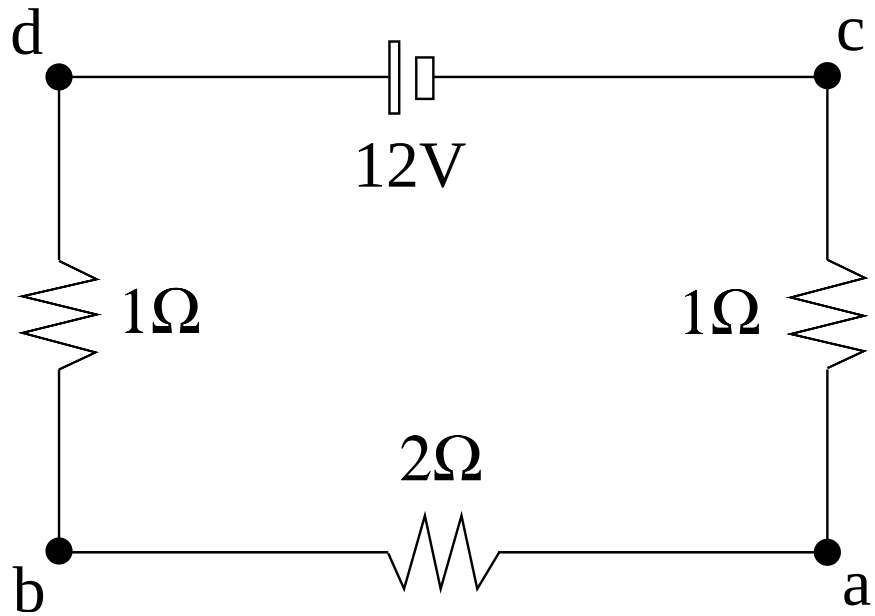
Consider the resistor circuit shown.

(a) Find the direction of the current I (cw/ccw).

(b) Find the magnitude of the current I.

(c) Find the voltage $V_{ab} = V_b - V_a$.

(d) Find the voltage $V_{cd} = V_d - V_c$.



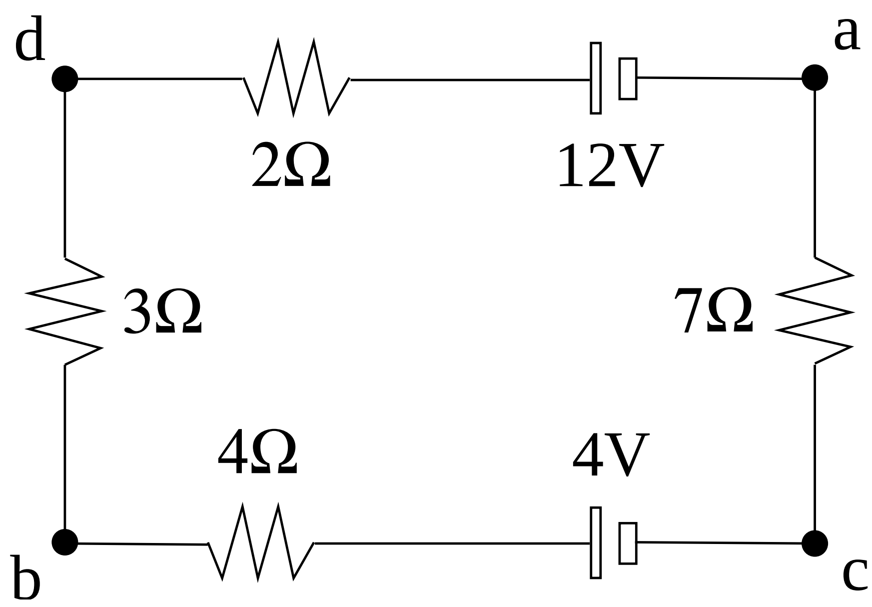
Consider the resistor circuit shown.

(a) Find the direction (cw/ccw) of the current I in the loop.

(b) Find the magnitude of the current I in the loop.

(c) Find the potential difference $V_{ab} = V_b - V_a$.

(d) Find the voltage $V_{cd} = V_d - V_c$.



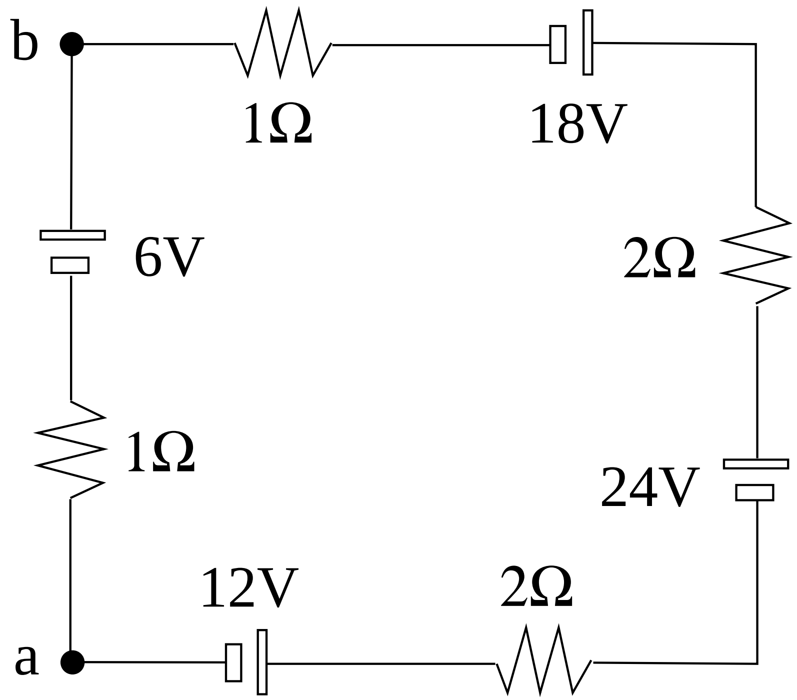
Resistor Circuit (6)

Consider the resistor circuit shown.

(a) Guess the current direction (cw/ccw).

(b) Use the loop rule to determine the current.

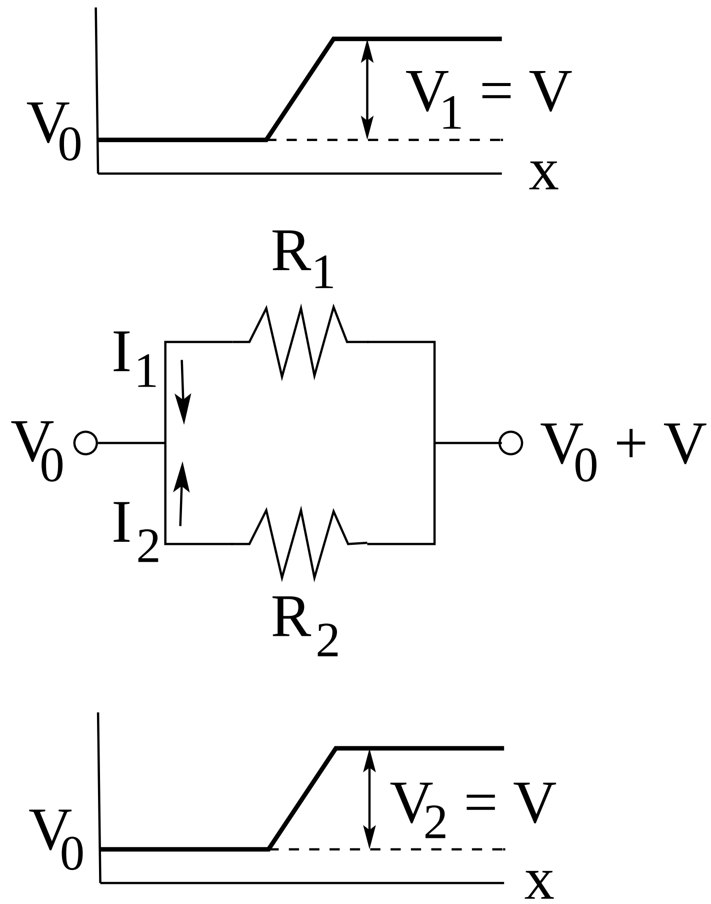
(c) Find $V_{ab} \equiv V_b - V_a$ along two different paths.



Resistors Connected in Parallel

Find the equivalent resistance of two resistors connected in parallel.

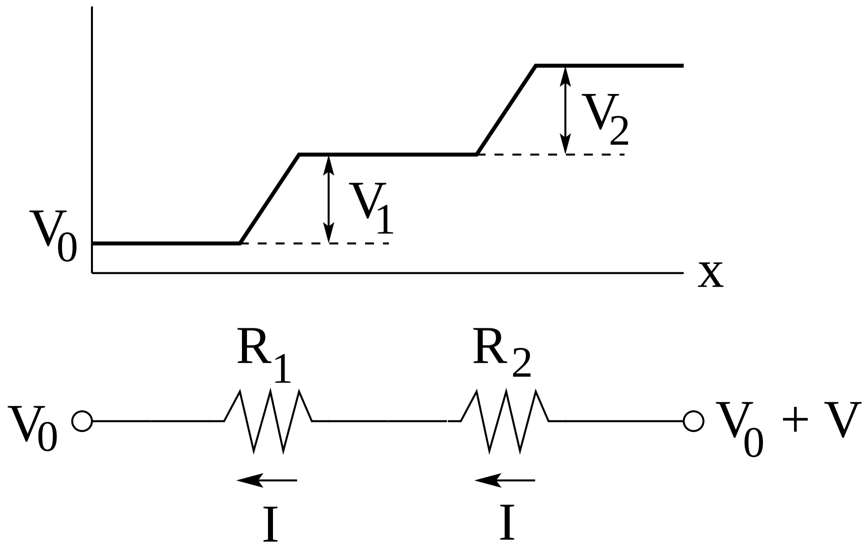
- Current through resistors: $I_1 + I_2 = I$
- Voltage across resistors: $V_1 = V_2 = V$
- Equivalent resistance: $\frac{1}{R} \equiv \frac{I}{V} = \frac{I_1}{V_1} + \frac{I_2}{V_2}$
- $\Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$



Resistors Connected in Series

Find the equivalent resistance of two resistors connected in series.

