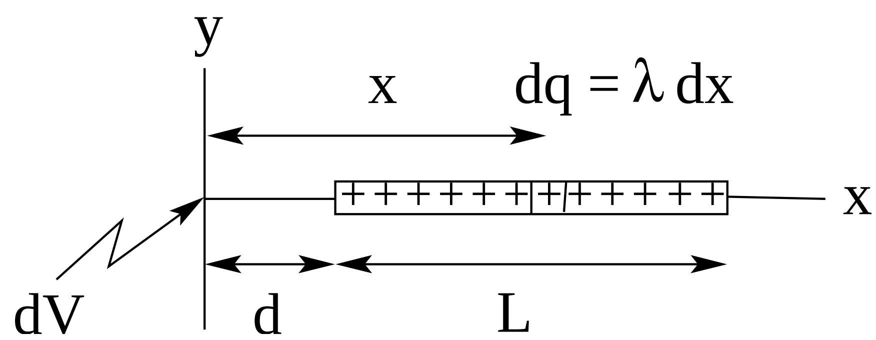
2015

06. Electric Potential II

Gerhard Müller
University of Rhode Island, gmuller@uri.edu

Abstract
Part six of course materials for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island. Documents will be updated periodically as more entries become presentable.
Some of the slides contain figures from the textbook, Paul A. Tipler and Gene Mosca. *Physics for Scientists and Engineers*, 5th/6th editions. The copyright to these figures is owned by W.H. Freeman. We acknowledge permission from W.H. Freeman to use them on this course web page. The textbook figures are not to be used or copied for any purpose outside this class without direct permission from W.H. Freeman.

Electric Potential of Charged Rod



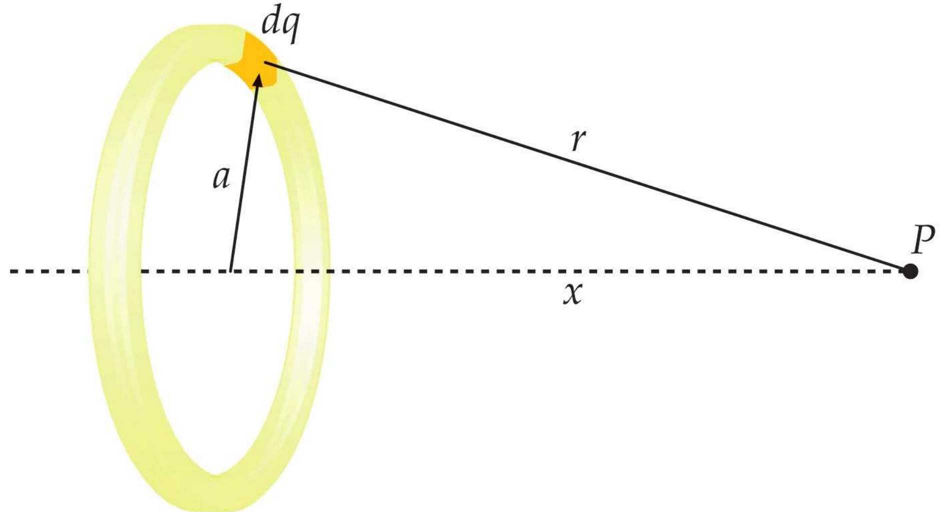
- Charge per unit length: $\lambda = Q/L$
- Charge on slice dx: $dq = \lambda dx$
- Electric potential generated by slice dx: $dV = \dfrac{kdq}{x} = \dfrac{k\lambda dx}{x}$
- Electric potential generated by charged rod:

$$V = k\lambda \int_d^{d+L} \frac{dx}{x} = k\lambda \left[\ln x\right]_d^{d+L} = k\lambda \left[\ln(d+L) - \ln d\right] = k\lambda \ln \frac{d+L}{d}$$

- Limiting case of very short rod $(L \ll d)$: $V = k\lambda \ln\left(1 + \dfrac{L}{d}\right) \simeq k\lambda \dfrac{L}{d} = \dfrac{kQ}{d}$

Electric Potential of Charged Ring

- Total charge on ring: Q
- Charge per unit length: $\lambda = Q/2\pi a$
- Charge on arc: dq

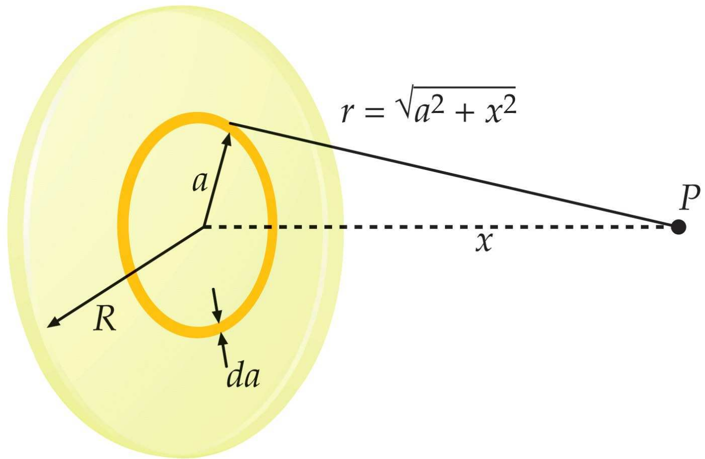


Find the electric potential at point P on the axis of the ring.

- $dV = k\frac{dq}{r} = \frac{kdq}{\sqrt{x^2+a^2}}$
- $V(x) = k\int\frac{dq}{\sqrt{x^2+a^2}} = \frac{k}{\sqrt{x^2+a^2}}\int dq = \frac{kQ}{\sqrt{x^2+a^2}}$

Electric Potential of Charged Disk

- Area of ring: $2\pi a da$
- Charge on ring: $dq = \sigma(2\pi a da)$
- Charge on disk: $Q = \sigma(\pi R^2)$



Find the electric potential at point P on the axis of the disk.

- $dV = k\dfrac{dq}{\sqrt{x^2+a^2}} = 2\pi\sigma k\dfrac{a da}{\sqrt{x^2+a^2}}$
- $V(x) = 2\pi\sigma k\displaystyle\int_0^R \frac{a da}{\sqrt{x^2+a^2}} = 2\pi\sigma k\left[\sqrt{x^2+a^2}\right]_0^R = 2\pi\sigma k\left[\sqrt{x^2+R^2} - |x|\right]$

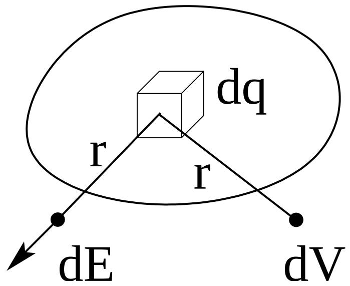
Electric potential at large distances from the disk $(|x| \gg R)$:

$$V(x) = 2\pi\sigma k|x|\left[\sqrt{1+\frac{R^2}{x^2}} - 1\right] \simeq 2\pi\sigma k|x|\left[1+\frac{R^2}{2x^2} - 1\right] = \frac{k\sigma\pi R^2}{|x|} = \frac{kQ}{|x|}$$

Electric Field and Electric Potential

Determine the field or the potential from the source (charge distribution):

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\hat{r}$$



$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

Determine the field from the potential: $\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$

Determine the potential from the field: $V = -\int_{\vec{r}_0}^{\vec{r}} \vec{E}\cdot d\vec{s}$

- Systems with $\vec{E} = E_x(x)\hat{i}$: $E_x = -\frac{dV}{dx} \quad \Leftrightarrow \quad V(x) = -\int_{x_0}^{x} E_x dx$
- Application to charged ring: $E_x = \frac{kQx}{(x^2+a^2)^{3/2}} \Leftrightarrow V = \frac{kQ}{\sqrt{x^2+a^2}}$
- Application to charged disk (at $x > 0$):

$$E_x = 2\pi\sigma k\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right] \Leftrightarrow V = 2\pi\sigma k\left[\sqrt{x^2+R^2} - x\right]$$

Electric Potential and Electric Field in One Dimension (1)

For given electric potential $V(x)$find the electric field

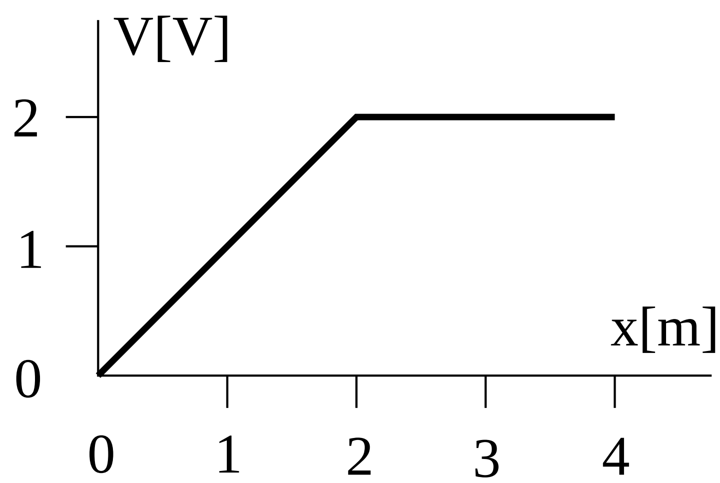
(a) $E_x(1\text{m})$,

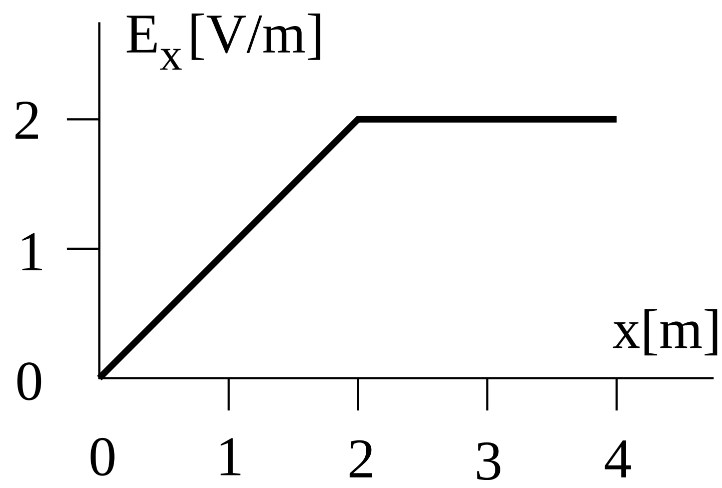
(b) $E_x(3\text{m})$.

