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08. Capacitors II

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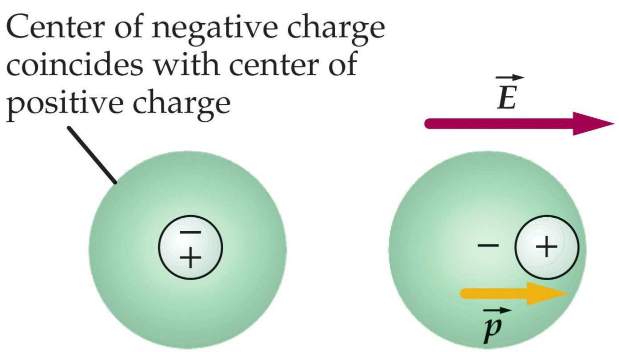
Abstract
Part eight of course materials for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island. Documents will be updated periodically as more entries become presentable.
Some of the slides contain figures from the textbook, Paul A. Tipler and Gene Mosca. *Physics for Scientists and Engineers*, 5th/6th editions. The copyright to these figures is owned by W.H. Freeman. We acknowledge permission from W.H. Freeman to use them on this course web page. The textbook figures are not to be used or copied for any purpose outside this class without direct permission from W.H. Freeman.

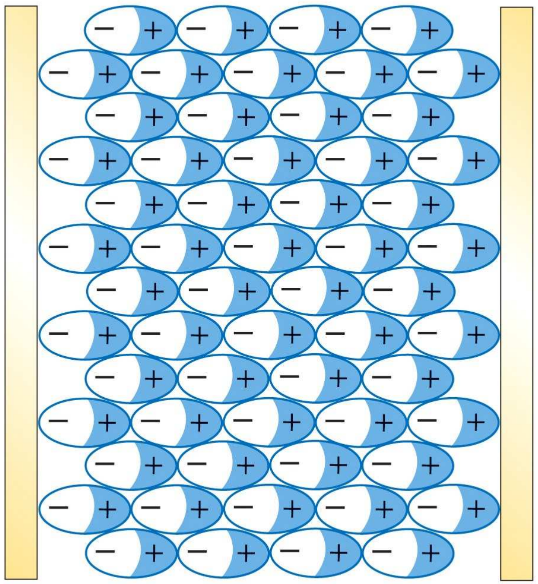
Capacitor with Dielectric

Most capacitors have a dielectric (insulating solid or liquid material) in the space between the conductors. This has several advantages:

- Physical separation of the conductors.
- Prevention of dielectric breakdown.
- Enhancement of capacitance.

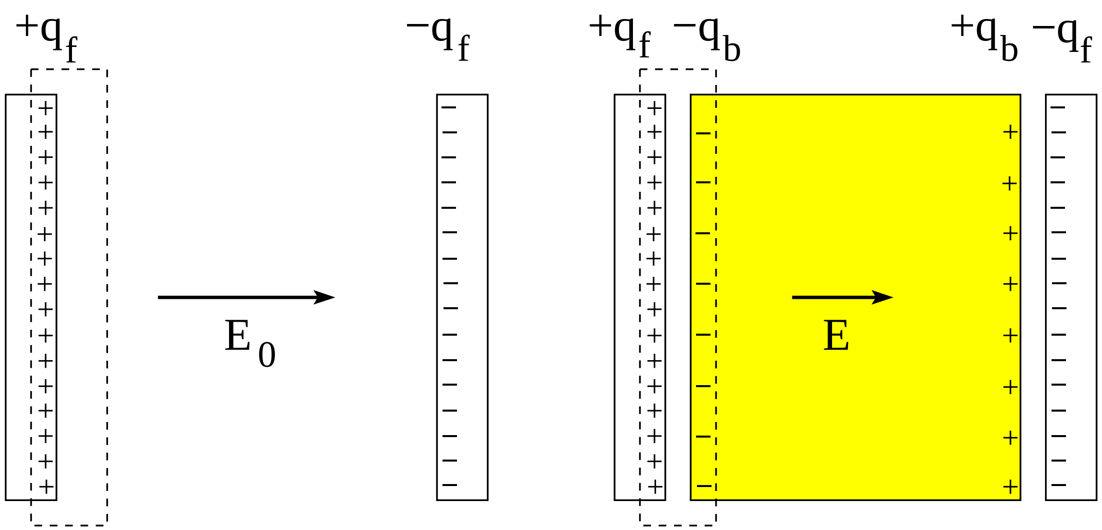
The dielectric is polarized by the electric field between the capacitor plates.





Parallel-Plate Capacitor with Dielectric (1)

The polarization produces a bound charge on the surface of the dielectric.

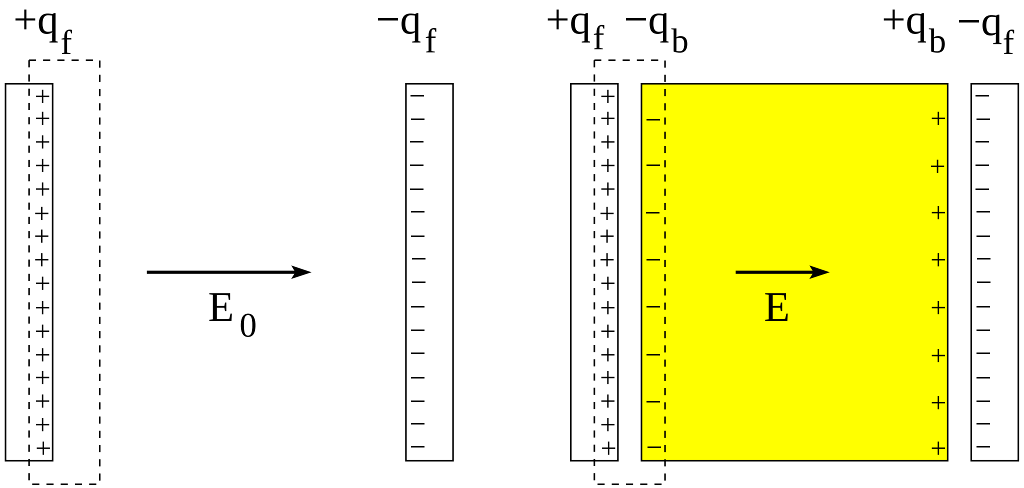


The bound surface charge has the effect of reducing the electric field between the plates from $\vec{E}_0$ to $\vec{E}$.