- Current through resistors: $I_1 = I_2 = I$
- Voltage across resistors: $V_1 + V_2 = V$
- Equivalent resistance: $R \equiv \frac{V}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2}$
- $\Rightarrow \; R = R_1 + R_2$



Resistor Circuit (1)

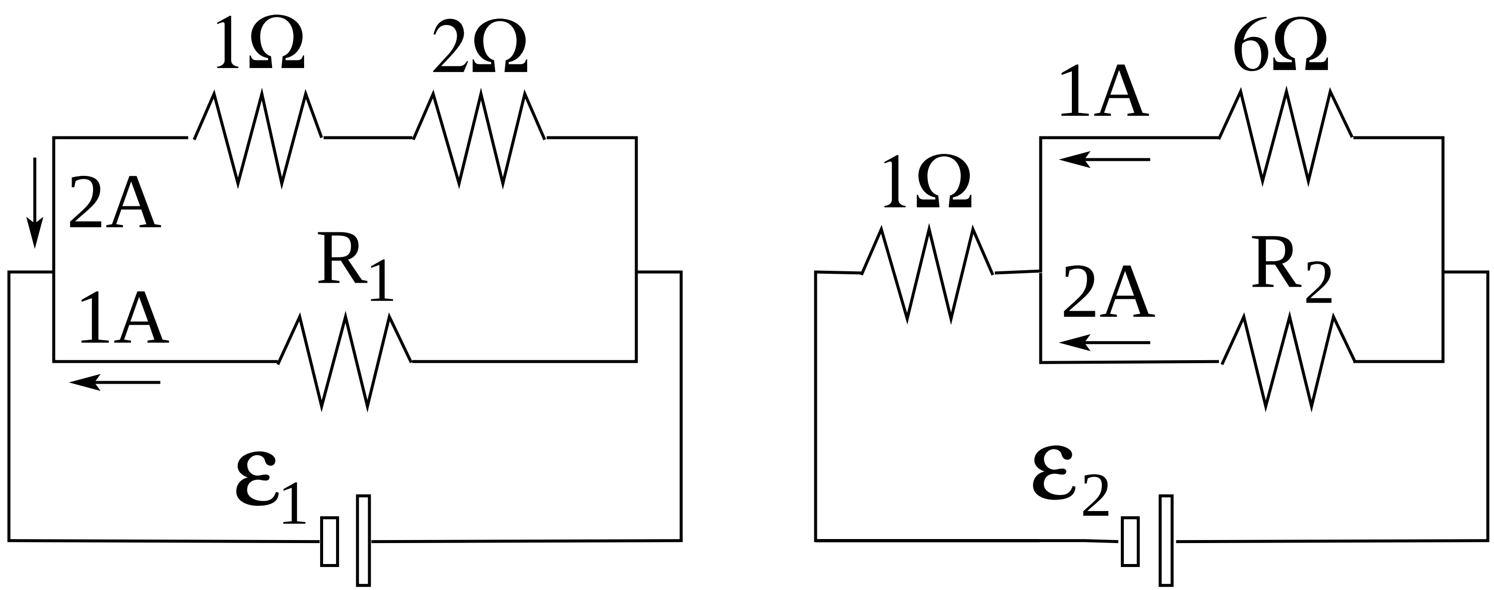
Consider the two resistor circuits shown.

(a) Find the resistance R_1.

(b) Find the emf $\mathcal{E}_1$.

(c) Find the resistance R_2.

(d) Find the emf $\mathcal{E}_2$.



Resistor Circuit (2)

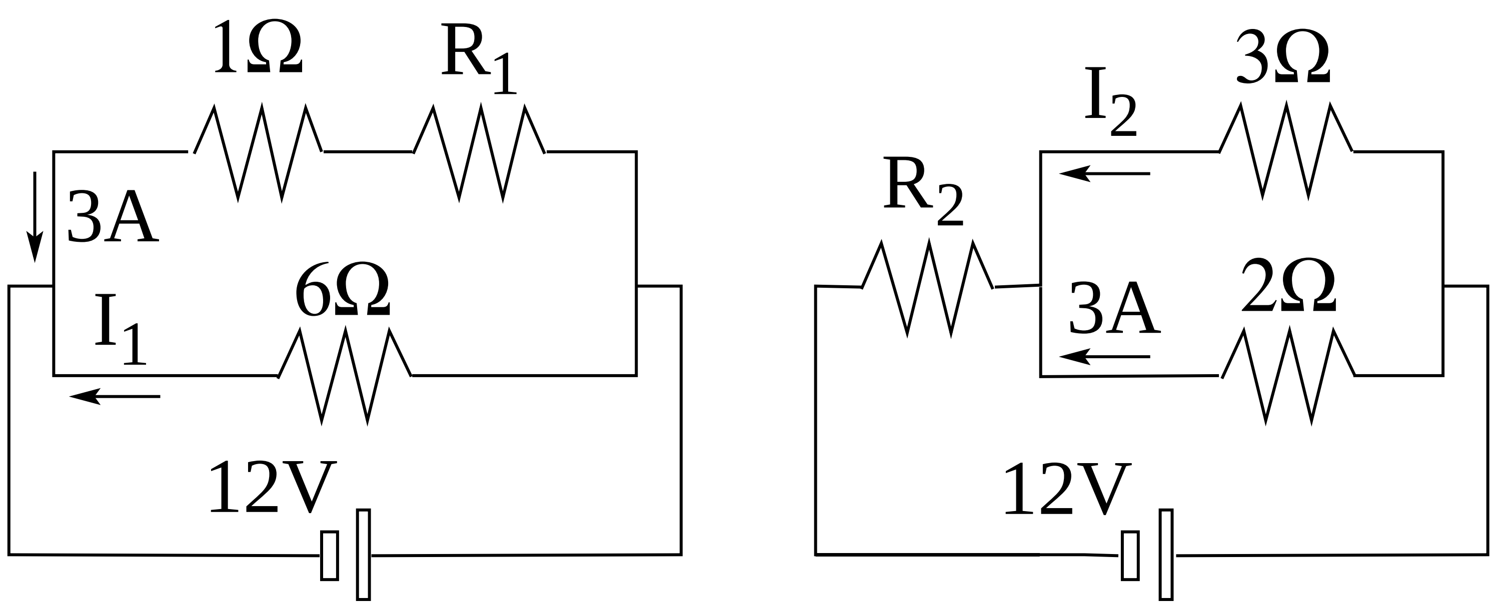
Consider the two resistor circuits shown.

(a) Find the resistance R_1.

(b) Find the current I_1.

(c) Find the resistance R_2.

(d) Find the current I_2.



Resistor Circuit (3)

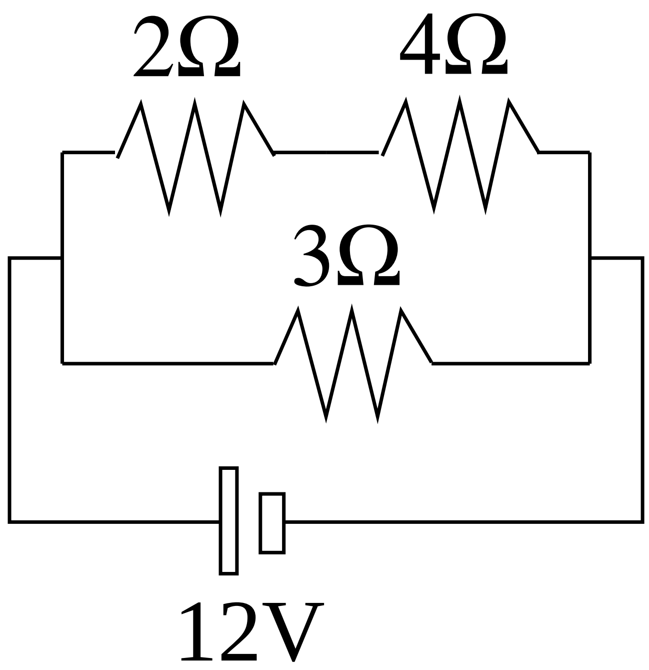
Consider the rsistor and capacitor circuits shown.

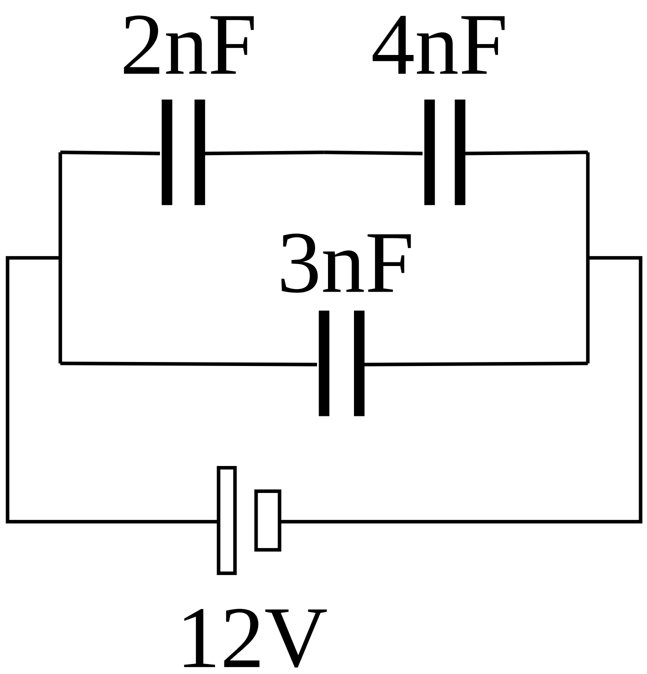
(a) Find the equivalent resistance R_{eq}.

(b) Find the power P_2, P_3, P_4 dissipated in each resistor.

(c) Find the equivalent capacitance C_{eq}.

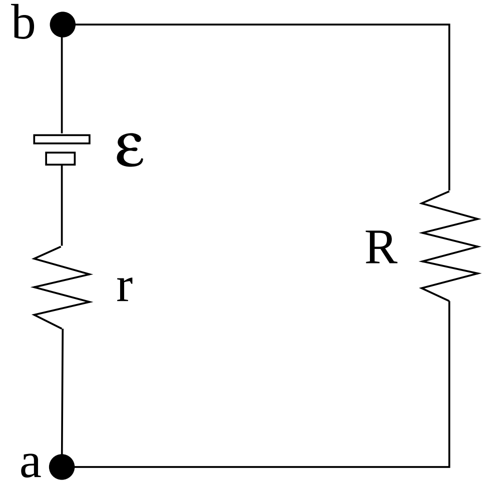
(d) Find the energy U_2, U_3, U_4 stored in each capacitor.