For given electric field $E_x(x)$ and given reference potential potential $V(0) = 0$ find the electric potential

(c) $V(2\text{m})$,

(d) $V(4\text{m})$.





Electric Potential and Electric Field in One Dimension (2)

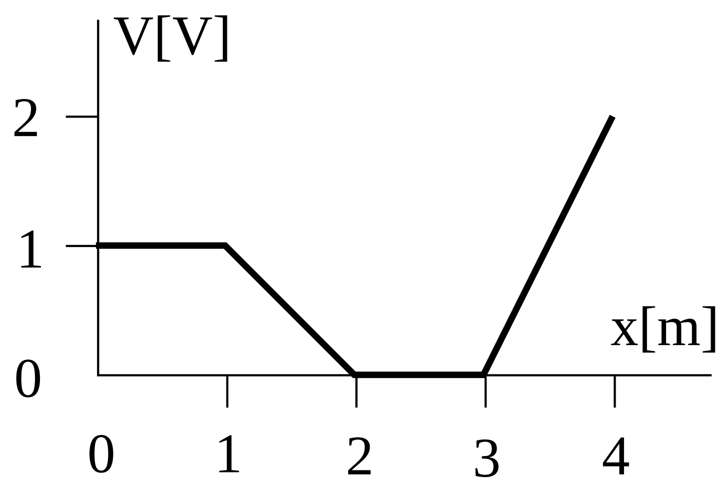
For given electric potential $V(x)$ find the electric field

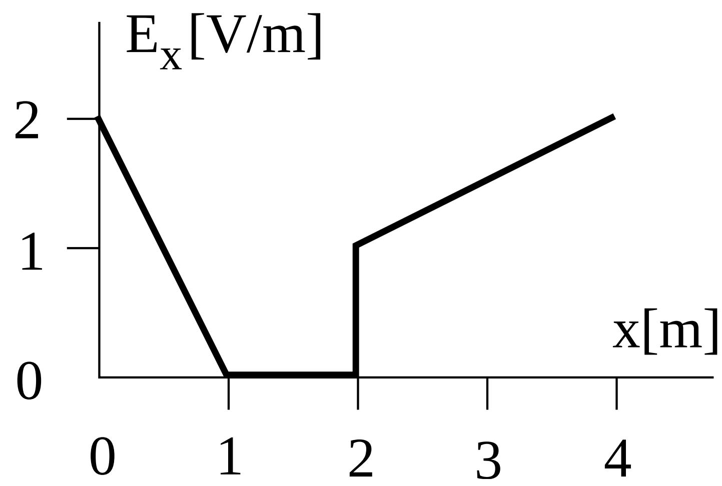
(a) $E_x(0.5\mathrm{m})$, (b) $E_x(1.5\mathrm{m})$,

(c) $E_x(2.5\mathrm{m})$, (d) $E_x(3.5\mathrm{m})$.

For given electric field $E_x(x)$ and given reference potential potential $V(0) = 0$ find the electric potential

(e) $V(1\mathrm{m})$, (f) $V(2\mathrm{m})$, (g) $V(4\mathrm{m})$.

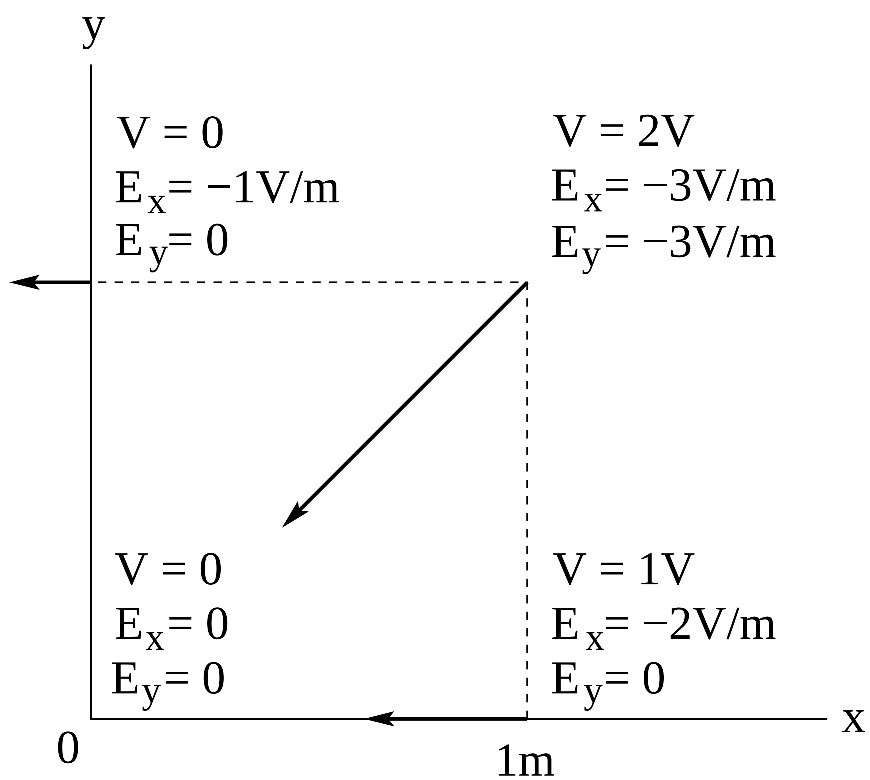




Electric Field from Electric Potential in Two Dimensions

- Given is the electric potential: $V(x,y) = ax^2 + bxy^3$ with $a = 1\text{V/m}^2$, $b = 1\text{V/m}^4$.
- Find the electric field: $\vec{E}(x,y) = E_x(x,y)\hat{i} + E_y(x,y)\hat{j}$ via partial derivatives.

$$E_x = -\frac{\partial V}{\partial x} = -2ax - by^3, \qquad E_y = -\frac{\partial V}{\partial y} = -3bxy^2$$



Electric Potential from Electric Field in Two Dimensions

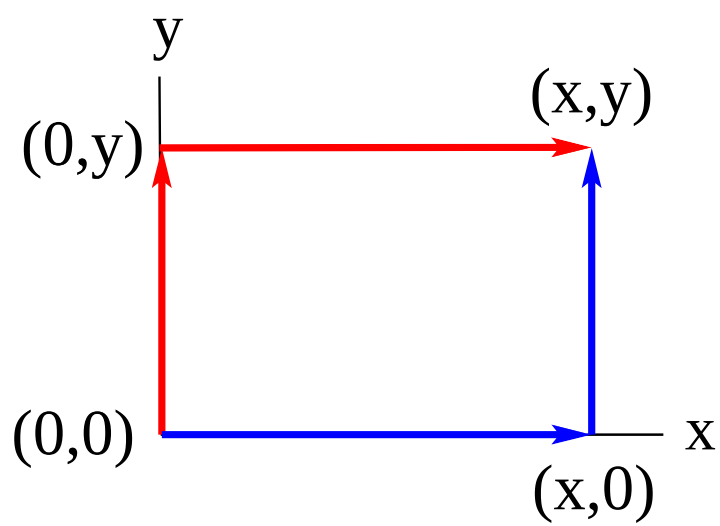
- Given is the electric field: $\vec{E} = -(2ax + by^3)\hat{i} - 3bxy^2\hat{j}$ with $a = 1$V/m^2, $b = 1$V/m^4.
- Find the electric potential $V(x, y)$ via integral along a specific path:

Red path $(0,0) \to (0,y) \to (x,y)$:

$$V(x,y) = -\int_0^y E_y(0,y)dy - \int_0^x E_x(x,y)dx$$
$$= 0 + \int_0^x (2ax + by^3)dx = ax^2 + bxy^3$$

Blue path $(0,0) \to (x,0) \to (x,y)$:

$$V(x,y) = -\int_0^x E_x(x,0)dx - \int_0^y E_y(x,y)dy$$
$$= \int_0^x (2ax)dx + \int_0^y (3bxy^2)dy = ax^2 + bxy^3$$

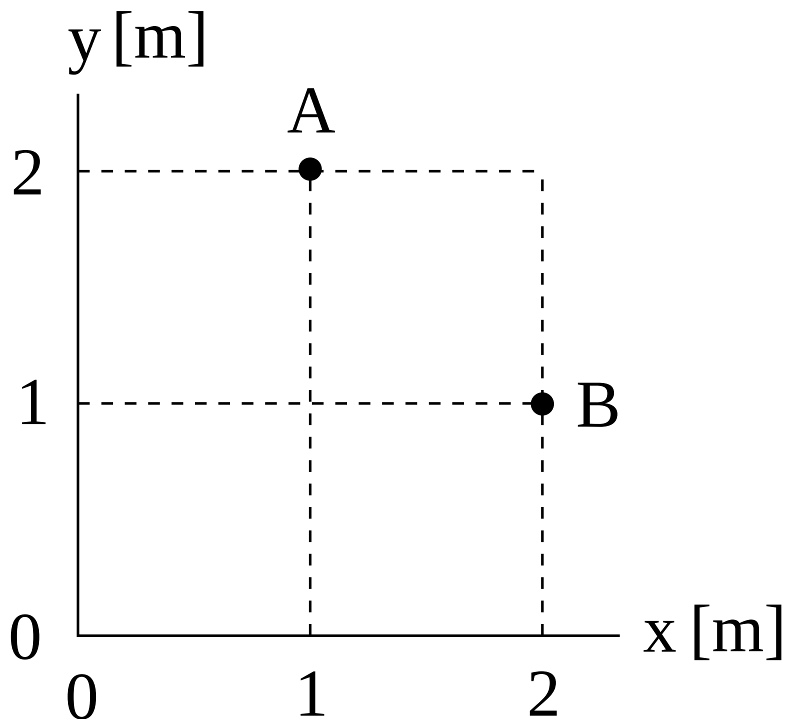


Electric Potential and Electric Field in Two Dimensions

Given is the electric potential $V(x, y) = cxy^2$ with $c = 1$V/m^3.