- A: area of plates
- d: separation between plates
- $\pm q_f$: free charge on plate
- $\pm q_b$: bound charge on surface of dielectric
- $\vec{E}_0$: electric field in vacuum
- $\vec{E}$: electric field in dielectric

Parallel-Plate Capacitor with Dielectric (2)

Use Gauss' law to determine the electric fields $\vec{E}_0$ and $\vec{E}$.



- Field in vacuum: $E_0 A = \frac{q_f}{\epsilon_0} \quad \Rightarrow \quad E_0 = \frac{q_f}{\epsilon_0 A}$
- Field in dielectric: $EA = \frac{q_f - q_b}{\epsilon_0} \quad \Rightarrow \quad E = \frac{q_f - q_b}{\epsilon_0 A} < E_0$
- Voltage: $V_0 = E_0 d$ (vacuum), $V = Ed = \frac{V_0}{\kappa} < V_0$ (dielectric)

Dielectric constant: $\kappa \equiv \frac{E_0}{E} = \frac{q_f}{q_f - q_b} > 1.$ Permittivity of dielectric: $\epsilon = \kappa\epsilon_0$.

Dielectric Materials

TABLE 24-1

Dielectric Constants and Dielectric Strengths of Various Materials

Material	Dielectric Constant κ	Dielectric Strength, kV/mm
Air	1.00059	3
Bakelite	4.9	24
Glass (Pyrex)	5.6	14
Mica	5.4	10–100
Neoprene	6.9	12
Paper	3.7	16
Paraffin	2.1–2.5	10
Plexiglas	3.4	40
Polystyrene	2.55	24
Porcelain	7	5.7
Transformer oil	2.24	12

- Dielectrics increase the capacitance: $C/C_0 = \kappa$.
- The capacitor is discharged spontaneously across the dielectric if the electric field exceeds the value quoted as dielectric strength.

Impact of Dielectric (1)

What happens when a dielectric is placed into a capacitor
with the **charge on the capacitor** kept constant?

	vacuum	dielectric
charge	Q_0	$Q = Q_0$
electric field	E_0	$E = \frac{E_0}{\kappa} < E_0$
voltage	V_0	$V = \frac{V_0}{\kappa} < V_0$
capacitance	$C_0 = \frac{Q_0}{V_0}$	$C = \frac{Q}{V} = \kappa C_0 > C_0$
potential energy	$U_0 = \frac{Q_0^2}{2C_0}$	$U = \frac{Q^2}{2C} = \frac{U_0}{\kappa} < U_0$
energy density	$u_E^{(0)} = \frac{1}{2}\epsilon_0 E_0^2$	$u_E = \frac{u_E^{(0)}}{\kappa} = \frac{1}{2}\kappa\epsilon_0 E^2 < u_E^{(0)}$

Impact of Dielectric (2)

What happens when a dielectric is placed into a capacitor
with the **voltage across the capacitor** kept constant?

	vacuum	dielectric
charge	Q_0	$Q = \kappa Q_0$
electric field	E_0	$E = E_0$
voltage	V_0	$V = V_0$
capacitance	$C_0 = \frac{Q_0}{V_0}$	$C = \frac{Q}{V} = \kappa C_0 > C_0$
potential energy	$U_0 = \frac{1}{2}C_0 V_0^2$	$U = \frac{1}{2}CV^2 = \kappa U_0 > U_0$
energy density	$u_E^{(0)} = \frac{1}{2}\epsilon_0 E_0^2$	$u_E = \kappa u_E^{(0)} = \frac{1}{2}\kappa\epsilon_0 E^2 > u_E^{(0)}$

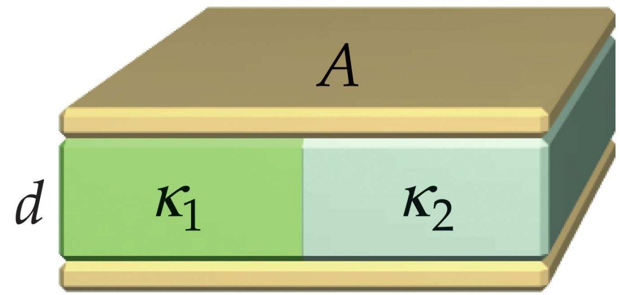
Stacked Dielectrics

Consider a parallel-plate capacitor with area A of each plate and spacing d.

- Capacitance without dielectric: $C_0 = \dfrac{\epsilon_0 A}{d}$.
- Dielectrics stacked in parallel: $C = C_1 + C_2$

  with $C_1 = \kappa_1 \epsilon_0 \dfrac{A/2}{d}$, $C_2 = \kappa_2 \epsilon_0 \dfrac{A/2}{d}$.

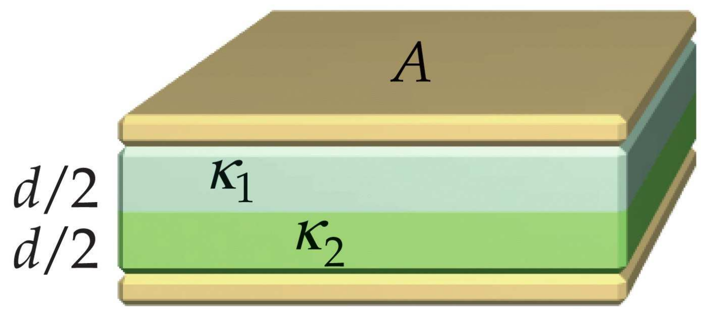
  $\Rightarrow C = \dfrac{1}{2}(\kappa_1 + \kappa_2) C_0$.