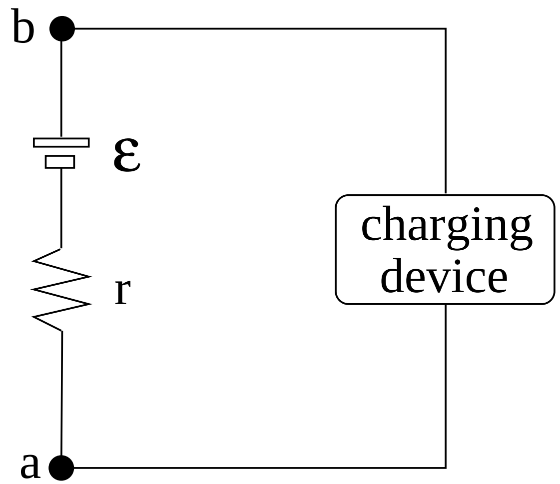
Battery in use

- Terminal voltage: $V_{ab} = \mathcal{E} - Ir = IR$
- Power output of battery: $P = V_{ab}I = \mathcal{E}I - I^2 r$
    - Power generated in battery: $\mathcal{E}I$
    - Power dissipated in battery: $I^2 r$
- Power dissipated in resistor: $P = I^2 R$



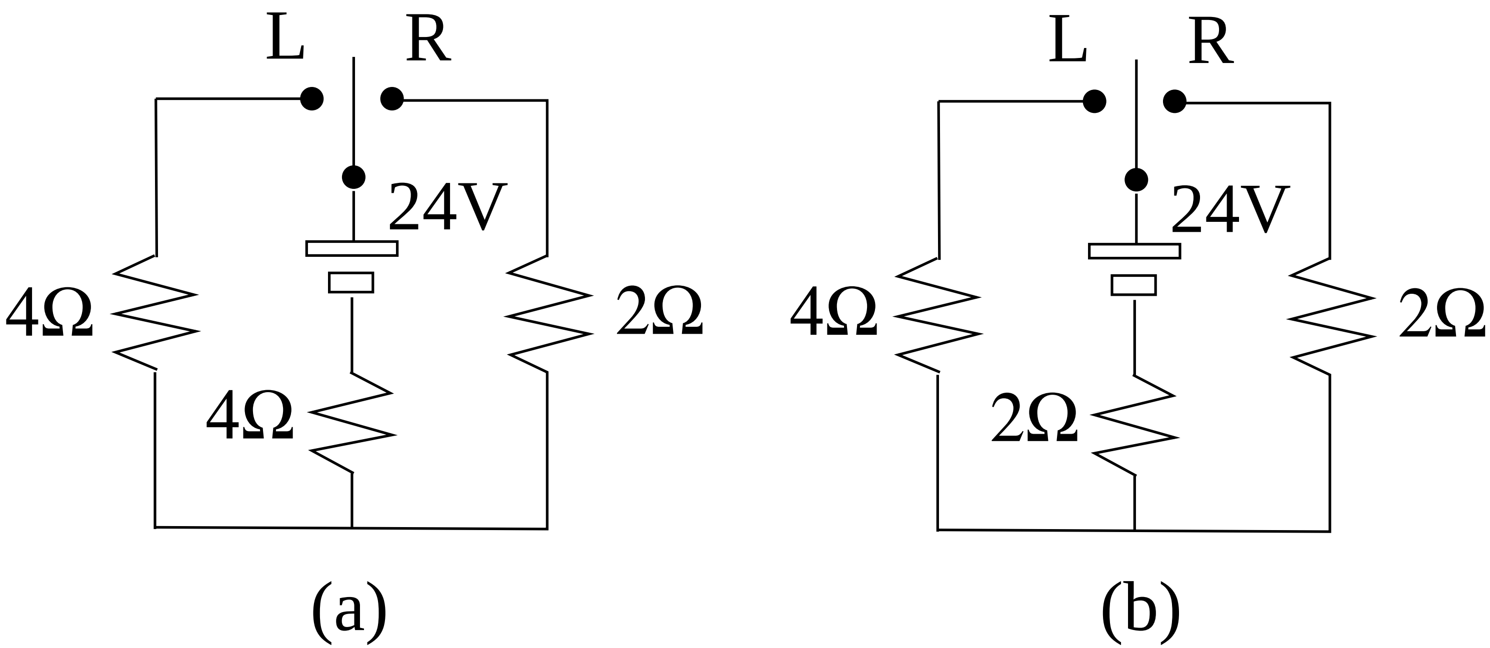
Battery being charged:

- Terminal voltage: $V_{ab} = \mathcal{E} + Ir$
- Power supplied by charging device: $P = V_{ab}I$
- Power input into battery: $P = \mathcal{E}I + I^2 r$
    - Power stored in battery: $\mathcal{E}I$
    - Power dissipated in battery: $I^2 r$



Consider two 24V batteries with internal resistances (a) $r = 4\Omega$, (b) $r = 2\Omega$.

- Which setting of the switch (L/R) produces the larger power dissipation in the resistor on the side?

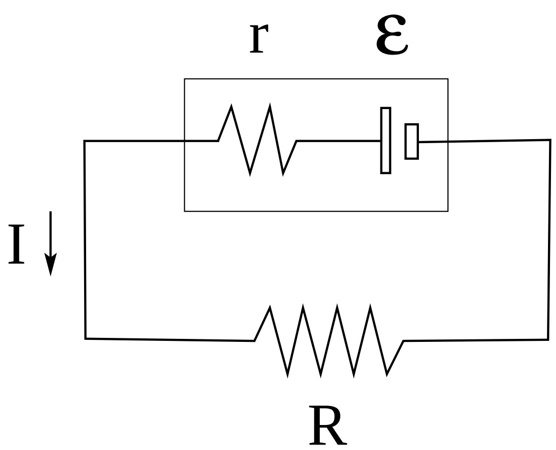


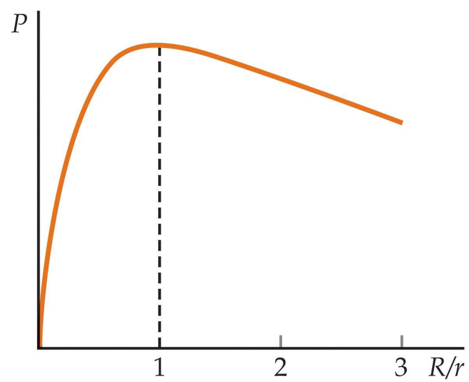
Impedance Matching

A battery providing an emf $\mathcal{E}$ with internal resistance r is connected to an external resistor of resistance R as shown.

For what value of R does the battery deliver the maximum power to the external resistor?

- Electric current: $\mathcal{E} - Ir - IR = 0 \quad \Rightarrow \; I = \dfrac{\mathcal{E}}{R+r}$
- Power delivered to external resistor: $P = I^2R = \dfrac{\mathcal{E}^2 R}{(R+r)^2}$
- Condition for maximum power: $\dfrac{dP}{dR} = 0 \quad \Rightarrow \; R = r$

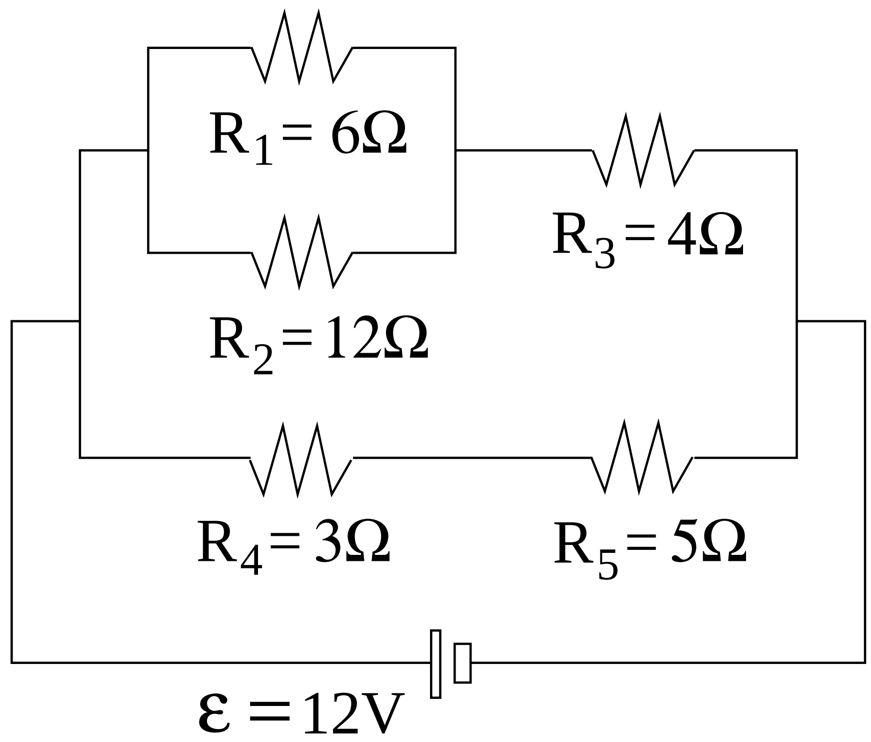




Resistor Circuit (8)

Consider the circuit of resistors shown.

- Find the equivalent resistance R_{eq}.
- Find the currents $I_1, \ldots, I_5$ through each resistor and the voltages $V_1, \ldots, V_5$ across each resistor.
- Find the total power P dissipated in the circuit.



Loop Rule

- When any closed-circuit loop is traversed, the algebraic sum of the changes in electric potential must be zero.

Junction Rule

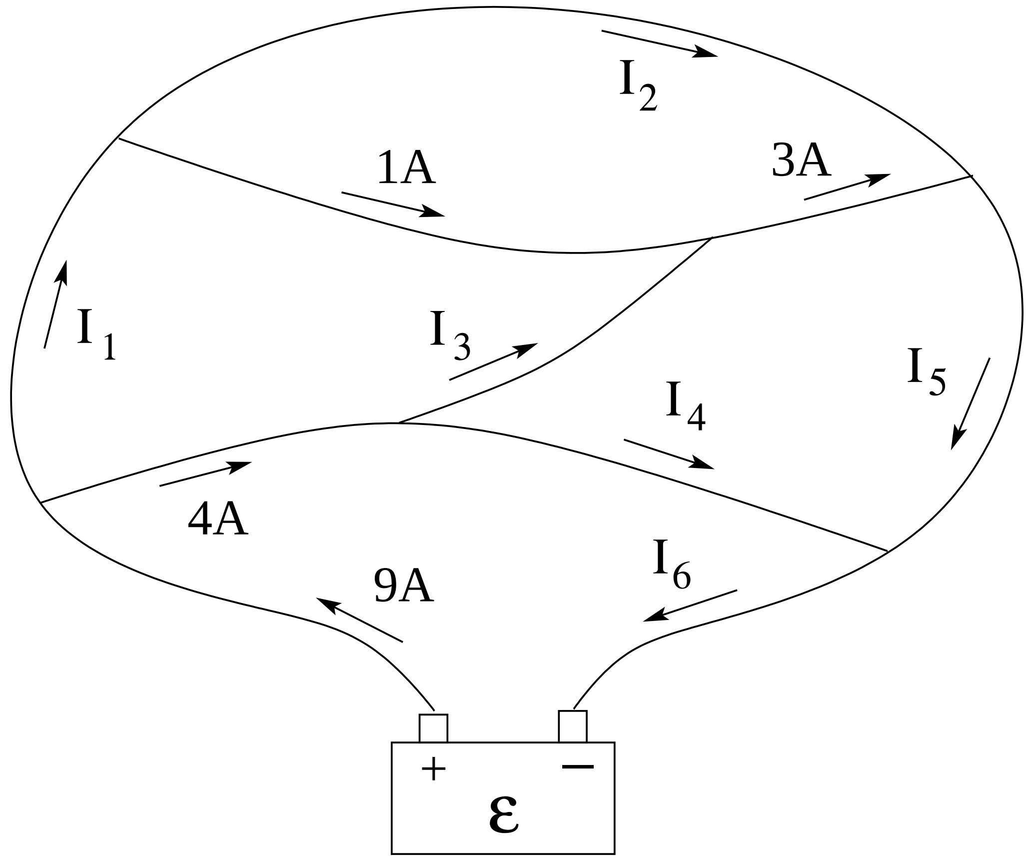
- At any junction in a circuit, the sum of the incoming currents must equal the sum of the outgoing currents.