(a) Find the value (in SI units) of the electric potential V at point A.

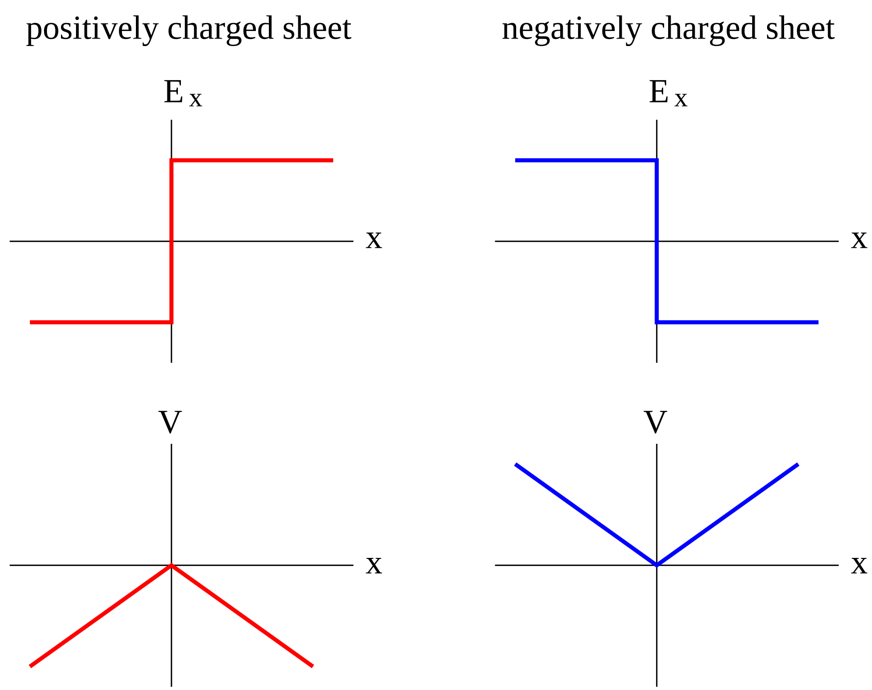
(b) Find the components E_x, E_y (in SI units) of the electric field at point B.



Electric Potential of a Charged Plane Sheet

Consider an infinite plane sheet perpendicular to the x-axis at $x = 0$.
The sheet is uniformly charged with charge per unit area σ.

- Electric field (magnitude): $E = 2\pi k|\sigma| = \dfrac{|\sigma|}{2\epsilon_0}$
- Direction: away from (toward) the sheet if $\sigma > 0$ $(\sigma < 0)$.
- Electric field (x-component):
  $E_x = \pm 2\pi k\sigma$.
- Electric potential:
  $V = -\int_0^x E_x dx = \mp 2\pi k\sigma x$.
- Here we have used $x_0 = 0$
  as the reference coordinate.



Electric Potential of a Uniformly Charged Spherical Shell

- Electric charge on shell: $Q = \sigma A = 4\pi\sigma R^2$

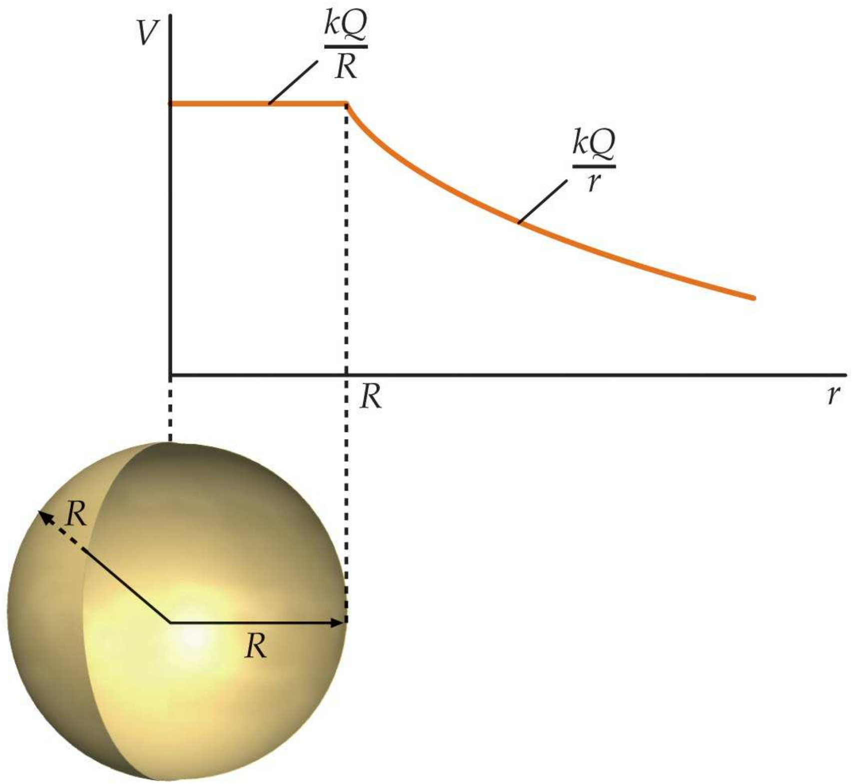
- Electric field at $r > R$: $E = \dfrac{kQ}{r^2}$
- Electric field at $r < R$: $E = 0$
- Electric potential at $r > R$:

$$V = -\int_{\infty}^{r} \frac{kQ}{r^2}\, dr = \frac{kQ}{r}$$

- Electric potential at $r < R$:

$$V = -\int_{\infty}^{R} \frac{kQ}{r^2}\, dr - \int_{R}^{r} (0) dr = \frac{kQ}{R}$$

- Here we have used $r_0 = \infty$ as the reference value of the radial coordinate.



Electric Potential of a Uniformly Charged Solid Sphere

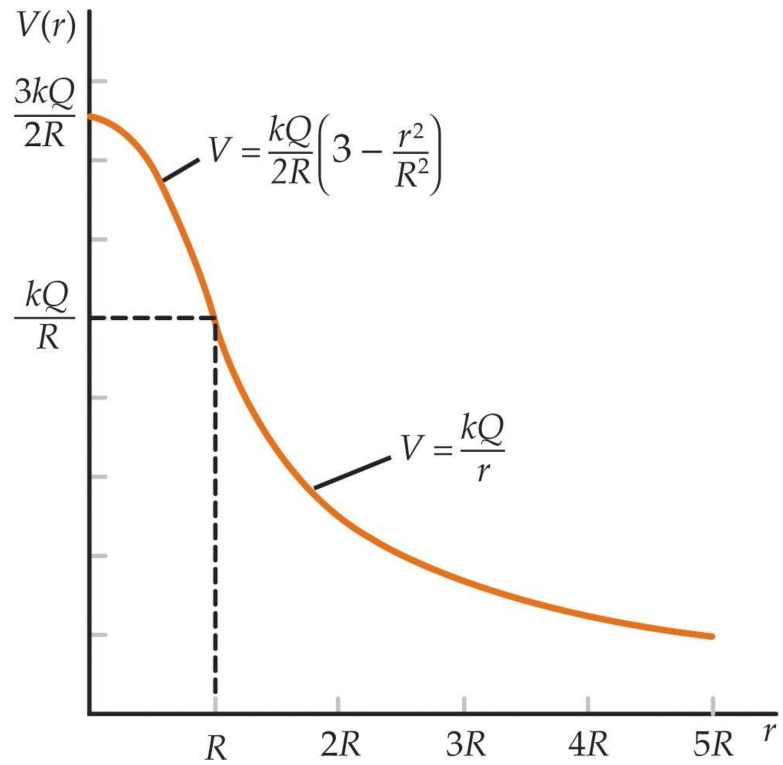
- Electric charge on sphere: $Q = \rho V = \frac{4\pi}{3}\rho R^3$
- Electric field at $r > R$: $E = \frac{kQ}{r^2}$
- Electric field at $r < R$: $E = \frac{kQ}{R^3}\,r$
- Electric potential at $r > R$:

$$V = -\int_{\infty}^{r} \frac{kQ}{r^2}\,dr = \frac{kQ}{r}$$

- Electric potential at $r < R$:

$$V = -\int_{\infty}^{R} \frac{kQ}{r^2}\,dr - \int_{R}^{r} \frac{kQ}{R^3}\,r\,dr$$

$$\Rightarrow V = \frac{kQ}{R} - \frac{kQ}{2R^3}\left(r^2 - R^2\right) = \frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)$$



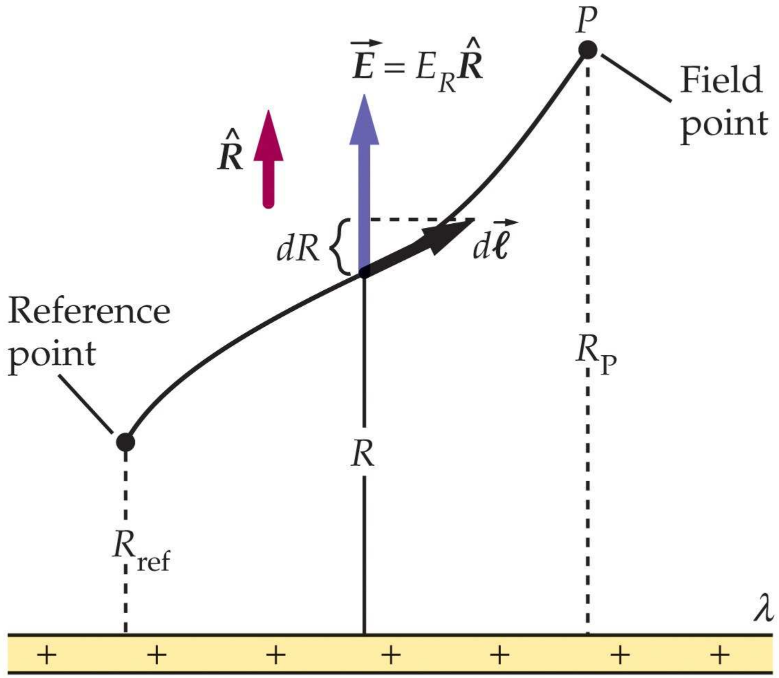
Electric Potential of a Uniformly Charged Wire

- Consider a uniformly charged wire of infinite length.
- Charge per unit length on wire: λ (here assumed positive).
- Electric field at radius r: $E = \dfrac{2k\lambda}{r}$.
- Electric potential at radius r:

$$V = -2k\lambda \int_{r_0}^{r} \frac{1}{r}\, dr = -2k\lambda \left[\ln r - \ln r_0\right]$$

$$\Rightarrow V = 2k\lambda \ln \frac{r_0}{r}$$

- Here we have used a finite, nonzero reference radius $r_0 \neq 0, \infty$.
- The illustration from the textbook uses R_{ref} for the reference radius, R for the integration variable, and R_p for the radial position of the field point.