- Dielectrics stacked in series: $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

  with $C_1 = \kappa_1 \epsilon_0 \dfrac{A}{d/2}$, $C_2 = \kappa_2 \epsilon_0 \dfrac{A}{d/2}$

  $\Rightarrow C = \dfrac{2\kappa_1\kappa_2}{\kappa_1 + \kappa_2} C_0$.



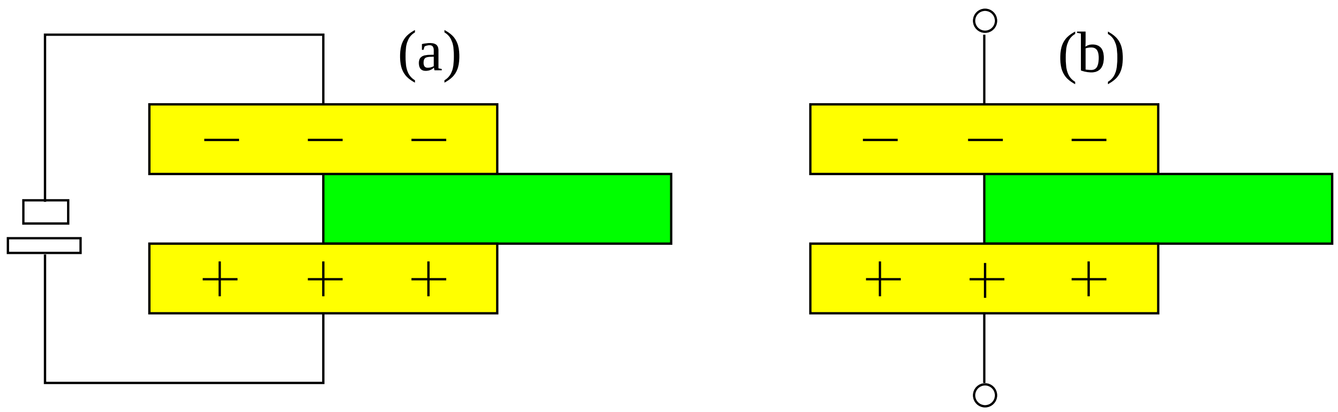
Lateral Force on Dielectric

Consider two charged capacitors with dielectrics only halfway between the plates.

In configuration (a) any lateral motion of the dielectric takes place
at **constant voltage** across the plates.

In configuration (b) any lateral motion of the dielectric takes place
at **constant charge** on the plates.

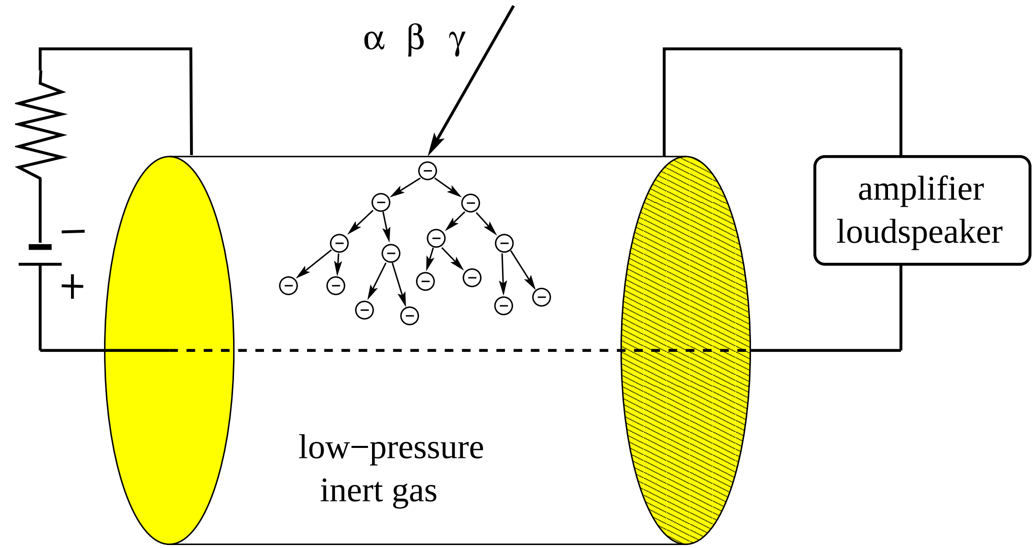
Determine in each case the direction (left/zero/right) of the lateral force experienced by the dielectric.



Geiger Counter

Radioactive atomic nuclei produce high-energy particles of three different kinds:

- α-particles are ^{4}He nuclei.
- β-particles are electrons or positrons.
- γ-particles are high-energy photons.



- Free electrons produced by ionizing radiation are strongly accelerated toward the central wire.
- Collisions with gas atoms produce further free electrons, which are accelerated in the same direction.
- An avalanche of electrons reaching the wire produces a current pulse in the circuit.

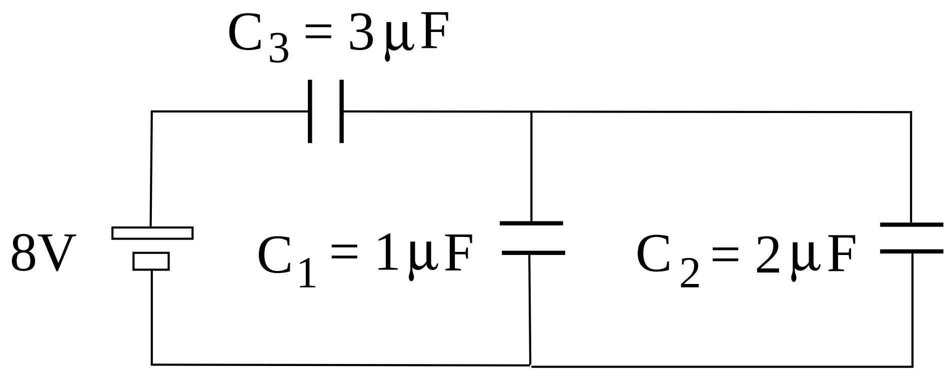
Intermediate Exam II: Problem #1 (Spring '05)

The circuit of capacitors connected to a battery is at equilibrium.