Strategy

- Use the junction rule to name all independent currents.
- Use the loop rule to determine the independent currents.

Applying the Junction Rule

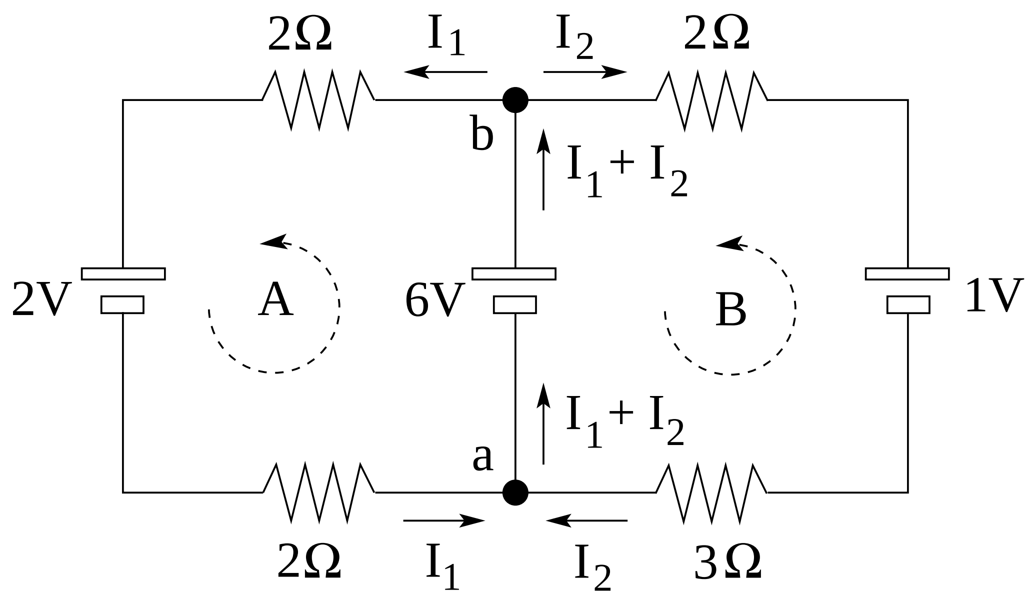
In the circuit of steady currents, use the junction rule to find the unknown currents $I_1, \ldots, I_6$.



Applying Kirchhoff's Rules

Consider the circuit shown below.

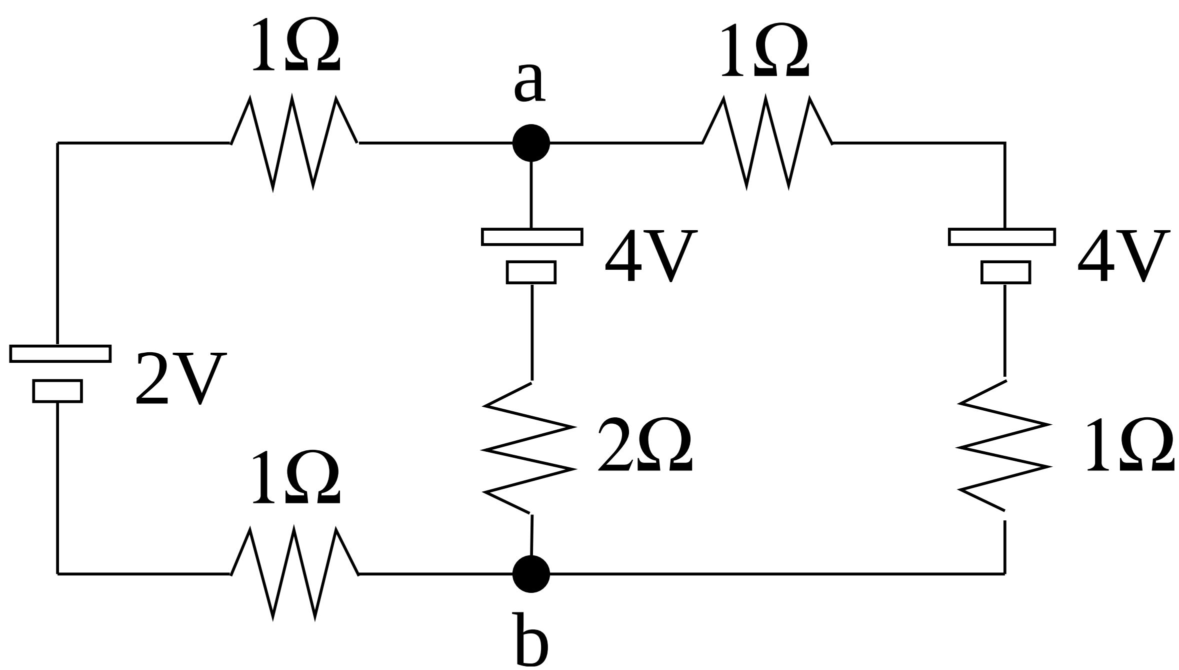
- Junction a: I_1, I_2 (in); $I_1 + I_2$ (out)
- Junction b: $I_1 + I_2$ (in); I_1, I_2 (out)
- Two independent currents require the use of two loops.
- Loop A (ccw): $6\text{V} - (2\Omega)I_1 - 2\text{V} - (2\Omega)I_1 = 0$
- Loop B (ccw): $(3\Omega)I_2 + 1\text{V} + (2\Omega)I_2 - 6\text{V} = 0$
- Solution: $I_1 = 1\text{A}, \quad I_2 = 1\text{A}$



Resistor Circuit (11)

Consider the electric circuit shown.

- Identify all independent currents via junction rule.
- Determine the independent currents via loop rule.
- Find the Potential difference $V_{ab} = V_b - V_a$.



Resistor Circuit (9)

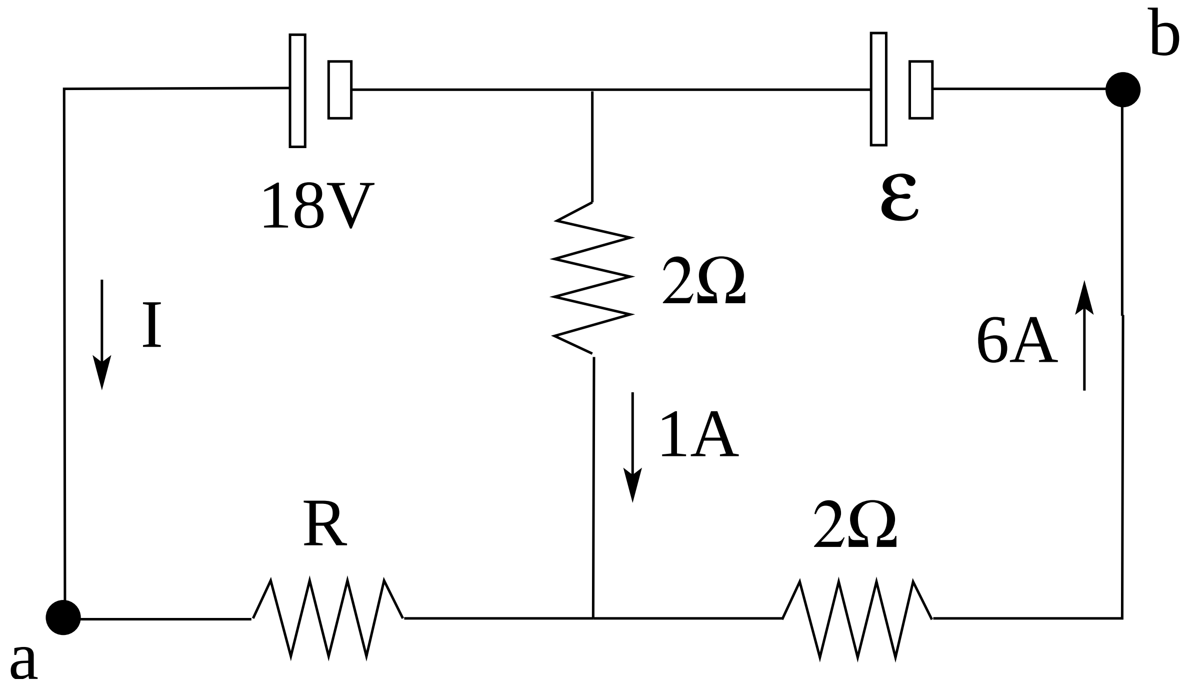
Use Kirchhoff's rules to find

(a) the current I,

(b) the resistance R,

(c) the emf $\mathcal{E}$,

(d) the voltage $V_{ab} \equiv V_b - V_a$.



Resistor Circuit (10)

Consider the electric circuit shown.

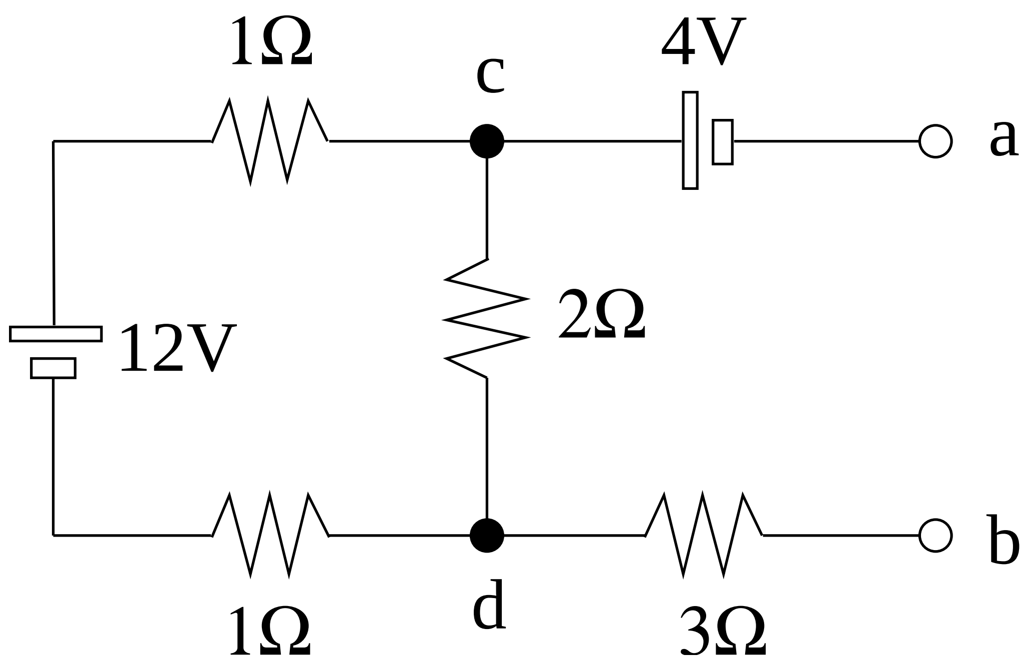
(a) Find the current through the 12V battery.