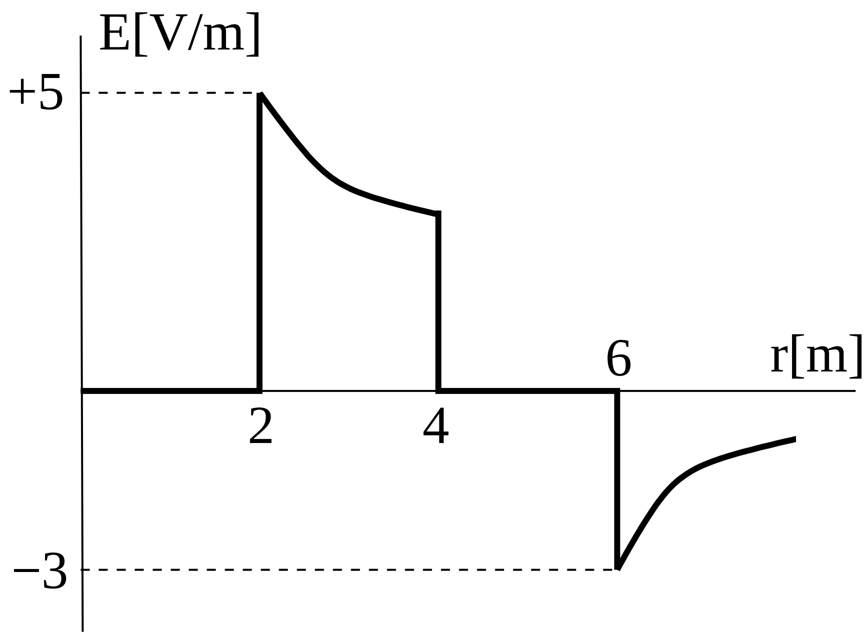
Electric Potential of Conducting Spheres (1)

A conducting sphere of radius $r_1 = 2$m is surrounded by a concentric conducting spherical shell of radii $r_2 = 4$m and $r_3 = 6$m. The graph shows the electric field $E(r)$.

(a) Find the charges q_1, q_2, q_3 on the three conducting surfaces.

(b) Find the values V_1, V_2, V_3 of the electric potential on the three conducting surfaces relative to a point at infinity.

(c) Sketch the potential $V(r)$.



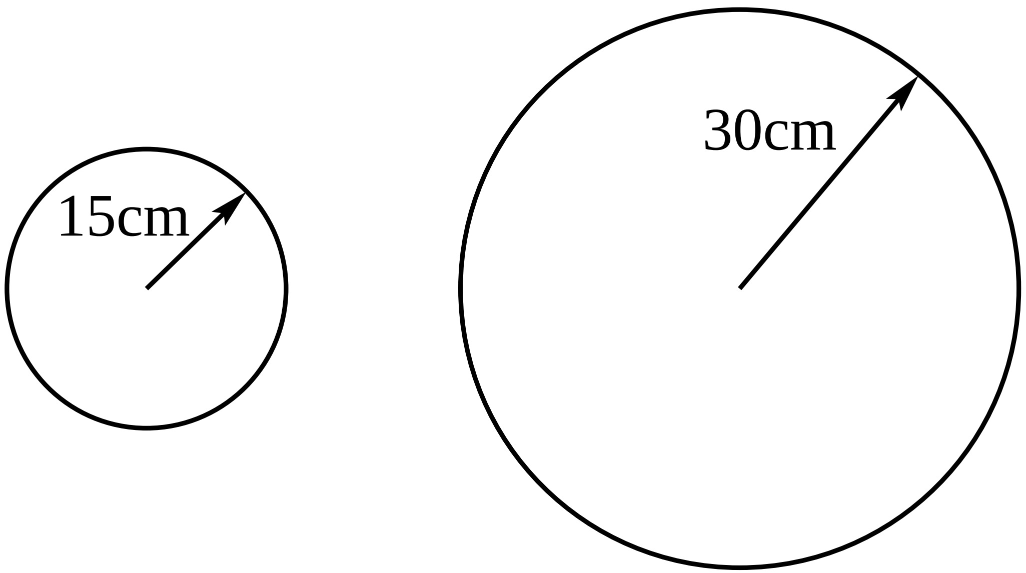
Electric Potential of Conducting Spheres (2)

Consider a conducting sphere with radius $r = 15$cm and electric potential $V = 200$V relative to a point at infinity.

(a) Find the charge Q and the surface charge density σ on the sphere.

(b) Find the magnitude of the electric field E just outside the sphere.

(c) What happens to the values of Q, V, σ, E when the radius of the sphere is doubled?

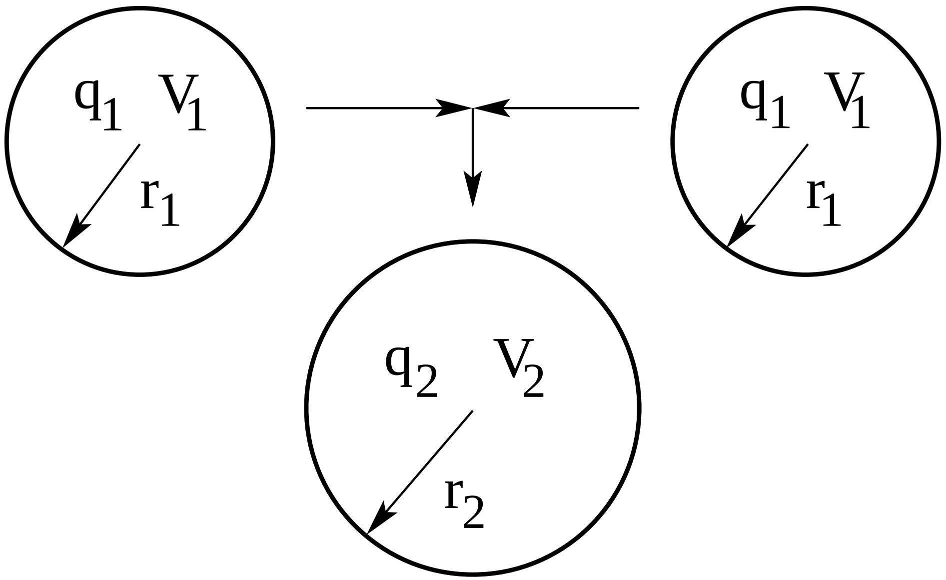


Electric Potential of Conducting Spheres (3)

A spherical raindrop of 1mm diameter carries a charge of 30pC.

(a) Find the electric potential of the drop relative to a point at infinity under the assumption that it is a conductor.

(b) If two such drops of the same charge and diameter combine to form a single spherical drop, what is its electric potential?



Electric Potential of Conducting Spheres (4)

A positive charge is distributed over two conducting spheres 1 and 2 of unequal size and connected by a long thin wire. The system is at equilibrium.

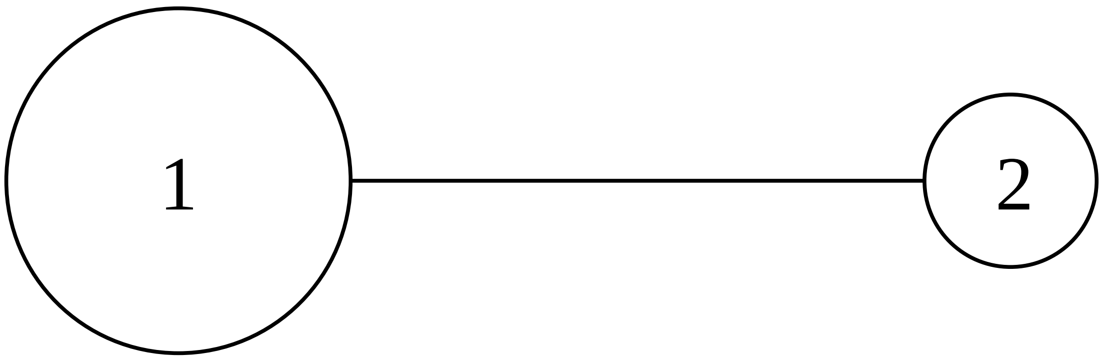
Which sphere (1 or 2)...

(a) carries more charge on its surface?

(b) has the higher surface charge density?

(c) is at a higher electric potential?

(d) has the stronger electric field next to it?



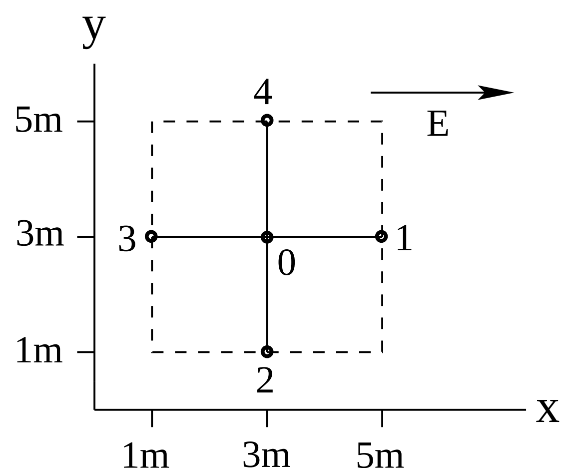
Unit Exam I: Problem #3 (Spring '11)

Consider a region of space with a uniform electric field $\mathbf{E} = 0.5\mathrm{V/m}\,\hat{\mathbf{i}}$. Ignore gravity.