(a) Find the equivalent capacitance C_{eq}.

(b) Find the voltage V_3 across capacitor C_3.

(c) Find the the charge Q_2 on capacitor C_2.



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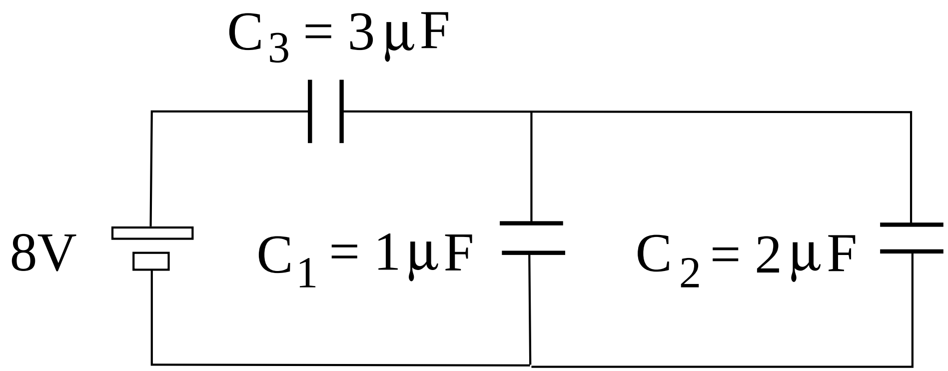

Solution:

(a) $C_{12} = C_1 + C_2 = 3\mu\text{F}, \quad C_{eq} = \left(\frac{1}{C_{12}} + \frac{1}{C_3}\right)^{-1} = 1.5\mu\text{F}.$

The circuit of capacitors connected to a battery is at equilibrium.

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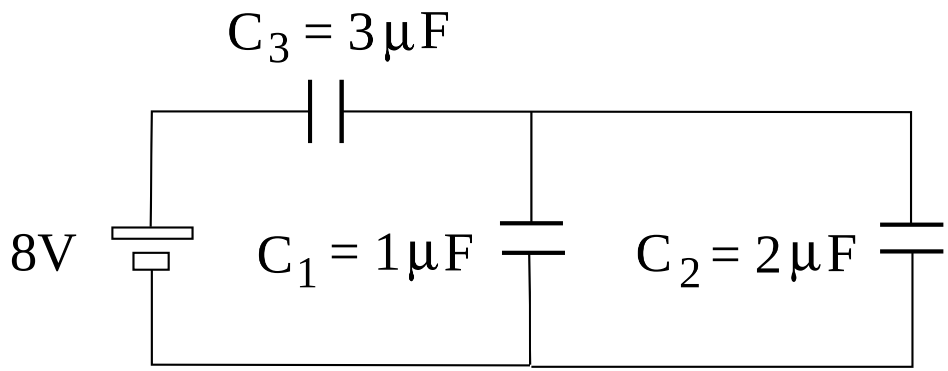


Solution:

(a) $C_{12} = C_1 + C_2 = 3\mu\mathrm{F}, \quad C_{eq} = \left(\frac{1}{C_{12}} + \frac{1}{C_3}\right)^{-1} = 1.5\mu\mathrm{F}.$

(b) $Q_3 = Q_{12} = Q_{eq} = C_{eq}(8\mathrm{V}) = 12\mu\mathrm{C}$

$\Rightarrow \; V_3 = \frac{Q_3}{C_3} = \frac{12\mu\mathrm{C}}{3\mu\mathrm{F}} = 4\mathrm{V}.$

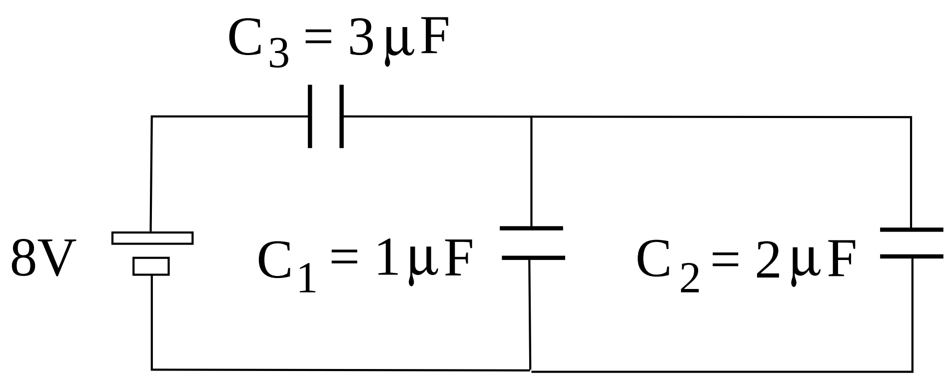
Intermediate Exam II: Problem #1 (Spring '05)

The circuit of capacitors connected to a battery is at equilibrium.

(a) Find the equivalent capacitance C_{eq}.

(b) Find the voltage V_3 across capacitor C_3.

(c) Find the the charge Q_2 on capacitor C_2.