(b) Find the current through the 2Ω resistor.

(c) Find the total power dissipated.

(d) Find the voltage $V_{cd} \equiv V_d - V_c$.

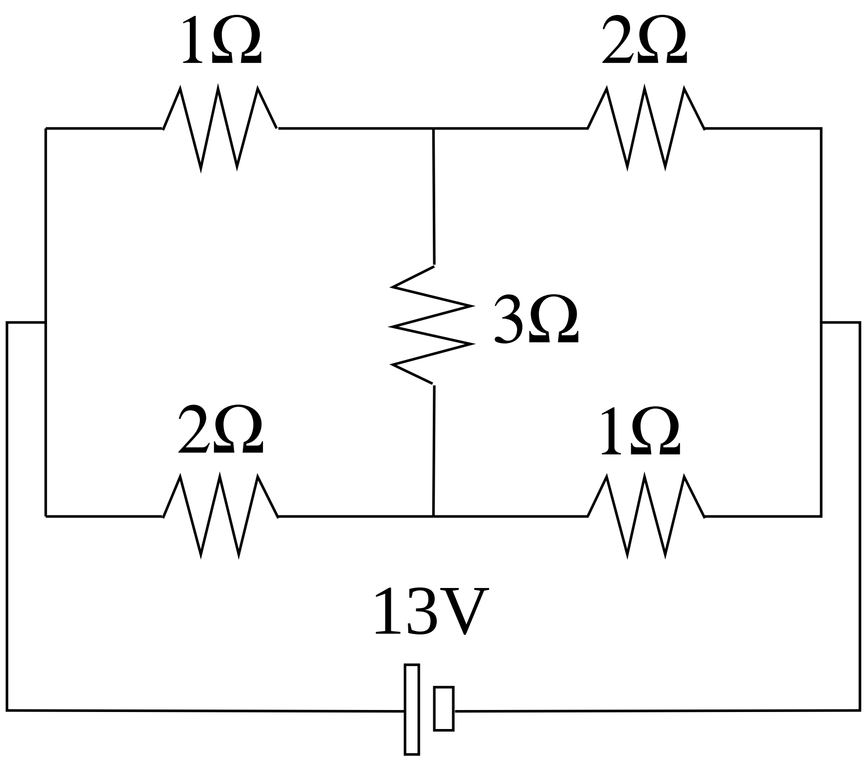
(e) Find the voltage $V_{ab} \equiv V_b - V_a$.



Resistor Circuit (12)

Consider the electric circuit shown.

- Find the equivalent resistance R_{eq} of the circuit.
- Find the total power P dissipated in the circuit.



More Complex Capacitor Circuit

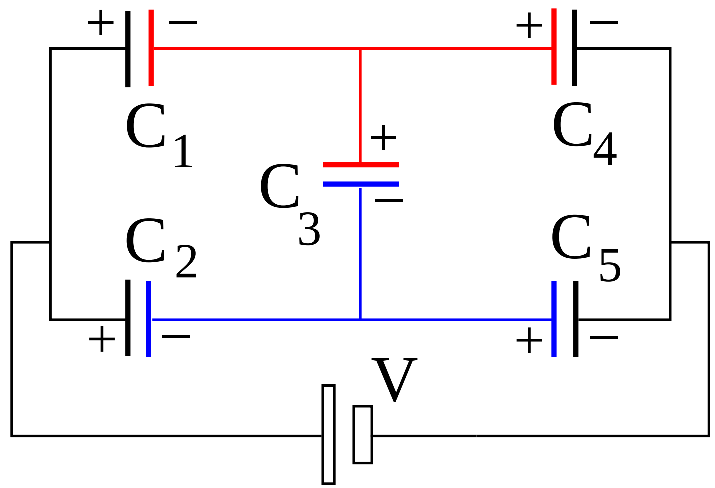
No two capacitors are in parallel or in series.
Solution requires different strategy:

- zero charge on each conductor (here color coded),
- zero voltage around any closed loop.

Specifications: $C_1, \ldots, Q_5, V$.
Five equations for unknowns $Q_1, \ldots, Q_5$:

- $Q_1 + Q_2 - Q_4 - Q_5 = 0$
- $Q_3 + Q_4 - Q_1 = 0$
- $\dfrac{Q_5}{C_5} + \dfrac{Q_3}{C_3} - \dfrac{Q_4}{C_4} = 0$
- $\dfrac{Q_2}{C_2} - \dfrac{Q_1}{C_1} - \dfrac{Q_3}{C_3} = 0$
- $V - \dfrac{Q_1}{C_1} - \dfrac{Q_4}{C_4} = 0$

Equivalent capacitance: $C_{eq} = \dfrac{Q_1 + Q_2}{V}$



(a) $C_m = 1\text{pF}, m = 1, \ldots, 5$ and $V = 1\text{V}$:

$$C_{eq} = 1\text{pF}, \; Q_3 = 0,$$

$$Q_1 = Q_2 = Q_4 = Q_5 = \frac{1}{2}\text{pC}.$$

(b) $C_m = m\,\text{pF}, m = 1, \ldots, 5$ and $V = 1\text{V}$:

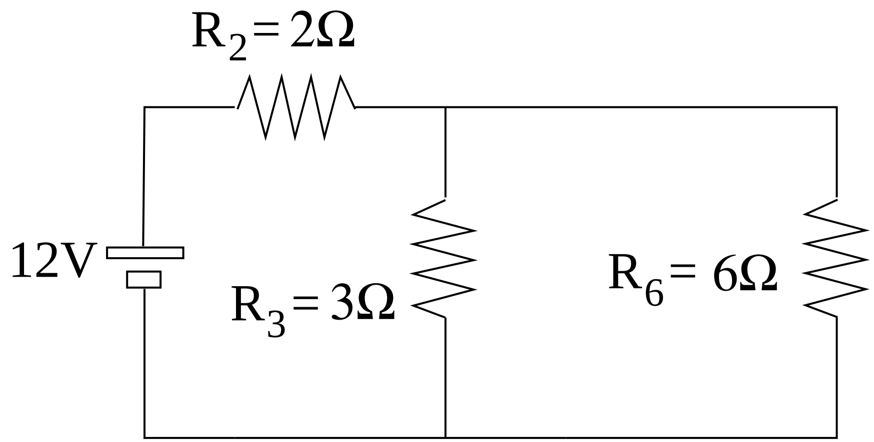
$$C_{eq} = \frac{159}{71}\text{pF}, \; Q_1 = \frac{55}{71}\text{pC}, \; Q_2 = \frac{104}{71}\text{pC},$$

$$Q_3 = -\frac{9}{71}\text{pC}, \; Q_4 = \frac{64}{71}\text{pC}, \; Q_5 = \frac{95}{71}\text{pC}.$$

Consider the electrical circuit shown.

(a) Find the equivalent resistance R_{eq}.

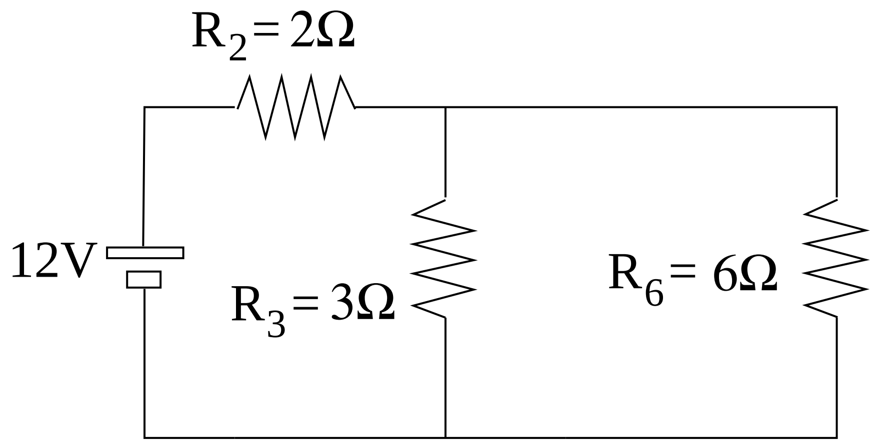
(b) Find the current I_3 through resistor R_3.



Consider the electrical circuit shown.

(a) Find the equivalent resistance R_{eq}.

(b) Find the current I_3 through resistor R_3.



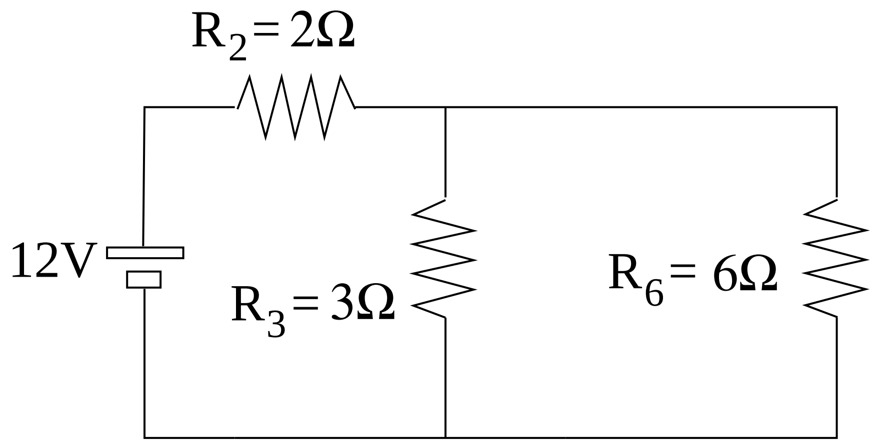
Solution:

(a) $R_{36} = \left(\frac{1}{R_3} + \frac{1}{R_6}\right)^{-1} = 2\Omega, \quad R_{eq} = R_2 + R_{36} = 4\Omega.$

Consider the electrical circuit shown.

(a) Find the equivalent resistance R_{eq}.

(b) Find the current I_3 through resistor R_3.