(a) If the electric potential vanishes at point 0, what are the electric potentials at points 1 and 2?

(b) If an electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) is released from rest at point 0, toward which point will it start moving?

(c) What will be the speed of the electron when it gets there?

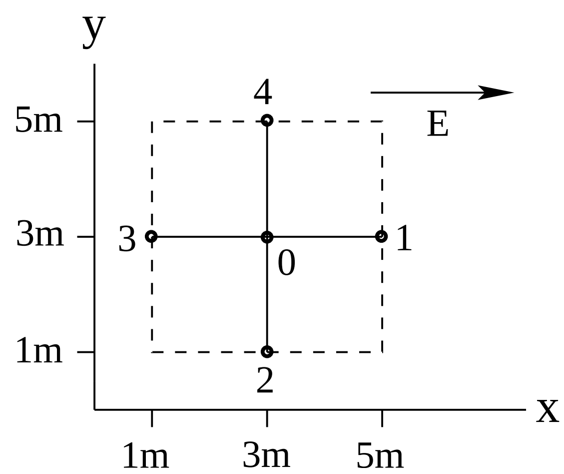


Consider a region of space with a uniform electric field $\mathbf{E} = 0.5\mathrm{V/m}\,\hat{\mathbf{i}}$. Ignore gravity.

(a) If the electric potential vanishes at point 0, what are the electric potentials at points 1 and 2?

(b) If an electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) is released from rest at point 0, toward which point will it start moving?

(c) What will be the speed of the electron when it gets there?



Solution:

(a) $V_1 = -(0.5\mathrm{V/m})(2\mathrm{m}) = -1\mathrm{V}, \quad V_2 = 0.$

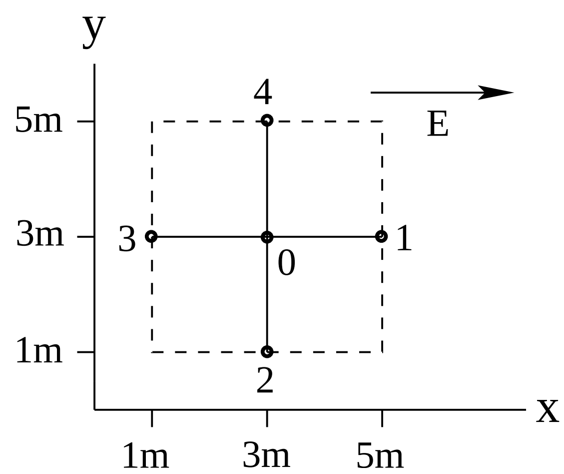
Unit Exam I: Problem #3 (Spring '11)

Consider a region of space with a uniform electric field $\mathbf{E} = 0.5\mathrm{V/m}\,\hat{\mathbf{i}}$. Ignore gravity.

(a) If the electric potential vanishes at point 0, what are the electric potentials at points 1 and 2?

(b) If an electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) is released from rest at point 0, toward which point will it start moving?

(c) What will be the speed of the electron when it gets there?



Solution:

(a) $V_1 = -(0.5\mathrm{V/m})(2\mathrm{m}) = -1\mathrm{V}, \quad V_2 = 0.$

(b) $\mathbf{F} = q\mathbf{E} = -|qE|\hat{\mathbf{i}}$ (toward point 3).

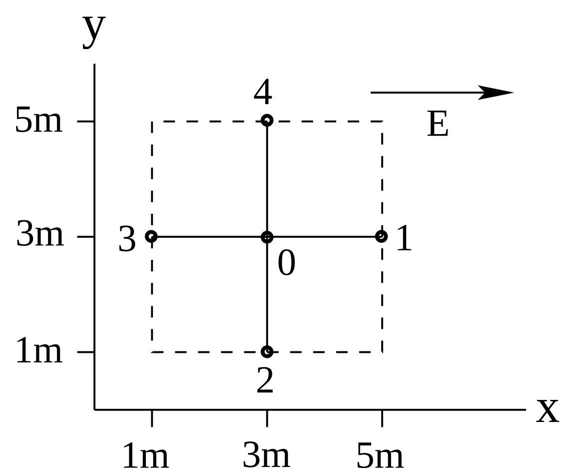
Unit Exam I: Problem #3 (Spring '11)

Consider a region of space with a uniform electric field $\mathbf{E} = 0.5\mathrm{V/m}\,\hat{\mathbf{i}}$. Ignore gravity.

(a) If the electric potential vanishes at point 0, what are the electric potentials at points 1 and 2?

(b) If an electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) is released from rest at point 0, toward which point will it start moving?

(c) What will be the speed of the electron when it gets there?



Solution:

(a) $V_1 = -(0.5\mathrm{V/m})(2\mathrm{m}) = -1\mathrm{V}, \quad V_2 = 0.$

(b) $\mathbf{F} = q\mathbf{E} = -|qE|\hat{\mathbf{i}}$ (toward point 3).

(c) $\Delta V = (V_3 - V_0) = 1\mathrm{V}, \quad \Delta U = q\Delta V = -1.60 \times 10^{-19}\mathrm{J},$

$$K = -\Delta U = 1.60 \times 10^{-19}\mathrm{J}, \quad v = \sqrt{\frac{2K}{m}} = 5.93 \times 10^5\mathrm{m/s}.$$

Alternatively:

$$F = qE = 8.00 \times 10^{-20}\mathrm{N}, \quad a = \frac{F}{m} = 8.78 \times 10^{10}\mathrm{m/s}^2,$$

$$|\Delta x| = 2\mathrm{m}, \quad v = \sqrt{2a|\Delta x|} = 5.93 \times 10^5\mathrm{m/s}.$$

An electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) and a proton ($m = 1.67 \times 10^{-27}$kg, $q = +1.60 \times 10^{-19}$C) are released from rest midway between oppositely charged parallel plates. The plates are at the electric potentials shown.

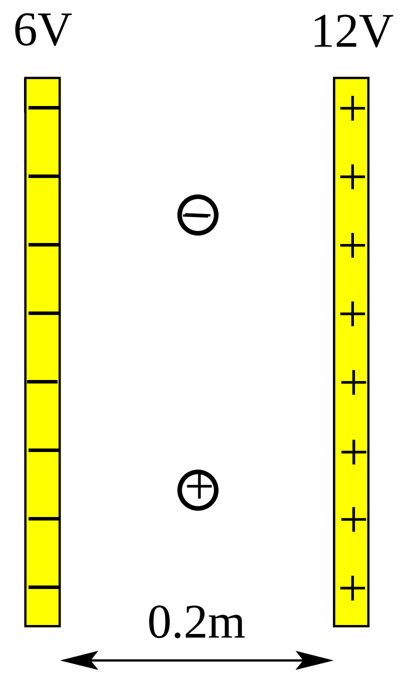
(a) Find the magnitude of the electric field between the plates.

(b) What direction (left/right) does the electric field have?

(c) Which particle (electron/proton/both) is accelerated to the left?

(d) Why does the electron reach the plate before the proton?

(e) Find the kinetic energy of the proton when it reaches the plate.



Unit Exam I: Problem #3 (Fall '10)

An electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) and a proton ($m = 1.67 \times 10^{-27}$kg, $q = +1.60 \times 10^{-19}$C) are released from rest midway between oppositely charged parallel plates. The plates are at the electric potentials shown.

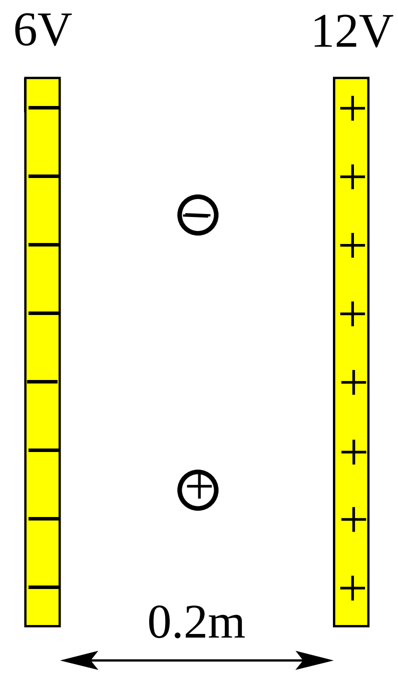
(a) Find the magnitude of the electric field between the plates.

(b) What direction (left/right) does the electric field have?

(c) Which particle (electron/proton/both) is accelerated to the left?

(d) Why does the electron reach the plate before the proton?

(e) Find the kinetic energy of the proton when it reaches the plate.



Solution:

(a) $E = 6\text{V}/0.2\text{m} = 30\text{V/m}.$

Unit Exam I: Problem #3 (Fall '10)

An electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) and a proton ($m = 1.67 \times 10^{-27}$kg, $q = +1.60 \times 10^{-19}$C) are released from rest midway between oppositely charged parallel plates. The plates are at the electric potentials shown.

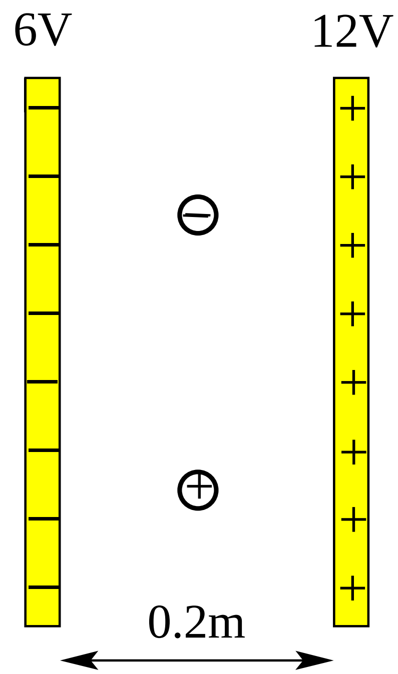
(a) Find the magnitude of the electric field between the plates.

(b) What direction (left/right) does the electric field have?

(c) Which particle (electron/proton/both) is accelerated to the left?