Solution:

(a) $C_{12} = C_1 + C_2 = 3\mu\text{F}, \quad C_{eq} = \left(\frac{1}{C_{12}} + \frac{1}{C_3}\right)^{-1} = 1.5\mu\text{F}.$

(b) $Q_3 = Q_{12} = Q_{eq} = C_{eq}(8\text{V}) = 12\mu\text{C}$

$\Rightarrow \; V_3 = \frac{Q_3}{C_3} = \frac{12\mu\text{C}}{3\mu\text{F}} = 4\text{V}.$

(c) $Q_2 = V_2 C_2 = 8\mu\text{C}.$

Unit Exam II: Problem #1 (Spring ’07)

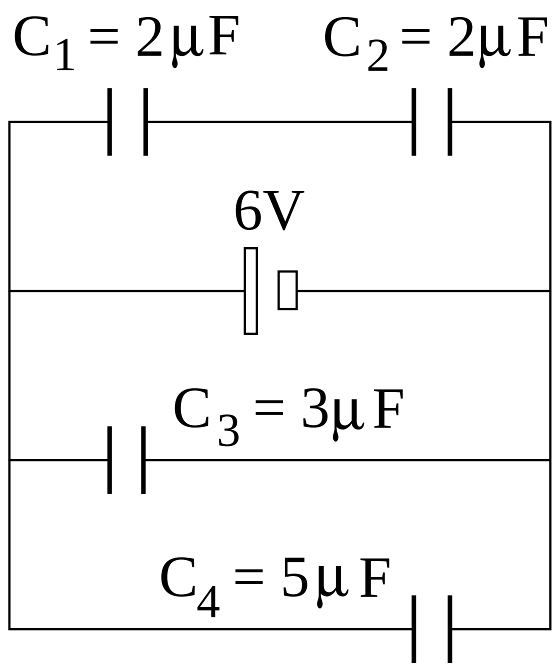
Consider the configuration of two point charges as shown.

(a) Find the energy U_3 stored on capacitor C_3.

(b) Find the voltage V_4 across capacitor C_4.

(c) Find the voltage V_2 across capacitor C_2.

(d) Find the charge Q_1 on capacitor C_1.



Unit Exam II: Problem #1 (Spring '07)

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(b) Find the voltage V_4 across capacitor C_4.

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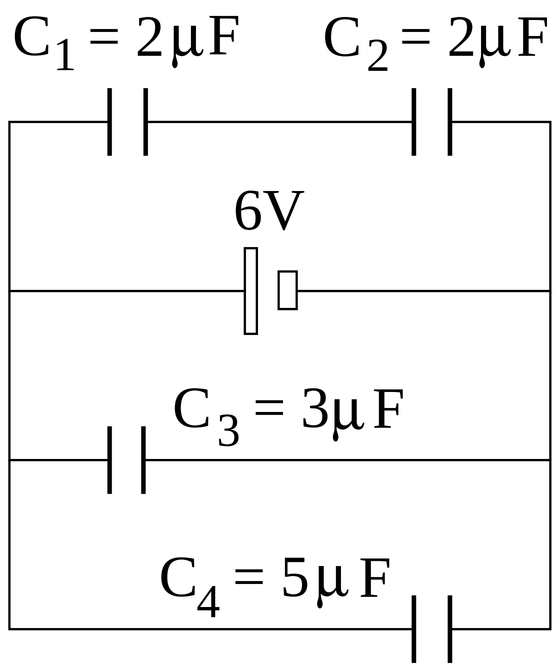

Solution:

(a) $U_3 = \frac{1}{2}(3\mu\mathrm{F})(6\mathrm{V})^2 = 54\mu\mathrm{J}.$

Unit Exam II: Problem #1 (Spring '07)

Consider the configuration of two point charges as shown.

(a) Find the energy U_3 stored on capacitor C_3.

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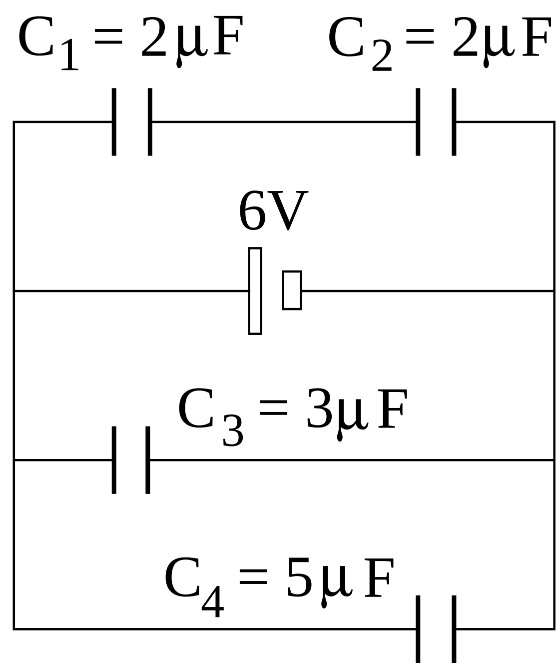


Solution:

(a) $U_3 = \frac{1}{2}(3\mu\text{F})(6\text{V})^2 = 54\mu\text{J}$.

(b) $V_4 = 6\text{V}$.

Unit Exam II: Problem #1 (Spring '07)

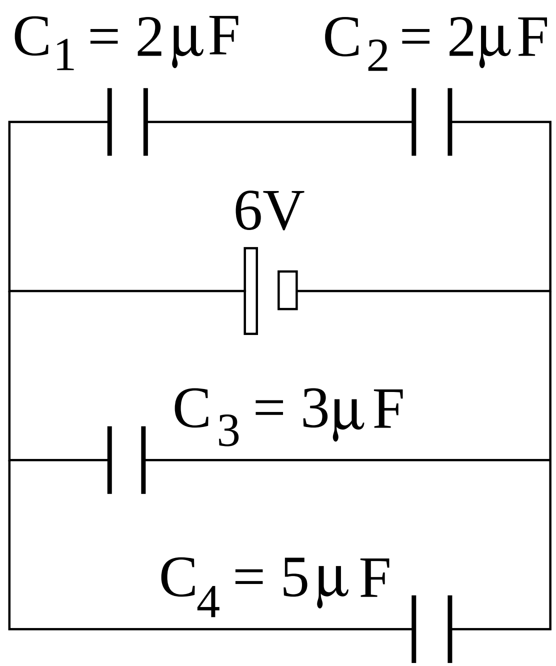
Consider the configuration of two point charges as shown.

(a) Find the energy U_3 stored on capacitor C_3.

(b) Find the voltage V_4 across capacitor C_4.

(c) Find the voltage V_2 across capacitor C_2.

(d) Find the charge Q_1 on capacitor C_1.