Solution:

(a) $R_{36} = \left(\frac{1}{R_3} + \frac{1}{R_6}\right)^{-1} = 2\Omega, \quad R_{eq} = R_2 + R_{36} = 4\Omega.$

(b) $I_2 = I_{36} = \frac{12\text{V}}{R_{eq}} = 3\text{A}$

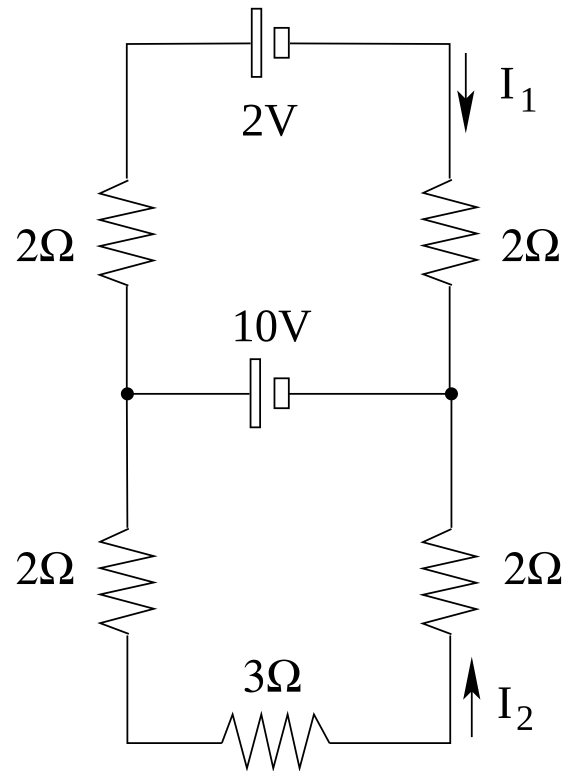
$\Rightarrow V_3 = V_{36} = I_{36}R_{36} = 6\text{V} \quad \Rightarrow I_3 = \frac{V_3}{R_3} = 2\text{A}.$

Intermediate Exam II: Problem #2 (Spring '06)

Consider the two-loop circuit shown.

(a) Find the current I_1.

(b) Find the current I_2.

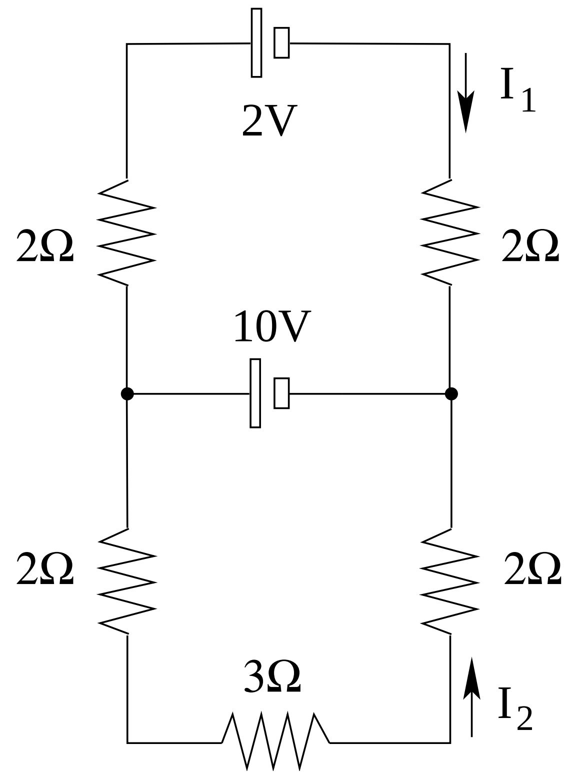


Intermediate Exam II: Problem #2 (Spring ’06)

Consider the two-loop circuit shown.

(a) Find the current I_1.

(b) Find the current I_2.



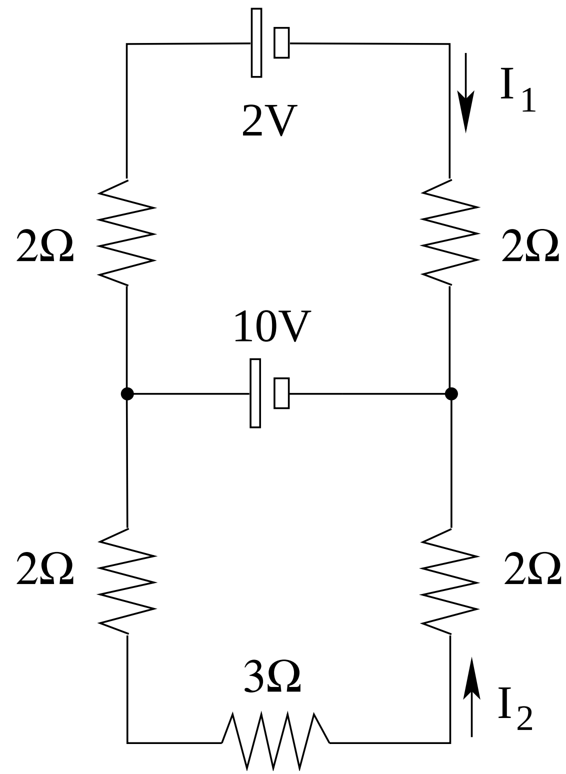
Solution:

(a) $-(2\Omega)(I_1) + 10\text{V} - (2\Omega)(I_1) - 2\text{V} = 0 \quad \Rightarrow \; I_1 = \dfrac{8\text{V}}{4\Omega} = 2\text{A}.$

Consider the two-loop circuit shown.

(a) Find the current I_1.

(b) Find the current I_2.



Solution:

(a) $-(2\Omega)(I_1) + 10\text{V} - (2\Omega)(I_1) - 2\text{V} = 0 \quad \Rightarrow I_1 = \dfrac{8\text{V}}{4\Omega} = 2\text{A}.$

(b) $-(2\Omega)(I_2) + 10\text{V} - (2\Omega)(I_2) - (3\Omega)(I_2) = 0 \quad \Rightarrow I_2 = \dfrac{10\text{V}}{7\Omega} = 1.43\text{A}.$

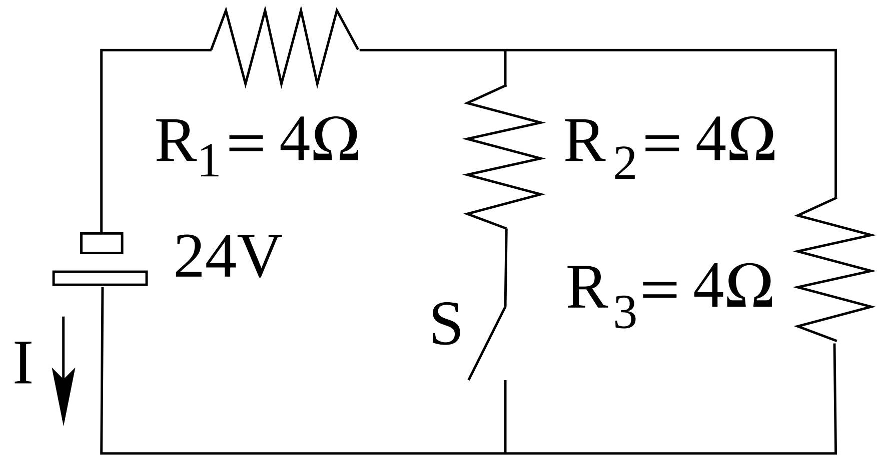
Consider the electric circuit shown.

(a) Find the current I when the switch S is open.

(b) Find the power P_3 dissipated in resisistor R_3 when the switch is open.

(c) Find the current I when the switch S is closed.

(d) Find the power P_3 dissipated in resisistor R_3 when the switch is closed.



Unit Exam II: Problem #2 (Spring '07)

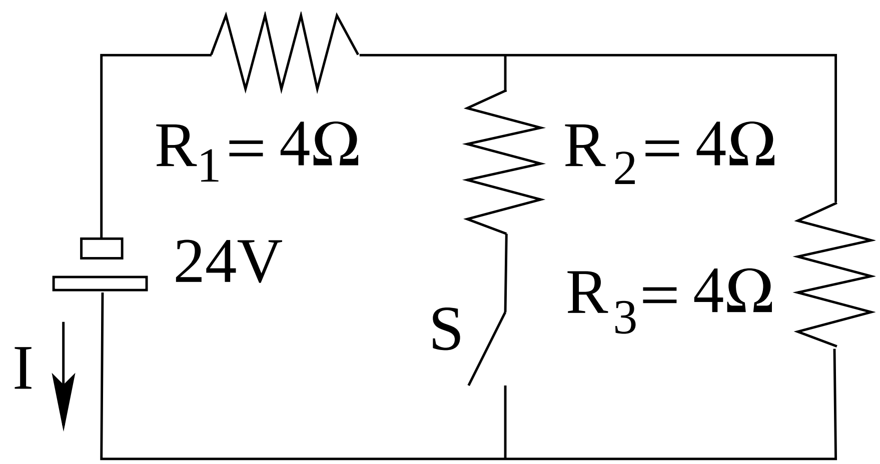
Consider the electric circuit shown.

(a) Find the current I when the switch S is open.

(b) Find the power P_3 dissipated in resisistor R_3 when the switch is open.

(c) Find the current I when the switch S is closed.

(d) Find the power P_3 dissipated in resisistor R_3 when the switch is closed.

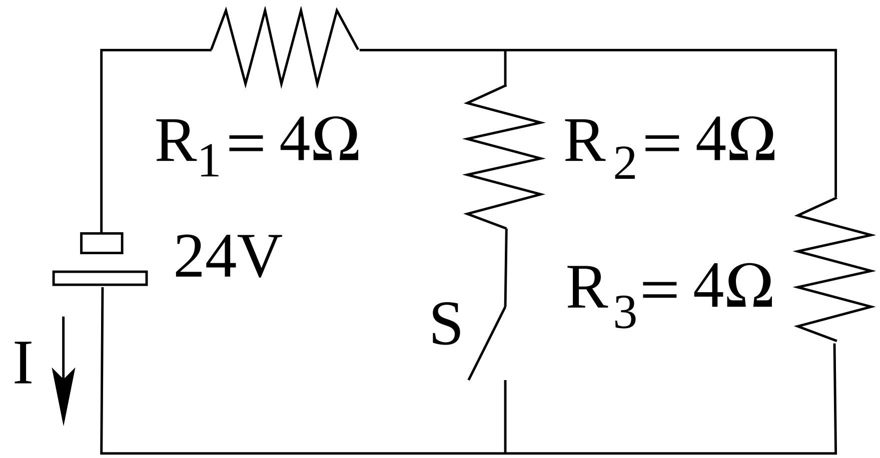


Solution:

(a) $I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$

Consider the electric circuit shown.

(a) Find the current I when the switch S is open.
(b) Find the power P_3 dissipated in resisistor R_3 when the switch is open.
(c) Find the current I when the switch S is closed.
(d) Find the power P_3 dissipated in resisistor R_3 when the switch is closed.



Solution:

(a) $I = \frac{24\mathrm{V}}{8\Omega} = 3\mathrm{A}.$
(b) $P_3 = (3\mathrm{A})^2(4\Omega) = 36\mathrm{W}.$

Unit Exam II: Problem #2 (Spring ’07)

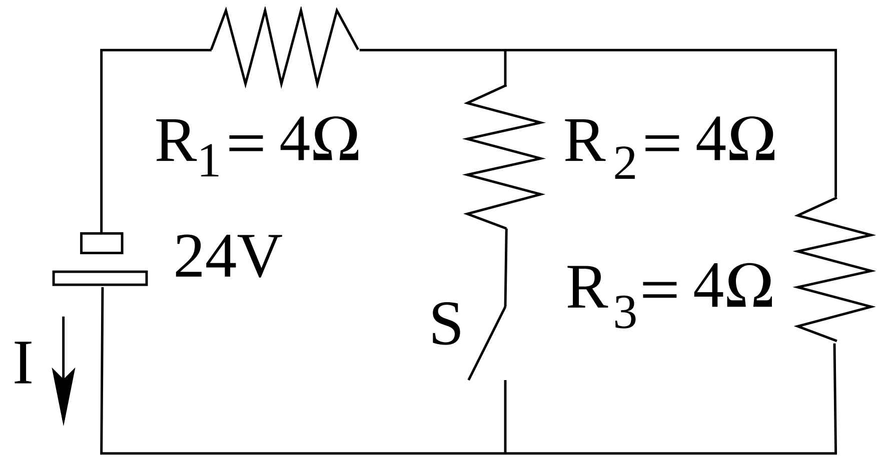
Consider the electric circuit shown.