(d) Why does the electron reach the plate before the proton?

(e) Find the kinetic energy of the proton when it reaches the plate.



Solution:

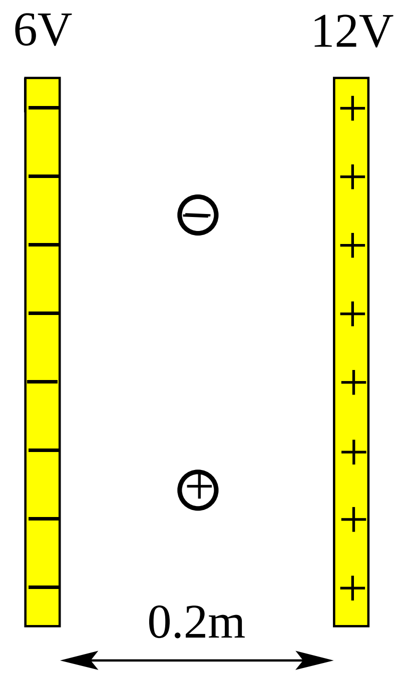
(a) $E = 6\text{V}/0.2\text{m} = 30\text{V/m}.$

(b) left

Unit Exam I: Problem #3 (Fall '10)

An electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) and a proton ($m = 1.67 \times 10^{-27}$kg, $q = +1.60 \times 10^{-19}$C) are released from rest midway between oppositely charged parallel plates. The plates are at the electric potentials shown.

(a) Find the magnitude of the electric field between the plates.
(b) What direction (left/right) does the electric field have?
(c) Which particle (electron/proton/both) is accelerated to the left?
(d) Why does the electron reach the plate before the proton?
(e) Find the kinetic energy of the proton when it reaches the plate.



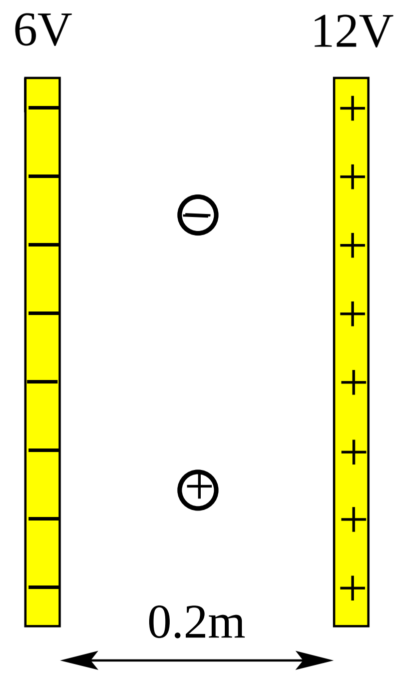
Solution:

(a) $E = 6\text{V}/0.2\text{m} = 30\text{V/m}.$
(b) left
(c) proton (positive charge)

Unit Exam I: Problem #3 (Fall '10)

An electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) and a proton ($m = 1.67 \times 10^{-27}$kg, $q = +1.60 \times 10^{-19}$C) are released from rest midway between oppositely charged parallel plates. The plates are at the electric potentials shown.

(a) Find the magnitude of the electric field between the plates.
(b) What direction (left/right) does the electric field have?
(c) Which particle (electron/proton/both) is accelerated to the left?
(d) Why does the electron reach the plate before the proton?
(e) Find the kinetic energy of the proton when it reaches the plate.

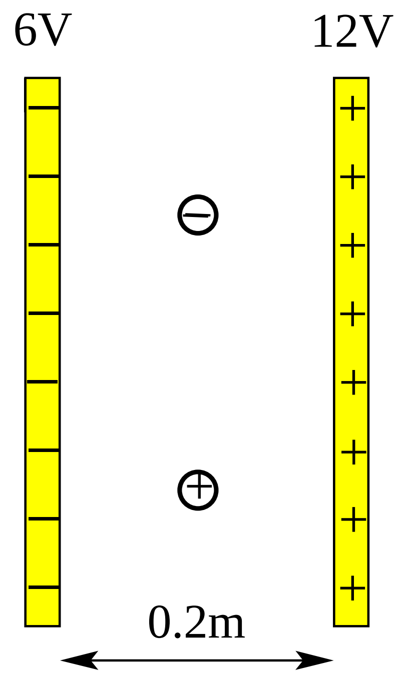


Solution:

(a) $E = 6\text{V}/0.2\text{m} = 30\text{V/m}$.
(b) left
(c) proton (positive charge)
(d) smaller m, equal $|q| \quad \Rightarrow$ larger $|q|E/m$

Unit Exam I: Problem #3 (Fall ’10)

An electron ($m = 9.11 \times 10^{-31}$kg, $q = -1.60 \times 10^{-19}$C) and a proton ($m = 1.67 \times 10^{-27}$kg, $q = +1.60 \times 10^{-19}$C) are released from rest midway between oppositely charged parallel plates. The plates are at the electric potentials shown.



(a) Find the magnitude of the electric field between the plates.

(b) What direction (left/right) does the electric field have?

(c) Which particle (electron/proton/both) is accelerated to the left?

(d) Why does the electron reach the plate before the proton?

(e) Find the kinetic energy of the proton when it reaches the plate.

Solution:

(a) $E = 6\text{V}/0.2\text{m} = 30\text{V/m}.$

(b) left

(c) proton (positive charge)

(d) smaller m, equal $|q|$ $\Rightarrow$ larger $|q|E/m$

(e) $K = |q\Delta V| = (1.6 \times 10^{-19}\text{C})(3\text{V}) = 4.8 \times 10^{-19}\text{J}.$

Intermediate Exam I: Problem #1 (Spring ’06)

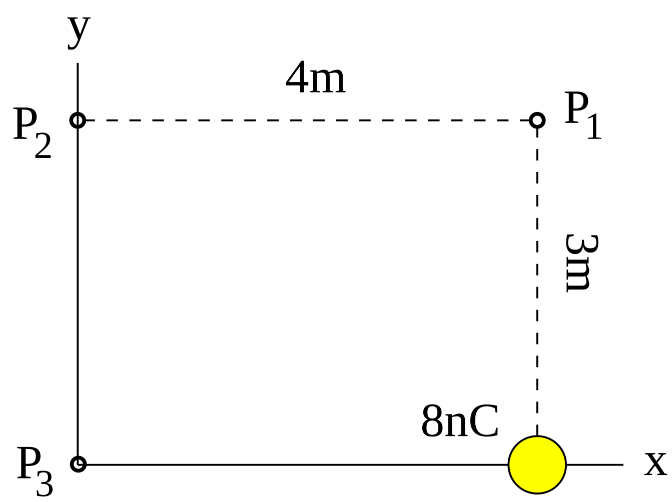
Consider a point charge $q = +8$nC at position $x = 4$m, $y = 0$ as shown.

(a) Find the electric field components E_x and E_y at point P_1.

(b) Find the electric field components E_x and E_y at point P_2.

(c) Find the electric potential V at point P_3.

(d) Find the electric potential V at point P_2.



Intermediate Exam I: Problem #1 (Spring '06)

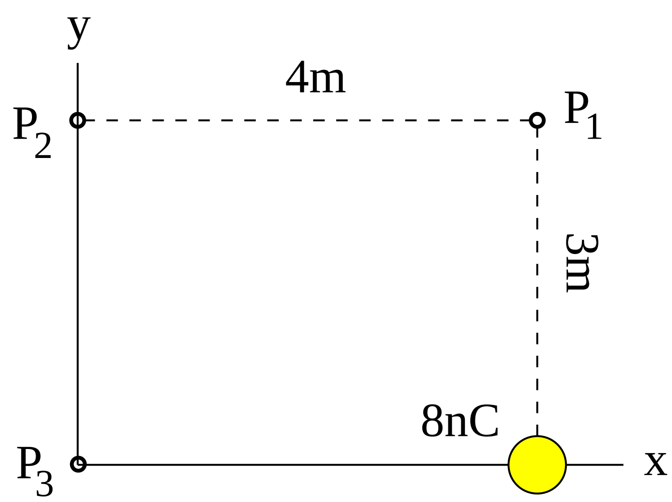
Consider a point charge $q = +8\text{nC}$ at position $x = 4\text{m}$, $y = 0$ as shown.

(a) Find the electric field components E_x and E_y at point P_1.

(b) Find the electric field components E_x and E_y at point P_2.

(c) Find the electric potential V at point P_3.

(d) Find the electric potential V at point P_2.



Solution:

(a) $E_x = 0, \qquad E_y = k\frac{8\text{nC}}{(3\text{m})^2} = 7.99\text{N/C}.$

Intermediate Exam I: Problem #1 (Spring ’06)

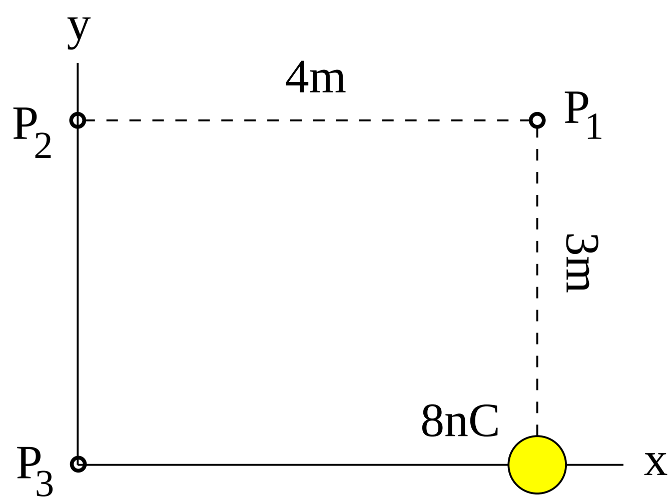
Consider a point charge $q = +8$nC at position $x = 4$m, $y = 0$ as shown.

(a) Find the electric field components E_x and E_y at point P_1.

(b) Find the electric field components E_x and E_y at point P_2.

(c) Find the electric potential V at point P_3.

(d) Find the electric potential V at point P_2.