Solution:

(a) $U_3 = \frac{1}{2}(3\mu\text{F})(6\text{V})^2 = 54\mu\text{J}$.

(b) $V_4 = 6\text{V}$.

(c) $V_2 = \frac{1}{2}6\text{V} = 3\text{V}$.

Unit Exam II: Problem #1 (Spring '07)

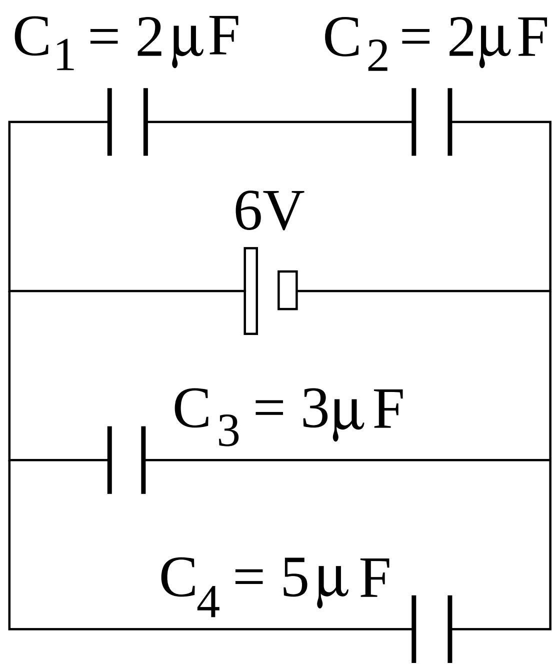
Consider the configuration of two point charges as shown.

(a) Find the energy U_3 stored on capacitor C_3.

(b) Find the voltage V_4 across capacitor C_4.

(c) Find the voltage V_2 across capacitor C_2.

(d) Find the charge Q_1 on capacitor C_1.



Solution:

(a) $U_3 = \frac{1}{2}(3\mu\text{F})(6\text{V})^2 = 54\mu\text{J}$.

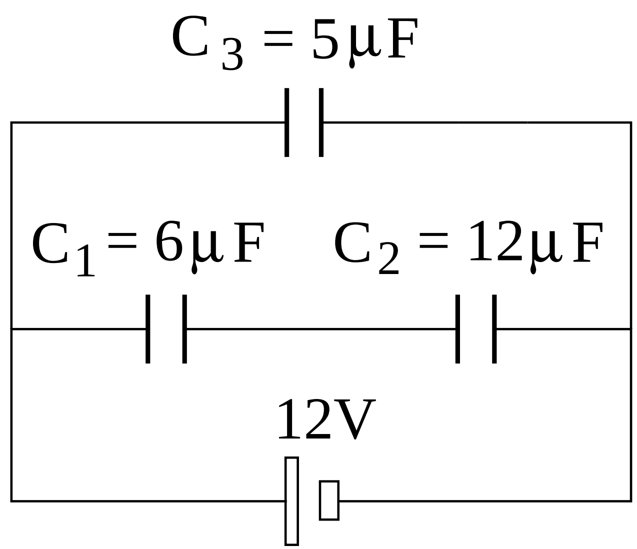
(b) $V_4 = 6\text{V}$.

(c) $V_2 = \frac{1}{2}6\text{V} = 3\text{V}$.

(d) $Q_1 = (2\mu\text{F})(3\text{V}) = 6\mu\text{C}$.

The circuit of capacitors is at equilibrium.

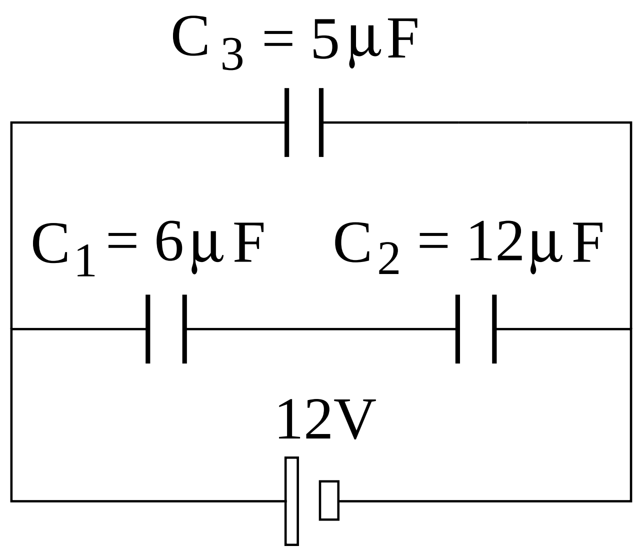
(a) Find the charge Q_1 on capacitor 1 and the charge Q_2 on capacitor 2.
(b) Find the voltage V_1 across capacitor 1 and the voltage V_2 across capacitor 2.
(c) Find the charge Q_3 and the energy U_3 on capacitor 3.



Unit Exam II: Problem #1 (Spring '08)

The circuit of capacitors is at equilibrium.

(a) Find the charge Q_1 on capacitor 1 and the charge Q_2 on capacitor 2.
(b) Find the voltage V_1 across capacitor 1 and the voltage V_2 across capacitor 2.
(c) Find the charge Q_3 and the energy U_3 on capacitor 3.