(a) Find the current I when the switch S is open.

(b) Find the power P_3 dissipated in resisistor R_3 when the switch is open.

(c) Find the current I when the switch S is closed.

(d) Find the power P_3 dissipated in resisistor R_3 when the switch is closed.



Solution:

(a) $I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$

(b) $P_3 = (3\text{A})^2(4\Omega) = 36\text{W}.$

(c) $I = \frac{24\text{V}}{6\Omega} = 4\text{A}.$

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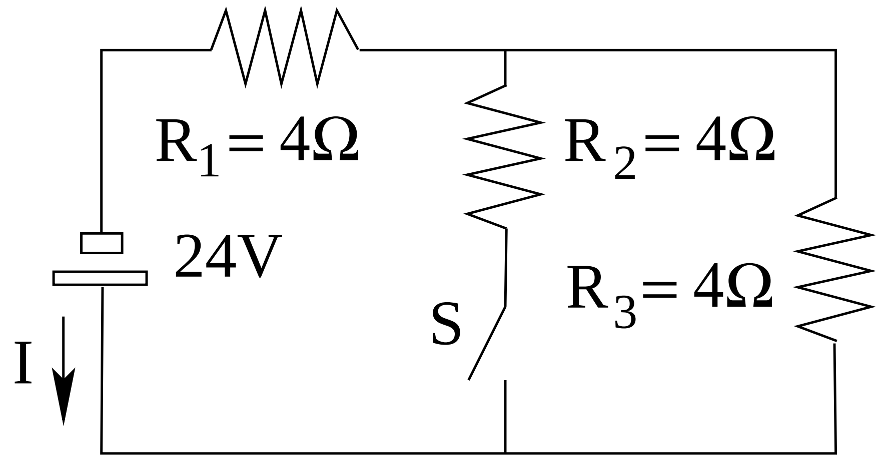


Solution:

(a) $I = \frac{24\text{V}}{8\Omega} = 3\text{A}.$

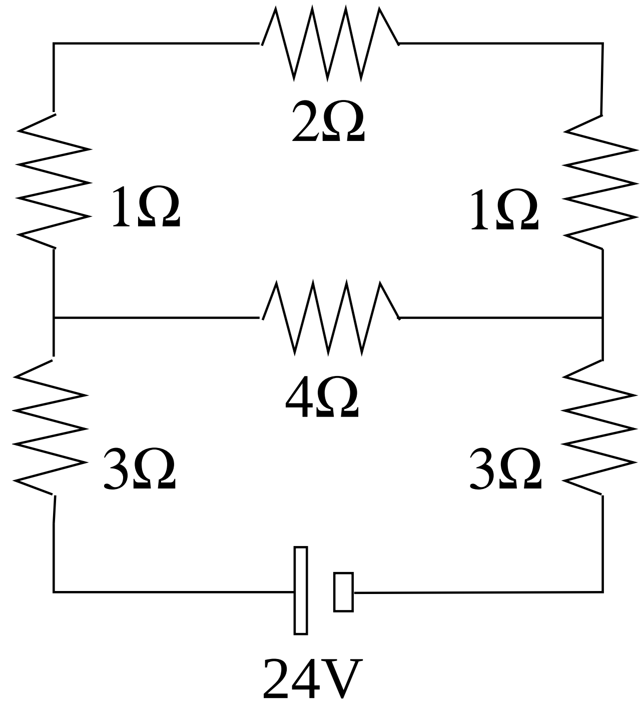
(b) $P_3 = (3\text{A})^2(4\Omega) = 36\text{W}.$

(c) $I = \frac{24\text{V}}{6\Omega} = 4\text{A}.$

(d) $P_3 = (2\text{A})^2(4\Omega) = 16\text{W}.$

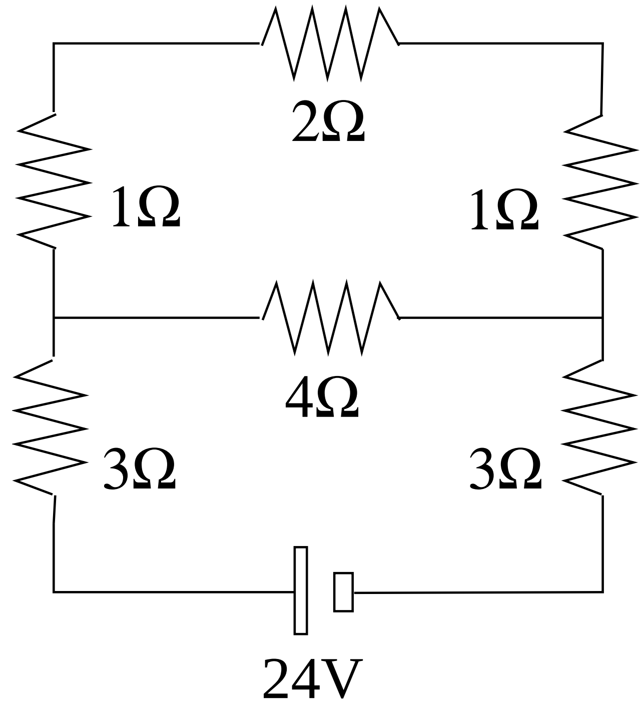
Unit Exam II: Problem #2 (Spring ’09)

Consider the resistor circuit shown.
(a) Find the equivalent resistance R_{eq}.
(b) Find the power P supplied by the battery.
(c) Find the current I_4 through the 4Ω-resistor.
(d) Find the voltage V_2 across the 2Ω-resistor.



Unit Exam II: Problem #2 (Spring '09)

Consider the resistor circuit shown.
(a) Find the equivalent resistance R_{eq}.
(b) Find the power P supplied by the battery.
(c) Find the current I_4 through the 4Ω-resistor.
(d) Find the voltage V_2 across the 2Ω-resistor.

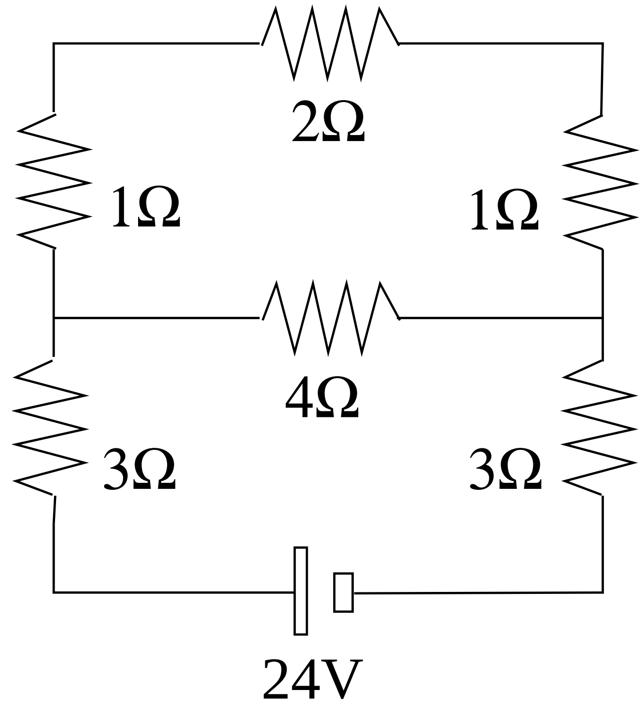


Solution:

(a) $R_{eq} = 8\Omega$.

Unit Exam II: Problem #2 (Spring '09)

Consider the resistor circuit shown.
(a) Find the equivalent resistance R_{eq}.
(b) Find the power P supplied by the battery.
(c) Find the current I_4 through the 4Ω-resistor.
(d) Find the voltage V_2 across the 2Ω-resistor.



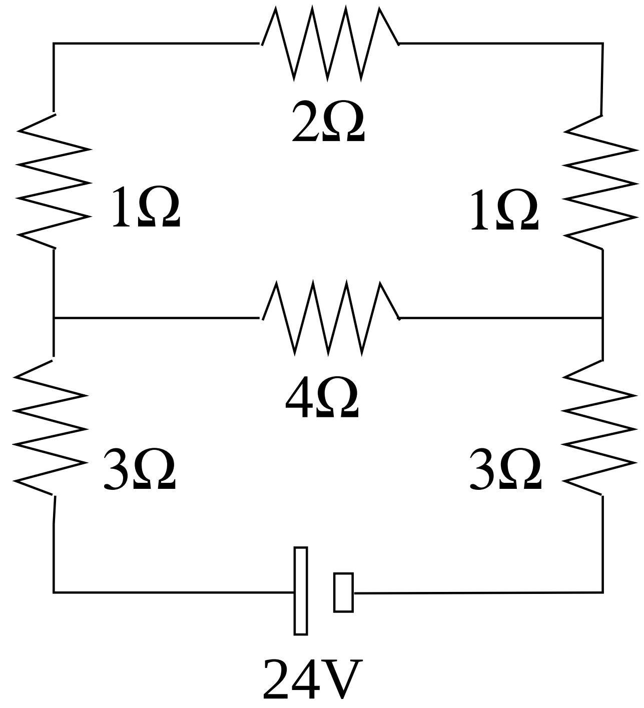
Solution:

(a) $R_{eq} = 8\Omega$.

(b) $P = \frac{(24\text{V})^2}{8\Omega} = 72\text{W}$.

Unit Exam II: Problem #2 (Spring '09)

Consider the resistor circuit shown.
(a) Find the equivalent resistance R_{eq}.
(b) Find the power P supplied by the battery.
(c) Find the current I_4 through the 4Ω-resistor.
(d) Find the voltage V_2 across the 2Ω-resistor.