Solution:

(a) $E_x = 0, \qquad E_y = k\frac{8\text{nC}}{(3\text{m})^2} = 7.99\text{N/C}.$

(b) $E_x = -k\frac{8\text{nC}}{(5\text{m})^2}\cos\theta = -2.88\text{N/C} \times \frac{4}{5} = -2.30\text{N/C}.$

$E_y = k\frac{8\text{nC}}{(5\text{m})^2}\sin\theta = 2.88\text{N/C} \times \frac{3}{5} = 1.73\text{N/C}.$

Intermediate Exam I: Problem #1 (Spring '06)

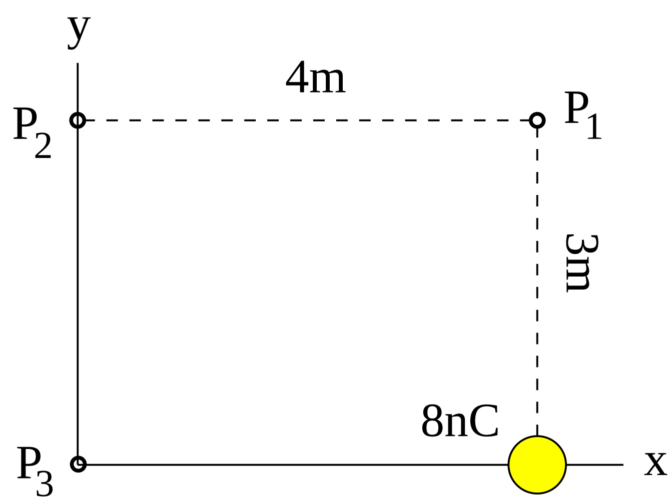
Consider a point charge $q = +8$nC at position $x = 4$m, $y = 0$ as shown.

(a) Find the electric field components E_x and E_y at point P_1.

(b) Find the electric field components E_x and E_y at point P_2.

(c) Find the electric potential V at point P_3.

(d) Find the electric potential V at point P_2.



Solution:

(a) $E_x = 0, \qquad E_y = k\dfrac{8\text{nC}}{(3\text{m})^2} = 7.99\text{N/C}.$

(b) $E_x = -k\dfrac{8\text{nC}}{(5\text{m})^2}\cos\theta = -2.88\text{N/C} \times \dfrac{4}{5} = -2.30\text{N/C}.$

$E_y = k\dfrac{8\text{nC}}{(5\text{m})^2}\sin\theta = 2.88\text{N/C} \times \dfrac{3}{5} = 1.73\text{N/C}.$

(c) $V = k\dfrac{8\text{nC}}{4\text{m}} = 17.98\text{V}.$

Intermediate Exam I: Problem #1 (Spring '06)

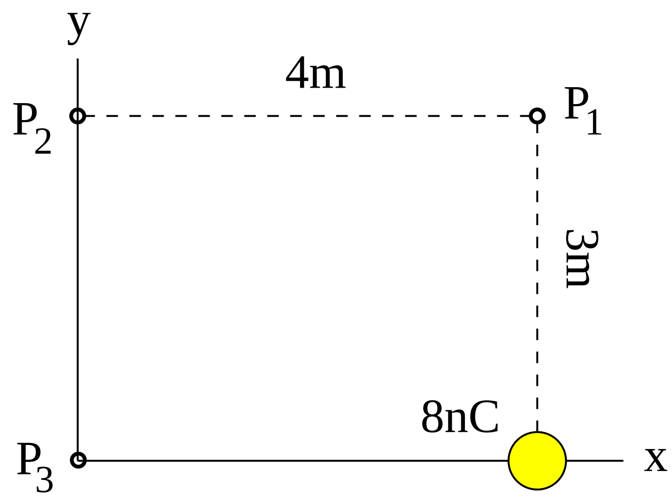
Consider a point charge $q = +8\text{nC}$ at position $x = 4\text{m}$, $y = 0$ as shown.

(a) Find the electric field components E_x and E_y at point P_1.

(b) Find the electric field components E_x and E_y at point P_2.

(c) Find the electric potential V at point P_3.

(d) Find the electric potential V at point P_2.



Solution:

(a) $E_x = 0, \qquad E_y = k\dfrac{8\text{nC}}{(3\text{m})^2} = 7.99\text{N/C}.$

(b) $E_x = -k\dfrac{8\text{nC}}{(5\text{m})^2}\cos\theta = -2.88\text{N/C} \times \dfrac{4}{5} = -2.30\text{N/C}.$

$E_y = k\dfrac{8\text{nC}}{(5\text{m})^2}\sin\theta = 2.88\text{N/C} \times \dfrac{3}{5} = 1.73\text{N/C}.$

(c) $V = k\dfrac{8\text{nC}}{4\text{m}} = 17.98\text{V}.$

(d) $V = k\dfrac{8\text{nC}}{5\text{m}} = 14.38\text{V}.$

Unit Exam I: Problem #1 (Spring '09)

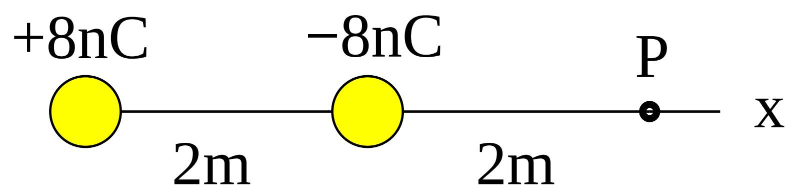
Consider two point charges positioned on the x-axis as shown.

(a) Find magnitude and direction of the electric field at point P.

(b) Find the electric potential at point P.

(c) Find the electric potential energy of an electron (mass $m = 9.1 \times 10^{-31}$kg, charge $q = -1.6 \times 10^{-19}$C) when placed at point P.

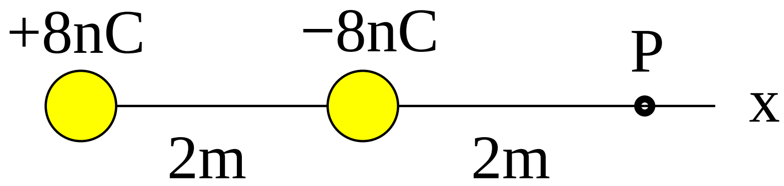
(d) Find magnitude and direction of the acceleration the electron experiences when released at point P.



Unit Exam I: Problem #1 (Spring ’09)

Consider two point charges positioned on the x-axis as shown.

(a) Find magnitude and direction of the electric field at point P.
(b) Find the electric potential at point P.
(c) Find the electric potential energy of an electron (mass $m = 9.1 \times 10^{-31}$kg, charge $q = -1.6 \times 10^{-19}$C) when placed at point P.
(d) Find magnitude and direction of the acceleration the electron experiences when released at point P.



Solution:

(a) $E_x = +k\frac{8\text{nC}}{(4\text{m})^2} + k\frac{(-8\text{nC})}{(2\text{m})^2} = 4.5\text{N/C} - 18\text{N/C} = -13.5\text{N/C}$ (directed left).

Unit Exam I: Problem #1 (Spring '09)

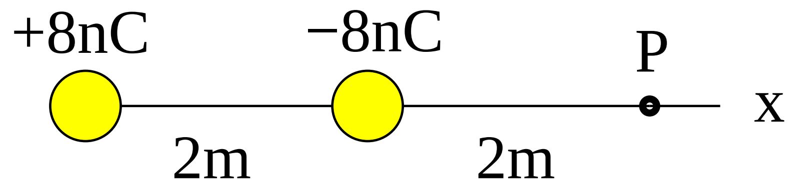
Consider two point charges positioned on the x-axis as shown.

(a) Find magnitude and direction of the electric field at point P.

(b) Find the electric potential at point P.

(c) Find the electric potential energy of an electron (mass $m = 9.1 \times 10^{-31}$kg, charge $q = -1.6 \times 10^{-19}$C) when placed at point P.

(d) Find magnitude and direction of the acceleration the electron experiences when released at point P.



Solution:

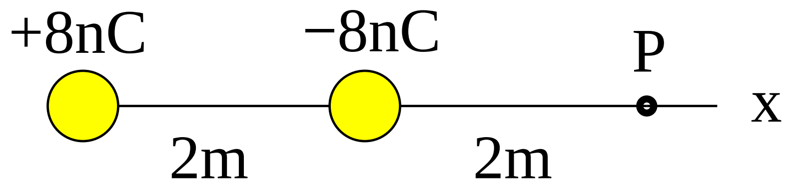
(a) $E_x = +k\frac{8\text{nC}}{(4\text{m})^2} + k\frac{(-8\text{nC})}{(2\text{m})^2} = 4.5\text{N/C} - 18\text{N/C} = -13.5\text{N/C}$ (directed left).

(b) $V = +k\frac{8\text{nC}}{4\text{m}} + k\frac{(-8\text{nC})}{2\text{m}} = 18\text{V} - 36\text{V} = -18\text{V}.$

Unit Exam I: Problem #1 (Spring '09)

Consider two point charges positioned on the x-axis as shown.

(a) Find magnitude and direction of the electric field at point P.
(b) Find the electric potential at point P.
(c) Find the electric potential energy of an electron (mass $m = 9.1 \times 10^{-31}$kg, charge $q = -1.6 \times 10^{-19}$C) when placed at point P.
(d) Find magnitude and direction of the acceleration the electron experiences when released at point P.



Solution:

(a) $E_x = +k\frac{8\text{nC}}{(4\text{m})^2} + k\frac{(-8\text{nC})}{(2\text{m})^2} = 4.5\text{N/C} - 18\text{N/C} = -13.5\text{N/C}$ (directed left).

(b) $V = +k\frac{8\text{nC}}{4\text{m}} + k\frac{(-8\text{nC})}{2\text{m}} = 18\text{V} - 36\text{V} = -18\text{V}.$

(c) $U = qV = (-18\text{V})(-1.6 \times 10^{-19}\text{C}) = 2.9 \times 10^{-18}\text{J}.$

Unit Exam I: Problem #1 (Spring '09)

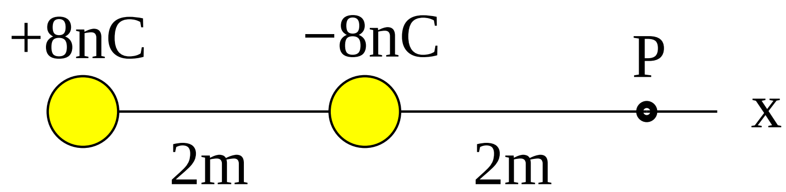
Consider two point charges positioned on the x-axis as shown.