Solution:

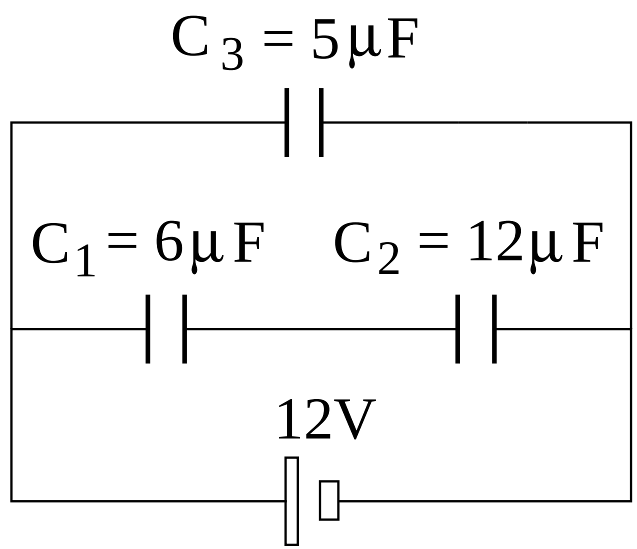
(a) $C_{12} = \left(\frac{1}{6\mu\text{F}} + \frac{1}{12\mu\text{F}}\right)^{-1} = 4\mu\text{F},$

$Q_1 = Q_2 = Q_{12} = (4\mu\text{F})(12\text{V}) = 48\mu\text{C}.$

Unit Exam II: Problem #1 (Spring ’08)

The circuit of capacitors is at equilibrium.

(a) Find the charge Q_1 on capacitor 1 and the charge Q_2 on capacitor 2.
(b) Find the voltage V_1 across capacitor 1 and the voltage V_2 across capacitor 2.
(c) Find the charge Q_3 and the energy U_3 on capacitor 3.



Solution:

(a) $C_{12} = \left(\frac{1}{6\mu\text{F}} + \frac{1}{12\mu\text{F}}\right)^{-1} = 4\mu\text{F},$

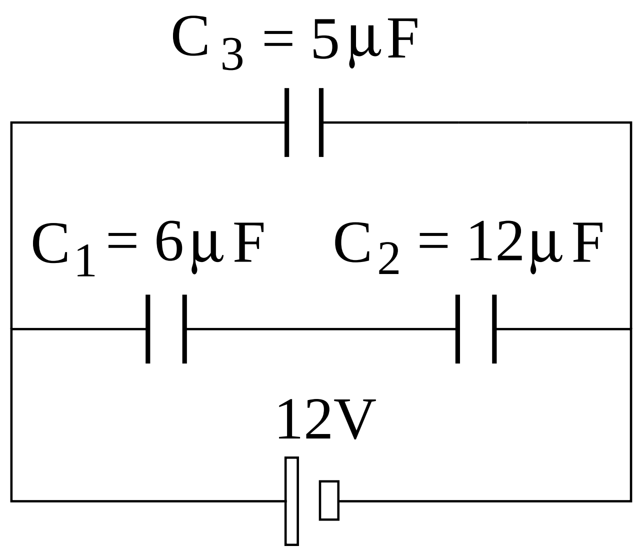
$Q_1 = Q_2 = Q_{12} = (4\mu\text{F})(12\text{V}) = 48\mu\text{C}.$

(b) $V_1 = \frac{Q_1}{C_1} = \frac{48\mu\text{C}}{6\mu\text{F}} = 8\text{V},$

$V_2 = \frac{Q_2}{C_2} = \frac{48\mu\text{C}}{12\mu\text{F}} = 4\text{V}.$

The circuit of capacitors is at equilibrium.

(a) Find the charge Q_1 on capacitor 1 and the charge Q_2 on capacitor 2.
(b) Find the voltage V_1 across capacitor 1 and the voltage V_2 across capacitor 2.
(c) Find the charge Q_3 and the energy U_3 on capacitor 3.



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(a) $C_{12} = \left(\frac{1}{6\mu\text{F}} + \frac{1}{12\mu\text{F}}\right)^{-1} = 4\mu\text{F},$

$Q_1 = Q_2 = Q_{12} = (4\mu\text{F})(12\text{V}) = 48\mu\text{C}.$

(b) $V_1 = \frac{Q_1}{C_1} = \frac{48\mu\text{C}}{6\mu\text{F}} = 8\text{V},$

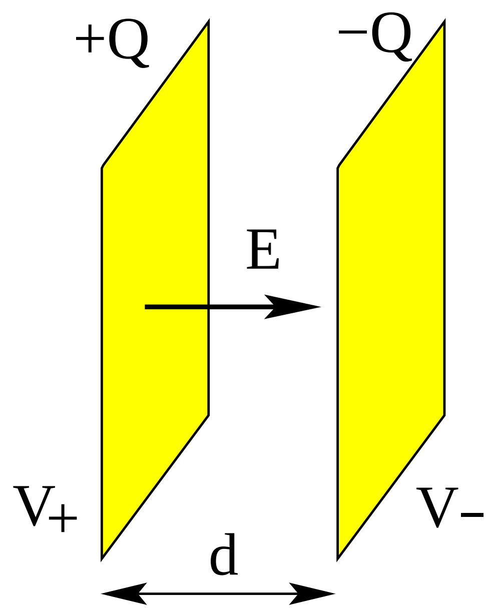
$V_2 = \frac{Q_2}{C_2} = \frac{48\mu\text{C}}{12\mu\text{F}} = 4\text{V}.$

(c) $Q_3 = (5\mu\text{F})(12\text{V}) = 60\mu\text{C},$

$U_3 = \frac{1}{2}(5\mu\text{F})(12\text{V})^2 = 360\mu\text{J}.$

Unit Exam II: Problem #2 (Spring '12)

Consider a parallel-plate capacitor of capacitance $C = 6$pF with plates separated a distance $d = 1$mm and a potential difference $V = V_+ - V_- = 3$V between them.
(a) Find the magnitude E of the electric field between the plates.
(b) Find the amount Q of charge on each plate.
(c) Find the energy U stored on the capacitor.
(d) Find the area A of each plate.