Solution:

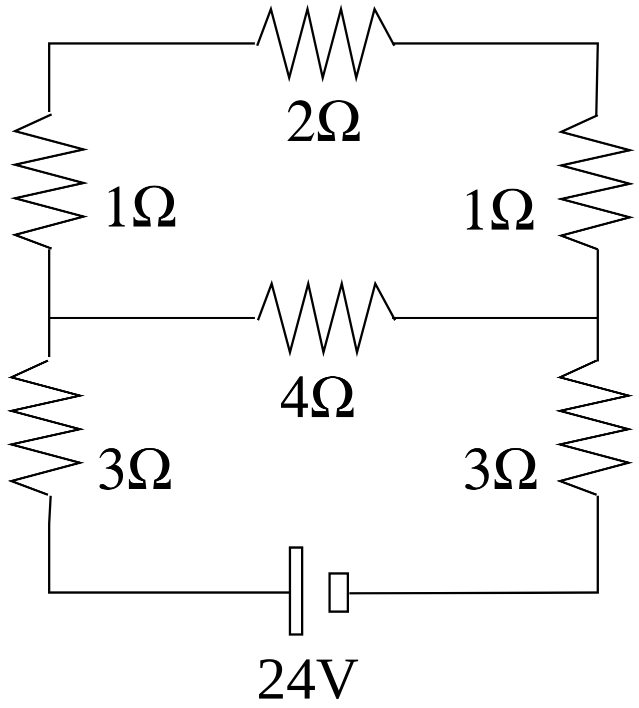
(a) $R_{eq} = 8\Omega.$

(b) $P = \dfrac{(24\text{V})^2}{8\Omega} = 72\text{W}.$

(c) $I_4 = \dfrac{1}{2}\dfrac{24\text{V}}{8\Omega} = 1.5\text{A}.$

Unit Exam II: Problem #2 (Spring ’09)

Consider the resistor circuit shown.
(a) Find the equivalent resistance R_{eq}.
(b) Find the power P supplied by the battery.
(c) Find the current I_4 through the 4Ω-resistor.
(d) Find the voltage V_2 across the 2Ω-resistor.



Solution:

(a) $R_{eq} = 8\Omega$.

(b) $P = \dfrac{(24\text{V})^2}{8\Omega} = 72\text{W}$.

(c) $I_4 = \dfrac{1}{2}\dfrac{24\text{V}}{8\Omega} = 1.5\text{A}$.

(d) $V_2 = (1.5\text{A})(2\Omega) = 3\text{V}$.

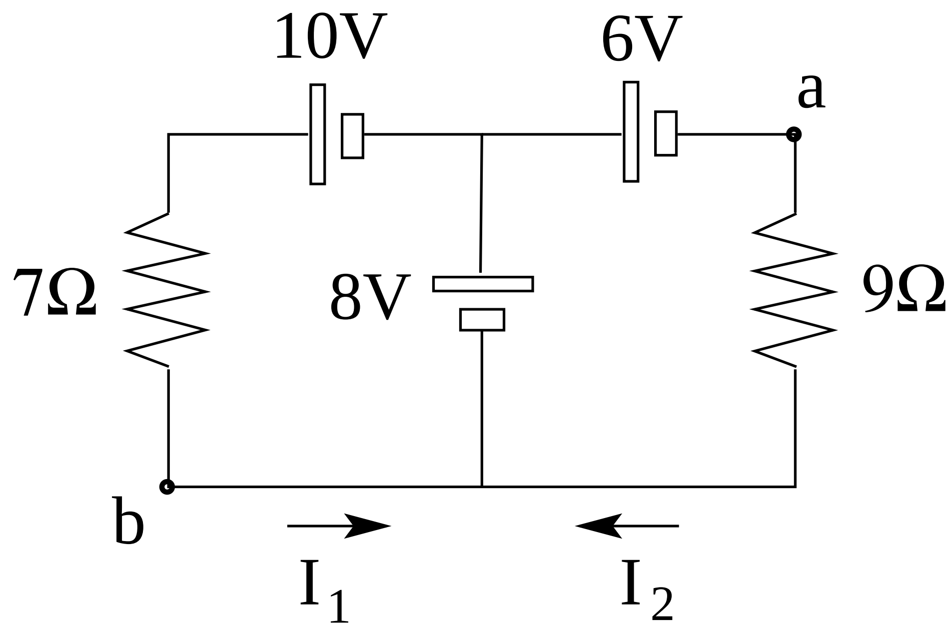
Unit Exam II: Problem #3 (Spring ’07)

Consider the two-loop circuit shown.

(a) Find the current I_1.

(b) Find the current I_2.

(c) Find the potential difference $V_a - V_b$.



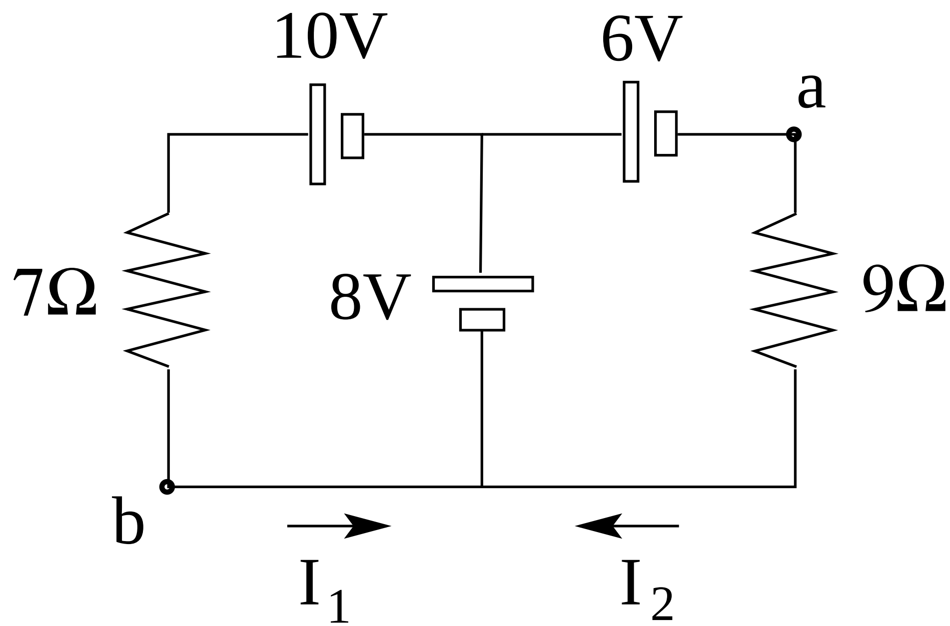
Unit Exam II: Problem #3 (Spring '07)

Consider the two-loop circuit shown.

(a) Find the current I_1.

(b) Find the current I_2.

(c) Find the potential difference $V_a - V_b$.



Solution:

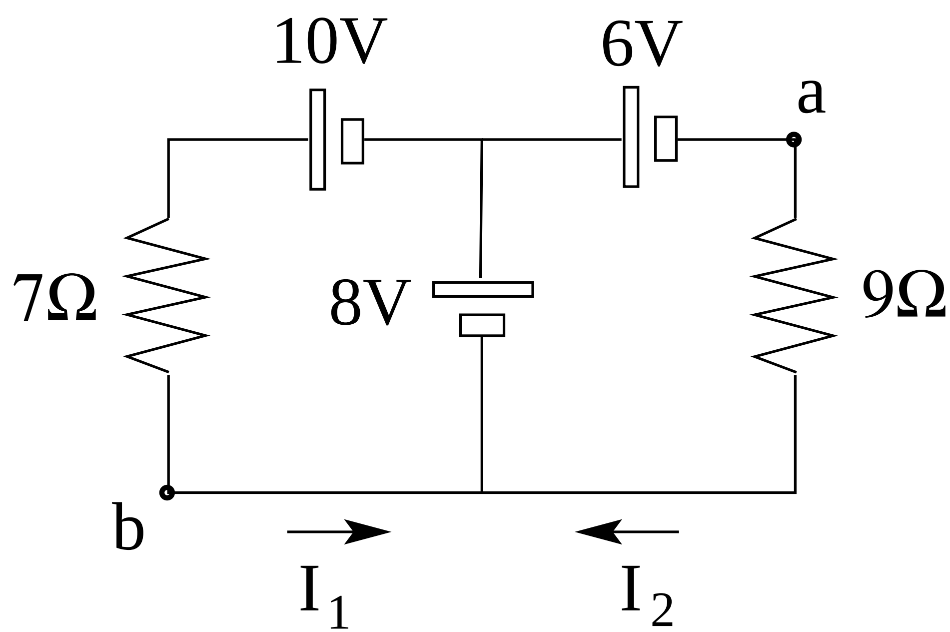
(a) $I_1 = \dfrac{8\text{V} + 10\text{V}}{7\Omega} = 2.57\text{A}.$

Consider the two-loop circuit shown.

(a) Find the current I_1.

(b) Find the current I_2.

(c) Find the potential difference $V_a - V_b$.



Solution:

(a) $I_1 = \dfrac{8\text{V} + 10\text{V}}{7\Omega} = 2.57\text{A}.$

(b) $I_2 = \dfrac{8\text{V} - 6\text{V}}{9\Omega} = 0.22\text{A}.$

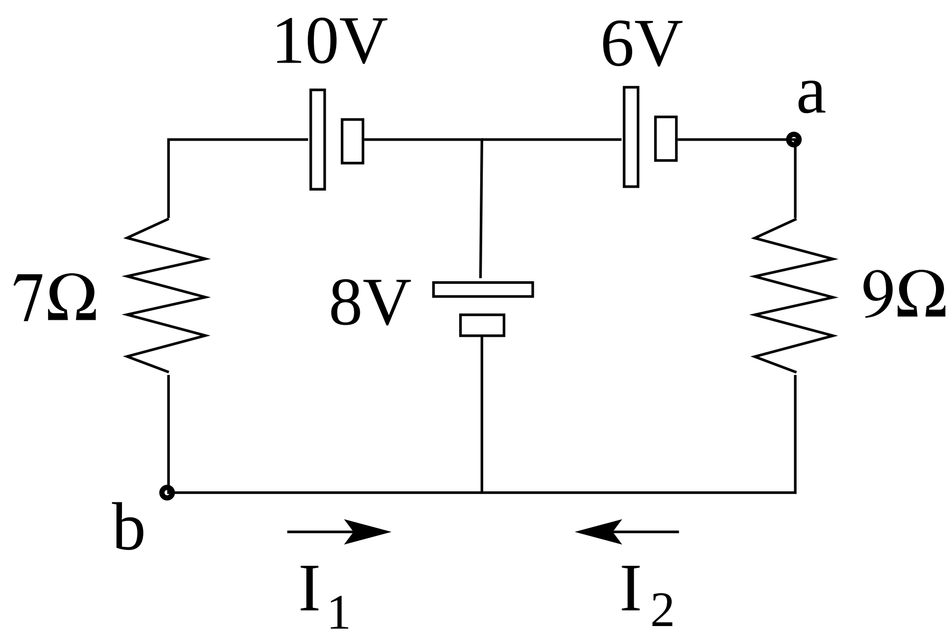
Unit Exam II: Problem #3 (Spring ’07)

Consider the two-loop circuit shown.

(a) Find the current I_1.

(b) Find the current I_2.

(c) Find the potential difference $V_a - V_b$.



Solution:

(a) $I_1 = \dfrac{8\text{V} + 10\text{V}}{7\Omega} = 2.57\text{A}.$

(b) $I_2 = \dfrac{8\text{V} - 6\text{V}}{9\Omega} = 0.22\text{A}.$

(c) $V_a - V_b = 8\text{V} - 6\text{V} = 2\text{V}.$