(a) Find magnitude and direction of the electric field at point P.

(b) Find the electric potential at point P.

(c) Find the electric potential energy of an electron (mass $m = 9.1 \times 10^{-31}$kg, charge $q = -1.6 \times 10^{-19}$C) when placed at point P.

(d) Find magnitude and direction of the acceleration the electron experiences when released at point P.



Solution:

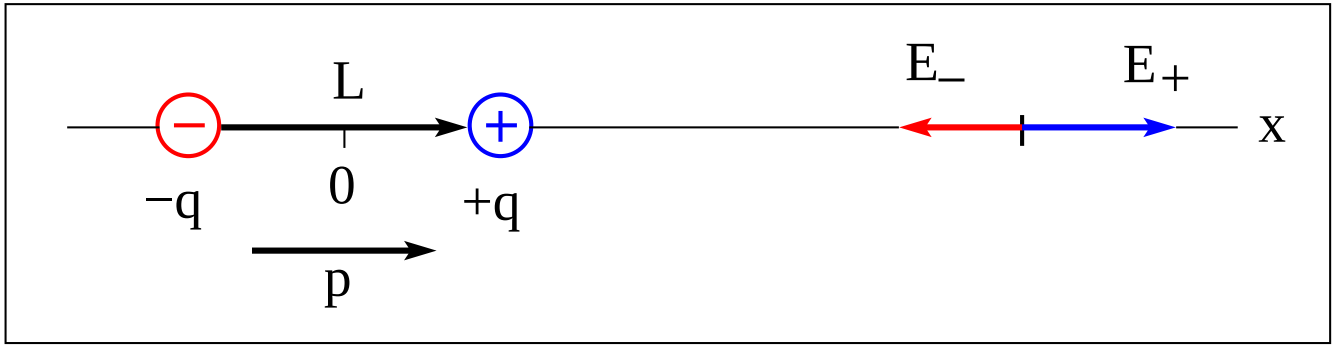
(a) $E_x = +k\dfrac{8\text{nC}}{(4\text{m})^2} + k\dfrac{(-8\text{nC})}{(2\text{m})^2} = 4.5\text{N/C} - 18\text{N/C} = -13.5\text{N/C}$ (directed left).

(b) $V = +k\dfrac{8\text{nC}}{4\text{m}} + k\dfrac{(-8\text{nC})}{2\text{m}} = 18\text{V} - 36\text{V} = -18\text{V}.$

(c) $U = qV = (-18\text{V})(-1.6 \times 10^{-19}\text{C}) = 2.9 \times 10^{-18}\text{J}.$

(d) $a_x = \dfrac{qE_x}{m} = \dfrac{(-1.6 \times 10^{-19}\text{C})(-13.5\text{N/C})}{9.1 \times 10^{-31}\text{kg}} = 2.4 \times 10^{12}\text{ms}^{-2}$ (directed right).

Electric Dipole Field



$$E = \frac{kq}{(x-L/2)^2} - \frac{kq}{(x+L/2)^2} = kq\left[\frac{(x+L/2)^2-(x-L/2)^2}{(x-L/2)^2(x+L/2)^2}\right] = \frac{2kqLx}{(x^2-L^2/4)^2}$$

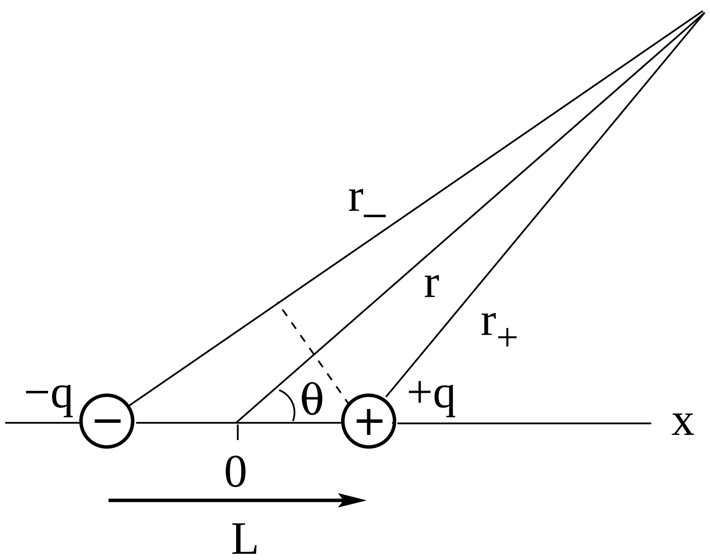
$$\simeq \frac{2kqL}{x^3} = \frac{2kp}{x^3} \qquad (\text{for } x \gg L)$$

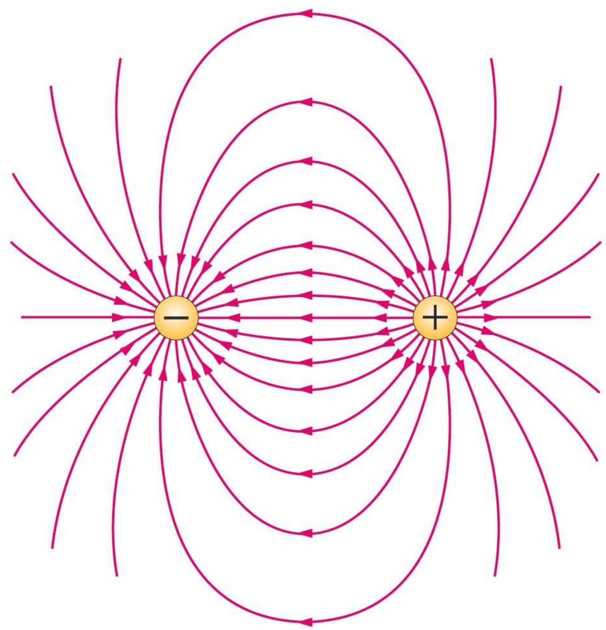
Electric dipole moment: $\vec{p} = q\vec{L}$

- Note the more rapid decay of the electric field with distance from an electric dipole ($\sim r^{-3}$) than from an electric point charge ($\sim r^{-2}$).
- The dipolar field is not radial.

Electric Dipole Potential

- Use spherical coordinates: $V = V(r,\theta)$ independent of azimuthal coordinate ϕ.
- Superposition principle: $V = V_+ + V_- = k\left(\frac{q}{r_+} + \frac{(-q)}{r_-}\right) = kq\,\frac{r_- - r_+}{r_- r_+}$
- Large distances $(r \gg L)$: $r_- - r_+ \simeq L\cos\theta,\ r_- r_+ \simeq r^2 \Rightarrow\ V(r,\theta) \simeq k\,\frac{qL\cos\theta}{r^2}$
- Electric dipole moment: $p = qL$ (magnitude)
- Electric dipole potential: $V(r,\theta) \simeq k\,\frac{p\cos\theta}{r^2}$





Electric Potential Energy of Two Point Charges

Consider two different perspectives:

#1a Electric potential when q_1 is placed: $V(\vec{r}_2) \doteq V_2 = k\frac{q_1}{r_{12}}$

Electric potential energy when q_2 is placed into potential V_2: $U = q_2 V_2 = k\frac{q_1 q_2}{r_{12}}$

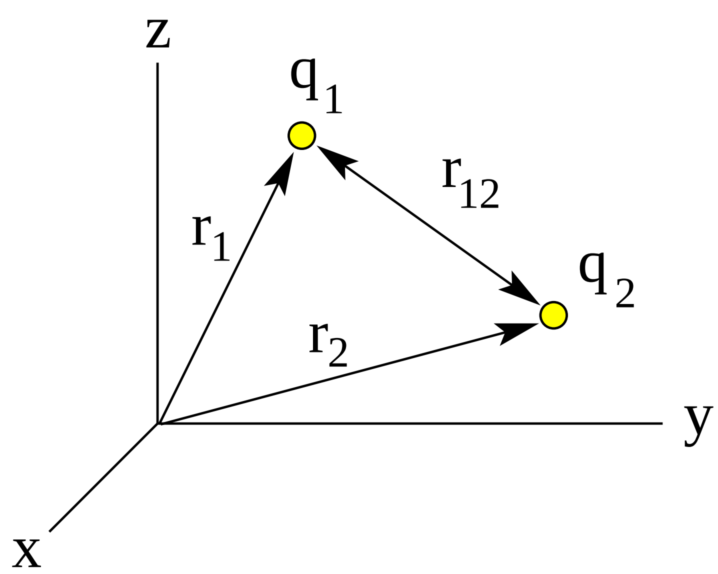
#1b Electric potential when q_2 is placed: $V(\vec{r}_1) \doteq V_1 = k\frac{q_2}{r_{12}}$

Electric potential energy when q_1 is placed into potential V_1: $U = q_1 V_1 = k\frac{q_1 q_2}{r_{12}}.$

#2 Electric potential energy of q_1 and q_2:

$$U = \frac{1}{2}\sum_{i=1}^{2} q_i V_i,$$

where $V_1 = k\frac{q_2}{r_{12}}, \quad V_2 = k\frac{q_1}{r_{12}}.$



Electric Potential Energy of Three Point Charges

#1 Place q_1, then q_2, then q_3, and add all changes in potential energy:

$$U = 0 + k\frac{q_1q_2}{r_{12}} + k\left(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right) = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right).$$

#2 Symmetric expression of potential energy U in terms of the potentials V_i experienced by point charges q_1:

$$U = \frac{1}{2}\sum_{i=1}^{3} q_iV_i = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right),$$

where

$$V_1 = k\left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{13}}\right),$$

$$V_2 = k\left(\frac{q_1}{r_{12}} + \frac{q_3}{r_{23}}\right),$$

$$V_3 = k\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right).$$