Consider a parallel-plate capacitor of capacitance $C = 6\text{pF}$ with plates separated a distance $d = 1\text{mm}$ and a potential difference $V = V_+ - V_- = 3\text{V}$ between them.
(a) Find the magnitude E of the electric field between the plates.
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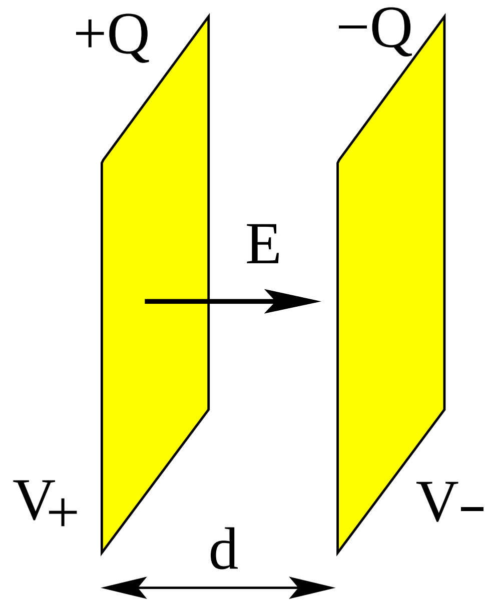

Solution:

(a) $E = \dfrac{V}{d} = \dfrac{3\text{V}}{1\text{mm}} = 3000\text{V/m}.$

Consider a parallel-plate capacitor of capacitance $C = 6$pF with plates separated a distance $d = 1$mm and a potential difference $V = V_+ - V_- = 3$V between them.
(a) Find the magnitude E of the electric field between the plates.
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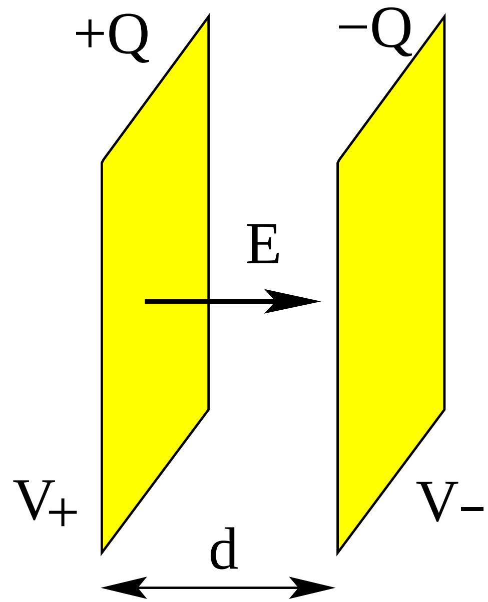

Solution:

(a) $E = \frac{V}{d} = \frac{3\text{V}}{1\text{mm}} = 3000\text{V/m}.$

(b) $Q = CV = (6\text{pF})(3\text{V}) = 18\text{pC}.$

Consider a parallel-plate capacitor of capacitance $C = 6$pF with plates separated a distance $d = 1$mm and a potential difference $V = V_+ - V_- = 3$V between them.
(a) Find the magnitude E of the electric field between the plates.
(b) Find the amount Q of charge on each plate.
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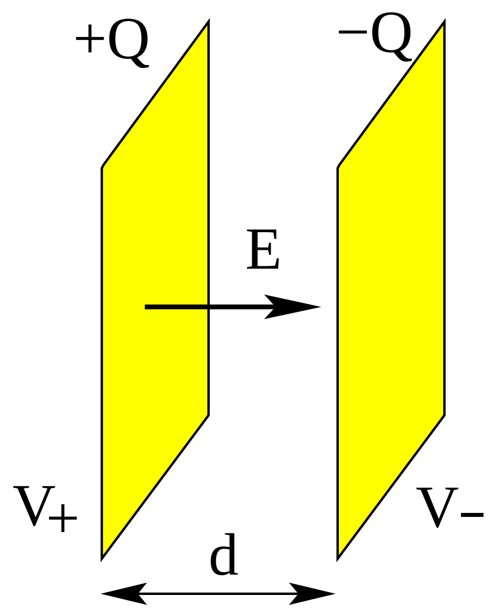

Solution:

(a) $E = \dfrac{V}{d} = \dfrac{3\mathrm{V}}{1\mathrm{mm}} = 3000\mathrm{V/m}.$

(b) $Q = CV = (6\mathrm{pF})(3\mathrm{V}) = 18\mathrm{pC}.$

(c) $U = \dfrac{1}{2}QV = 0.5(18\mathrm{pC})(3\mathrm{V}) = 27\mathrm{pJ}.$

Unit Exam II: Problem #2 (Spring '12)

Consider a parallel-plate capacitor of capacitance $C = 6$pF with plates separated a distance $d = 1$mm and a potential difference $V = V_+ - V_- = 3$V between them.
(a) Find the magnitude E of the electric field between the plates.
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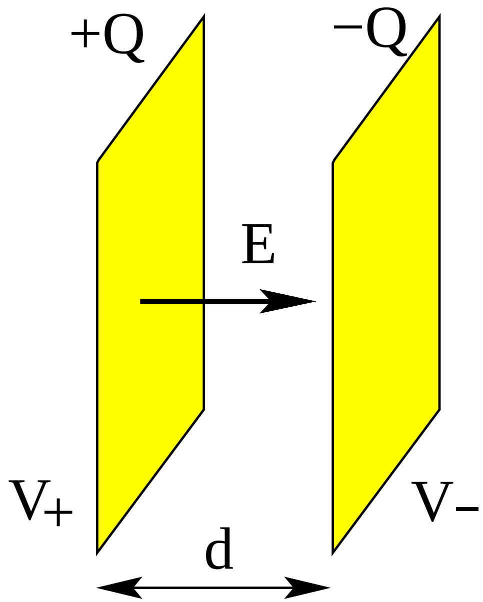
(c) Find the energy U stored on the capacitor.
(d) Find the area A of each plate.



Solution:

(a) $E = \frac{V}{d} = \frac{3\text{V}}{1\text{mm}} = 3000\text{V/m}.$

(b) $Q = CV = (6\text{pF})(3\text{V}) = 18\text{pC}.$

(c) $U = \frac{1}{2}QV = 0.5(18\text{pC})(3\text{V}) = 27\text{pJ}.$

(d) $A = \frac{Cd}{\epsilon_0} = \frac{(6\text{pF})(1\text{mm})}{8.85 \times 10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2}} = 6.78 \times 10^{-4}\text{m}^2.$